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Fuzzy (r, s)-pre-semicontinuous mappings

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Abstract

In this paper, we introduce the concept of fuzzy (r, s)-pre-semicontinuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense. The concepts of fuzzy (r, s)-pre-semineighborhood and fuzzy (r, s)-quasi-pre-semineighborhood are given. The characterizations for the fuzzy (r, s)-pre-semicontinuous mappings are obtained. Also, we introduce the notions of fuzzy (r, s)-pre-semiopen and fuzzy (r, s)-pre-semiclosed mappings, and then we investigate some of their characteristic properties.

Key Words : fuzzy (r, s)-pre-semicontinuous mapping, fuzzy (r, s)-pre-semiopen mapping, fuzzy (r, s)-pre-semiclosed mapping

1. Introduction

Chang [4] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [17], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay and his colleagues [5], and by Ramadan [16].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Çoker [6] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [8] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. S. Z. Bai [2] introduced the concepts of fuzzy pre-semiopen sets and fuzzy pre-semicontinuous mappings, and S. Z. Bai and W. L. Wang [3] established some other properties of fuzzy pre-semicontinuous mappings on Chang's fuzzy toplogical spaces. S. J. Lee and Y. S. Eoum [11] considered these concepts on smooth topological spaces.

In this paper, we introduce the concept of fuzzy (r, s)-pre-semicontinuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense. The concepts of fuzzy (r, s)-pre-semineighborhood and fuzzy (r, s)-quasi-pre-semineighborhood are given. The characterizations for the fuzzy (r, s)-pre-semicontinuous mappings are obtained. Also, we introduce the notions of fuzzy (r, s)-pre-semiopen and fuzzy (r, s)-pre-semiclosed mappings, and then we investigate some of their characteristic properties.

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2. Preliminaries

For the nonstandard definitions and notations we refer to [10, 12, 14, 15].

Definition 2.1. ([8]) Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense*(SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a mapping $\mathcal{T} : I(X) \to I \otimes I$ which satisfies the following properties:

(1) $\mathcal{T}_1(\underline{0}) = \mathcal{T}_1(\underline{1}) = 1$ and $\mathcal{T}_2(\underline{0}) = \mathcal{T}_2(\underline{1}) = 0$.

(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.

(3) $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$.

The $(X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense*(SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a *gradation of openness* of A and $\mathcal{T}_2(A)$ a *gradation of nonopenness* of A.

Definition 2.2. ([7, 9]) Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS and $(r, s) \in I \otimes I$. Then

(1) an intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X is said to be *quasi-coincident* with the intuitionistic fuzzy set A in X, denoted by $x_{(\alpha,\beta)}\mathrm{q}A$, if and only if $\mu_A(x) > \beta$ or $\gamma_A(x) < \alpha$.

(2) two intuitionistic fuzzy sets A and B in X are said to be *quasi-coincident*, denoted by $A\text{q}B$, if and only if there exists an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$.

The word 'not quasi-coincident' will be abbreviated as $\tilde{\text{q}}$.

Definition 2.3. ([13]) Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) *fuzzy (r, s)-pre-semiopen* if $A \subseteq \text{sint}(\text{cl}(A, r, s), r, s)$,

(2) *fuzzy (r, s)-pre-semiclosed* if $\text{scl}(\text{int}(A, r, s), r, s) \subseteq A$.

Definition 2.4. ([13]) Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy (r, s)-pre-semiinterior* is defined by
$\text{psint}(A, r, s) = \bigcup\{B \in I(X) \mid B \subseteq A,$ B is fuzzy (r, s)-pre-semiopen$\}$
and the *fuzzy (r, s)-pre-semiclosure* is defined by
$\text{pscl}(A, r, s) = \bigcap\{B \in I(X) \mid A \subseteq B,$ B is fuzzy (r, s)-pre-semiclosed$\}$.

