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Takagi-Sugeno Model-Based Non-Fragile Guaranteed Cost Control for Uncertain Discrete-Time Systems with State Delay

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Abstract

A non-fragile guaranteed cost control (GCC) problem is presented for a class of discrete time-delay nonlinear systems described by Takagi-Sugeno (T-S) fuzzy model. The systems are assumed to have norm-bounded time-varying uncertainties in the matrices of state, delayed state and control gains. Sufficient conditions are first obtained which guarantee that the closed-loop system is asymptotically stable and the closed-loop cost function value is not more than a specified upper bound. Then the design method of the non-fragile guaranteed cost controller is formulated in terms of the linear matrix inequality (LMI) approach. A numerical example is given to illustrate the effectiveness of the proposed design method.

Key Words : T-S model; discrete-time; state delay; non-fragile; guaranteed cost control; linear matrix inequality

1. Introduction

Guaranteed cost control (GCC) for linear systems with parameter uncertainty has been an active area in control system community for several decades [1-3]. In particular, GCC for discrete-time uncertain systems has attracted great attention [4, 5]. Recently, the GCC scheme has been extended to uncertain nonlinear discrete-time systems [6-7]. For example, the problem of guaranteed cost analysis and control for a class of nonlinear discrete-time systems is discussed in [6] and the nonlinear discrete-time uncertain systems with state delay is considered using linear matrix inequality (LMI) techniques [7]. Because of the existence of time delays and the complexity of the nonlinear systems, there still remain some difficulties in designing of guaranteed cost control for the general nonlinear systems with time delays.

One approach to deal with these difficulties is the Takagi-Sugeno (T-S) fuzzy approach [8]. Studies have shown that T-S fuzzy model can be used to approximate global behaviors of some kinds of highly complex nonlinear systems [9]. T-S model-based controller can combine the merits of both fuzzy controller and conventional linear theory. Furthermore, the controller guarantees stability in the sense of Lyapunov and control performance theoretically [9-11]. Based on the T-S model, many significant results have been proposed for the discrete time-delay systems [12, 13]. Therefore, T-S model-based technique is an effective scheme to deal with the GCC problem of nonlinear discrete time-delay systems.

Usually, the synthesized controllers are robust with respect to system uncertainty. However, the robustness with respect to controller uncertainty is not considered. The controller robustness issue with respect to controller uncertainty has been presented by Keel & Bhattacharyya [14]. This motivates the so-called non-fragile control in which a controller must tolerate a certain degree of controller uncertainty as well as system uncertainty. This controller fragility issue has since attracted some research interest [15-17]. In [16], the problem of non-fragile GCC state feedback design for discrete-time uncertain linear systems is studied by using LMI techniques. And in [17], a non-fragile linear quadratic fuzzy control problem for a class of nonlinear time-delayed descriptor systems is discussed.

To our knowledge, the design and analysis of non-fragile GCC by using T-S model approach for nonlinear discrete time-delay systems remains untouched. In this paper, we study the design and analysis of non-fragile GCC state-feedback for a class of discrete time-delay nonlinear uncertain systems using T-S fuzzy model.

The paper is organized as follows: section 2 discusses the T-S fuzzy systems and the problem under consideration. Section 3 presents conditions for the existence of the non-fragile guaranteed cost controllers and design methods of such controllers. A numerical example is given to illustrate the proposed design methods in Section 4, which is followed by conclusions in Section 5.

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2. Problem Statement

Consider a class of uncertain discrete time-delay nonlinear systems described by the following T-S fuzzy model

$$\begin{aligned} x(k+1) = \sum_i^l h_i(z(k))\{(A_i + \Delta A_i)x(k) \\ +(A_{di} + \Delta A_{di})x(k-d) + (B_i + \Delta B_i)u(k)\} \end{aligned} \quad (1)$$

where $z(k) = [z_1(k) \cdots z_g(k)]^T$ is the premise variable vector, $d > 0$ is the time delay, $x(k) \in R^n$ is the state vector, $u(k) \in R^m$ is the control input vector, l is the number of model rules. $h_i(z(k)) \geq 0$ is the normalized weight for each rule and satisfying

