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FUZZY PAIRWISE γ-IRRESOLUTE HOMEOMORPHISMS

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ABSTRACT. We define and characterize a fuzzy pairwise γ-irresolute open mapping (fuzzy pairwise γ-irresolute closed mapping) on a fuzzy bitopological space. We also characterize a fuzzy pairwise γ-irresolute homeomorphism on a fuzzy bitopological space.

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1. Introduction

Azad [1], Singal and Prakash [9] introduced a fuzzy semiopen set and a fuzzy preopen set and studied the characteristic properties of a fuzzy semicontinuous mapping and a fuzzy precontinuous mapping on a fuzzy topological space. Later, Sampath Kumar [7, 8] defined a (τ_i, τ_j)-fuzzy semiopen set and a (τ_i, τ_j)-fuzzy preopen set and characterized a fuzzy pairwise semicontinuous mapping and a fuzzy pairwise precontinuous mapping on a fuzzy bitopological space as a natural generalization of a fuzzy topological space.

Hanafy [2] defined a fuzzy γ-open set and studied a fuzzy γ-continuous mapping on a fuzzy topological space. The first author and others [4, 5] developed Hanafy's results. In particular, they defined and characterized a fuzzy γ-irresolute mapping and a fuzzy γ-irresolute open mapping on a fuzzy topological space. They also [3, 6] extended their results to a fuzzy bitopological space, that is, they defined and characterized a fuzzy pairwise γ-continuous mapping

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and a fuzzy pairwise γ-irresolute continuous mapping on a fuzzy bitopological space.

In this paper, we define a fuzzy pairwise γ-irresolute open mapping (a fuzzy pairwise γ-irresolute closed mapping) on a fuzzy bitopological space and study their properties. And we characterize a fuzzy pairwise γ-irresolute homeomorphism on a fuzzy bitopological space.

2. Preliminaries

Let X be a set and let τ_1 and τ_2 be fuzzy topologies on X. Then we call (X, τ_1, τ_2) a *fuzzy bitopological space* [*fbts*].

A mapping $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ is *fuzzy pairwise continuous* [*fpc*] if and only if the induced mapping $f : (X, \tau_k) \to (Y, \tau_k^*)$ is fuzzy continuous for $k = 1, 2$.

Notations. (1) Throughout this paper, we take an ordered pair (τ_i, τ_j) with $i, j \in \{1, 2\}$ and $i \neq j$.

(2) For simplicity, we abbreviate a τ_i-fuzzy open set μ and a τ_j-fuzzy closed set μ with a $\tau_i - fo$ set μ and a $\tau_j - fc$ set μ respectively. Also, we denote the interior and the closure of μ for a fuzzy topology τ_i with $\tau_i - \operatorname{Int} \mu$ and $\tau_i - \operatorname{Cl} \mu$ respectively.

Definition 2.1. [6, 7, 8] Let μ be a fuzzy set on a *fbts* X. Then we call μ;

(1) a (τ_i, τ_j)–fuzzy semiopen [$(\tau_i, \tau_j) - fso$] set on X if

$$\mu \leq \tau_j - \operatorname{Cl}(\tau_i - \operatorname{Int} \mu),$$

(2) a (τ_i, τ_j)–fuzzy semiclosed [$(\tau_i, \tau_j) - fsc$] set on X if

$$\tau_j - \operatorname{Int}(\tau_i - \operatorname{Cl} \mu) \leq \mu,$$

(3) a (τ_i, τ_j)–fuzzy preopen [$(\tau_i, \tau_j) - fpo$] set on X if

$$\mu \leq \tau_i - \operatorname{Int}(\tau_j - \operatorname{Cl} \mu),$$

(4) a (τ_i, τ_j)–fuzzy preclosed [$(\tau_i, \tau_j) - fpc$] set on X if

$$\tau_i - \operatorname{Cl}(\tau_j - \operatorname{Int} \mu) \leq \mu,$$

(5) a (τ_i, τ_j)–fuzzy γ–open [$(\tau_i, \tau_j) - f\gamma o$] set on X if

$$\mu \leq \tau_j - \operatorname{Cl}(\tau_i - \operatorname{Int} \mu) \vee \tau_i - \operatorname{Int}(\tau_j - \operatorname{Cl} \mu) \text{ and}$$