Theorem 2.5. ([13]) Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then we have

(1) $\text{psint}(A, r, s)^c = \text{pscl}(A^c, r, s)$,

(2) $\text{pscl}(A, r, s)^c = \text{psint}(A^c, r, s)$.

3. Fuzzy (r, s)-pre-semicontinuous mappings

We define the notions of fuzzy (r, s)-pre-semicontinuous, fuzzy (r, s)-pre-semiopen, and fuzzy (r, s)-pre-semiclosed mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and then we investigate some of their characteristic properties.

Definition 3.1. Let A be an intuitionistic fuzzy set and $x_{(\alpha,\beta)}$ an intuitionistic fuzzy point in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is called

(1) a *fuzzy (r, s)-pre-semineighborhood* of $x_{(\alpha,\beta)}$ if there is a fuzzy (r, s)-pre-semiopen set B in X such that $x_{(\alpha,\beta)} \in B \subseteq A$,

(2) a *fuzzy (r, s)-quasi-pre-semineighborhood* of $x_{(\alpha,\beta)}$ if there is a fuzzy (r, s)-pre-semiopen set B in X such that $x_{(\alpha,\beta)}\text{q}B \subseteq A$.

Theorem 3.2. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS and $(r, s) \in I \otimes I$. Then an intuitionistic fuzzy set A in X is fuzzy (r, s)-pre-semiopen if and only if A is a fuzzy (r, s)-pre-semineighborhood of $x_{(\alpha,\beta)}$ for each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in A.

Proof. Let A be a fuzzy (r, s)-pre-semiopen set in X and $x_{(\alpha,\beta)} \in A$. Put $B = A$. Then B is fuzzy (r, s)-pre-semiopen in X and $x_{(\alpha,\beta)} \in B \subseteq A$. Thus A is a fuzzy (r, s)-pre-semineighborhood of $x_{(\alpha,\beta)}$.

Conversely, let $x_{(\alpha,\beta)} \in A$. Since A is a fuzzy (r, s)-pre-semineighborhood of $x_{(\alpha,\beta)}$, there is a fuzzy (r, s)-pre-semiopen set $B_{x_{(\alpha,\beta)}}$ in X such that $x_{(\alpha,\beta)} \in B_{x_{(\alpha,\beta)}} \subseteq A$. Thus we have

$$\begin{aligned} A &= \bigcup\{x_{(\alpha,\beta)} \mid x_{(\alpha,\beta)} \in A\} \\ &\subseteq \bigcup\{B_{x_{(\alpha,\beta)}} \mid x_{(\alpha,\beta)} \in A\} \\ &\subseteq A. \end{aligned}$$

Hence $A = \bigcup\{B_{x_{(\alpha,\beta)}} \mid x_{(\alpha,\beta)} \in A\}$. Therefore A is a fuzzy (r, s)-pre-semiopen set. □

Theorem 3.3. Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then $x_{(\alpha,\beta)} \in \text{pscl}(A, r, s)$ if and only if $B\text{q}A$ for any fuzzy (r, s)-quasi-pre-semineighborhood B of $x_{(\alpha,\beta)}$.

Proof. Suppose that there is a fuzzy (r, s)-quasi-pre-semineighborhood B of $x_{(\alpha,\beta)}$ such that $B\tilde{\text{q}}A$. Since B is a fuzzy (r, s)-quasi-pre-semineighborhood of $x_{(\alpha,\beta)}$, there is a fuzzy (r, s)-pre-semiopen set C such that $x_{(\alpha,\beta)}\text{q}C \subseteq B$. Thus $C\tilde{\text{q}}A$ and hence $A \subseteq C^c$. Since C^c is fuzzy (r, s)-pre-semiclosed, we have $x_{(\alpha,\beta)} \in \text{pscl}(A, r, s) \subseteq \text{pscl}(C^c, r, s) = C^c$. However $x_{(\alpha,\beta)} \notin C^c$, because $x_{(\alpha,\beta)}\text{q}C$. This is a contradiction.

