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유통기한이 있는 제품의 할인정책에 관한 연구*

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A Discount Policy for Perishable Items Sold from Two Shops*

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■ Abstract ■

This paper deals with two shops dealing with single perishable product : the fresh items are sold at a list price in the primary shop and the unsold items that have reached a certain allowed age are transferred to the secondary shop to be sold at a discounted price. It is assumed that the demand rates in two shops are independent each other and can be expressed as a function of inventory level and price. With the objective of maximizing the profit under a Last-In-first-Out (LIFO) issuing policy, we develop mathematical models for the following two cases : (1) opening primary shop only and (2) opening both primary shop and secondary shop. There are three decision variables, i.e., the reduced price in the secondary shop, the allowed age at the primary shop, and the order quantities at the primary shop. A solution procedure is developed based on tabu search and its validity is illustrated through a comparative study.

Keyword : Perishable Item, Two Shops, LIFO Issuing Policy

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1. Introduction

This paper is motivated by commercial practice of grocery shops in some department stores where perishable products are sold by two-bin (shop) system. The products are purchased and received initially at the primary shop where they are sold at a list price. It is assumed that all ordered units arrive new and fresh, that is, their age equals zero. The units remaining unsold until they reach a certain age (will be called the allowed age, hereafter) at the primary shop are transferred to the secondary shop to be sold at a reduced price. Those units reaching their expiry date are removed from the inventory of the secondary shop and then discarded with disposal cost.

Perishable items can be divided into two categories, one with a fixed product lifetime and the other with a random lifetime. Milk is a representative example of fixed lifetime item. In the case of a random product lifetime, it comes from deterioration which is defined as decay, damage, spoilage, evaporation or loss of utility. Alcohol and gasoline are typical examples of the random lifetime item. There are many papers about perishable items. Cohen and Peckelman [6] dealt with the inventory cost minimization problem of items with fixed lifetime under stochastic demand. Adachi et al. [2] proposed an inventory model that considers different selling prices of perishable commodities under stochastic demand. They discriminated selling prices by different lifetimes. Fujiwara et al. [9] developed an optimal ordering and issuing policy for a two-stage inventory system for perishable items. They considered two types of inventories in which the product is stored in the cool-room (stage I) before being issued to the display shelves (stage II). In the case of deteriorating item, Pal et al. [16] developed an inventory model for the items that deteriorate at a constant demand rate, assuming that the demand rate is stock-dependent. Goyal and Gunasekaran [11] dealt with an integrated production-inventory-marketing model for deteriorating items in order to determine the economic production quantity (EPQ) and economic order quantity (EOQ).

A retailer usually considers the effects that the amount of inventory displayed has on the sales volume when he determines his procurement policy. Levin et al. [15] reported that sales at the retail level tend to be proportional to inventory level and a large pile of goods displayed will lead the customer to buy more. Baker and Urban [4] studied a basic model in which the demand rate $D(i)$ of an item is a function of the inventory level i, i.e. $D(i) = ai^{\beta}$ where $a > 0$, $0 < \beta < 1$. Datta and Pal [8] extended the model to the case in which the demand rate of an item is dependent upon the instantaneous inventory level until a given inventory level is achieved, after which the demand rate becomes constant. They assumed that at the end of each cycle the inventory level is zero. Recognizing that it can be more profitable to have higher inventory levels, Urban [19] relaxed the terminal condition of zero ending-inventory. Hwang and Hahn [13] evaluated an optimal procurement policy for the perishable items with an inventory-level-dependent demand rate in FIFO issuing policy.

Another important element related to the variation of sales volume is retail price. Many research papers examined inventory models assuming that the customer's demand rate is a decreasing function of selling price. Abad [1] dealt with the joint determination of price and lot size when the supplier offers all-unit quantity discounts. He used a constant price elasticity demand function with the form of $D = aP^{-\gamma}$, $\gamma > 1$. Also, Shinn et al. [18] introduced the retailer's price and lot size determination problem under the condition of permissible delay in payment.

There are several studies related to two shops (or bins). Kar et al. [14] proposed an inventory model for several deteriorating items (which have random lifetime). They assumed that the items are sold from two shops under single management with limitations on investment and total floor-space area. Das and Maiti [7] studied the model for the case in which shortages are allowed and fully backlogged. Chun [5] determined the optimal product price for perishable commodities with a given demand rate, buyers' preferences, and length of the sales period. He also proposed a multi-period pricing model for the case where the seller can divide the sales period into several short periods. Sezen [17] introduced a simple methodology that utilizes the probability obtained from the past experiences and puts forward an expected profit for each alternative discount policy.

This study deals with a discount policy for a single perishable product sold from two different shops (bins) in a grocery store. We adopt an EOQ-type model where an order is assumed to be placed automatically at the end of each unit period. Demands for the item in the two shops are assumed to be independent each other and expressed as a function of both inventory level and price. We want to decide the order quantities and allowed age at the primary shop, and the reduced price in the secondary shop so that the profit is maximized. Some literature on fixed lifetime item assumed that retailer enforces a FIFO issuing policy with the interest of minimizing outdates. In reality, many retailers arrange their goods from the oldest up or front to the new items down or back hoping that customers may pick the oldest goods. But many customers select the newest items at the stack, which will generally results in LIFO policy. The remainder of the paper is organized as follows. In Section 2, we describe the assumptions and notations adopted for this study, and then develop mathematical models under a LIFO issuing policy with the objective of maximizing the retailer's average net profit based on a periodic-review inventory model. We consider two cases; (1) opening primary shop only and (2) opening both primary and secondary shops. In Section 3, a solution procedure is developed based on the Tabu search algorithm. Numerical examples and sensitivity analysis are also provided. Finally, concluding remarks appear in Section 4.

2. Model Development

2.1 Notations and assumptions

The following notations and assumptions are introduced to develop the model.