$$\sum_i^l h_i(z(k)) = 1 \quad (2)$$

A_i, $A_{di} \in R^{n \times n}$ and $B_i \in R^{n \times m}$ are known constant matrices, $\Delta A_i, \Delta A_{di} \in R^{n \times n}$, and $\Delta B_i \in R^{n \times m}$ represent the time-varying parametric uncertainties having the following structure:

$$[\Delta A_i \quad \Delta A_{di} \quad \Delta B_i] = DF(k)[E_{ai} \quad E_{di} \quad E_{bi}] \quad (3)$$

where D, E_{ai}, E_{di}, and E_{bi} are known real constant matrices of appropriate dimensions, and $F(k)$ is an unknown matrix function with Lebesgue-measurable elements and satisfying $F^T(k)F(k) \leq I$.

The cost function associated with this system is

$$J = \sum_{k=1}^{\infty}(x^T(k)Qx(k) + u^T(k)Ru(k)) \quad (4)$$

where $Q > 0$ and $R > 0$ are given weighting matrices.

Based on the parallel distributed compensation scheme [10], the following non-fragile fuzzy controller is proposed to deal with the GCC problem for system (1):

$$u(k) = \sum_i^l h_i(z(k))(K_i + \Delta K_i)x(k) \quad (5)$$

where $K_i \in R^{m \times n}$ is the local nominal feedback gain of the controller, $\Delta K_i \in R^{m \times n}$ denotes the norm-bounded additive form uncertainty in controller and has the following structure:

$$\Delta K_i = GH(k)E_{ki} \quad (6)$$

where G, E_{ki} are known real constant matrices with appropriate dimensions, and $H(k)$ is an unknown matrix function with Lebesgue-measurable elements and satisfying $H^T(k)H(k) \leq I$.

Substituting (5) into (1), the closed-loop system can be written as (7),

$$\begin{aligned} x(k+1) &= \sum_i^l \sum_j^l h_i(z(k))h_j(z(k))\{(A_i + \Delta A_i \\ &+(B_i + \Delta B_i)(K_j + \Delta K_j))x(k) \\ &+(A_{di} + \Delta A_{di})x(k-d)\} \\ &= (\overline{A} + \Delta\overline{A})x(k) + (\overline{B} + \Delta\overline{B})(\overline{K} + \Delta\overline{K})x(k) \\ &+(\overline{A}_d + \Delta\overline{A}_d)x(k-d) \end{aligned} \quad (7)$$

where $\overline{A} = \sum_i^l h_i(z(k))A_i$, $\overline{A}_d = \sum_i^l h_i(z(k))A_{di}$, $\overline{B} = \sum_i^l h_i(z(k))B_i$

$$\Delta\overline{A} = \sum_i^l h_i(z(k))\Delta A_i, \quad \Delta\overline{A}_d = \sum_i^l h_i(z(k))\Delta A_{di}$$

$$\Delta\overline{B} = \sum_i^l h_i(z(k))\Delta B_i$$

$$\overline{K} = \sum_j^l h_j(z(k))K_j, \quad \Delta\overline{K} = \sum_j^l h_j(z(k))\Delta K_j \quad (8)$$

Lemma 1 : Given matrices Y, H, E of appropriate dimensions and with Y symmetric, then for all F satisfying $F^T F \leq I$ and $Y + HFE + E^T F^T H^T < 0$, if and only if there exists $\varepsilon > 0$ such that $Y + \varepsilon HH^T + \varepsilon^{-1}E^T E < 0$ [4].

3. Main Results

In this section, sufficient condition that guarantees the asymptotical stability of the closed-loop system, as well as the smaller of closed-loop cost function value than a specified upper bound, is presented in Theorem 1. The design method of such controller in terms of the feasible solutions to the LMIs is discussed in Theorem 2.

