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Welfare Effects of the Tax Reforms in Two Vertically-Related Oligopolies with Environmental Externality

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국 문 요 약

본 논문에서는 산업구조상 수직적으로 연결된 과점산업들을 상정한 뒤, 상류 및 하류 산업의 과점으로 인해 발생하는 최적에 미달하는 산출수준, 하류부문의 투입물 사용에서 발생하는 비효율성, 그리고 상류부문 과점산업의 환경오염으로 인해 발생하는 네 가지 비효율성을 모두 고려할 수 있는 이론 모형으로 정식화하고, 정책당국이 세수중립적인 세제개혁을 통해서 어떻게 후생증진을 도모할 수 있는지 논증한다. 이를 위해서 상류부문이 생산하는 중간재와 하류부문이 생산하는 최종재에 각각 산출세를 부과하는 경우와 환경오염 자체에 피구稅를 부과하는 경우를 분리 상정하고, 각각의 후생변화를 수식으로 도출한다. 그 결과, 오염에 부과되는 생산요소간 대체를 가능하게하는 피구稅가 생산감축을 통한 환경오염감소를 허용하는 산출세에 비해서 더 효과적으로 후생을 증진시킬 수 있는 논거가 경쟁적 단일산업에서뿐만 아니라 수직적으로 연결된 과점산업구조하에서도 여전히 타당함을 증명하고, 세수중립적인 조세-보조금 정책조합을 이용한 조세개혁을 통해 정책당국이 후생증진을 도모할 수 있는 조건을 명시적으로 도출한다. 또한 상하류 과점산업들의 한계적 이윤비용구조 및 한계적 환경오염의 상대적 크기가 각 산업에 대한 조세-보조금 정책조합의 방향을 결정하는데 매우 중요한 역할을 한다는 점을 밝힌다.

❙ 주제어 ❙ 조세 및 보조금, 환경세, 수직적 과점, 피구세, 산출세

Abstract

In this paper, I examine the welfare effects of various revenue-neutral tax reforms in the case of two vertically-related oligopolies (downstream and upstream), where the upstream industry is polluting. I show analytically when and how government can improve welfare by initiating various tax reforms, regardless of either the feasibility of a lump sum transfer or the availability of a tax on pollution. The profit wedge that is the difference between the unit price and the unit cost and the marginal environmental damages (MED) becomes important to decidethe direction of a tax reform and is crucial to determine the direction of welfare-improving tax-subsidy schemes. I also show that a tax on pollution (Pigouvian tax) is superior to a tax on intermediate good even in the case of vertically-related oligopolies, because the former always brings in positive welfare effect from the upstream firms'input substitutability, which a tax on intermediate good cannot provide. Some policy implications for 'reducing environmentally-harmful subsidies' are also discussed.

❙ Keywords ❙ environmental tax policies, vertical oligopolies, emissions tax, output tax
JEL classifications: Q58, H23, D43

I Introduction

In a single imperfectly competitive industry, the output level is less than the competitive level due to the divergence between price and marginal cost (so called 'marginalization' distortion). If this imperfectly competitive industry is polluting, the second distortion arises from environmental externality. If this polluting oligopolistic (upstream) industry supplies its product to another (downstream) industry — in other words, if both industries are 'vertically-related' — then an input-mix distortion in the downstream industry arises due to the marginalization distortion from the upstream industry. Furthermore, if the downstream industry is also imperfectly competitive, then another marginalization distortion arises.

It is not difficult to find markets that have these conditions. Many industries consist of only a few firms, including automobiles, iron and steel, and petroleum refining industries. Among them, many are polluting and vertically-related to other oligopolistic industries. For instance, it is well known that coal is one of the most polluting fossil fuels, and the iron and steel industry heavily relies on coal and emits various pollutants.1) Furthermore, the iron and steel industry supplies its product to several oligopolistic final goods producers such as automobiles and electrical and machinery equipment industries.

This article can be considered as an extension from — as well as a link among — various previous studies on taxation, market imperfection, and environmental externalities. However, previous studies have been mostly interested in a single oligopolistic industry.2) When vertically-related oligopolies were considered, the

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1) As of 1994, the U.S. iron and steel industry accounts for approximately 9 percent of all U.S. manufacturing use of energy. Nearly half of the industry's energy is derived from coal (US DOE, 2000). The situation is not much different in Korea: The close relation between the oil and coal industries and the iron and steel industries has been strengthened further in the 1995-2000 period (Rhee and Kim, 2006).

2) For example, Konishi (1990) examines the case where an oligopolistic industry producing a final good uses many intermediate goods but the intermediate goods are produced competitively. Although not an exhaustive list, the following different aspects of the topic have been studied: the case of symmetric oligopoly with a fixed number of firms and various types of technology (Ebert, 1991), the case of endogenous market structure (Katsoulacos and Xepapadeas, 1995a),

focuses were on the other aspects of market imperfection other than environmental externalities. For example, Myles (1989) considers two vertically-related industries, but only one of them is imperfectly competitive. Panzar and Sibley (1989) consider vertically-related industries, but they focus on the optimal two-part tariff that corrects for the marginalization distortion in the downstream industry due to the upstream monopoly. Colangelo and Galmarini (2001) also examine the vertically-relatedoligopoly case, but their focus is on the relative advantages of value-added taxation over cascade taxation when the firms in the upstream industry produce intermediate goods.Furthermore, they only consider the importance of the downstream input substitutability, because the upstream producers have a single input and, therefore, cannot substitute between inputs. Though some papers have examined environmental problems in the context of vertically-related oligopolies, they have usually focused on other aspects of externalities. For instance, Ki-Dong Lee (2007) examines strategic incentives to distort the use of emissions tax on intermediate good production in successively oligopolistic industries, when trans-boundary pollution problem exists. On the other hand, Canton, et al. (2008) examine the upstream eco-industry and its market structure on the effectiveness of emissions tax, which is the only available policy instrument for environmental authority.

In this paper, I construct an analytical partial equilibrium model of two vertically-related oligopolies where the upstream industry is polluting, and examine the welfare effects of various revenue-neutral tax reforms. The upstream industry produces an intermediate good using labor, and it generates pollution. The downstream industry produces a final good using the intermediate good and labor.

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endogenous entry/exit decisions (Katsoulacos and Xepapadeas, 1995b), dynamic tax/subsidy schemes when the stock of pollution accumulates over time (Benchekroun and Long, 1998), endogenous product quality (Cremer and Thisse, 1999; Goering and Boyce, 1999), strategy-proof optimal tax schemes (Kim and Chang, 1993; Shaffer, 1995), general functional forms for demand (S-H Lee, 1999), the link between pollution taxes and firms' financial decisions (Damania, 2000), asymmetric cost functions among producers (Levin, 1985; Simpson, 1995; Carlsson, 2000), the effects of a shift from specific to ad valorem taxation (Okuguchi and Yamazaki, 1994), and the second-best environmental taxation with both monopoly and distorting labor taxes (Fullerton and Metcalf, 2002). See also Requate (2007) for broad discussion about optimal taxation on emissions in non-competitive markets.

I incorporate the above mentioned four distortions into a single framework and introduce three different taxes (or subsidies) for purposes of various tax reforms: (i) a tax on the final good, (ii) a tax on the intermediate good, and (iii) a tax on pollution. In my model, both the upstream and downstream producers can substitute one input for another. As I will show later, these extensions are important in that they yield significantly different results from previous studies.

I derive an analytical expression for welfare change from various combinations of tax instruments and examine how environmental authority can use a general expression of welfare change to improve welfare. I focus on two cases: the case for an ad valorem tax on the intermediate good and another case for an emissions tax. For these two different cases, I show that the changes in welfare from these tax reforms are functions of the degree of market power as well as input substitutability in both industries, and the demand and cost structures in both industries.

A tax on the intermediate good changes welfare through two separate channels: (i) by changing the final good price and (ii) by changing the intermediate good price. The marginal change in welfare is the sum of the marginal changes in consumer's surplus as well as producers' surplus through these two channels. And it comes from the output effect on welfare from the change in intermediate good production. I present the third channel that a tax on pollution provides: substitution possibility between inputs by upstream producers. This input substitutability provides the potential strength of a tax on pollution over a tax on the intermediate good, which cannot correctly target the source of the environmental externality when pollution is variable per unit of polluting activities. And this relative superiority of a tax on pollution over a tax on the intermediate good is maintained regardless of whether a lump sum transfer to consumers is available.