Conversely, suppose $x_{(\alpha,\beta)} \notin \text{pscl}(A, r, s)$. Then there is a fuzzy (r, s)-pre-semiclosed set D such that $A \subseteq D$ and $x_{(\alpha,\beta)} \notin D$. Thus D^c is fuzzy (r, s)-pre-semiopen and $x_{(\alpha,\beta)}\text{q}D^c$, and hence D^c is a fuzzy (r, s)-quasi-pre-semineighborhood of $x_{(\alpha,\beta)}$. By hypothesis, $D^c\text{q}A$ and hence $A \nsubseteq (D^c)^c = D$. This is a contradiction. □

Definition 3.4. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is called

(1) a *fuzzy (r, s)-pre-semicontinuous* mapping if $f^{-1}(B)$ is a fuzzy (r, s)-pre-semiopen set in X for each fuzzy (r, s)-open set B in Y,

(2) a *fuzzy (r, s)-pre-semiopen* mapping if $f(A)$ is a fuzzy (r, s)-pre-semiopen set in Y for each fuzzy (r, s)-open set A in X,

(3) a *fuzzy (r, s)-pre-semiclosed* mapping if $f(A)$ is a fuzzy (r, s)-pre-semiclosed set in Y for each fuzzy (r, s)-closed set A in X.

Remark 3.5. It is clear that every fuzzy (r,s)-precontinuous and every fuzzy (r,s)-semicontinuous mapping is fuzzy (r,s)-pre-semicontinuous for each $(r,s) \in I \otimes I$. Also, every fuzzy (r,s)-preopen(resp. fuzzy (r,s)-preclosed) and every fuzzy (r,s)-semiopen(resp. fuzzy (r,s)-semiclosed) mapping is fuzzy (r,s)-pre-semiopen(resp. fuzzy (r,s)-pre-semiclosed). However, the following example shows that all of the converses need not be true.

Example 3.6. Let $X = \{x\}$ and let A_1 and A_2 be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0.2, 0.7), A_2(x) = (0.3, 0.5).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a mapping $f : (X, \mathcal{T}) \to (X, \mathcal{U})$ defined by $f(x) = x$. Since $A_2 \subseteq \text{sint}(\text{cl}(A_2, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_1^c$ in $(X, \mathcal{T})$, $f^{-1}(A_2) = A_2$ is fuzzy $(\frac{1}{2}, \frac{1}{3})$-pre-semiopen in $(X, \mathcal{T})$. Thus f is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-pre-semicontinuous mapping. However, f is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-precontinuous mapping, because $A_2 \nsubseteq \text{int}(\text{cl}(A_2, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_1$ in $(X, \mathcal{T})$. Since $A_1 \subseteq \text{sint}(\text{cl}(A_1, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_2^c$ in $(X, \mathcal{U})$, $f(A_1) = A_1$ is fuzzy $(\frac{1}{2}, \frac{1}{3})$-pre-semiopen in $(X, \mathcal{U})$. Thus f is a fuzzy $(\frac{1}{2}, \frac{1}{3})$-pre-semiopen mapping. However, f is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen mapping, because $\text{cl}(\text{int}(A_1, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = \underline{0} \not\supseteq A_1$ in $(X, \mathcal{U})$. On the other hand, consider a mapping $g : (X, \mathcal{U}) \to (X, \mathcal{T})$ defined by $g(x) = x$. Then g is fuzzy $(\frac{1}{2}, \frac{1}{3})$-pre-semicontinuous and not fuzzy $(\frac{1}{2}, \frac{1}{3})$-semicontinuous. Also, g is fuzzy $(\frac{1}{2}, \frac{1}{3})$-pre-semiopen and not fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen.

Theorem 3.7. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is fuzzy (r,s)-pre-semicontinuous.

(2) For each fuzzy (r,s)-closed set B in Y, $f^{-1}(B)$ is a fuzzy (r,s)-pre-semiclosed set in X.