Theorem 1: Given the controller gain matrix K_i, (5) is a guaranteed cost fuzzy controller, if there exist symmetric positive-definite matrices $\hat{P}$, $\hat{S}$, and scalar $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, such that LMI (9) holds,

$$\begin{bmatrix} -\hat{P}+\hat{S} & * & * & * & * & * & * \\ 0 & -\hat{S} & * & * & * & * & * \\ \overline{A}+\overline{B}\overline{K}P^{-1} & \overline{A}_d\hat{P} & -\hat{P}+\varepsilon_1 DD^T+\varepsilon_2\overline{B}GG^T\overline{B}^T & * & * & * & * \\ \hat{P} & 0 & 0 & -Q^{-1} & * & * & * \\ \overline{K}\hat{P} & 0 & \varepsilon_2 GG^T\overline{B}^T & 0 & -R^{-1}+\varepsilon_2 GG^T & * & * \\ \overline{E}_a+\overline{E}_b\overline{K}\hat{P} & \overline{E}_d\hat{P} & \varepsilon_2\overline{E}_b GG^T\overline{B}^T & 0 & \varepsilon_2\overline{E}_b GG^T & -\varepsilon_1 I+\varepsilon_2\overline{E}_b GG^T\overline{E}_b^T & * \\ \overline{E}_k\hat{P} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_1 I \end{bmatrix} < 0 \quad (9)$$

where $\bar{K}$ is defined in (8) and

$$\bar{E}_a = \sum_i^l h_i(z(k))E_{ai},\ \bar{E}_b = \sum_i^l h_i(z(k))E_{bi}$$
$$\bar{E}_d = \sum_i^l h_i(z(k))E_{di},\ \bar{E}_k = \sum_j^l h_j(z(k))E_{kj} \quad (10)$$

Moreover, cost function (4) satisfies the following upper bound:

$$J \le J^* = \sum_{k=1}^{\infty}(x^T(0)\hat{P}^{-1}x(0) + \sum_i^d x^T(-i)\hat{P}^{-1}\hat{S}\hat{P}^{-1}x(-i)) \quad (11)$$

Proof : Define the following Lyapunov-krasovskii function

$$V(k) = x^T(k)Px(k) + \sum_{i=1}^d x^T(k-i)Sx(k-i)$$

where P and S are positive-definite matrices. It follows from (7) that

$$\Delta V(k) = V(k+1) - V(k) = \begin{bmatrix} x(k) \\ x(k-d) \end{bmatrix}^T \begin{bmatrix} A_c^T PA_c - P + S & A_c^T PA_{dc} \\ A_{dc}^T PA_c & A_c^T PA_c - S \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-d) \end{bmatrix}$$

where

$$A_c = \bar{A} + \Delta\bar{A} + (\bar{B} + \Delta\bar{B})K$$
$$A_{dc} = \bar{A}_d + \Delta\bar{A}_d$$
$$K = \bar{K} + \Delta\bar{K} \quad (12)$$

Suppose that with given weighting matrices $Q > 0$ and $R > 0$, (13) holds for all admissible uncertainties

$$\begin{bmatrix} A_c^T PA_c - P + S + Q + K^T RK & A_c^T PA_{dc} \\ A_{dc}^T PA_c & A_c^T PA_c - S \end{bmatrix} < 0 \quad (13)$$

Then, we have

$$\Delta V(k) < -x^T(k)(Q + K^T RKx(k) < 0 \quad (14)$$

Thus, it follows from Lyapunov stability theory that the closed-loop system (7) is asymptotically stable. Summing both sides of (14) from 0 to infinity and using the system stability, it is easy to show that the cost function satisfies (11).

By Schur complement, (13) is equivalent to

$$\begin{bmatrix} -P + S + Q + K^T RK & * & * \\ 0 & -S & * \\ A_c & A_{dc} & -P^{-1} \end{bmatrix} < 0$$

By (3), the above inequality can be rewritten as

$$\begin{bmatrix} -P + S + Q + K^T RK & * & * \\ 0 & -S & * \\ \bar{A} + \bar{B}K & \bar{A}_d & -P^{-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ D \end{bmatrix} F(k)\begin{bmatrix} \bar{E}_a + \bar{E}_b K & \bar{E}_d & 0 \end{bmatrix} + \begin{bmatrix} (\bar{E}_a + \bar{E}_b K)^T \\ \bar{E}_d^T \\ 0 \end{bmatrix} F^T(k)\begin{bmatrix} 0 & 0 & D^T \end{bmatrix} < 0$$

In light of Lemma 1 and using the Schur complement, the above inequality is true for all admissible uncertain matrices $F(k)$ if and only if there exists a scalar $\varepsilon_1 > 0$ such that

$$\begin{bmatrix} -P+S & * & * & * & * & * \\ 0 & -S & * & * & * & * \\ \bar{A}+\bar{B}K & \bar{A}_d & -P^{-1}+\varepsilon_1 DD^T & * & * & * \\ I & 0 & 0 & -Q^{-1} & * & * \\ K & 0 & 0 & 0 & -R^{-1} & * \\ \bar{E}_a+\bar{E}_b K & \bar{E}_d & 0 & 0 & 0 & -\varepsilon_1 I \end{bmatrix} < 0 \quad (15)$$