I also examine the directions of various welfare-improving tax reforms in the presence of the market distortions. I derive the condition that determines the direction of a welfare-improving tax reform. In my model, the profit wedges between the unit price and the unit cost in both industries can be interpreted as a simple barometer that shows how serious are the corresponding industry's inefficiencies. When the marginal environmental damages (MED) is included in the model, these marginality conditions for profit decides which tax instruments should be used to improve welfare. Suppose that the distortion from pollution is more serious than the distortions from market imperfection in this economy. Then, the intermediate good production should be discouraged, even if that would aggravate the pre-existing distortions from market imperfection. If a lump sum transfer is feasible and government can only use a tax on the output of the upstream industry —with no possibility of a tax on pollution—then it should levy a tax on the intermediate good and use the revenue to give a subsidy to consumers. On the contrary, if the size of environmental damages is smaller than the size of the upstream industry's profit, then it means that the inefficiency problems arising from the upstream industry's market imperfection are more serious than the pollution problem. Then, government should tax consumers and use this revenue to subsidize the upstream industry to increase the production of intermediate good.

In Section 2, I present the model and discuss the impact of taxation on the two oligopolistic industries. In Section 3, I derive the general welfare expression for various revenue-neutral tax reforms. The welfare effects of the tax reforms when lump sum transfer is available are discussed in Section 4. Section 5 examines the cases of tax reforms when a tax on the final good is used in revenue-neutral way instead of a lump sum transfer. Section 6 is the conclusion.

Ⅱ The Model

My model is based on those of Colangelo and Galmarini (2001) and Fullerton, et al. (2001). My model consists of a representative consumer, the government, and two vertically-related oligopolistic industries (downstream and upstream). And it is a partial equilibrium model. The upstream industry produces an intermediate good

(X) using labor (L^X) and aggregate pollution (E) with a constant returns to scale technology. The downstream industry produces the final consumption good (Y) using the intermediate good (X) and labor (L^Y), also with a constant returns to scale technology. I assume that the product in each industry is homogeneous and that the numbers of firms in each industry is fixed. Labor is assumed untaxed and competitively supplied.

The representative consumer demands the final consumption good. I assume that the representative consumer's overall utility is separable and linear in both labor and pollution: $U(Y,\ L,\ E) = U(Y) - L - D(E)$, where $L = L^X + L^Y$. The damage function $D(E)$ denotes the disutility to consumer from the aggregate emissions. The consumer suffers from E, but she ignores the effect of her own purchases on the aggregate pollution of producers.[3] Then, the inverse demand for Y is:

$$p = p\left(Y,\ w\right), \tag{1}$$

where w is the wage rate, p is the consumer price for Y, and $p_Y \equiv \partial p/\partial Y < 0$. I use subscripts to denote the first derivatives as in p_Y except for the indices that denote individual producers, as in x_j and y_i. In contrast, a superscript denotes the industry that a variable represents, as in L^X and L^Y.

1. Downstream Industry

Each downstream firm i $\left(i = 1, \ldots, m\right)$ produces its final good $\left(y_i\right)$ using labor and the intermediate good in a constant returns to scale technology.[4] Total industry

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[3] This assumption is appropriate if the number of consumers is large. In my model, the consumer represents the choice of many price-taking consumers. This can be interpreted to mean that many consumers lie on a continuum from zero to one, so that the aggregate size of the population is normalized to one.

[4] As in Konishi (1990), I implicitly assume that market inefficiency due to imperfect competition comes from the fixed number of firms and their conjectural behaviors, not from the existence of fixed costs and decreasing average costs.

output is defined as $Y = \sum_{i=1}^{m} y_i$. Each downstream firm maximizes its own profit function:

$$\pi_i^Y = \left[\left(1 - t^Y\right) \cdot p\left(Y,\ w\right) - c^Y\left(q,\ w\right)\right] y_i, \tag{2}$$

where q is the price of the intermediate good, t^Y is the ad valorem tax rate levied on the consumer price of Y, and $c^Y\left(q,\ w\right)$ is the unit cost function. Downstream profit is assumed to be non-negative and untaxed.5) And the downstream firms take q as fixed in choosing their outputs (and inputs).

I also assume that each downstream firm shares a common conjecture (v^Y) about how the other firms in its industry will respond to a change in its output level. The conjecture can be defined formally as:

$$v^Y \equiv \frac{dY}{dy_i}, \tag{3}$$

where $Y = y_i + \sum_{i' \neq i} y_{i'}$ and $0 \le v^Y \le m$. For example, $v^Y = 0$ generates the Bertrand equilibrium with competitive marginal cost pricing, $v^Y = 1$ represents Cournot behavior, and $v^Y = m$ corresponds to perfectly collusive behavior.

Given these conjectures, the first-order necessary condition for the firm's optimizing choice of $y_i > 0$ is that the downstream firm's perceived marginal profit must be equal to zero:6)

$$\left(1 - t^Y\right)\left(p + \frac{dp}{dY}\frac{dY}{dy_i} y_i\right) - c^Y = 0. \tag{4}$$

The solution to (4) yields an industry with m equal-sized firms. Aggregate all m downstream firms in (4) and divide it by $(1 - t^Y)$ to get:

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5) However, a lump sum tax on consumer is equivalent to a profit tax.

6) Note that this first-order condition is not for any $y_i > 0$, but for optimizing y_i.

$$p \cdot \left(1 - \frac{\beta^Y}{\varepsilon^Y}\right) = \frac{c^Y}{1 - t^Y}, \tag{5}$$

where $\varepsilon^Y(p,\ w) \equiv -pY_p/Y$ is the elasticity of product demand, and $\beta^Y \equiv v^Y/m \in [0,\ 1]$ is the degree of conjectural variation of the downstream firm, normalized between zero and one. Thus, implicit collusion among firms increases as β^Y becomes closer to one. This parameter can be interpreted as the aggregate conjectural variation or the market power parameter.[7] The left-hand side (LHS) of (5) is a downstream firm's perceived marginal revenue.

For $y_i > 0$, I assume that the following existence conditions hold, as in Colangelo and Galmarini (2001):

$$\varepsilon^Y > \beta^Y, \qquad t^Y < 1, \qquad \text{and} \qquad c^Y < (1 - t^Y) \cdot p(0,\ w). \tag{6}$$

To guarantee the second-order condition, I also assume that the well-known Stern's condition is satisfied (Stern, 1987):

$$G^Y \equiv 1 - \frac{\beta^Y}{\varepsilon^Y}\left(1 - \frac{p\varepsilon_p^Y}{\varepsilon^Y}\right) > 0, \tag{7}$$

where the term $p\varepsilon_p^Y/\varepsilon^Y$ in the bracketis the price elasticity of the elasticity of demand for Y and is always positive.[8] Equation (7) is the effect on perceived marginal revenue of a marginal increase in price. Then, the solution function for the final product price is:

$$p = \varphi(q,\ \ w,\ \ t^Y). \tag{8}$$

In order to see how the inverse demand for Y changes as q and t^Y vary, apply

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[7] Therefore, market imperfection comes from the two different sources: first, from the conjectural behaviors of firms (v^Y) and second, from the number of firms (m). I use the normalized aggregate conjectural variation, however, because my paper does not distinguish between these two factors. This normalization is also useful in that β^Y can be compared to ε^Y in clarifying the existence conditions presented below.

[8] To obtain (7), differentiate the LHS of (5) with respect to p.

the implicit function theorem to (5) and use the stability condition (7) to get:

$$p_q \equiv \frac{d\varphi}{dq} = \frac{c_q^Y}{\left(1-t^Y\right)G^Y} > 0 \tag{9a}$$

and

$$p_{t^Y} \equiv \frac{d\varphi}{dt^Y} = \frac{c^Y}{\left(1-t^Y\right)^2 G^Y} = \frac{p\left(\varepsilon^Y - \beta^Y\right)}{\varepsilon^Y\left(1-t^Y\right)G^Y} > 0 \tag{9b}$$

The final good price is raised either by an increase in the price of the intermediate good or by an increase in the ad valorem tax on the final good.

2. Upstream Industry

Using a constant returns to scale technology, each upstream firm j $\left(j = 1,\ldots,n\right)$ produces intermediate good (x_j) using labor and generates pollution (e_j).9) For simplicity, I refer to the labor as any inputs employed by the upstream firm, but under some conditions it can be interpreted more generally as a fixed total amount of labor, capital, land, and any other resource that can be sold in the market. Distinction among these inputs is not necessary for any of the points I make below. Following the conventional approach widely used in the environmental economics literature, I also model the aggregate production function for the intermediate good as $X = F\left(L^X,\ E\right)$, where $X = \sum_{j=1}^{n} x_j$. Aggregate pollution, $E = \sum_{j=1}^{n} e_j$, has a harmful effect on overall environmental quality and can be disposal of gaseous, liquid, and solid waste used to produce output. This external effect can be captured in the environmental damage function $D\left(E\right)$, where $D_E > 0$. I assume that the government as an environmental regulator can monitor firms' pollution in some cases. I also assume that the enforcement of its environmental policies is effective.