(3) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r,s)-open set B in Y such that $f(x_{(\alpha,\beta)}) \in B$, there is a fuzzy (r,s)-pre-semiopen set A in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.

(4) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r,s)-neighborhood B of $f(x_{(\alpha,\beta)})$, $f^{-1}(B)$ is a fuzzy (r,s)-pre-semineighborhood of $x_{(\alpha,\beta)}$.

(5) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r,s)-neighborhood B of $f(x_{(\alpha,\beta)})$, there is a fuzzy (r,s)-pre-semineighborhood A of $x_{(\alpha,\beta)}$ such that $f(A) \subseteq B$.

(6) $f(\text{pscl}(A,r,s)) \subseteq \text{cl}(f(A),r,s)$ for each intuitionistic fuzzy set A in X.

(7) $\text{pscl}(f^{-1}(B),r,s) \subseteq f^{-1}(\text{cl}(B,r,s))$ for each intuitionistic fuzzy set B in Y.

(8) $f^{-1}(\text{int}(B,r,s)) \subseteq \text{psint}(f^{-1}(B),r,s)$ for each intuitionistic fuzzy set B in Y.

Proof. (1) $\Leftrightarrow$ (2) It is obvious.

(1) $\Rightarrow$ (3) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r,s)-open set in Y such that $f(x_{(\alpha,\beta)}) \in B$. Then $x_{(\alpha,\beta)} \in f^{-1}(B)$. Put $A = f^{-1}(B)$. Then by (1), A is a fuzzy (r,s)-pre-semiopen set in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(3) $\Rightarrow$ (1) Let B be a fuzzy (r,s)-open set in Y and $x_{(\alpha,\beta)} \in f^{-1}(B)$. Then $f(x_{(\alpha,\beta)}) \in B$. By (3), there is a fuzzy (r,s)-pre-semiopen set $A_{x_{(\alpha,\beta)}}$ in X such that $x_{(\alpha,\beta)} \in A_{x_{(\alpha,\beta)}}$ and $f(A_{x_{(\alpha,\beta)}}) \subseteq B$. Thus $x_{(\alpha,\beta)} \in A_{x_{(\alpha,\beta)}} \subseteq f^{-1}(f(A_{x_{(\alpha,\beta)}})) \subseteq f^{-1}(B)$. So we have

$$\begin{aligned} f^{-1}(B) &= \bigcup \{x_{(\alpha,\beta)} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\} \\ &\subseteq \bigcup \{A_{x_{(\alpha,\beta)}} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\} \\ &\subseteq f^{-1}(B). \end{aligned}$$

Thus $f^{-1}(B) = \bigcup \{A_{x_{(\alpha,\beta)}} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\}$ and hence $f^{-1}(B)$ is fuzzy (r,s)-pre-semiopen in X. Therefore f is a fuzzy (r,s)-pre-semicontinuous mapping.

(1) $\Rightarrow$ (4) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r,s)-neighborhood of $f(x_{(\alpha,\beta)})$. Then there is a fuzzy (r,s)-open set C in Y such that $f(x_{(\alpha,\beta)}) \in C \subseteq B$ and hence $x_{(\alpha,\beta)} \in f^{-1}(C) \subseteq f^{-1}(B)$. Since f is fuzzy (r,s)-pre-semicontinuous, $f^{-1}(C)$ is a fuzzy (r,s)-pre-semiopen set in X. Thus $f^{-1}(B)$ is a fuzzy (r,s)-pre-semineighborhood of $x_{(\alpha,\beta)}$.