(6) a (τ_i, τ_j)–fuzzy γ–closed [$(\tau_i, \tau_j) - f\gamma c$] set on X if

$$\tau_i - \operatorname{Cl}(\tau_j - \operatorname{Int} \mu) \wedge \tau_j - \operatorname{Int}(\tau_i - \operatorname{Cl} \mu) \leq \mu.$$

It is clear that every $(\tau_i, \tau_j) - fso$ set is a $(\tau_i, \tau_j) - f\gamma o$ set and every $(\tau_i, \tau_j) - fpo$ set is a $(\tau_i, \tau_j) - f\gamma o$ set from the above definition. But the converses need not be true in general [6].

Proposition 2.2. [6] (1) *The union of* $(\tau_i, \tau_j) - f\gamma o$ *sets is a* $(\tau_i, \tau_j) - f\gamma o$ *set.*
(2) *The intersection of* $(\tau_i, \tau_j) - f\gamma c$ *sets is a* $(\tau_i, \tau_j) - f\gamma c$ *set.*

But the intersection (the union) of two $(\tau_i, \tau_j) - f\gamma o$ sets ($(\tau_i, \tau_j) - f\gamma c$ sets) need not be a $(\tau_i, \tau_j) - f\gamma o$ set ($(\tau_i, \tau_j) - f\gamma c$ set) [6].

Proposition 2.3. [6] *Let* μ *be a fuzzy set on a* $fbts$ X.
(1) *If* μ *is a* $(\tau_i, \tau_j) - f\gamma o$ *and* $\tau_j - fc$ *set, then* μ *is a* $(\tau_i, \tau_j) - fso$ *set.*
(2) *If* μ *is a* $(\tau_i, \tau_j) - f\gamma c$ *and* $\tau_j - fo$ *set, then* μ *is a* $(\tau_i, \tau_j) - fsc$ *set.*

Definition 2.4. [6] Let μ be a fuzzy set on a $fbts$ X.
(1) The $(\tau_i, \tau_j) - \gamma-$interior of μ, $(\tau_i, \tau_j) - \gamma\mathrm{Int}\,\mu$, is

$$\bigvee \left\{\nu \,|\, \nu \leq \mu,\ \nu \text{ is a } (\tau_i, \tau_j) - f\gamma o \text{ set}\right\}.$$

(2) The $(\tau_i, \tau_j) - \gamma-$closure of μ, $(\tau_i, \tau_j) - \gamma\mathrm{Cl}\,\mu)$, is

$$\bigwedge \left\{\nu \,|\, \nu \geq \mu,\ \nu \text{ is a } (\tau_i, \tau_j) - f\gamma c \text{ set}\right\}.$$

Obviously, $(\tau_i, \tau_j) - \gamma\mathrm{Cl}\mu$ is the smallest $(\tau_i, \tau_j) - f\gamma c$ set which contains μ, and $(\tau_i, \tau_j) - \gamma\mathrm{Int}\mu$ is the largest $(\tau_i, \tau_j) - f\gamma o$ set which is contained in μ. Therefore, $(\tau_i, \tau_j) - \gamma\mathrm{Cl}\mu = \mu$ for every $(\tau_i, \tau_j) - f\gamma c$ set μ and $(\tau_i, \tau_j) - \gamma\mathrm{Int}\mu = \mu$ for every $(\tau_i, \tau_j) - f\gamma o$ set μ.

Moreover, we have

$$\tau_i - \mathrm{Int}\mu \leq (\tau_i, \tau_j) - \mathrm{sInt}\mu \leq (\tau_i, \tau_j) - \gamma\mathrm{Int}\mu \leq \mu,$$

$$\mu \leq (\tau_i, \tau_j) - \gamma\mathrm{Cl}\mu \leq (\tau_i, \tau_j) - \mathrm{sCl}\mu \leq \tau_i - \mathrm{Cl}\mu$$

and

$$\tau_i - \mathrm{Int}\mu \leq (\tau_i, \tau_j) - \mathrm{pInt}\mu \leq (\tau_i, \tau_j) - \gamma\mathrm{Int}\mu \leq \mu,$$

$$\mu \leq (\tau_i, \tau_j) - \gamma\mathrm{Cl}\mu \leq (\tau_i, \tau_j) - \mathrm{pCl}\mu \leq \tau_i - \mathrm{Cl}\mu.$$

We also have the following lemma from the above definition, which will be used later.