By (6) and (12), (15) is equivalent to (16).

$$\begin{bmatrix} -P+S & * & * & * & * & * \\ 0 & -S & * & * & * & * \\ \bar{A}+\bar{B}\bar{K} & \bar{A}_d & -P^{-1}+\varepsilon_1 DD^T & * & * & * \\ I & 0 & 0 & -Q^{-1} & * & * \\ \bar{K} & 0 & 0 & 0 & -R^{-1} & * \\ \bar{E}_a+\bar{E}_b\bar{K} & \bar{E}_d & 0 & 0 & 0 & -\varepsilon_1 I \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \bar{B}G \\ 0 \\ G \\ \bar{E}_b G \end{bmatrix} H(k)\begin{bmatrix} \bar{E}_k & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \bar{E}_k^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} H^T(k)\begin{bmatrix} 0 & 0 & G^T\bar{B}^T & 0 & G^T & G^T\bar{E}_b^T \end{bmatrix} < 0 \quad (16)$$

Using Lemma 1 and Schur complement, (16) is true for all admissible uncertain matrices $H(k)$ if and only if there exists a scalar $\varepsilon_2 > 0$ such that following matrix inequality holds:

$$\begin{bmatrix} -P+S & * & * \\ 0 & -S & * \\ \bar{A}+\bar{B}\bar{K} & \bar{A}_d & -P^{-1}+\varepsilon_1 DD^T+\varepsilon_2\bar{B}GG^T\bar{B}^T \\ I & 0 & 0 \\ \bar{K} & 0 & \varepsilon_2 GG^T\bar{B}^T \\ \bar{E}_a+\bar{E}_b\bar{K} & \bar{E}_d & \varepsilon_2\bar{E}_b GG^T\bar{B}^T \\ \bar{E}_k & 0 & 0 \end{bmatrix}$$

$$\begin{array}{cccc} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ -Q^{-1} & * & * & * \\ 0 & -R^{-1}+\varepsilon_2 GG^T & * & * \\ 0 & \varepsilon_2 \overline{E}_b GG^T & -\varepsilon_1 I+\varepsilon_2 \overline{E}_b GG^T \overline{E}_b^T & * \\ 0 & 0 & 0 & -\varepsilon_1 I \end{array}\Bigg] < 0 \tag{17}$$

Pre- and post-multiplying both sides of (17) by $diag\{-P^{-1},-P^{-1},I,I,I,I\}$, and denoting $\hat{P}=P^{-1}$, and $\hat{S}=P^{-1}SP^{-1}$, it is clear that (17) is equivalent to the LMI condition (9). Thus, (13) is fulfilled, which implies that (14) and (11) hold.□

Remark 1 : In Theorem 1, an LMI condition has been given which guarantees that the closed-loop system is asymptotically stable and guaranteed cost performance is achieved. However, (9) is not the form of LMI for $K_i,\ i=1,2\cdots l$ before the determination of these control gains. Therefore, Theorem 1 can not be used directly to design a controller. In the sequel, the controller design problem will be transformed to a feasibility problem of a set of LMIs that can be solved easily.

Theorem 2 : non-fragile fuzzy controller (5) is a guaranteed cost fuzzy non-fragile controller for system (1), if there exist symmetric positive-definite matrices $\hat{P}\in R^{n\times n}$, $\hat{S}\in R^{n\times n}$, and real matrix $W_j\in R^{m\times n}$, scalar $\varepsilon_1>0$, $\varepsilon_2>0$, such that the following LMIs holds:

$$U_{ii}<0, \quad i=1,2,\cdots,l \tag{18}$$

$$U_{ij}+U_{ji}<0, \quad 1\le i<j\le l \tag{19}$$

where U_{ij} is given by (20).