Notations

M : lifetime of a perishable item expressed in integer multiple M of unit time period, i.e., day or week

Q : the order size

C_p : unit purchasing cost

C_0 : ordering cost per order

C_h : unit inventory holding cost per unit time

C_w : disposal cost per unit outdated. A negative C_w indicates salvage value for outdated units

For the kth shop where k = 1 (2) implies the primary shop (secondary shop),

S_k : order up to level at the *k*th shop (S_1 is decision variable)

m_k : allowed age for items at *k*th shop (m_1 is decision variable)

P_k : unit selling price at the *k*th shop (P_2 is decision variable)

H_k : number of the items carried as inventory at the *k*th shop during an order cycle

Assumptions

1) A periodic review inventory model is considered for a single perishable item. At the end of each period, the inventory level is reviewed and an order is placed.
2) All ordered units arrive new and fresh, that is, their age equals zero.
3) Demand rates in the primary and secondary shop are independent each other.
4) Demand is deterministic and decreasing with respect to the retailer's unit retail price and increasing with respect to the instantaneous inventory level *i*. Thus, the demand rate $D_k(i, P_k)$ of the item at the *k*th shop, *k* = 1, 2, is expressed as

$$D_k(i, P_k) = \alpha_k \frac{i^{\beta_k}}{P_k^{\gamma_k}}, \quad 0 < \beta_k < 1, \quad \gamma_k > 1,$$

where α_k is a scale parameter, β_k is a shape parameter which measures the responsiveness of the demand rate to changes in the level of on-hand inventory, and γ_k is the index of price elasticity at the *k*th shop.

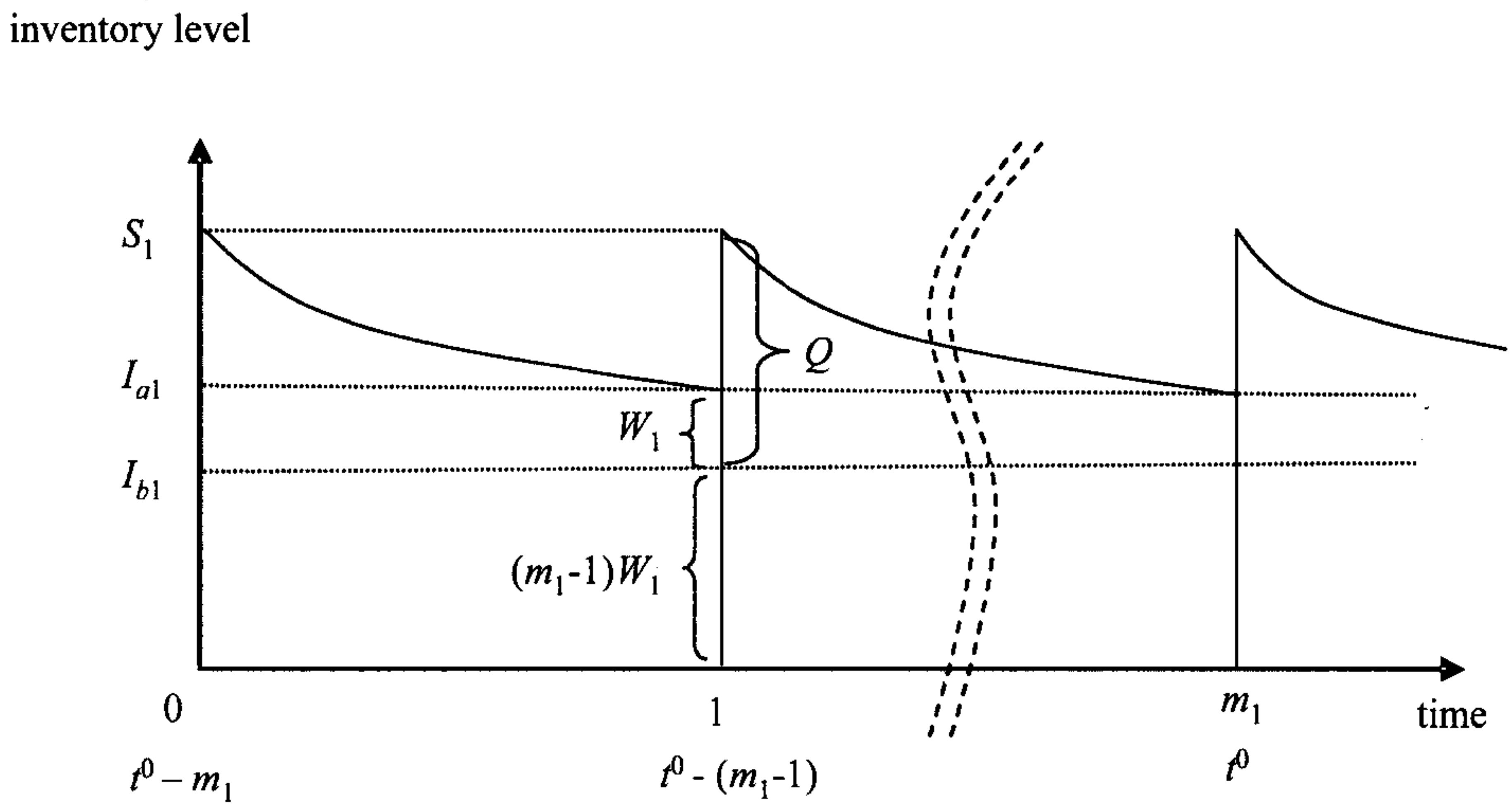
5) The lifetime of the perishable item is assumed as some integer multiple M of a unit time period.
6) All items in the primary shop are sold at a list price, and those at the secondary shop are sold at a same reduced price irrespective of remaining lifetime.
7) Inventory is depleted according to a LIFO issuing policy.
8) Replenishments are instantaneous with zero lead-time.
9) The time horizon of the inventory model is infinite.
10) Transferring cost of unsold item from the primary shop to the secondary shop is ignored.

2.2 Inventory Model

In the primary and secondary shops, the inventory level is reviewed, and a new order is placed at the end of each unit period (will be called review point, hereafter). The quantity demanded becomes greater at a high level of inventory due to the assumption of dependency of sales volume on the amount of inventory displayed. Consequently, the inventory level decreases rapidly at the beginning of a cycle. And the quantity demanded decreases at a decreasing rate as the inventory is depleted. Considering the profit expected from a high demand rate, it might be desirable to have somewhat large lot size allowing unsold stocks to be outdated. We adopt the results of Hwang and Hahn [13] to develop the mathematical expression of inventory level.

2.2.1 Primary shop

[Figure 1] shows a pictorial representation of



[Figure 1] An inventory graph at the primary shop under LIFO policy

the inventory levels in the primary shop. Under LIFO issuing policy, customers buy items from the most recently delivered lot. At each review point the unsold items W_1 that have reached the allowed age (m_1) are transferred, if exist, to the secondary shop and then a new order is placed. The order quantity Q is the difference between the inventory level and order up-to level S_1.