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9) Pollution e_j and aggregate pollution E could be measured in tons of emissions such as SO_2, NO_X, or CO_2. Then damages $D(E)$ are measured in units of utility. Emission E could be taxed if each firm's tons of SO_2 or CO_2 can be monitored accurately. For some pollutants, such monitoring is more difficult.

Therefore, my paper does not concern administrative, monitoring, or enforcement problems.

Note that the production function for X has variable pollution per unit of output. If the environmental externality is fixed per unit of polluting activities, then the choice of environmental tax instruments becomes trivial since an output tax becomes identical to an emissions tax (Fullerton, et al., 2001)[10]. By modeling explicitly the variable relationship between pollution and per unit of output, however, I can show how an output tax affects welfare differently compared to an emissions tax.

An upstream firm acts simultaneously with every other intermediate good producer tomaximize the following profit function with respect to its own output x_j:

$$\pi_j^X = \left[\left(1-t^X\right)\cdot q\left(X,\ w,\ t^Y\right)-c^X\left(r,\ w\right)\right]x_j, \tag{10}$$

where t^X is the ad valorem tax rate levied on the price of the intermediate good and $c^X\left(r,\ w\right)$ is the unit cost function. The function $q\left(X,\ w,\ t^Y\right)$ is the inverse function of the derived demand for X, which can be derived from the equilibrium aggregate demand for X using the solution function for the final product price.[11]

The unit cost function is a function of the full price of polluting the environment (r), which is the sum of the hidden, internal price of using the environment as an input (r^0) and a specific tax rate on pollution (t^E). The polluting firms cannot completely ignore any environmental consequences from their operation, even when there exist neither explicit environmental regulations nor any legal requirements for the environmentally harmful production activities. Firms might simply want to project an environmentally friendly image to the public (Gangadharan, 2001), or voluntary pollution abatement might be used as a barrier to potential entry (Helland and Matsuno, 2003). Environmental regulation tends to change frequently, and anticipating a stricter regulation, firms might prepare themselves by investing

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[10] In this case, the output tax rate is equal to the emissions tax adjusted to the proportion of emissions per unit output.

[11] See (15) and (16) below the inverse function of the derived demand for the intermediate good.

pollution abatement equipments (Lee and Alm, 2004).12) Furthermore, normative and social motivations are as influential as economic motivations in initiating an effort to reduce pollution (Winter and May, 2001; Lai, et al., 2003).13) Therefore, the term r^0 can be viewed as the price that the upstream producers pay for using the environment without any environmental tax or any explicit environmental regulations.14) I assume that $r^0 > 0$. Then, t^E serves as one of many possible environmental policy instruments to curtail the discrepancy between r and r^0. I also assume that the relationship between r^0 and t^E is independent from each other for analytical simplicity: $dr^0/dt^E = 0$ and, therefore, $dr = dt^E$.

Again, I assume that each upstream firm conjectures that a change in its own output alters total industry output by a constant: $v^X \equiv dX/dx_j$, where $0 \leq v^X \leq n$ and $X = x_j + \sum_{j' \neq j}^{n} x_{j'}$.

Define the elasticity of the derived demand for the intermediate good as $\varepsilon^X(q,\ w,\ t^Y) \equiv -qX_q/X$, define the normalized conjectural variation of the upstream industry as $\beta^X \equiv v^X/n \in [0,\ 1]$, and aggregate the first-order condition from the maximization of (10) to obtain:

$$q \cdot \left[1 - \frac{\beta^X}{\varepsilon^X(q,\ w,\ t^Y)}\right] = \frac{c^X}{1 - t^X}. \tag{11}$$

The elasticity of the derived demand for the intermediate good (ε^X) can be

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12) This kind of motive would be weakened if the grandfathering environmental policies are expected by the polluting firms.

13) As Earnhart (2004) shows, community characteristics can affect regulatory decisions to intervene against specific facilities with inspections and penalties. If the polluting firms anticipate this, they might be willing to initiate pollution abatement, even in a limited magnitude.

14) However, this internal price (r^0) might be sometimes perceived as non-environmental by firms. For example, Joshi, et al. (2002) report that firms' accounting systems tend to classify incorrectly the substantial amount of costs incurred due to environmental reasons, and they are often unaware of existence and magnitude of these costs.

rewritten as follows:[15]

$$\varepsilon^X = \frac{\sigma^Y w c^Y_w}{c^Y} + \frac{\varepsilon^Y q c^Y_q}{c^Y G^Y}\left(1 - \frac{\beta^Y}{\varepsilon^Y}\right), \tag{12}$$

where σ^Y denotes the input substitution elasticity between L^Y and X in production of Y.

The upstream industry's existence and stability conditions are analogous to those of the downstream industry, namely:

$$\varepsilon^X > \beta^X, \quad t^X < 1, \quad t^E < 1, \quad \text{and} \quad w < \left(1 - t^X\right) \cdot q\left(0,\ w,\ t^Y\right), \tag{13}$$

and

$$G^X \equiv 1 - \frac{\beta^X}{\varepsilon^X}\left(1 - \frac{q\varepsilon^X_q}{\varepsilon^X}\right) > 0, \tag{14}$$

where the term $q\varepsilon^X_q / \varepsilon^X$ in the bracket is the price elasticity of the elasticity of the derived demand for X and is always positive.[16]

By Shepard's lemma, the equilibrium aggregate demand for X is given by:

$$X = c^Y_q\left(q,\ w\right) \cdot Y\left(p,\ w\right).$$

Rewrite it, using (8), to obtain:

$$X = c^Y_q\left(q,\ w\right) \cdot Y\left(\varphi\left(q,\ w,\ t^Y\right),\ w\right) = X\left(q,\ w,\ t^Y\right). \tag{15}$$

Define $c^Y_{qq} \equiv \partial^2 c^Y / \left(\partial q\right)^2$. Then, since $X_q \equiv \partial X / \partial q = c^Y_{qq} Y + c^Y_q Y_p \varphi_q$ is negative and finite as long as c^Y_{qq} is negative and finite, I can invert (9) to obtain thederived demand function:[17]

$$q = q\left(X,\ w,\ t^Y\right), \tag{16}$$

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[15] See Appendix A for derivation of (12).

[16] To obtain (14), differentiate the LHS of (11) with respect to q.

[17] If inputs are not perfect substitutes for each other, then $c^Y_q Y_p \varphi_q$ is negative and finite.

where $q_X < 0$ and $q_{t^Y} < 0$.[18]

From (11), the equilibrium price function for the intermediate good can be rewritten by using the corresponding existence and stability conditions (since $r = r^0 + t^E$):

$$q = \xi\left(r,\ w,\ t^X,\ t^Y\right) = \xi\left(r^0,\ w,\ t^X,\ t^Y,\ t^E\right). \tag{17}$$

Note that q is endogenous, because the upstream sector is explicitly analyzed in my model.

Substitute (17) into (8) to obtain:

$$p = \varphi\left(\xi\left(r,\ w,\ t^X,\ t^Y\right),\ w,\ t^Y\right) = \varphi\left(r^0,\ w,\ t^X,\ t^Y,\ t^E\right). \tag{18}$$

To see how taxes affect prices, totally differentiate (11) and use the definition of G^X in (14) to get:

$$G^X\ dq + \frac{\beta^X q \varepsilon^X_{t^Y}}{\left(\varepsilon^X\right)^2}\ dt^Y = \frac{c^X}{\left(1-t^X\right)^2}\ dt^X + \frac{c^X_r}{1-t^X}\ dt^E \quad , \tag{19}$$

where $\varepsilon^X_{t^Y} \equiv \partial\varepsilon^X / \partial t^Y$. The partial derivatives of (17) with respect to t^X, t^Y, and t^E are obtained by setting the corresponding terms equal to zero as follows:[19]

$$\frac{dq}{dt^X} \equiv \xi_{t^X} = \frac{\left(\varepsilon^X - \beta^X\right)q}{\varepsilon^X\left(1-t^X\right)G^X} > 0 \quad , \tag{20a}$$

$$\frac{dq}{dt^Y} \equiv \xi_{t^Y} = -\frac{\beta^X q \varepsilon^X_{t^Y}}{\left(\varepsilon^X\right)^2 G^X}\ , \tag{20b}$$

and

$$\frac{dq}{dt^E} \equiv \xi_{t^E} = \frac{c^X_r}{\left(1-t^X\right)G^X} > 0 \quad . \tag{20c}$$

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[18] See (20b) below and the subsequent discussion about the sign of q_{t^Y}.