$$U_{ij}=\begin{bmatrix} -\hat{P}+\hat{S} & * & * & * & * & * & * & * & * \\ 0 & -\hat{S} & * & * & * & * & * & * & * \\ A_i\hat{P}+B_iW_j & A_{di}\hat{P} & -\hat{P} & * & * & * & * & * & * \\ \hat{P} & 0 & 0 & -Q^{-1} & * & * & * & * & * \\ W_j & 0 & 0 & 0 & -R^{-1} & * & * & * & * \\ E_{ai}\hat{P}+E_{bi}W_j & E_{di}\hat{P} & 0 & 0 & 0 & -\varepsilon_1 I & * & * & * \\ E_{kj}\hat{P} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 I & * & * \\ 0 & 0 & \varepsilon_2 G^T B_i^T & 0 & \varepsilon_2 G^T & \varepsilon_2 G^T E_{bi}^T & 0 & -\varepsilon_2 I & * \\ 0 & 0 & \varepsilon_1 D^T & 0 & 0 & 0 & 0 & 0 & -\varepsilon_1 I \end{bmatrix} \tag{20}$$

Furthermore, the non-fragile fuzzy GCC can be given by

$$K_i=W_i\hat{P}^{-1} \tag{21}$$

Proof : From Theorem 1, matrix inequality (9) can be rewritten as

$$\Omega_1+\Omega_3^T\Omega_2\Omega_3<0 \tag{22}$$

where

$$\Omega_1=\begin{bmatrix} -\hat{P}+\hat{S} & * & * & * & * & * & * \\ 0 & -\hat{S} & * & * & * & * & * \\ \overline{A}+\overline{BK}P^{-1} & \overline{A}_d\hat{P} & -\hat{P} & * & * & * & * \\ \hat{P} & 0 & 0 & -Q^{-1} & * & * & * \\ \overline{K}\hat{P} & 0 & 0 & 0 & -R^{-1} & * & * \\ \overline{E}_a+\overline{E}_b\overline{K}\hat{P} & \overline{E}_d\hat{P} & 0 & 0 & 0 & -\varepsilon_1 I & * \\ \overline{E}_k\hat{P} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_1 I \end{bmatrix}$$

$$\Omega_2=\begin{bmatrix} \varepsilon_2 I & 0 \\ 0 & \varepsilon_1 I \end{bmatrix}$$

$$\Omega_3=\begin{bmatrix} 0 & 0 & G^T\overline{B}^T & 0 & G^T & G^T\overline{E}_B^T & 0 \\ 0 & 0 & D^T & 0 & 0 & 0 & 0 \end{bmatrix}$$

Using the Schur complement, (22) is equivalent to

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2^{-1} \end{bmatrix}<0 \tag{23}$$

Pre- and post-multiplying both sides of (23) by $diag\{I,I,I,I,I,I,I,\varepsilon_2 I,\varepsilon_1 I\}$, and denoting, $\overline{W}=\overline{K}\hat{P}$, yield the following matrix inequality

$$\begin{bmatrix} -\hat{P}+\hat{S} & * & * & * & * & * & * & * & * \\ 0 & -\hat{S} & * & * & * & * & * & * & * \\ \overline{A}\hat{P}+\overline{BW} & \overline{A}_d\hat{P} & -\hat{P} & * & * & * & * & * & * \\ \hat{P} & 0 & 0 & -Q^{-1} & * & * & * & * & * \\ \overline{W} & 0 & 0 & 0 & -R^{-1} & * & * & * & * \\ \overline{E}_a\hat{P}+\overline{E}_b\overline{W} & \overline{E}_d\hat{P} & 0 & 0 & 0 & -\varepsilon_1 I & * & * & * \\ \overline{E}_k\hat{P} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 I & * & * \\ 0 & 0 & \varepsilon_2 G^T\overline{B}^T & 0 & \varepsilon_2 G^T & \varepsilon_2 G^T\overline{E}_b^T & 0 & -\varepsilon_2 I & * \\ 0 & 0 & \varepsilon_1 D^T & 0 & 0 & 0 & 0 & 0 & -\varepsilon_1 I \end{bmatrix}<0 \tag{24}$$

The above inequality is equivalent to

$$\sum_i^l \sum_j^l h_i(z(k))h_j(z(k))U_{ij} = \sum_i^l h_i(z(k))h_i(z(k))U_{ii} + \sum_i^l \sum_j^l h_i(z(k))h_j(z(k))(U_{ij} + U_{ji}) < 0 \quad (25)$$