Now, we investigate the changes of inventory level during a repetitive order cycle. The differential equation describing the instantaneous state of inventory level $i(t)$ in the cycle is given by

$$\frac{\mathrm{d}i(t)}{\mathrm{d}t} = -\frac{\alpha_1[i(t)]^{\beta_1}}{P_1^{\gamma_1}}, \quad 0<t<1. \qquad (1)$$

Solving the differential equation (1) with the initial condition $i(t) = S_1$ at $t = 0$, we have

$$i(t) = \left[S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)t}{P_1^{\gamma_1}} \right]^{\frac{1}{1-\beta_1}}, \quad 0<t<1.$$

The inventory level I_{a1} at the end of period before transferring is

$$I_{a1} = i(1) = \left[S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}} \right]^{\frac{1}{1-\beta_1}}.$$

And the number of items carried as inventory during the period is

$$H_1 = \int_0^1 i(t)\mathrm{d}t = \frac{P_1^{\gamma_1}}{\alpha_1(2-\beta_1)} \{ S_1^{2-\beta_1} - I_{a1}^{2-\beta_1} \}.$$

Let S_{01} be the order up to level which makes $W_1 = 0$, i.e., $I_{a1} = 0$. Note that S_{01} can be a lower bound of S_1. From the definition of S_{01},

$$I_{a1} = \left[S_{01}^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}} \right]^{\frac{1}{1-\beta_1}} = 0, \text{ and}$$

$$S_{01} = \left[\frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}} \right]^{\frac{1}{1-\beta_1}}.$$

In the primary shop, at each review point the unsold items that have reached the allowed age (m_1) are transferred to the secondary shop and then a new order is placed. Due to the LIFO policy, products unsold during the first period of an arriving lot remains unsold until they reach the allowed age (m_1) to be transferred to the secondary shop. It can be seen that products transferred to the secondary shop at t^0 are part of the lot received at t^0-m_1. Let Q^i be the lot received at reorder point i. Also, let W_1^i be the amount unsold from Q^i. The inventory of the primary shop at t^0 after transferring but before receiving Q^{t_0} consists of W_1^i, $i = t^0-(m_1-1), t^0-(m_1-2), \cdots, t^0-1$. Note that under the deterministic nature of the problem W_1^i becomes identical for each i, i.e., $W_1^i = W_1$. Similarly, $Q^i = Q$. Let I_{a1} and I_{b1} be the inventory level at a review point before transferring and after transferring, respectively. Then, we can conclude that $I_{a1} = m_1 W_1$ and $I_{b1} = I_{a1} - W_1 = (m_1-1)W_1$.

From $I_{a1} = m_1 W_1$,

$$W_1 = \frac{1}{m_1} I_{a1} = \frac{1}{m_1}\left[S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\right]^{\frac{1}{1-\beta_1}}.$$

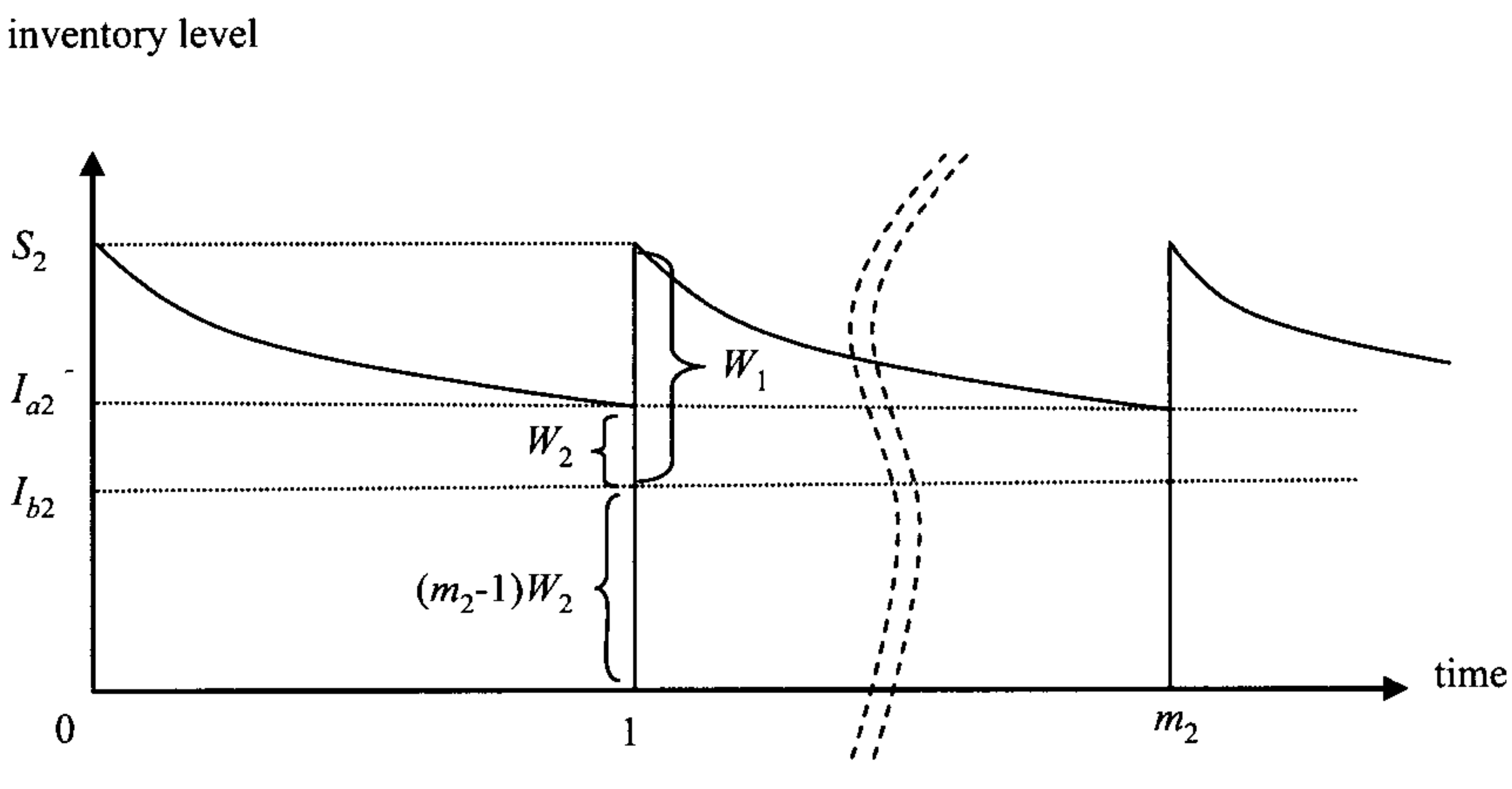
2.2.2 Secondary shop

[Figure 2] shows a pictorial representation of the inventory levels in the secondary shop under the proposed policy. Replenishments come from the primary shop and W_1 can be interpreted as the difference between the inventory level after outdating and order up-to level S_2. At each review point, the unsold items W_2 that have reached the allowed age of m_2, i.e., M time period, are outdated.