Lemma 2.5. *Let* μ *be a fuzzy set on a* $fbts$ X. *Then*

$$(\tau_i, \tau_j) - \gamma Int(\mu^c) = ((\tau_i, \tau_j) - \gamma Cl\mu)^c$$

and

$$(\tau_i, \tau_j) - \gamma Cl(\mu^c) = ((\tau_i, \tau_j) - \gamma Int\mu)^c.$$

Proof. Let μ be a fuzzy set on a $fbts$ X. Then

$$\begin{aligned}(\tau_i,\tau_j)-\gamma\mathrm{Cl}\mu &= \bigwedge\left\{\nu^c \mid \nu^c \geq \mu,\ \nu \text{ is a } (\tau_i,\tau_j)-f\gamma o \text{ set}\right\}\\ &= \left(\bigvee\left\{\mu^c \mid \mu^c \geq \mu,\ \nu \text{ is a } (\tau_i,\tau_j)-f\gamma o \text{ set}\right\}\right)^c\\ &= \Big((\tau_i,\tau_j)-\gamma\mathrm{Int}(\mu^c)\Big)^c.\end{aligned}$$

Hence $(\tau_i,\tau_j)-\gamma\mathrm{Int}(\mu^c) = \Big(((\tau_i,\tau_j)-\gamma\mathrm{Cl}\mu\Big)^c$. Similarly we can prove the second equality. □

Definition 2.6. [6] Let $f:(X,\tau_1,\tau_2)\to(Y,\tau_1^*,\tau_2^*)$ be a mapping. Then f is called a fuzzy pairwise γ−continuous [$fp\gamma c$] mapping if $f^{-1}(\nu)$ is a $(\tau_i,\tau_j)-f\gamma o$ set on X for each τ_i^*-fo set ν on Y.

Definition 2.7. [6] Let $f:(X,\tau_1,\tau_2)\to(Y,\tau_1^*,\tau_2^*)$ be a mapping. Then f is called;

(1) a fuzzy pairwise γ−open [$fp\gamma$ *open*] mapping if $f(\mu)$ is a $(\tau_i^*,\tau_j^*)-f\gamma o$ set on Y for each τ_i-fo set μ on X and

(2) a fuzzy pairwise γ−closed [$fp\gamma$ *closed*] mapping if $f(\mu)$ is a $(\tau_i^*,\tau_j^*)-f\gamma c$ set on Y for each τ_i-fc set μ on X.

Definition 2.8. [3] Let $f:(X,\tau_1,\tau_2)\to(Y,\tau_1^*,\tau_2^*)$ be a mapping. Then f is called a fuzzy pairwise γ-irresolute continuous [$fp\gamma$-irresolute continuous] mapping if $f^{-1}(\nu)$ is a $(\tau_i,\tau_j)-f\gamma o$ set on X for each $(\tau_i^*,\tau_j^*)-f\gamma o$ set ν on Y.

It is clear that every $fp\gamma$-irresolute continuous mapping is $fp\gamma c$ from the above definitions. But the converse is not true in general [3].