Obviously, (18) and (19) are the sufficient conditions for (25). Thus, if (18) and (19) hold, (9) can be obtained. Thus, (5) is a guaranteed cost fuzzy controller for system (1). □

Remark 2 : In conditions (18) and (19), $\hat{P}, \hat{S}, W_j, \varepsilon_1 I \;\; \varepsilon_2 I$, are the matrix variables, and A_i, B_i, A_{di}, Q, R, D, G, E_{ai}, E_{bi}, E_{di}, E_{kj} are the known real constant matrices. Thus, Theorem 2 presents an LMI-based method for the non-fragile GCC design. It is a feasibility problem of LMIs (18)-(19), which can be solved by using Matlab LMI Toolbox.

4. A numerical Example

To illustrate the effectiveness of the proposed method, a number example is presented and compared with a conventional GCC approach presented in [18] in which a backing-up control of the computer-simulated truck-trailer model is used.

The model is formulated as

$$x_1(k+1) = (1 - \frac{vt}{L})x_1(k) + 0.1x_1(k-d) + \frac{vt}{l}u(k) \quad (26)$$

$$x_2(k+1) = x_2(k) + \frac{vt}{L}x_1(k) \quad (27)$$

$$x_3(k+1) = \Delta a_{31}x_1(k) + \Delta a_{32}x_2(k) + x_3(k) + vt\sin(x_2(k) + \frac{vt}{2L}x_1(k)) \quad (28)$$

where

$$\Delta a_{31} = a\frac{vt}{2L}\sin(k)$$

and

$$\Delta a_{32} = a\sin(k)$$

is the uncertainty. Let

$$z(k) = x_2(k) + \frac{vt}{2L}x_1(k)$$

and the membership functions as

$$h_1(z(k)) = \begin{cases} \dfrac{\sin(z(k)) - m\cdot z(k)}{z(k)\cdot(1-m)}, & z(k) \neq 0 \\ 1, & z(k) \neq 0 \end{cases} \quad (29)$$

$$h_2(z(k)) = \begin{cases} \dfrac{z(k) - \sin(z(k))}{z(k)\cdot(1-m)}, & z(k) \neq 0 \\ 0, & z(k) \neq 0 \end{cases} \quad (30)$$

Thus the uncertain nonlinear system (26)-(28) can be represented by the T-S fuzzy model (1) with $l = 2$, $n = 3$ and

$$A_1 = \begin{bmatrix} 1-\frac{vt}{L} & 0 & 0 \\ \frac{vt}{L} & 1 & 0 \\ \frac{v^2t^2}{2L} & vt & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1-\frac{vt}{L} & 0 & 0 \\ \frac{vt}{L} & 1 & 0 \\ \frac{mv^2t^2}{2L} & mvt & 1 \end{bmatrix},$$

$$A_{d1} = A_{d2} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} \frac{vt}{l} \\ 0 \\ 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 & a \end{bmatrix}^T$$

$$E_{a1} = E_{a2} = \begin{bmatrix} \frac{vt}{2L} & 1 & 0 \end{bmatrix}$$

Set the same model parameters from [18] as $l = 2.8$, $L = 5.5$, $v = -1.0$, $t = 2.0$, $d = 1$, $m = (0.01/\pi)$, $a = 0.0023$, and

$$Q = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, R = 1$$

For the controller uncertainty, we set

$$G = 2.5, \; E_{k1} = E_{k2} = \begin{bmatrix} 0.035 & 0.035 & 0.0010 \end{bmatrix}$$

By Theorem 2, feedback gains $K_i, i = 1,2$ can be obtain as follows

$$K_1 = \begin{bmatrix} 1.8924 & -1.0799 & 0.0918 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} 1.8584 & -0.8976 & 0.0915 \end{bmatrix}$$

The corresponding upper bound of cost function is given as $J^* = 59.24$.

The same problem is solved by the method in [18] without considering the uncertainties in the controller and the controller gain matrices are obtained as

$$K_1 = \begin{bmatrix} 0.9290 & -0.0915 & 0.0016 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} 0.8795 & -0.0643 & 0.0015 \end{bmatrix}$$

The corresponding upper bound of cost function is given as $J^* = 13.93$.