Now, we investigate the changes of inventory level during a repetitive order cycle. The differential equation describing the instantaneous states of inventory level $i(t)$ in the cycle is given by

$$\frac{di(t)}{dt} = -\frac{\alpha_2[i(t)]^{\beta_2}}{P_2^{\gamma_2}}, \quad 0 < t < 1. \tag{2}$$

Solving the differential equation (2) with the initial condition $i(t) = S_2$ at $t = 0$, we have



[Figure 2] An inventory graph at the secondary shop under LIFO policy

$$i(t) = \left[S_2^{1-\beta_2} - \frac{\alpha_2(1-\beta_2)t}{P_2^{\gamma_2}} \right]^{\frac{1}{1-\beta_2}}, \quad 0 < t < 1.$$

The inventory level I_{a2} at the end of period before outdating is

$$I_{a2} = i(1) = \left[S_2^{1-\beta_2} - \frac{\alpha_2(1-\beta_2)}{P_2^{\gamma_2}} \right]^{\frac{1}{1-\beta_2}}.$$

And the number of items carried as inventory during the period is

$$H_2 = \int_0^1 i(t)dt = \frac{P_2^{\gamma_2}}{\alpha_2(2-\beta_2)}\{S_2^{2-\beta_2} - I_{a2}^{2-\beta_2}\}.$$

Let S_{02} be the order up to level which makes $W_2 = 0$, i.e., $I_{a2} = 0$. It becomes a lower bound of S_2. From the definition of S_{02},

$$I_{a2} = \left[S_{02}^{1-\beta_2} - \frac{\alpha_2(1-\beta_2)}{P_2^{\gamma_2}} \right]^{\frac{1}{1-\beta_2}} = 0, \text{ and}$$

$$S_{02} = \left[\frac{\alpha_2(1-\beta_2)}{P_2^{\gamma_2}} \right]^{\frac{1}{1-\beta_2}}.$$

From $I_{a2} = m_2W_2$ and $I_{b2} = (m_2-1)W_2$, W_2 can be expressed as

$$W_2 = \frac{1}{m_2} I_{a2} = \frac{1}{m_2}\left[S_2^{1-\beta_2} - \frac{\alpha_2(1-\beta_2)}{P_2^{\gamma_2}} \right]^{\frac{1}{1-\beta_2}}.$$

And the following relation also holds.

$S_2 = W_1 + (m_2 - 1)W_2$

2.3 Model with primary shop only

We consider the model with primary shop only where $m_1 = M$ and $m_2 = 0$. An optimal S_1 can be found by examining the following two cases: $S_1 > S_{01}$ and $S_1 = S_{01}$

2.3.1 Case with S1>S01

$S_1 > S_{01}$ implies that the outdating quantity becomes nonzero. The revenue and cost elements of the objective function are as follow :

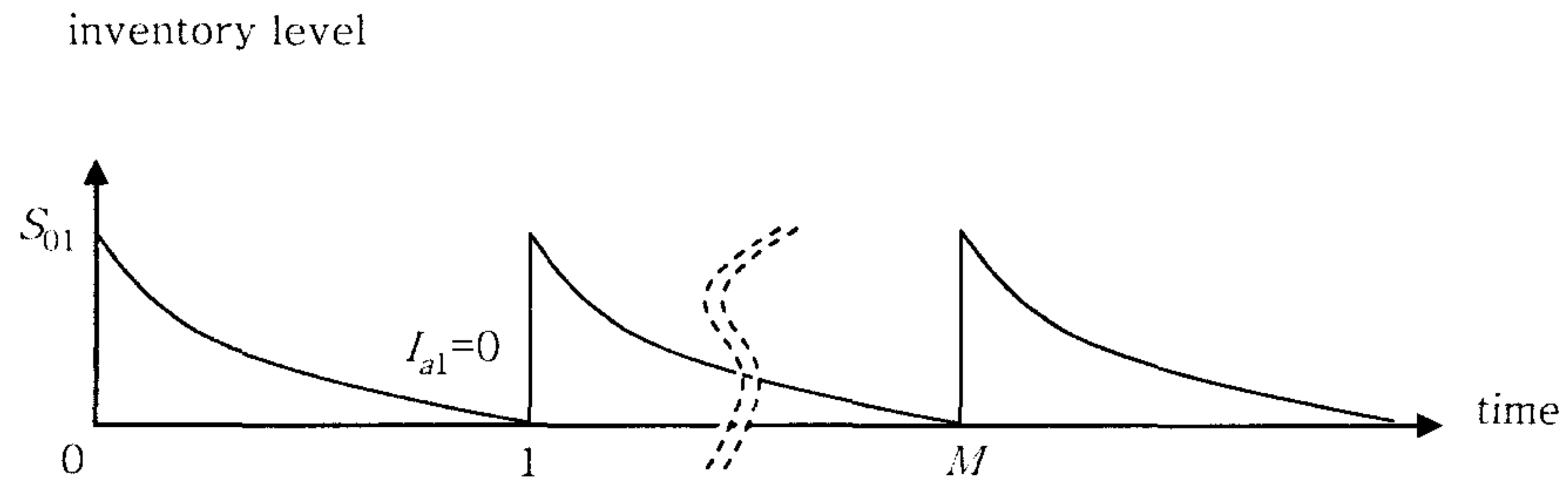
1) Average sales revenue = $P_1(Q-W_1)$
2) Average purchasing cost = C_pQ
3) Average holding cost = C_hH_1
4) Average ordering cost = C_0
5) Average outdating cost = C_wW_1

The average net profit can be formulated as follows :