Proposition 2.9. [3] *Let $f:(X,\tau_1,\tau_2)\to(Y,\tau_1^*,\tau_2^*)$ be a mapping. Then the following statements are equivalent:*

(1) *f is $fp\gamma$-irresolute continuous.*

(2) *The inverse image of each $(\tau_i^*,\tau_j^*)-f\gamma c$ set on Y is a $(\tau_i,\tau_j)-f\gamma c$ set on X.*

(3) $f\Big((\tau_i,\tau_j)-\gamma Cl\mu\Big)\leq(\tau_i^*,\tau_j^*)-\gamma Cl\Big(f(\mu)\Big)$ *for each fuzzy set μ on X.*

(4) $(\tau_i,\tau_j)-\gamma Cl\Big(f^{-1}(\nu)\Big)\leq f^{-1}((\tau_i^*,\tau_j^*)-\gamma Cl\nu)$ *for each fuzzy set ν on Y.*

(5) $f^{-1}((\tau_i^*,\tau_j^*)-\gamma Int\nu)\leq(\tau_i,\tau_j)-\gamma Int(f^{-1}(\nu))$ *for each fuzzy set ν on Y.*

Proposition 2.10. [3] *Let $f:(X,\tau_1,\tau_2)\to(Y,\tau_1^*,\tau_2^*)$ be a bijection. Then f is $fp\gamma$-irresolute continuous if and only if for each fuzzy set μ on X,*

$$(\tau_i^*,\tau_j^*)-\gamma Int(f(\mu))\leq f\Big((\tau_i,\tau_j)-\gamma Int\mu\Big).$$

3. Fuzzy pairwise γ-irresolute homeomorphisms

In this section, we introduce a fuzzy pairwise γ-irresolute open mapping (a fuzzy pairwise γ-irresolute closed mapping) which is stronger than a fuzzy pairwise γ-open mapping (a fuzzy pairwise γ-closed mapping). And we characterize a fuzzy pairwise γ-irresolute open mapping (a fuzzy pairwise γ-irresolute closed mapping) and a fuzzy pairwise γ-irresolute homeomorphism.

Definition 3.1. Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then f is called

(1) a fuzzy pairwise γ-irresolute open [$fp\gamma$-irresolute open] mapping if $f(\mu)$ is a $(\tau_i^*, \tau_j^*) - f\gamma o$ set on Y for each $(\tau_i, \tau_j) - f\gamma o$ set μ on X and

(2) a fuzzy pairwise γ-irresolute closed [$fp\gamma$-irresolute closed] mapping if $f(\mu)$ is a $(\tau_i^*, \tau_j^*) - f\gamma c$ set on Y for each $(\tau_i, \tau_j) - f\gamma c$ set μ on X.

It is clear that every $fp\gamma$-irresolute open mapping and every $fp\gamma$-irresolute closed mapping are $fp\gamma$ *open* and $fp\gamma$ *closed* respectively. But the converses are not true in general as the following example shows.

Example 3.2. Let μ_1, μ_2, μ_3 and μ_4 be fuzzy sets on $X = \{a, b, c\}$ with

$$\mu_1(a) = 0.9, \mu_1(b) = 0.9, \mu_1 = 0.9,$$
$$\mu_2(a) = 0.4, \mu_2(b) = 0.4, \mu_2 = 0.4 \text{ and}$$
$$\mu_3(a) = 0.7, \mu_3(b) = 0.7, \mu_3 = 0.7.$$

Let

$$\tau_1 = \{0_X, \mu_1, 1_X\}, \tau_2 = \{0_X, 1_X\} \text{ and}$$
$$\tau_1^* = \{0_X, \mu_3, 1_X\}, \tau_2^* = \{0_X, \mu_2, 1_X\}.$$

be fuzzy topologies on X.

Then we can show that the identity mapping $i_X : (X, \tau_1, \tau_2) \to (X, \tau_1^*, \tau_2^*)$ is $fp\gamma$ *open*. But i_X is not $fp\gamma$-irresolute open since μ_3^c is not a $(\tau_i^*, \tau_j^*) - f\gamma o$ set.