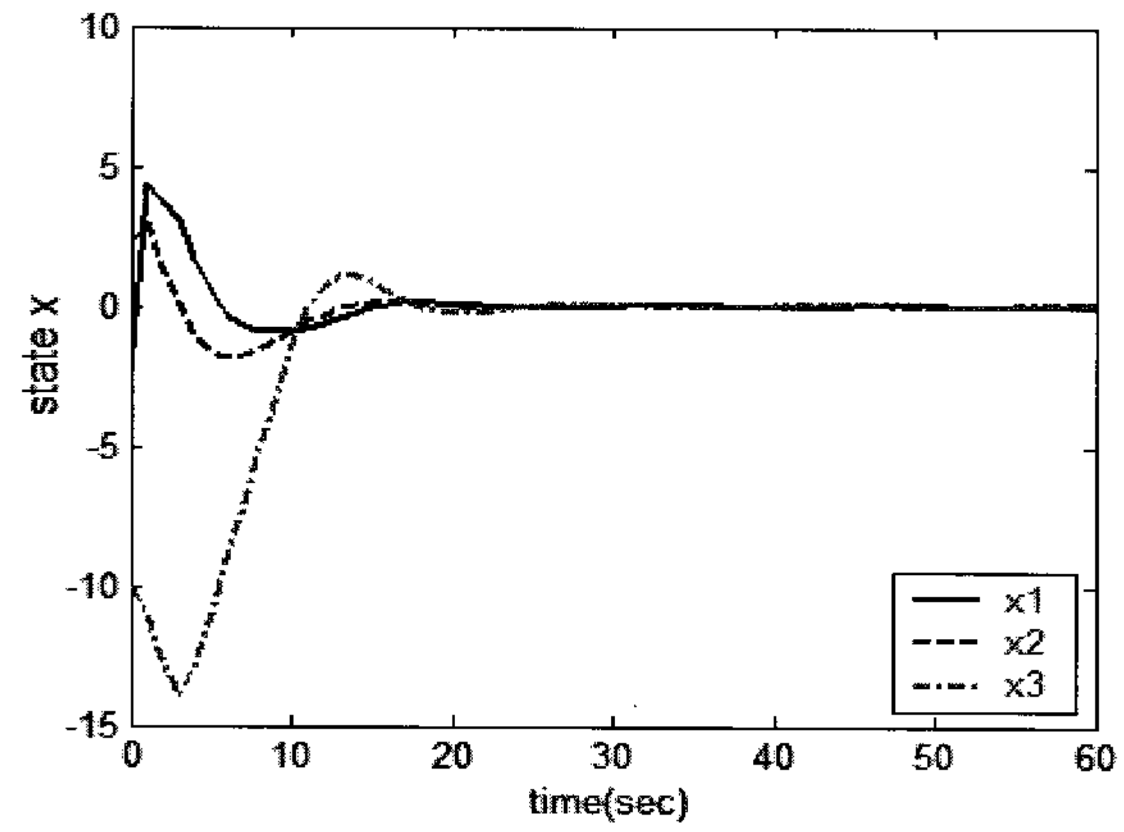

Fig.1 Trajectories of the state variables of the truck-trailer system for non-fragile fuzzy GCC design