Max $\pi pa(S_1)$

$$= P_1(Q - W_1) - C_pQ - C_hH_1 - C_wW_1 - C_0$$

$$= P_1(S_1 - I_{a1}) - C_p\{S_1 - (M-1)W_1\} - C_hH_1 - \frac{C_wI_{a1}}{M} - C_0$$



[Figure 3] An inventory graph at the primary shop with $S_1 = S_{01}$

$$= P_1[S_1 - \{S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\}^{\frac{1}{1-\beta_1}}]$$
$$- C_p[S_1 - \frac{M-1}{M}\{S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\}^{\frac{1}{1-\beta_1}}]$$
$$- C_h \cdot \frac{P_1^{\gamma_1}}{\alpha_1(2-\beta_1)}[S_1^{2-\beta_1} - \{S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\}^{\frac{2-\beta_1}{1-\beta_1}}]$$
$$- \frac{C_w}{M}\{S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\}^{\frac{1}{1-\beta_1}} - C_0$$

subject to

$$S_1 > S_{01} = [\frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}]^{\frac{1}{1-\beta_1}}$$

2.3.2 Case with $S_1 = S_{01}$

$S_1 = S_{01}$ implies that no item is outdated and thus W_1 is zero. The average net profit becomes :

$$\pi pb = (P_1 - C_p)S_{01} - C_h H_1 - C_0$$
$$= (P_1 - C_p)\{\frac{\alpha_1(1-\beta_1)}{P_1}\}^{\frac{1}{1-\beta_1}} - \frac{C_h P_1^{\gamma_1} S_{01}}{\alpha_1(2-\beta_1)} - C_0$$
$$= (P_1 - C_p)\{\frac{\alpha_1(1-\beta_1)}{P_1}\}^{\frac{1}{1-\beta_1}} - \frac{C_h P_1^{\gamma_1}}{\alpha_1(2-\beta_1)}\{\frac{\alpha_1(1-\beta_1)}{P_1}\}^{\frac{1}{1-\beta_1}} - C_0$$

Note that πpb is constant and provides a lower bound of the optimal objective function value.

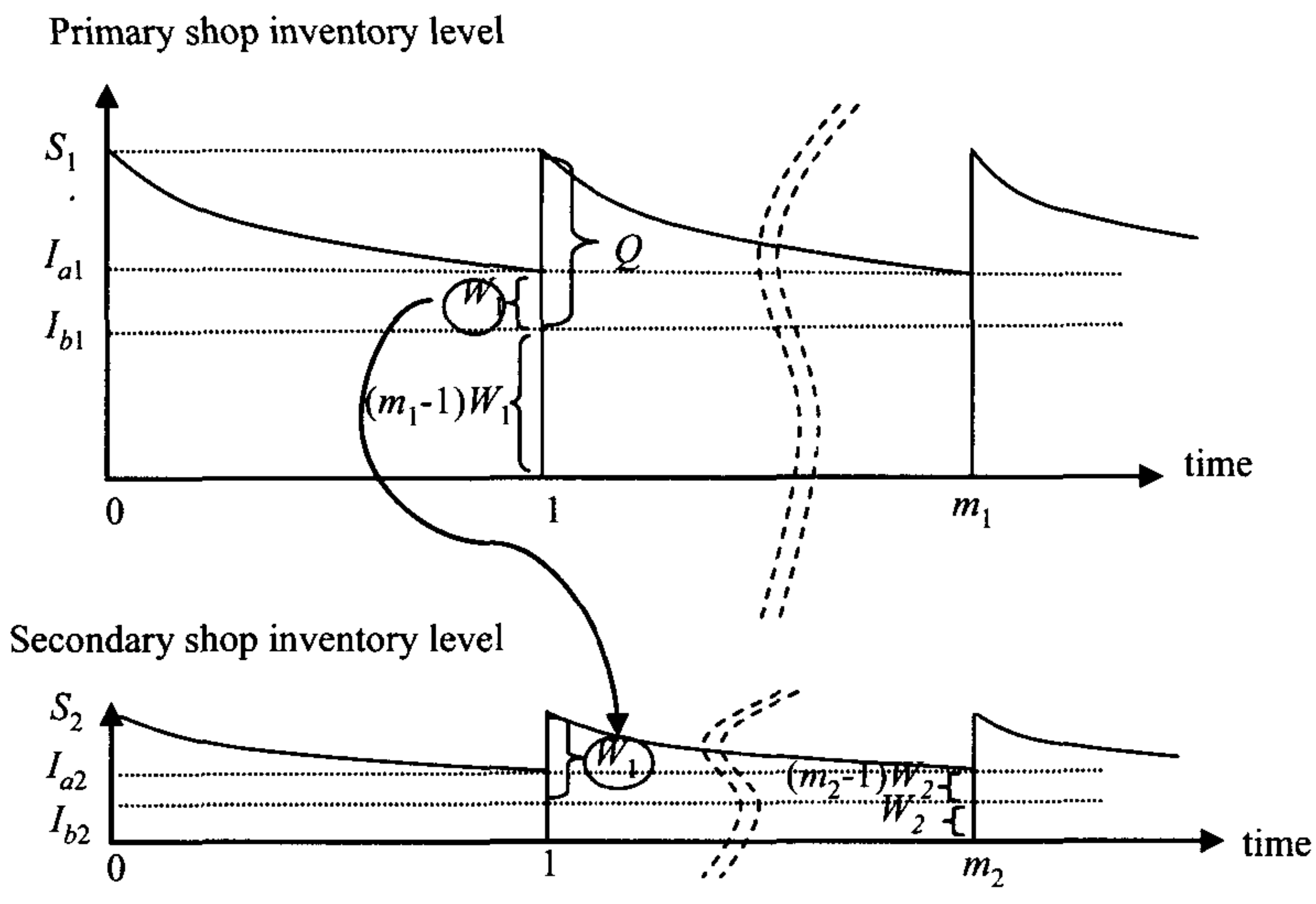
2.4 Model with two shops

We consider the case where both the primary and secondary shops are open. Let π_1 and π_2 be the profit of the primary shop and secondary shop, respectively. The purchasing and ordering costs occur only at the primary shop and the outdating cost occurs only at the secondary shop. Note that W_1 is required to be nonzero, i.e., $S_1 > S_{01}$, to have the secondary shop open. The following two cases are examined : $S_2 > S_{02}$ and $S_2 = S_{02}$.