(4) $\Rightarrow$ (5) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r,s)-neighborhood of $f(x_{(\alpha,\beta)})$. By (4), $A = f^{-1}(B)$ is a fuzzy (r,s)-pre-semineighborhood of $x_{(\alpha,\beta)}$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(5) $\Rightarrow$ (3) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r,s)-open set in Y such that $f(x_{(\alpha,\beta)}) \in B$. Then B is a fuzzy (r,s)-neighborhood of $f(x_{(\alpha,\beta)})$. By (5), there is a fuzzy (r,s)-pre-semineighborhood A of $x_{(\alpha,\beta)}$ in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$. Thus there is a fuzzy (r,s)-pre-semiopen set C in X such that $x_{(\alpha,\beta)} \in C \subseteq A$ and hence $f(C) \subseteq f(A) \subseteq B$.

(2) $\Rightarrow$ (6) Let A be an intuitionistic fuzzy set in X. Since $\text{cl}(f(A),r,s)$ is a fuzzy (r,s)-closed set in Y, $f^{-1}(\text{cl}(f(A),r,s))$ is a fuzzy (r,s)-pre-semiclosed set in X. Thus we have

$$\begin{aligned}\text{pscl}(A,r,s) &\subseteq \text{pscl}(f^{-1}(f(A)),r,s)\\ &\subseteq \text{pscl}(f^{-1}(\text{cl}(f(A),r,s)),r,s)\\ &= f^{-1}(\text{cl}(f(A),r,s)).\end{aligned}$$

Hence $f(\text{pscl}(A,r,s)) \subseteq f(f^{-1}(\text{cl}(f(A),r,s))) \subseteq \text{cl}(f(A),r,s)$.

(6) $\Rightarrow$ (7) Let B be an intuitionistic fuzzy set in Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy set in X. By (6), we have

$$\begin{aligned}f(\text{pscl}(f^{-1}(B),r,s)) &\subseteq \text{cl}(f(f^{-1}(B)),r,s)\\ &\subseteq \text{cl}(B,r,s).\end{aligned}$$

Hence

$$\begin{aligned}\text{pscl}(f^{-1}(B),r,s) &\subseteq f^{-1}(f(\text{pscl}(f^{-1}(B),r,s)))\\ &\subseteq f^{-1}(\text{cl}(B,r,s)).\end{aligned}$$

(7) $\Rightarrow$ (8) Let B be an intuitionistic fuzzy set in Y. By (7),

$$\begin{aligned}\text{psint}(f^{-1}(B),r,s)^c &= \text{pscl}(f^{-1}(B^c),r,s)\\ &\subseteq f^{-1}(\text{cl}(B^c,r,s)).\end{aligned}$$

Thus

$$\begin{aligned}f^{-1}(\text{int}(B,r,s)) &= f^{-1}(\text{cl}(B^c,r,s))^c\\ &\subseteq \text{psint}(f^{-1}(B),r,s).\end{aligned}$$

(8) $\Rightarrow$ (1) Let B be a fuzzy (r,s)-open set in Y. By (8),

$$\begin{aligned}f^{-1}(B) = f^{-1}(\text{int}(B,r,s)) &\subseteq \text{psint}(f^{-1}(B),r,s)\\ &\subseteq f^{-1}(B).\end{aligned}$$

Thus $f^{-1}(B) = \text{psint}(f^{-1}(B),r,s)$. Hence $f^{-1}(B)$ is a fuzzy (r,s)-pre-semiopen set in X. Therefore f is fuzzy (r,s)-pre-semicontinuous. □

Theorem 3.8. Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \rightarrow (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a bijective mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is fuzzy (r,s)-pre-semicontinuous.

(2) For each fuzzy (r,s)-closed set B in Y, $f^{-1}(B)$ is a fuzzy (r,s)-pre-semiclosed set in X.

(3) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r,s)-open set B in Y such that $f(x_{(\alpha,\beta)}) \in B$, there is a fuzzy (r,s)-pre-semiopen set A in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.

(4) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r,s)-neighborhood B of $f(x_{(\alpha,\beta)})$, $f^{-1}(B)$ is a fuzzy (r,s)-pre-semineighborhood of $x_{(\alpha,\beta)}$.