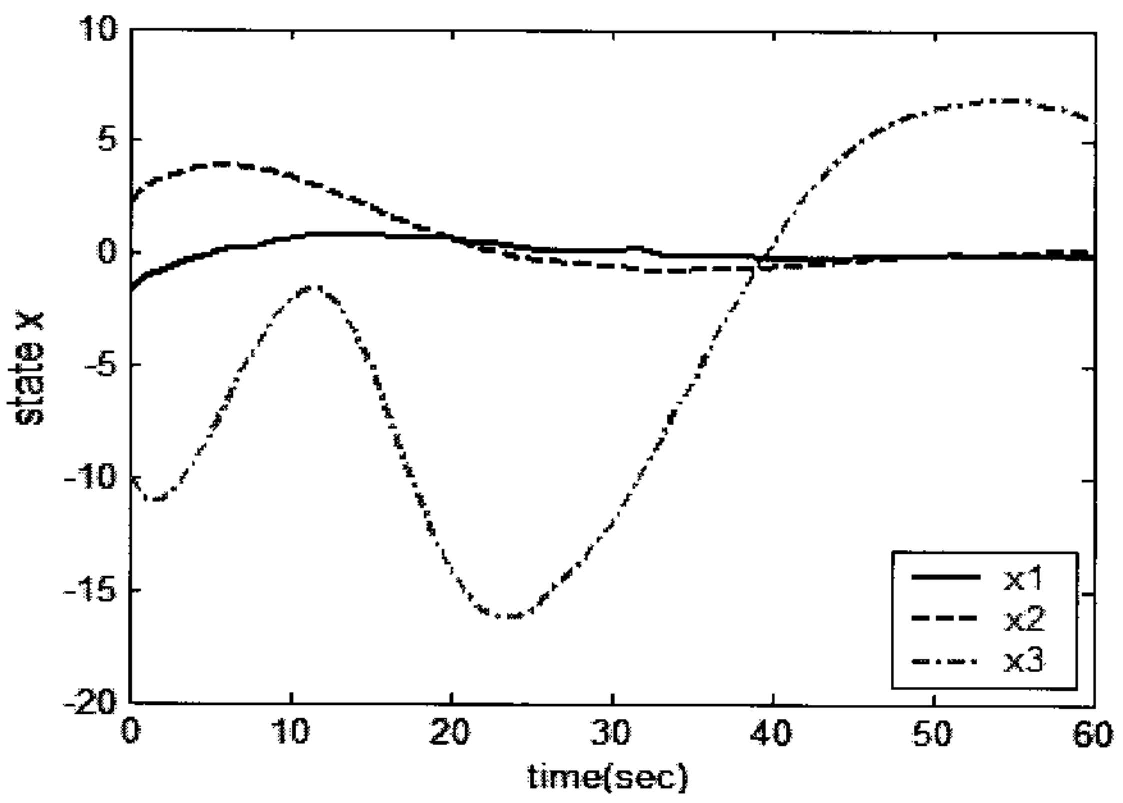

Fig. 2 Trajectories of the state variables of the truck-trailer system for conventional GCC design

The proposed non-fragile GCC method and conventional GCC method [18] are compared under initial state $x(-1)=\begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$ and $x(0)=\begin{bmatrix}-0.5\pi & 0.75\pi & -10\end{bmatrix}^T$. The control results are shown in Fig. 1 and Fig. 2, respectively. From these results, it can be seen that the closed-loop system becomes unstable in the conventional GCC design which does not take into account the uncertainties in the controller. On the contrary, the closed-loop response under the controller from the non-fragile GCC design is stable. It should be noted that the upper bound of cost function value for the non-fragile GCC is larger than that of the conventional GCC design. Therefore, there exists a trade-off between controller non-fragility and the guaranteed cost performance.

5. Conclusion

Based on the T-S fuzzy model, the non-fragile control scheme is extended to nonlinear systems and the non-fragile GCC problem for a class of nonlinear discrete-time uncertain systems with state delay is studied. The stability analysis and the design method of the non-fragile guaranteed cost controllers have been proposed in terms of the feasible solutions to LMIs. The numerical simulation results of a truck-trailer demonstrate that the proposed fuzzy non-fragile controller can tolerate a certain degree of controller uncertainty as well as system uncertainty. Furthermore the simulation results also show that there exists a trade-off between controller non-fragility and system performance.

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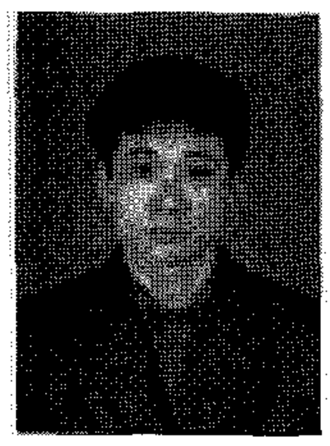

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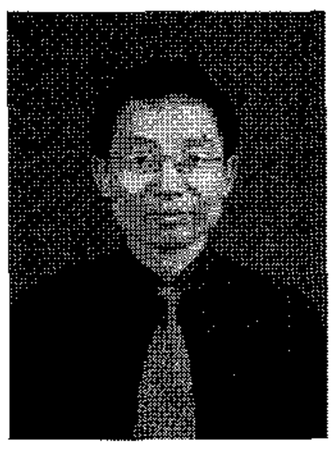

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