2.4.1 Case with $S_2 > S_{02}$

The average net profit of the primary shop consists of the following revenue and cost elements:

1) Average sales revenue = $P_1(Q-W_1)$
2) Average purchasing cost = C_pQ
3) Average holding cost = C_hH_1
4) Average ordering cost = C_0



[Figure 4] Inventory graph at the primary and secondary shop with $S_2 > S_{02}$

Thus the average net profit of the primary shop can be formulated as

$$\pi ta_1(S_1, m_1) = P_1(Q - W_1) - C_pQ - C_hH_1 - C_0$$
$$= P_1(S_1 - I_{a1}) - C_p\{S_1 - (m_1 - 1)W_1\} - C_hH_1 - C_0$$
$$= P_1[S_1 - \{S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\}^{\frac{1}{1-\beta_1}}]$$
$$- C_p[S_1 - \frac{m_1-1}{m_1}\{S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\}^{\frac{1}{1-\beta_1}}]$$
$$- C_h \cdot \frac{P_1^{\gamma_1}}{\alpha_1(2-\beta_1)}[S_1^{2-\beta_1} - \{S_1^{1-\beta_1}$$
$$- \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\}^{\frac{2-\beta_1}{1-\beta_1}}] - C_0$$

Whereas for the secondary shop, the revenue and cost elements are :

1) Average sales revenue = $P_2(W_1-W_2)$
2) Average holding cost = C_hH_2
3) Average outdating cost = C_wW_2

The average net profit of the secondary shop becomes :

$$\pi ta_2(S_2, P_2, m_1) = P_2(W_1 - W_2) - C_hH_2 - C_wW_2$$
$$= P_2(S_2 - I_{a2}) - C_hH_2 - C_wW_2$$
$$= P_2[S_2 - \{S_2^{1-\beta_2} - \frac{\alpha_2(1-\beta_2)}{P_2^{\gamma_2}}\}^{\frac{1}{1-\beta_2}}] - C_h \cdot$$
$$\frac{P_2^{\gamma_2}}{\alpha_2(2-\beta_2)}[S_2^{2-\beta_2} - \{S_2^{1-\beta_2} - \frac{\alpha_2(1-\beta_2)}{P_2^{\gamma_2}}\}^{\frac{2-\beta_2}{1-\beta_2}}]$$
$$- \frac{C_w}{M - m_1}\{S_2^{1-\beta_2} - \frac{\alpha_2(1-\beta_2)}{P_2^{\gamma_2}}\}^{\frac{1}{1-\beta_2}}$$

S_2 can be expressed as a function of S_1, m_1 and P_2 as follows :

$$S_2 = W_1 + (m_2 - 1)W_2$$
$$= \frac{1}{m_1}[S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}]^{\frac{1}{1-\beta_1}}$$
$$+ \frac{(m_2-1)}{m_2}[S_2^{1-\beta_2} - \frac{\alpha_2(1-\beta_2)}{P_2^{\gamma_2}}]^{\frac{1}{1-\beta_2}}$$

The average profit π with the decision variables S_1, m_1 and P_2 can be formulated as

Max πta (S_1, m_1, P_2)

$$= \pi ta_1 + \pi ta_2$$
$$= P_1[S_1 - \{S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\}^{\frac{1}{1-\beta_1}}] - C_p[S_1 - \frac{(m_1-1)}{m_1}$$
$$\{S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\}^{\frac{1}{1-\beta_1}}]$$
$$-C_h \cdot \frac{P_1^{\gamma_1}}{\alpha_1(2-\beta_1)}[S_1^{2-\beta_1} - \{S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\}^{\frac{2-\beta_1}{1-\beta_1}}] - C_0$$
$$+P_2[S_2 - \{S_2^{1-\beta_2} - \frac{\alpha_2(1-\beta_2)}{P_2^{\gamma_2}}\}^{\frac{1}{1-\beta_2}}] - C_h \cdot$$
$$\frac{P_2^{\gamma_2}}{\alpha_2(2-\beta_2)}[S_2^{2-\beta_2} - \{S_2^{1-\beta_2} - \frac{\alpha_2(1-\beta_2)}{P_2^{\gamma_2}}\}^{\frac{2-\beta_2}{1-\beta_2}}]$$
$$- \frac{C_w}{m_2}\{S_2^{1-\beta_2} - \frac{\alpha_2(1-\beta_2)}{P_2^{\gamma_2}}\}^{\frac{1}{1-\beta_2}}$$

where

$$S_2 = \frac{1}{m_1}[S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}]^{\frac{1}{1-\beta_1}}$$
$$+ \frac{(m_2-1)}{m_2}[S_2^{1-\beta_2} - \frac{\alpha_2(1-\beta_2)}{P_2^{\gamma_2}}]^{\frac{1}{1-\beta_2}}$$