(5) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r,s)-neighborhood B of $f(x_{(\alpha,\beta)})$, there is a fuzzy (r,s)-pre-semineighborhood A of $x_{(\alpha,\beta)}$ such that $f(A) \subseteq B$.

(6) $f(\text{pscl}(A,r,s)) \subseteq \text{cl}(f(A),r,s)$ for each intuitionistic fuzzy set A in X.

(7) $\text{pscl}(f^{-1}(B),r,s) \subseteq f^{-1}(\text{cl}(B,r,s))$ for each intuitionistic fuzzy set B in Y.

(8) $f^{-1}(\text{int}(B,r,s)) \subseteq \text{psint}(f^{-1}(B),r,s)$ for each intuitionistic fuzzy set B in Y.

(9) $\text{int}(f(A),r,s) \subseteq f(\text{psint}(A,r,s))$ for each intuitionistic fuzzy set A in X.

Proof. By the above theorem, it suffices to show that (8) is equivalent to (9). Let A be an intuitionistic fuzzy set in X. Then $f(A)$ is an intuitionistic fuzzy set in Y. Since f is one-to-one, we have

$$\begin{aligned}f^{-1}(\text{int}(f(A),r,s)) &\subseteq \text{psint}(f^{-1}(f(A)),r,s)\\ &= \text{psint}(A,r,s).\end{aligned}$$

Since f is onto, we have

$$\begin{aligned}\text{int}(f(A),r,s) &= f(f^{-1}(\text{int}(f(A),r,s)))\\ &\subseteq f(\text{psint}(A,r,s)).\end{aligned}$$

Conversely, let B be any intuitionistic fuzzy set in Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy set in X. Since f is onto, we have

$$\begin{aligned}\text{int}(B,r,s) &= \text{int}(f(f^{-1}(B)),r,s)\\ &\subseteq f(\text{psint}(f^{-1}(B),r,s)).\end{aligned}$$

Since f is one-to-one, we have

$$\begin{aligned}f^{-1}(\text{int}(B,r,s)) &\subseteq f^{-1}(f(\text{psint}(f^{-1}(B),r,s)))\\ &= \text{psint}(f^{-1}(B),r,s).\end{aligned}$$

Hence the theorem follows. □

Theorem 3.9. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is a fuzzy (r, s)-pre-semiopen mapping.

(2) $f(\text{int}(A, r, s)) \subseteq \text{psint}(f(A), r, s)$ for each intuitionistic fuzzy set A in X.

(3) $\text{int}(f^{-1}(B), r, s) \subseteq f^{-1}(\text{psint}(B, r, s))$ for each intuitionistic fuzzy set B in Y.

Proof. (1) $\Rightarrow$ (2) Let A be an intuitionistic fuzzy set in X. Then $\text{int}(A, r, s)$ is a fuzzy (r, s)-open set in X. Since f is fuzzy (r, s)-pre-semiopen, $f(\text{int}(A, r, s))$ is a fuzzy (r, s)-pre-semiopen in Y. Thus we have

$$\begin{aligned} f(\text{int}(A, r, s)) &= \text{psint}(f(\text{int}(A, r, s)), r, s) \\ &\subseteq \text{psint}(f(A), r, s). \end{aligned}$$

(2) $\Rightarrow$ (3) Let B be an intuitionistic fuzzy set in Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy set in X. By (2),

$$\begin{aligned} f(\text{int}(f^{-1}(B), r, s)) &\subseteq \text{psint}(f(f^{-1}(B)), r, s) \\ &\subseteq \text{psint}(B, r, s). \end{aligned}$$

Thus we have

$$\begin{aligned} \text{int}(f^{-1}(B), r, s) &\subseteq f^{-1}(f(\text{int}(f^{-1}(B), r, s))) \\ &\subseteq f^{-1}(\text{psint}(B, r, s)). \end{aligned}$$