[19] Recall that $dr = dt^E$ since I assumed that $r^0 > 0$ but $dr^0 = 0$.

Equations (20a) and (20c) show that either a tax on upstream producers' output (t^X) or a tax on their pollution (t^E) increase the intermediate good price (q), since both increase the intermediate good's marginal cost of production. However, the sign of (20b) is ambiguous. If the upstream industry applies competitive marginal cost pricing $(\beta^X = 0)$, then the change in downstream tax t^Y has no effect on the price of upstream intermediate good. On the other hand, if the upstream industry has some degree of market power $(\beta^X > 0)$, then the sign of (20b) critically depends on the sign of $\varepsilon_{t^Y}^X$, which denotes how the downstream taxation changes the elasticity of the derived demand for X. In particular, if $\varepsilon_{t^Y}^X > 0$, that is, ε^X becomes more elastic as t^Y is raised, then t^Y would decrease q. For example, if the curvature of demand function for Y is concave or weakly convex (i.e., linear or exponential), then $\varepsilon_{t^Y}^X > 0$, and an increase in t^Y would decrease q.[20]

To see the effects of tax rates on the final good price, differentiate (18) with respect to the corresponding taxes:

$$\frac{dp}{dt^X} \equiv \varphi_q \cdot \xi_{t^X} = \frac{c_q^Y}{\left(1-t^Y\right)G^Y}\frac{\partial q}{\partial t^X} > 0, \tag{21a}$$

$$\frac{dp}{dt^Y} \equiv \varphi_{t^Y} + \varphi_q \cdot \xi_{t^Y} = \frac{\left(\varepsilon^Y - \beta^Y\right)p}{\varepsilon^Y \cdot \left(1-t^Y\right)G^Y} + \frac{c_q^Y}{\left(1-t^Y\right)G^Y}\frac{\partial q}{\partial t^Y}, \tag{21b}$$

and

$$\frac{dp}{dt^E} \equiv \varphi_q \cdot \xi_{t^E} = \frac{c_q^Y}{\left(1-t^Y\right)G^Y}\frac{\partial q}{\partial t^E} > 0. \tag{21c}$$

Equations (21a) and (21c) show that both upstream t^X and t^E increase final output price p. However, the sign of (21b) is ambiguous. Unlike t^X and t^E, a tax

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[20] However, if $\varepsilon_{t^Y}^X = 0$ (i.e., isoelastic demand), then t^Y would have no effect on q even if $\beta^X > 0$.

on the final good $\left(t^{Y}\right)$ has two channels through which it affects p. The first term in the right hand side (RHS) of (21b) denotes the direct effect that makes p increase with t^{Y}, while the second term denotes the indirect effect that works through the change in q. The sign of this indirect effect is determined by the sign of $\varepsilon_{t^{Y}}^{X}$ as already shown in (20b). Therefore, the final sign of (21b) is determined by the relative size of the direct and indirect effects. A sufficient condition for (21b) to be positive is that $\varepsilon_{t^{Y}}^{X} \leq 0$. Even if $\varepsilon_{t_{Y}}^{X} > 0$, however, as is the case for most product demand specifications, it would be difficult to find a case where the indirect effect outweighs the direct effect such that the final sign of (21b) is negative (Colangelo and Galmarini, 2001).

Ⅲ Welfare Changes from Revenue-Neutral Tax Reforms

The economy is assumed to have no revenue requirement, so taxes are used merely to lessen the distortions caused by imperfect competition as well as by the externality. This section does not examine the problems of optimal taxation. Solving an optimal taxation problem is more difficult than solving a tax reform problem. It is possible to obtain analytical solution for optimal taxes only in very restrictive cases. Since the main interest of my paper is about how the government can improve welfare by introducing a revenue-neutral tax-subsidy reform in the presence of vertically-related oligopolies, I focus on tax system rather than optimal taxation.

I start from an arbitrary tax system, where rates are not necessarily set optimally. Then, I solve for the effects on welfare of a proposed tax reform that is, an increase in one tax rate with a revenue-neutral decrease in some other tax rate. First, I examine what factors determine the direction of welfare change from that particular tax reform. Then, if the effect on welfare is positive, which is interpreted as a good tax reform, I examine the factors that determine the magnitude of welfare change.[21]

Given the assumptions on consumer preferences and other agents in this economy, social welfare is defined as the unweighted sum of consumers' and producers' surplus as follows:

$$W = W\left(t^X,\ t^Y,\ t^E,\ T\right) = CS + \frac{\Pi^X + \Pi^Y + T}{w}, \tag{22}$$

where consumer surplus is defined as $CS \equiv U\left(Y\left(p,\ w\right)\right) - p \cdot Y\left(p,\ w\right)/w - D\left(E\right)$. The producers' surplus (PS) consists of the upstream and downstream industries' profit functions: $\Pi^X = \left[\left(1 - t^X\right) \cdot q\left(X,\ w,\ t^Y\right) - c^X\left(r,\ w\right)\right] X$ and $\Pi^Y = \left[\left(1 - t^Y\right) \cdot p\left(Y,\ w\right) - c^Y\left(q,\ w\right)\right] Y$, respectively. The government tax revenue is given as:[22]

$$T = t^X qX + t^Y pY + t^E E = \left[t^Y p + c_q^Y \left(t^X q + t^E c_r^X\right)\right] Y. \tag{23}$$

Totally differentiate (23), assuming no pre-existing taxes $(t^X = t^Y = t^E = T = 0)$, to get:

$$dT = \left(q c_q^Y\, dt^X + p\, dt^Y + c_r^X c_q^Y\, dt^E\right) Y. \tag{24}$$

This is the change in the lump sum transfer necessary for government to balance the budget when introducing other new taxes. Totally differentiate (22), substitute (24) into it, and use $dW/dT = 1/w$ from (22) to get:

$$dW = \left(\frac{\partial W}{\partial t^X} + \frac{qX}{w}\right) dt^X + \left(\frac{\partial W}{\partial t^Y} + \frac{pY}{w}\right) dt^Y + \left(\frac{\partial W}{\partial t^E} + \frac{E}{w}\right) dt^E. \tag{25}$$

Rewrite (25) as:[23]

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[21] It is uncertain whetherthat is the best possible small tax reform, however, because there might be a larger increase in welfare by raising some other tax rate and/or lowering some other tax rate. If my result happened to be at the optimum, then any small revenue-neutral change in tax rates will yield no change in welfare.

[22] By Shepard's lemma, $X = c_q^Y Y$ and $E = c_r^X X = c_r^X c_q^Y Y$.

[23] See Appendix B for derivation of (26).

$$dW = -\left[\frac{\varepsilon^Y\left(\Pi^X + \Pi^Y - wD_E E\right)}{wp}\frac{dp}{dt^X} + \frac{\sigma^Y c_w^Y\left(\Pi^X - wD_E E\right)}{qc^Y}\frac{dq}{dt^X}\right]dt^X$$

$$-\left[\frac{\varepsilon^Y\left(\Pi^X + \Pi^Y - wD_E E\right)}{wp}\frac{dp}{dt^Y} + \frac{\sigma^Y c_w^Y\left(\Pi^X - wD_E E\right)}{qc^Y}\frac{dq}{dt^Y}\right]dt^Y$$

$$-\left[\begin{array}{c}\dfrac{\varepsilon^Y\left(\Pi^X + \Pi^Y - wD_E E\right)}{wp}\dfrac{dp}{dt^E} + \dfrac{\sigma^Y c_w^Y\left(\Pi^X - wD_E E\right)}{qc^Y}\dfrac{dq}{dt^E} \\ -\dfrac{\sigma^X c_w^X wD_E E}{r^0 c^X}\end{array}\right]dt^E. \tag{26}$$

Equation (26) shows how the governmentcan improve welfare by initiating a revenue-neutral tax reform when a lump sum transfer is available. The government can use any (or all) of three possible tax instruments: a tax on pollution $\left(t^E\right)$, and two taxes on outputs (t^X and t^Y). All tax reforms have in common two separate channels through which change welfare. The first term containing ε^Y in each large bracket denotes how the introduction of a small t^X (or t^Y or t^E) and the corresponding change in T affect welfare through p. This marginal change in welfare is the sum of the marginal changes in CS and PS, which is closely related to the change in consumer demand for Y due to the change in p (i.e., ε^Y). Therefore, this is the "output effect" on welfare from the change in Y.[24)]

On the other hand, the second term containing σ^Y in each large bracket denotes how the introduction of a small t^X (or t^Y or t^E) and the matching change in T

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24) The reason that $D_\varepsilon E$ is multiplied by w is as follows. The utility function is defined as $U = U(Y) - L - D(E)$, which essentially means that utility is measured in labor hours $(\partial U/\partial L = -1)$. If utility is measured in labor hours, then multiplication by w yields dollars. Since D_ε is in utils $(\partial U/\partial E)$, multiply by w to get hours. Equivalently in (26), divide $(\Pi^X + \Pi^Y)$ by w to get utils for dW/dt^E (which is measured in utils).

alter welfare through q. Again, this marginal change in welfare is the sum of the marginal changes in CS and PS, which is closely related to the change in cost of Y and the downstream firms' technological ability to accommodate the effects of a tax on cost by substituting L^Y for X (i.e., σ^Y). Therefore, this is the output effect on welfare from the change in X. (Or, this could be called a substitution effect in Y, since output of X is an input to Y.)