subject to

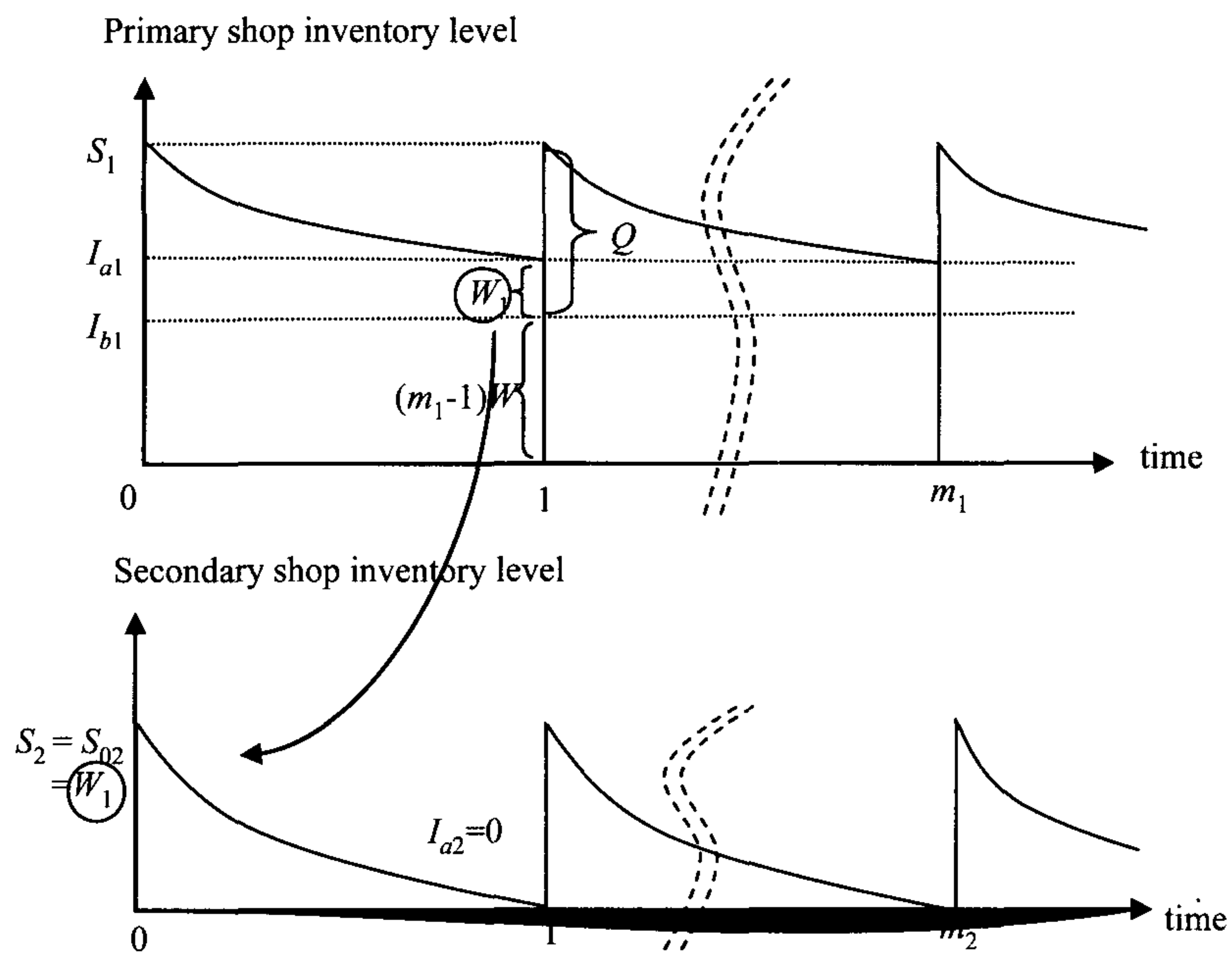
$$S_1 > S_{01} = [\frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}]^{\frac{1}{1-\beta_1}}$$

$$S_2 > S_{02} = [\frac{\alpha_2(1-\beta_2)}{P_2^{\gamma_2}}]^{\frac{1}{1-\beta_2}}$$

2.4.2 Case with $S_2 = S_{02}$

In this case, W_2 is zero and the outdating cost does not occur at the secondary shop. Consequently, S_2 equals W_1. From

$S_2 = S_{02} = [\frac{\alpha_2(1-\beta_2)}{P_2^{\gamma_2}}]^{\frac{1}{1-\beta_2}}$ and



[Figure 5] Inventory graph at the primary and secondary shop with $S_2 = S_{02}$

$$W_1 = \frac{1}{m_1} I_{a1} = \frac{1}{m_1}\left[S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\right]^{\frac{1}{1-\beta_1}}$$, we find that

$$S_2 = W_1$$

$$= \frac{1}{m_1}\left[S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\right]^{\frac{1}{1-\beta_1}} = \left[\frac{\alpha_2(1-\beta_2)}{P_2^{\gamma_2}}\right]^{\frac{1}{1-\beta_2}}$$

Rearranging the above,

$$P_2 = \frac{\{m_1\alpha_2(1-\beta_2)\}^{\frac{1}{\gamma_2}}}{\left\{S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\right\}^{\frac{1-\beta_2}{\gamma_2(1-\beta_1)}}}$$

The average sales revenue and average holding cost are P_2S_2 and C_hH_2, respectively. Thus the average net profit of the secondary shop can be found as follows :

$$\pi tb_2(S_1, m_1) = P_2S_2 - C_hH_2 = P_2W_1 - C_hH_2$$

$$= P_2 \cdot \frac{1}{m_1}\left\{S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\right\}^{\frac{1}{1-\beta_1}}$$

$$- C_h \cdot \frac{P_2^{\gamma_2}}{\alpha_2(2-\beta_2)} S_2^{2-\beta_2}$$

$$= \alpha_2(1-\beta_2)\left\{S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\right\}^{\frac{\beta_2}{1-\beta_1}}$$

$$- \frac{C_h m_1^{\beta_2}(1-\beta_2)}{2-\beta_2}$$

Combining $\pi ta_1(S_1, m_1)$ in 2.4.1 and $\pi tb_2(S_1, m_1)$, the average profit πtb can be formulated as follows :

Max πtb (S_1, m_1)

$$= \pi ta_1 + \pi tb_2$$

$$= P_1[S_1 - \{S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\}^{\frac{1}{1-\beta_1}}] - C_p[S_1 - \frac{(m_1-1)}{m_1}$$

$$\{S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\}^{\frac{1}{1-\beta_1}}]$$

$$-C_h \cdot \frac{P_1^{\gamma_1}}{\alpha_1(2-\beta_1)}[S_1^{2-\beta_1} - \{S_1^{1-\beta_1} - \frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}\}^{\frac{2-\beta_1}{1-\beta_1}}] - C_0$$

subject to

$$S_1 > S_{01} = [\frac{\alpha_1(1-\beta_1)}{P_1^{\gamma_1}}]^{\frac{1}{1-\beta_1}}$$

3. Solution Algorithm

3.1 Tabu search

The decision variables in our models include one integer, m_1 and two real numbers, S_1 and P_2. Even though the objective function is partially differentiable with respect to S_1 and P_2, the resulting equation is mathematically intractable, i.e., even with a given value of m_1 it seems very complicated to find a general explicit solution for S_1 and P_2. There have been attempts to develop algorithms to find solutions of nonconvex function minimization problems. Among others, simulated annealing-based algorithm, genetic algorithms and tabu search algorithm are mostly well known in the literature. In this study we adopt the Sultan-Fawzan Tabu search algorithm [3] to solve our problem. Glover [10] introduced tabu search where search starts at an initial point and then moves successively among neighboring points. During each iteration, a move is made to the best point in the neighborhood of the current point. In order to guide the search toward unexplored solution space without cycling, the method forbids points (called tabu) that satisfy certain attributes. Elements in a tabu list include the latest moves made with the goal of no generation of recently visited points again. The basic ingredients in tabu search are initial point, neighborhood generation method, tabu list, long-term memory, aspiration criterion and stopping criterion.