(3) $\Rightarrow$ (1) Let A be a fuzzy (r, s)-open set in X. By (3),

$$\begin{aligned} A = \text{int}(A, r, s) &\subseteq \text{int}(f^{-1}(f(A)), r, s) \\ &\subseteq f^{-1}(\text{psint}(f(A), r, s)). \end{aligned}$$

Thus we have

$$\begin{aligned} f(A) \subseteq f(f^{-1}(\text{psint}(f(A), r, s))) &\subseteq \text{psint}(f(A), r, s) \\ &\subseteq f(A). \end{aligned}$$

Thus $f(A) = \text{psint}(f(A), r, s)$. Hence $f(A)$ is a fuzzy (r, s)-pre-semiopen set in Y. Therefore f is fuzzy (r, s)-pre-semiopen. □

Theorem 3.10. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is fuzzy (r, s)-pre-semiclosed.

(2) $\text{pscl}(f(A), r, s) \subseteq f(\text{cl}(A, r, s))$ for each intuitionistic fuzzy set A in X.

Proof. (1) $\Rightarrow$ (2) Let A be an intuitionistic fuzzy set in X. Then $\text{cl}(A, r, s)$ is a fuzzy (r, s)-closed set in X. Since f is (r, s)-pre-semiclosed, $f(\text{cl}(A, r, s))$ is a fuzzy (r, s)-pre-semiclosed set in Y. Thus we have

$$\begin{aligned} \text{pscl}(f(A), r, s) &\subseteq \text{pscl}(f(\text{cl}(A, r, s)), r, s) \\ &= f(\text{cl}(A, r, s)). \end{aligned}$$

(2) $\Rightarrow$ (1) Let A be a fuzzy (r, s)-closed set in X. By (2),

$$f(A) \subseteq \text{pscl}(f(A), r, s) \subseteq f(\text{cl}(A, r, s)) = f(A).$$

Thus $f(A) = \text{pscl}(f(A), r, s)$. Hence $f(A)$ is a fuzzy (r, s)-pre-semiclosed set in Y. Therefore f is fuzzy (r, s)-pre-semiclosed. □

Theorem 3.11. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a bijective mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is fuzzy (r, s)-pre-semiclosed.

(2) $\text{pscl}(f(A), r, s) \subseteq f(\text{cl}(A, r, s))$ for each intuitionistic fuzzy set A in X.

(3) $f^{-1}(\text{pscl}(B, r, s)) \subseteq \text{cl}(f^{-1}(B), r, s)$ for each intuitionistic fuzzy set B in Y.

Proof. By the above theorem, it suffices to show that (2) is equivalent to (3). Let B be any intuitionistic fuzzy set in Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy set in X. Since f is onto, we have

$$\begin{aligned} \text{pscl}(B, r, s) &= \text{pscl}(f(f^{-1}(B)), r, s) \\ &\subseteq f(\text{cl}(f^{-1}(B), r, s)). \end{aligned}$$

Since f is one-to-one, we have

$$\begin{aligned} f^{-1}(\text{pscl}(B, r, s)) &\subseteq f^{-1}(f(\text{cl}(f^{-1}(B), r, s))) \\ &= \text{cl}(f^{-1}(B), r, s). \end{aligned}$$

Conversely, let A be an intuitionistic fuzzy set in X. Then $f(A)$ is an intuitionistic fuzzy set in Y. Since f is one-to-one, we have

$$\begin{aligned} f^{-1}(\text{pscl}(f(A), r, s)) &\subseteq \text{cl}(f^{-1}(f(A)), r, s) \\ &= \text{cl}(A, r, s). \end{aligned}$$

Since f is onto, we have

$$\begin{aligned} \text{pscl}(f(A), r, s) &= f(f^{-1}(\text{pscl}(f(A), r, s))) \\ &\subseteq f(\text{cl}(A, r, s)). \end{aligned}$$

Hence the theorem follows. □

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