In addition to these two common channels, the combination of t^E and T (i.e., dW/dt^E) has another way to change welfare: the third, last term in the large bracket containing σ^X. This change comes not from the changes of either p or q, but from the marginal change in cost of X. A specific tax on pollution directly affects the upstream producers' cost by changing the relative input price. This is the "substitution effect" on welfare from the change in E.

Market imperfection usually prescribes a subsidy for (rather than a tax on) the market output because a tax would decrease the already suboptimal level of production due to market imperfection. On the other hand, the presence of environmental externality prescribes policy makers to levy a tax.[25] Simultaneous presence of market imperfection and environmental externality, however, could offer policy makers conflicting recommendations, especially when taxes or subsidies are employed for their corrective roles in lessening inefficiencies.

While many studies of the optimal taxation provide relatively clear rules and answers on the effects of tax policies in various market conditions, it is difficult to find actual taxes based on these principles. Many taxes are not correctly targeted to the sources of inefficiencies, due to administrative and technological difficulties, and few environmental taxes are believed to set optimally (OECD, 1995 and 1999; Fullerton, et al., 2001). Political considerations or the practical problems of design and implementation such as who is to be taxed are often the most important factors

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[25] This tax would apply to output if pollution is fixed per unit of output. More generally, as here, it is a tax per unit of pollution created by production or consumption activities, set equal to the MED (Pigou, 1932; Baumol and Oates, 1988).

that determine the types of policy tools employed (Barthold, 1994). Therefore, it is important to know welfare-improving directions of a tax reform and what factors decide it before the government initiates a tax reform. And this might be an important starting point to implement a full-fledged optimal tax policy, especially in the case of vertically-related oligopolies with pollution, where various distortions are interlinked.

The above welfare expression (26) does exactly that. It provides the conditions that decide the direction of a welfare-improving tax reform. However, (26) is expressed in 'total' terms containing industry profits (Π^X and Π^Y), which make it difficult interpret. So I rearrange (26), using (B6), to obtain:

$$wW_p = wU_Y Y_p - Y - pY_p - wD_E c_r^X c_q^Y Y_p + \left[\left(1-t^Y\right)p - c^Y\right]Y_p$$

$$+\left(1-t^Y\right)Y + \left[\left(1-t^X\right)q - c^X\right]c_q^Y Y_p.$$

Set $t^X = t^Y = 0$, then:

$$W_p\Big|_{t^X=t^Y=0} = -\frac{\varepsilon^Y Y}{wp}\left[\left(q - c^X - wD_E c_r^X\right)c_q^Y + \left(p - c^Y\right)\right]. \tag{26a}$$

The term $\left(q - c^X\right)$ represents the upstream industry's profit wedge, which is the difference between the product price (q) and the unit (marginal) cost $\left(c^X\right)$. Note that $\left(q - c^X\right) \geq 0$ since $q\left(1 - \beta^X / \varepsilon^X\right) = c^X$, $\beta^X < \varepsilon^X$, and $\beta^X \in [0, 1]$. Similarly, $\left(p - c^Y\right) \geq 0$ represents the downstream industry's profit wedge. Another term $wD_E c_r^X$ in (26a) is the MED from the pollutionemitted by the upstream industry. Note that there is no term for the MED in the downstream industry's profit wedge, since the downstream industry does not emit pollution.

Similarly,

$$W_q\Big|_{t^X=t^Y=0} = -\frac{\sigma^Y c_w^Y X}{qc^Y}\left(q - c^X - wD_E c_r^X\right). \tag{26b}$$

IV Welfare Improving Tax Reforms When a Lump Sum Transfer Is Available

1. Ad Valorem Tax on the Intermediate Good

Suppose that government uses a tax on the intermediate good $\left(t^X\right)$ and a lump sum transfer to consumer $\left(T\right)$. Set $dt^Y = dt^E = 0$ in (26) and use (20a), (21a), (26a), and (26b) to obtain:

$$\frac{dW}{dt^X} = -\frac{\varepsilon^Y q c_q^Y Y\left[\left(q - c^X - wD_E c_r^X\right)c_q^Y + \left(p - c^Y\right)\right]}{wpG^Y}\frac{1}{G^X}\left(1 - \frac{\beta^X}{\varepsilon^X}\right)$$

$$-\frac{\sigma^Y c_w^Y X\left(q - c^X - wD_E c_r^X\right)}{c^Y}\frac{1}{G^X}\left(1 - \frac{\beta^X}{\varepsilon^X}\right). \tag{27}$$

With no environmental damages $(D_E = 0)$, the above (27) is always non-positive, which means that the government can improve welfare by using $t^X < 0$ (and $T > 0$). This extends the results from previous environmental studies to the case of two vertically-related industries: the marginalization distortion can be lessened by a subsidy (Buchanan, 1969; Barnett, 1980).

The above result shows that there are two separate channels through which the government can improve welfare by using t^X and T: (i) through affecting the price of final good $(W_p dp/dt^X)$ and (ii) through affecting the price of intermediate good $(W_q dq/dt^X)$. The above result also shows that the magnitude of welfare change depends on several important industry parameters: the elasticities of demand (ε^X and ε^Y), the market powers in both industries (β^X and β^Y), the cost structure of the downstream industry (c_q^Y and c_w^Y), the input substitutability of the downstream firms (σ^Y), the both industries' profit wedges along with the MED included:

$\left(q - c^X - wD_E c^X_r\right)c^Y_q + \left(p - c^Y\right)$.

Recall that from (26), the marginal change in welfare comes from the two separate 'marginal' changes. The first source of the welfare change is $W_p dp/dt^X$. This consists of various parameters such as the elasticity of demand (ε^Y) and the marginal cost of the downstream industry (c^Y_q). Multiplied by these 'marginal' parameters, the first term in (26), as a whole, represents the 'marginal' change in welfare by the introduction of a small tax on the intermediate good (t^X) through affecting the price of final good. The same logic is applied to the second term in (26).

Keeping this point in mind, the expression (27) provides an interesting condition that decides the direction of the tax reform: $\left(q - c^X - wD_E c^X_r\right)c^Y_q + \left(p - c^Y\right)$. These terms can be interpreted as how serious the corresponding distortion is to social welfare. In my model, the upstream industry is the source of three inefficiencies: the environmental damages, the marginalization distortion, and the downstream industry's input-mix distortion caused by this marginalization distortion. Therefore, the upstream industry's profit wedge along with the MED included can be interpreted as a signal to the government that shows how serious are the distortions from market imperfection in the upstream industry as well as from pollution. Similarly, the downstream industry's profit wedge represents how serious the distortion from the downstream market imperfection is.

If the wedges from the distortions in both industries are negative, that is, if $\left(q - c^X - wD_E c^X_r\right)c^Y_q + \left(p - c^Y\right) < 0$, which makes the sign of (27) positive, then intermediate good production should be discouraged in this second-best world with no t^E, even if doing so would aggravate the distortions from market imperfection. And the policy recommendation should be a tax on the intermediate good $(t^X > 0)$.

On the contrary, if $\left(q - c^X - wD_E c^X_r\right)c^Y_q > 0$, which makes the sign of (27) negative, then a welfare-improving, revenue-neutral tax reform consists of a subsidy for the upstream industry $(t^X < 0)$ and a lump sum tax on consumers $(T > 0)$. The subsidy for X makes production increase and price fall in the upstream industry. The decrease in price reduces profits, but it raises consumer surplus. At the same time, the lump sum tax on consumersreduces consumer income, while the subsidy increases profits. The final effect would be the increase in both profits and welfare.

However, if $\left(q - c^X - wD_E c_r^X\right) c_q^Y + \left(p - c^Y\right) > 0$ but $\left(q - c^X - wD_E c_r^X\right) < 0$, then the final sign of (27) becomes ambiguous. The input substitutability of the downstream industry (σ^Y) emerges as an important factor in this case. If the downstream firms can freely substitute inputs X and L^Y, the upstream industry's market power substantially weakens. Then, a tax on X would greatly improve welfare by reducing environmental damages, while it would not so much aggravate the distortions from the upstream industry's imperfect competition. If σ^Y is sufficiently high, then the sign of the whole welfare expression (27) becomes positive. In that case, government can improve welfare by taxing intermediate good.