3.2 The proposed algorithm

For the development of the algorithm we utilize the optimization technique of Hooke and Jeeves[12]. The algorithm starts at an initial point and then goes through NI number of iterations. At each iteration i, the algorithm performs NC exploratory searches (cycles) and one pattern search. We set the point x_i of each iteration at the starting point z_1 for the cycles and obtain the points $z_2, z_3, \cdots, z_{NC+1}$. In each cycle, we randomly generate ND directions and perform a line search along each direction. We select the direction with the maximum functional value under the condition that it is nontabu or it is tabu but better than the best solution found so far. Then the direction is stored in the tabu list. The negative of any direction in the tabu list is also stored in the tabu list. After NC cycles, we perform a line search along the direction $z_{NC+1}-z_1$ to generate the next point x_{i+1}, which constitutes the pattern step. The algorithm repeats the above procedure at next iteration starting from the point x_{i+1}. At each iteration the second best value is stored among the neighborhood points. This memory structure, denoted as long-term memory, is updated at each exploration of a neighborhood and it contains a suitable number of the second best solutions found during the search. When one of the following three conditions holds, we remove from the long-term memory the solution with the best objective function value and continue the search from that solution :

(1) Every neighboring point is in tabu without satisfying an aspiration criterion.

(2) No improvement in the current function value for a sequence of consecutive moves.

(3) For a sequence of consecutive moves no improvement in the best solution found.

Regarding to the aspiration criterion, we allow a tabu move if it leads to a solution with an objective function value better than that of the best solution found so far. Now, we present the proposed algorithm as follows :

Step 1 : Initialization

Let NL be a suitable size for the tabu list. Declare ND, NC, NL, and NI. Choose a starting point x_1. Let $z_1 = x_1$. Let tabu list = T = $\emptyset$, Best_value= $\pi(x_1)$, $j=k=1$ where π represents the objective function.

Step 2 : Perform NC number of cycles.

2.1 Generate ND different number of random directions, $R(1)$, $R(2)$, $\cdots$,$R(ND)$.

Let $\lambda*$ and $R*$ be such that

$\pi(z_j + \lambda^* R^*) = \max_{1 \le i \le ND} \pi(z_j + \lambda_i R(i))$.

2.2 Check the tabu status.

If ($R^* \notin T$) or ($R^* \in T$ and $\pi(z_j + \lambda^* R^*) >$ Best_value), go to step 2.3.

Otherwise, choose the next best solutions. Repeat Step 2.2.

If there is no more solution, we remove the solution with the best objective function value from the long-term memory and go to step 2.3.

2.3 Update the current point.

Let $z_{j+1} = z_j + \lambda^* R^*$.

Store $R*$ in T and update it accordingly.

If $\pi(z_j + \lambda^* R^*) >$ Best_value, then Best_value = $\pi(z_j + \lambda^* R^*)$.

2.4 If $j = NC$, go to Step 3. Otherwise, replace j by j+1 and go to Step 2.

Step 3 : Check the long-term memory.

Check the second and the third of the long-term memory conditions.

If one of them is satisfied, we remove the solution with the best objective function value from the long-term memory and continue the search from that solution.

Step 4 : Perform pattern search

Let $R^0 = z_{NC+1} - z_1$. Let λ^0 be the optimal solution to the problem satisfying

$\max_{\lambda} \pi(z_{NC+1} + \lambda R^0)$.

Let $x_{k+1} = z_{NC+1} + \lambda^0 R^0$, $z_1 = x_{k+1}$, replace k by k+1, and go to Step 2.

Step 5 : Check stopping criterion.

If k = NI, stop.

Else, go to Step 2.

3.3 Example problem

The performance of the proposed heuristic was examined through a comparative study with near-optimal solutions obtained by grid search method. An example problem has the following parameter values : $\alpha_1 = 500$, $\alpha_2 = 400$, $\beta_2 = 0.5$, $\gamma_1 = 1.4$, $P_1 = 20$, $C_p = 10$, $C_h = 0.5$, $C_0 = 5$, $C_w = 1$, $M = 7$. With two different values of β_1 and three different values of γ_2 , we solved six problems. The computation results are listed in <Table 1>.

We find that the performance of the proposed heuristic is satisfactory since the maximum deviation of the solutions from those of grid search is 1.39%, whereas the computation time of the proposed algorithm is substantially smaller.

〈Table 1〉 Performance of the Proposed Algorithm

β_1	γ_2	Tabu Search		Grid Search		Deviation(%)[c]
		π_T^a	running time (second)	π_G^b	running time (second)	
0.4	1.4	997.3	≤1	1011.3	494	-1.3900
	1.5	722.7	≤1	722.8	499	-0.0032
	1.6	580.5	≤1	580.5	496	0.0000
0.5	1.4	1771.3	≤1	1771.3	495	0.0000
	1.5	1488.7	≤1	1488.7	496	-0.0007
	1.6	1351.8	≤1	1351.9	498	-0.0022

주) [a] : Solution from the tabu search algorithm.
[b] : Solution from grid search method.
[c] : 100 $(\pi_T - \pi_G)/\pi_G$.

4. Conclusions

Discount pricing policy has been an important issue for large-scale retailers who deal with perishable products. When customers buy perishable products with known lifetime, they are less likely to purchase the items whose expiry dates are closer. Thus, the item having a short time left until its expiry date can be sold at a discounted price to stimulate customer's demand. This paper examines analytically a commercial practice of selling perishable products through two shop system where fresh items are sold at a list price in the primary shop while unsold items that have reached a certain allowed age are transferred to the secondary shop to be sold at a discounted price. We assume that the demand rate in each shop is independent each other and can be expressed as a function of inventory level and price. Since there is no guarantee that opening the secondary shop is always more desirable than having only one shop, we develop the mathematical models for both cases under LIFO issuing policy. Tabu search algorithm is developed to find the solution of the model. The validity of the algorithm is shown by a comparative study with grid search method. There could be interesting opportunities for future research in this subject. When there is a price difference between product segments, customer can migrate from high priced segments to low priced segments. In order to capture the intuition that more customers are willing to migrate to low priced segments as the price difference increases, Zhang and Bell [20] modeled the demand leakage as a linear function of the difference between the prices. Thus the demand function adopted in this study could be extended such that it integrates demand leakage triggered by a lower price in the secondary shop. Also, a stochastic rather than deterministic demand function seems more realistic.

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