Would a subsidy for the intermediate good $(t^X < 0)$ generate more pollution due to the decrease in price (q) and the resulting increase in demand by the downstream industry? Should the government reduce the subsidy level if it takes into account the possible additional environmental damages that might result from the subsidy? The answer is no. Consider the nature of the lump sum transfer to consumers in this case. With the initial condition of no pre-existing taxes, a subsidy for X results in more X, more E, lower q, and more Y. Revenue from consumers exactly compensates these effects because a lump sum tax on consumers is not really a different instrument but a compensated income for the subsidy for X.

2. A Tax on Pollution

The case of using only a tax on pollution $\left(t^E\right)$ can be obtained by setting $dt^X = dt^Y = 0$ in (26):

$$\frac{dW}{dt^E} = -\frac{\varepsilon^Y c_r^X c_q^Y Y\left[\left(q - c^X - wD_E c_r^X\right)c_q^Y + \left(p - c^Y\right)\right]}{wpG^X G^Y}$$

$$-\frac{\sigma^Y c_r^X c_w^Y X\left(q - c^X - wD_E c_r^X\right)}{qc^Y G^X} + \frac{\sigma^X c_w^X wD_E E}{r^0 c^X}. \tag{28}$$

Note that the first two terms in (28) are similar to those in (27). If the government introduces a tax on pollution $(t^E > 0)$, then the price of the intermediate good (q) increases, which in turn raises the production cost of Y as well as p. This causes the product demand for Y to decrease. Then, the downstream industry's derived demand for X falls. The first term containing ε^Y in (28) shows this output effect of t^E through the downstream industry. On the other hand, the increase in q makes the downstream firms substitute from X into L^Y, which has now become relatively cheaper due to the introduction of t^E. This output effect on X due to the input substitution by the downstream industry is captured in the second term containing σ^Y in (28). Note that these two effects are also present in (27) and come about from a single source: the decrease in production of X.

Ignoring the last σ^Y term for the time being, the signs of the first two terms in (28) depends on the signs of the profit wedge, as already explained in (27). For example, if $\left(q - c^X - wD_E c_r^X\right) + \left(p - c^Y\right) < 0$, then the sign of (28) becomes positive. And government can improve welfare by using $t^E > 0$ and $T < 0$. On the other hand, if $\left(q - c^X - wD_E c_r^X\right) > 0$, then the signs of the first two terms in (28) are both negative. It means that if the government has only t^E at its disposal (t^X must be zero), then government should introduce $t^E < 0$ and $T > 0$ to improve welfare. This particular policy recommendation appears against common sense because the polluting activity is being subsidized rather than penalized. However, it is perfectly understandable if considered in the context of social welfare. If $\left(q - c^X - wD_E c_r^X\right) > 0$, it means that the price wedge in the upstream industry is big even considering the MED. Hence, the distortions from the upstream industry's imperfect competition are most pressing. Therefore, the production of X should be encouraged, even if the only way to do that is a subsidy for E. By doing so, the price of X would fall and the downstream firms' derived demand for X would increase. As a result, the double marginalization as well as the input-mix distortions lessens.

However, note that the last term that has σ^Y in (28) is always positive, which denotes the input substitution effect of t^E. This effect comes from the fact that a tax on pollution correctly targets the source of distortion (Fullerton, et al., 2001). Equation (28) shows that this superiority of t^E to t^X is also valid in the case of vertically-related oligopolies with pollution. To see this point more clearly, assume Bertrand competition in both industries. Set $G^X = G^Y = 1$, $p = c^Y$, $q = c^X$ and $\beta^X = \beta^Y = 0$ in both (27) and (28) to get:

$$\frac{dW}{dt^X} = \frac{\varepsilon^Y q c_q^Y w D_E E}{wp} + \frac{\sigma^Y c_w^Y w D_E E}{c^Y} > 0 \tag{29a}$$

and

$$\frac{dW}{dt^E} = \frac{\varepsilon^Y c_r^X c_q^Y w D_E E}{wp} + \frac{\sigma^Y c_r^X c_w^Y w D_E E}{q c^Y} + \frac{\sigma^X c_w^X w D_E E}{r^0 c^X} > 0 . \tag{29b}$$

With no distortions from imperfect competition in either industry, there remains only one inefficiency in my model: the distortion from pollution $\left(D_E\right)$. In this case, the government can always improve welfare by introducing either t^X or t^E incrementally as shown in (29a) and (29b), respectively. The first two terms in both expressions reflect the welfare gain from the output effect due to decrease in X. However, (29b) has the third, positive term, which is the input substitution effect between E and L^X in the upstream industry.

V Welfare Improving Tax Reforms without Lump Sum Transfer

Now, I examine the case that a lump sum transfer is not feasible nor any other tax to raise revenue that can be used to pay for subsidies or to rebate revenue. Again, I start at an initial equilibrium with no pre-existing taxes. Government uses a tax on (or a subsidy for) Y to balance the budget when it introduces a new tax on (or a subsidy for) the upstream industry (t^X or t^E). In this case, two different combinations of tax reforms are available to the government: (t^X, t^Y) and (t^E, t^Y).[26]

Set $dT = 0$ in (25), assuming no pre-existing taxes, to get:

$$dt^Y = -\left(\frac{qX}{pY}\right)dt^X - \left(\frac{E}{pY}\right)dt^E \quad . \tag{30}$$

This is the change in t^Y necessary for government to balance the budget when introducing either t^X or t^E.

Totally differentiate the social welfare function (22) and use (30) to get:[27]

$$\begin{aligned} dW = &\left[W_p\left(\frac{dp}{dt^X} - \frac{dp}{dt^Y}\frac{qX}{pY}\right) + W_q\left(\frac{dq}{dt^X} - \frac{dq}{dt^Y}\frac{qX}{pY}\right)\right]dt^X \\ &+ \left[W_p\left(\frac{dp}{dt^E} - \frac{dp}{dt^Y}\frac{E}{pY}\right) + W_q\left(\frac{dq}{dt^E} - \frac{dq}{dt^Y}\frac{E}{pY}\right) + \frac{\sigma^X c_w^X w D_E E}{r^0 c^X}\right]dt^E. \end{aligned} \tag{31}$$

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[26] I do not consider the case of (t^X, t^E), since double taxation on a single industry is unrealistic.

[27] See Appendix C for derivation of (31).

Equation (31) shows how the government can improve welfare by using various tax reforms when a lump sum transfer is infeasible.

1. Ad Valorem Taxes on Both Goods

In this section, I consider the case that the government can only use output taxation for both industries: t^Y and t^X. Set $dt^E = 0$ in (31) to get:

$$\frac{dW}{dt^X} = W_p\left(\frac{dp}{dt^X} - \frac{dp}{dt^Y}\frac{qX}{pY}\right) + W_q\left(\frac{dq}{dt^X} - \frac{dq}{dt^Y}\frac{qX}{pY}\right). \tag{32}$$

When a lump sum transfer is available, only t^X affects prices dp/dt^X and dq/dt^X as shown earlier in (27), since a lump sum transfer does not change the relative price. However, when a lump sum transfer is not available, not only t^X but also t^Y affects both p and q. Moreover, unlike a lump sum transfer, t^Y changes the prices in more complicated ways. While t^Y changes q directly, it affects p in two ways: the direct effect on the final good price $(\partial p/\partial t^Y)$ and the indirect one through the intermediate price $(\partial p/\partial q)(dq/dt^Y)$.

Substitute (20a), (20b), (21a), (21b), (26a), (26b), (B6) and (B7) into (32) to obtain:

$$\frac{dW}{dt^X} = -\frac{\varepsilon^Y qc_q^Y Y\left[\left(q - c^X - wD_E c_r^X\right)c_q^Y + \left(p - c^Y\right)\right]}{wpG^Y}\left[\begin{array}{c}\dfrac{1}{G^X}\left(1 - \dfrac{\beta^X}{\varepsilon^X}\right) - \left(1 - \dfrac{\beta^Y}{\varepsilon^Y}\right)\\ + \dfrac{\beta^X qc_q^Y \varepsilon_{t^Y}^X}{\left(\varepsilon^X\right)^2 pG^X}\end{array}\right]$$

$$-\frac{\sigma^Y c_w^Y X\left(q - c^X - wD_E c_r^X\right)}{qc^Y}\left[\frac{1}{G^X}\left(1 - \frac{\beta^X}{\varepsilon^X}\right) + \frac{\beta^X qc_q^Y \varepsilon_{t^Y}^X}{\left(\varepsilon^X\right)^2 pG^X}\right]. \tag{33}$$

The first component in the large brackets $\frac{1}{G^X}\left(1-\frac{\beta^X}{\varepsilon^X}\right)$ measures the upstream industry's degree of market power. The second term in the large bracket $\left(1-\frac{\beta^Y}{\varepsilon^Y}\right)$ reflects the downstream industry's degree of market power. And the third term $\beta^X q\varepsilon_{t^Y}^X \Big/ \left(\varepsilon^X\right)^2 pG^X$ represents the effect of downstream taxation on the price of the final good through the change of the price of intermediate good. Combining these terms, the first line of (33) represents the welfare effect generated by the change in the product demand for Y, while the second line captures the welfare effect resulting from the change in the derived demand for X.

Using the definitions of G^X from (14), the first term in the large brackets can be rewritten as follows:

$$\frac{1}{G^X}\left(1-\frac{\beta^X}{\varepsilon^X}\right)=\frac{\left(\varepsilon^X-\beta^X\right)}{\left(\varepsilon^X-\beta^X\right)+\frac{\beta^X q\varepsilon_q^X}{\varepsilon^X}}. \tag{34}$$

If the elasticity of the derived demand for X by the downstream industry is not increasing in q, that is, if $\varepsilon_q^X \leq 0$, then (34) is greater than or equal to one.[28] The second component of the large bracket in the first line of (33), $\left[1-\left(\beta^Y/\varepsilon^Y\right)\right]$, is always less than one because it is already assumed that $\varepsilon^Y > \beta^Y$ in (6). Therefore, the difference between these two components is positive. Furthermore, if the elasticity of the derived demand for X is not decreasing in t^Y, that is, if $\varepsilon_{t^Y}^X \geq 0$, which is a reasonable assumption for most demand profiles, the third component of the large bracket in the first line of (33) is also positive. Hence, the sign of the large bracket in the first line of (33) is positive. In the same fashion, it can be shown

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[28] Both $G^X > 0$ and $\left[1-\left(\beta^X/\varepsilon^X\right)\right] > 0$. Hence, $\left[1-\left(\beta^X/\varepsilon^X\right)\right]/G^X > 0$.

that the sign of the large bracket in the second line of (33) is also positive. Therefore, the overall sign of (33) again depends only on the industry profit conditions.

The above two conditions that $\varepsilon_q^X \le 0$ and $\varepsilon_{t^Y}^X \ge 0$ ensure that the corresponding tax/subsidy scheme will be more effective, by strengthening or weakening the upstream industry's market power in a certain direction. For example, if $\left(q - c^X - wD_E c_r^X\right) > 0$, which denotes the situation where the market imperfection problem in the upstream industry is more damaging to welfare than the environmental problem, then the government could improve welfare using $t^X < 0$ and $t^Y > 0$. A subsidy for X reduces q. However, the first condition $\varepsilon_q^X \le 0$ keeps the downstream producers' elasticity of the derived demand for X from becoming less elastic due to this decrease in q, which means that the upstream industry's market power weakens. On the other hand, a tax on Y raises q, but the second condition $\varepsilon_{t^Y}^X \ge 0$ ensures that this tax on Y does not increase q further through changing ε^X, which keeps the upstream firms' market power weak.

However, if $\left(p - c^Y\right) + \left(q - c^X - wD_E c_r^X\right) c_q^Y < 0$, which means that the environmental problem is more severe than the market imperfection problem in both industries, then a tax reform should be reversed to $t^X > 0$ and $t^Y < 0$. First, $t^X > 0$ increases q, but $\varepsilon_q^X \le 0$ means that this increase in q does not make ε^X more elastic. Second, $\varepsilon_{t^Y}^X \ge 0$ ensures that $t^Y < 0$ does not decrease q. Therefore, the upstream market power is at least maintained same or strengthened. However, government levies $t^X > 0$ and tolerates the higher q, since the higher q means the less X and consequently, the smaller $wD_E E$, which lessens the more pressing pollution problem. Again, this is all where t^E is not available, and where the revenue effect of a change in t^Y must offset the revenue effect of a change in t^X.

2. A Tax on Pollution

In this subsection, I consider the case where government uses the combination of t^E and t^Y. Set $dt^X = 0$ in (31) and use (B6), (B7), (20b), (20c), (21b), (21c), (26a), and (26b) to get:

$$\frac{dW}{dt^E} = -\frac{\varepsilon^Y c_r^X c_q^Y Y}{wpG^Y}\begin{bmatrix}\left(q - c^X - wD_E c_r^X\right)c_q^Y \\ +\left(p - c^Y\right)\end{bmatrix}\left[\frac{1}{G^X} - \left(1 - \frac{\beta^Y}{\varepsilon^Y}\right) + \frac{\beta^X q c_q^Y \varepsilon_{t^Y}^X}{\left(\varepsilon^X\right)^2 pG^X}\right]$$

$$-\frac{\sigma^Y c_w^Y c_r^X X}{qc^Y}\left(q - c^X - wD_E c_r^X\right)\left[\frac{1}{G^X} + \frac{\beta^X q c_q^Y \varepsilon_{t^Y}^X}{\left(\varepsilon^X\right)^2 pG^X}\right] + \frac{\sigma^X c_w^X wD_E E}{r^0 c^X}. \qquad (40)$$

As t^E is levied on the upstream industry, the input substitutability of the upstream industry $\left(\sigma^X\right)$ appears in the welfare expression in addition to other factors affecting the magnitude of welfare change. Note that the sign of (40) is now determined by two factors: the profit wedges in both industries as well as the input substitutability of the upstream producers.

Only a tax on pollution $\left(t^E\right)$ can bring in not only the downstream firms' input substitution effect but also the upstream firms' input substitution effect. The existence of the upstream firms'input substitutability strengthens the reason to tax pollution of the upstream industry, and subsidize the downstream industry, if $\left(p - c^Y\right) + \left(q - c^X - wD_E c_r^X\right)c_q^Y < 0$. Moreover, the upstream firms' input substitutability weakens the need to subsidize pollution of the upstream industry, if $\left(q - c^X - wD_E c_r^X\right) > 0$. If the size of environmental damages to welfare is greater than the upstream industry's profit but is smaller than the sum of both industries' profits (i.e., $\left(p - c^Y\right) + \left(q - c^X - wD_E c^X_r\right) c^Y_q < 0 < \left(q - c^X - wD_E c^X_r\right)$), then the first term in (40) remains negative while the second becomes positive. Meanwhile, the third term is always positive. In this case, the downstream firms' input substitutability, together with the upstream firms' input substitutability has power to change the direction of a tax reform from $t^E > 0$ and $t^Y < 0$ to $t^E < 0$ and $t^Y > 0$. Therefore, in my model where the distortion from pollution is simultaneously considered along with other distortions from imperfect competition in both industries, σ^Y cannot solely determine the direction of tax reforms in general. Only the profit wedges and the upstream firms' input substitutability can change the direction of tax reforms in general.

Ⅵ Conclusion

In this paper, I have examined the welfare effects of various revenue-neutral tax reforms in the case of two vertically-related oligopolies (downstream and upstream), where the upstream industry is polluting. I have shown analytically when and how government can improve welfare by initiating various tax reforms, regardless of either the feasibility of a lump sum transfer or the availability of a tax on pollution.

The profit wedge that is the difference between the unit price and the unit cost and the marginal environmental damages (MED) becomes important to decide the direction of a tax reform. In general, if the MED is more damaging to social welfare than the marginalization problem due to under-production, government should levy a tax on pollution (or on the intermediate good) and use this revenue to subsidize the downstream industry. On the other hand, if the environmental damages are less pressing, the direction of a tax reform should be reversed to the combination of a subsidy for pollution (or for the intermediate good) and a tax on the downstream output.

If the pollution problem is less severe than the upstream industry's marginalization problem but is more severe than the double marginalization problems in both industries, the direction of a tax reform can be determined only when information on parametersis available. In this case, the downstream firms' input substitutability becomes important, because it weakens the upstream industry's market power. If it is strong enough, then the direction of a tax reform can be restored to a tax on pollution (or on the intermediate good) and a subsidy for the downstream output. A tax on the intermediate good has this input substitution effect by the downstream firms and, therefore, can be used as a corrective tax instrument even in vertically-related oligopolies with a pollution problem.

However, a tax on pollution is superior to a tax on intermediate good, because the former always brings in positive welfare effect from the upstream firms' input substitutability, which a tax on intermediate good cannot provide. And if this input substitution effect by the upstream firms is strong enough, then the direction of a tax reform can be restored to a tax on pollution (or on the intermediate good) and a subsidy for the downstream output even if the relative size of industry profits to environmental damages alone does not give policy makers a clear-cut conclusion about the direction of a tax reform.

The findings of this paper may shed some policy implications for 'Green Tax Reform' that have been being promoted to reduce environmentally-harmful subsidies for the energy and electricity sectors, which are increasing rapidly in many OECD countries (OECD, 2005). In Korea, for example, the annual subsidy for electricity sector only amounts to more than 2,000 billion won (nearly 2 billion US dollars) (Kang, et al., 2007). Though undesirable subsidies should be eliminated, my results suggest that it is important to consider the pertinent industry's as well as the related industries' market structures and cost and demand characteristics along with the adverse effects from perverse incentives of environmentally-harmful subsidies..

I have used some simplifying assumptions in my model to obtain the analytically tractable results. First, in order to analyze the upstream producers' input substitutability resulting from a tax on pollution, I have postulated the upstream cost function as a function of the full price of polluting the environment, which is assumed to be the sum of the hidden, internal price of using the environment as an input and a tax on pollution. And I have used a simplifying assumption that the relationship between these two prices is linear and that the implicit internal price for using the environment does not change as a tax on pollution is introduced. However, if these two price factors are related to each other, the results might turn out differently. Second, I have not examined the optimal taxation problem here since government does not have any revenue requirement in my model. But it would be more difficult to solve an optimal taxation problem, and it might be possible to derive closed form solutions only for some special cases.

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Appendices

Appendix A : Derivation of Equation (12)

Since the aggregate demand $X = c_q^Y(q,\ w)\cdot Y(p,\ w)$ by Shepard's lemma, $X_q = c_{qq}^Y Y + c_q^Y Y_p \varphi_q$. Then, the elasticity of the derived demand for X is:

$$\varepsilon^X \equiv -\frac{qX_q}{X} = -\frac{q\cdot\left(c_{qq}^Y Y + c_q^Y Y_p \varphi_q\right)}{c_q^Y Y}. \tag{A1}$$

Rewrite it using $\varphi_q = \dfrac{c_q^Y}{\left(1-t^Y\right)G^Y}$ from (9) to get:

$$\varepsilon^X = -\frac{qc_{qq}^Y}{c_q^Y} - \frac{qc_q^Y Y_p}{\left(1-t^Y\right)G^Y Y}. \tag{A2}$$

Define the substitution elasticity between labor and intermediate good in downstream production as:

$$\sigma^Y \equiv -\frac{d\left(X^{DY}/L^{DY}\right)/\left(X^{DY}/L^{DY}\right)}{d\left(q/w\right)/\left(q/w\right)} = -\left(\frac{dX^{DY}/X^{DY} - dL^{DY}/L^{DY}}{dq/q - dw/w}\right), \tag{A3}$$

where X^{DY} and L^{DY} denote the input demands for the intermediate good and for labor by the downstream firm, respectively.

By Shepard's lemma, $X^{DY} = c_q^Y Y$ and $L^{DY} = c_w^Y Y$. Totally differentiate these to obtain $dX^{DY} = \left(c_{qq}^Y dq + c_{qw}^Y dw\right)Y$ and $dL^{DY} = \left(c_{wq}^Y dq + c_{ww}^Y dw\right)Y$, respectively. Therefore,

$$\frac{dX^{DY}}{X^{DY}} - \frac{dL^{DY}}{L^{DY}} = \frac{c_{qq}^Y dq + c_{qw}^Y dw}{c_q^Y} - \frac{c_{wq}^Y dq + c_{ww}^Y dw}{c_w^Y}. \tag{A4}$$

Since the cost function is homogeneous of degree one in $(q,\ w)$, $c^Y = c_q^Y q + c_w^Y w$ by Euler's formula. Differentiate it with respect to q and w to obtain:

$$c^Y_{wq} = -\frac{q}{w}c^Y_{qq}, \qquad c^Y_{ww} = -\frac{q}{w}c^Y_{qw} = \left(\frac{q}{w}\right)^2 c^Y_{qq}, \qquad c^Y_{wq} = c^Y_{qw}. \tag*{A5)}$$

Substitute the above expressions into (A4) to get:

$$\frac{dX^{DY}}{X^{DY}} - \frac{dL^{DY}}{L^{DY}} = \frac{qc^Y_{qq}\left(qc^Y_q + wc^Y_w\right)}{wc^Y_q c^Y_w}\left(\frac{dq}{q} - \frac{dw}{w}\right). \tag{A6}$$

Substitute (A6) into (A3) to get:

$$\sigma^Y = -\frac{qc^Y_{qq}c^Y}{wc^Y_q c^Y_w}. \tag{A7}$$

Simplify (A2) using (A7) to finally get (12).

Similarly, the substitution elasticity between E and L^X by the upstream industry can be obtained as follows:

$$\sigma^X = -\frac{r^0 c^X_{rr} c^X}{wc^X_r c^X_w}. \tag{A8}$$

Appendix B: Derivation of Equation (26)

Substitute the definitions for CS, Π^X, and Π^Y into (22) to get:

$$W = U(Y) - \frac{pY}{w} - D(E) + \frac{\left[\left(1-t^X\right)q - c^X\right]c^Y_q Y + \left[\left(1-t^Y\right)p - c^Y\right]Y + T}{w}, \tag{B1}$$

where $X = X(q,\ w) = c^Y_q(q,\ w)\cdot Y(p,\ w)$, $Y = Y(p,\ w)$, $c^X = c^X(r,\ w)$, $c^X_r = c^X_r(r,\ w)$, $c^Y = c^Y(q,\ w)$, $c^Y_q = c^Y_q(q,\ w)$, $p = p(t^X,\ t^Y,\ t^E)$, $q = q(t^X,\ t^Y,\ t^E)$, and $D = D(E) = D(c^X_r X) = D(c^X_r c^Y_q Y)$.

From (B1),

$$\frac{\partial W}{\partial t^X} = W_p\frac{\partial p}{\partial t^X} + W_q\frac{\partial q}{\partial t^X} - \frac{qX}{w}, \tag{B2}$$

$$\frac{\partial W}{\partial t^Y} = W_p\frac{\partial p}{\partial t^Y} + W_q\frac{\partial q}{\partial t^Y} - \frac{pY}{w}, \tag{B3}$$

and
$$\frac{\partial W}{\partial t^E} = W_p \frac{\partial p}{\partial t^E} + W_q \frac{\partial q}{\partial t^E} - \frac{E}{w} - c^X_{rr} c^Y_q D_E Y, \tag{B4}$$

where $W_p \equiv \partial W / \partial p$, $W_q \equiv \partial W / \partial q$, and $c^X_{rr} \equiv \partial^2 c^X / (\partial r)^2$.

Substitute (B2), (B3), and (B4) into (25) and use $\sigma^X = -r c^X_{rr} c^X / w c^X_r c^X_w$ from (A8) to get:

$$dW = \left(W_p \frac{\partial p}{\partial t^X} + W_q \frac{\partial q}{\partial t^X} \right) dt^X + \left(W_p \frac{\partial p}{\partial t^Y} + W_q \frac{\partial q}{\partial t^Y} \right) dt^Y$$

$$+ \left(W_p \frac{\partial p}{\partial t^E} + W_q \frac{\partial q}{\partial t^E} + \frac{\sigma^X c^X_w w D_E E}{r^0 c^X} \right) dt^E. \tag{B5}$$

From (B1),

$$w W_p = -\frac{\varepsilon^Y \left(\Pi^X + \Pi^Y - w D_E E \right)}{p} - t^Y Y \tag{B6}$$

and

$$w W_q = -\frac{\sigma^Y w c^Y_w \left(\Pi^X - w D_E E \right)}{q c^Y} - t^X X. \tag{B7}$$

Plug (B6) and (B7) into (B5) and set $t^X = t^Y = t^E = T = 0$, then (26) is obtained.

Appendix C: Derivation of Equation (31)

Totally differentiate (22) to get:

$$dW = \frac{\partial W}{\partial t^X} dt^X + \frac{\partial W}{\partial t^Y} dt^Y + \frac{\partial W}{\partial t^E} dt^E. \tag{C1}$$

Rewrite it using (30):

$$dW = \left(\frac{\partial W}{\partial t^X} - \frac{\partial W}{\partial t^Y} \frac{qX}{pY} \right) dt^X + \left(\frac{\partial W}{\partial t^E} - \frac{\partial W}{\partial t^Y} \frac{E}{pY} \right) dt^E. \tag{C2}$$

Substitute (B2), (B3), (B4) and (A8) into (C2), then (31) is obtained.