Theorem 3.3. *Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then the following statements are equivalent:*

(1) *f is $fp\gamma$-irresolute open.*

(2) *$f\Big((\tau_i, \tau_j) - \gamma\text{Int}\mu\Big) \leq (\tau_i^*, \tau_j^*) - \gamma\text{Int}(f(\mu))$ for each fuzzy set μ on X.*

(3) *$(\tau_i, \tau_j) - \gamma\text{Int}\Big(f^{-1}(\nu)\Big) \leq f^{-1}\Big((\tau_i^*, \tau_j^*) - \gamma\text{Int}\nu\Big)$ for each fuzzy set ν on Y.*

Proof. (1) implies (2): Let μ be a fuzzy set on X. Then $f((\tau_i, \tau_j) - \gamma\text{Int}\mu)$ is a $(\tau_i^*, \tau_j^*) - f\gamma o$ set on Y and $f((\tau_i, \tau_j) - \gamma\text{Int}\mu) \leq f(\mu)$. Hence

$$\begin{aligned} f\Big((\tau_i, \tau_j) - \gamma\text{Int}\mu\Big) &= (\tau_i^*, \tau_j^*) - \gamma\text{Int}\Big(f((\tau_i, \tau_j) - \gamma\text{Int}\mu)\Big) \\ &\leq (\tau_i^*, \tau_j^*) - \gamma\text{Int}\Big(f(\mu)\Big). \end{aligned}$$

(2) implies (3): Let ν be a fuzzy set on Y. Then

$$\begin{aligned} f\Big((\tau_i,\tau_j)-\gamma\mathrm{Int}(f^{-1}(\nu))\Big) &\le (\tau_i^*,\tau_j^*)-\gamma\mathrm{Int}(f(f^{-1}(\nu))) \\ &\le (\tau_i^*,\tau_j^*)-\gamma\mathrm{Int}\nu. \end{aligned}$$

Therefore,

$$\begin{aligned} (\tau_i,\tau_j)-\gamma\mathrm{Int}(f^{-1}(\nu)) &\le f^{-1}\Big(f((\tau_i,\tau_j)-\gamma\mathrm{Int}(f^{-1}(\nu)))\Big) \\ &\le f^{-1}((\tau_i^*,\tau_j^*)-\gamma\mathrm{Int}\nu). \end{aligned}$$

(3) implies (1): Let μ be a $(\tau_i,\tau_j)-f\gamma o$ set on X. Then

$$\begin{aligned} \mu &= (\tau_i,\tau_j)-\gamma\mathrm{Int}\mu \\ &\le (\tau_i,\tau_j)-\gamma\mathrm{Int}\Big(f^{-1}(f(\mu))\Big) \\ &\le f^{-1}((\tau_i^*,\tau_j^*)-\gamma\mathrm{Int}(f(\mu))). \end{aligned}$$

Therefore,

$$\begin{aligned} f(\mu) &\le f\Big(f^{-1}((\tau_i,\tau_j)-\gamma\mathrm{Int}(f(\mu)))\Big) \\ &\le (\tau_i^*,\tau_j^*)-\gamma\mathrm{Int}(f(\mu)). \end{aligned}$$

Hence $f(\mu)=(\tau_i^*,\tau_j^*)-\gamma\mathrm{Int}(f(\mu))$. Consequently, $f(\mu)$ is a $(\tau_i^*,\tau_j^*)-f\gamma o$ set on Y and therefore f is $fp\gamma$-irresolute open. □

Proposition 3.4. *A mapping $f:(X,\tau_1,\tau_2)\to(Y,\tau_1^*,\tau_2^*)$ is $fp\gamma$-irresolute closed if and only if $(\tau_i^*,\tau_j^*)-\gamma Cl(f(\mu))\le f\Big((\tau_i,\tau_j)-\gamma Cl\mu\Big)$ for each fuzzy set μ on X.*

Proof. Let μ be a fuzzy set on X. Then $f((\tau_i,\tau_j)-\gamma\mathrm{Cl}\mu)$ is a $(\tau_i^*,\tau_j^*)-f\gamma c$ set on Y and $f(\mu)\le f((\tau_i,\tau_j)-\gamma\mathrm{Cl}\mu)$. Hence

$$\begin{aligned} (\tau_i^*,\tau_j^*)-\gamma\mathrm{Cl}(f(\mu)) &\le (\tau_i^*,\tau_j^*)-\gamma\mathrm{Cl}\Big(f((\tau_i,\tau_j)-\gamma\mathrm{Cl}\mu)\Big) \\ &= f((\tau_i,\tau_j)-\gamma\mathrm{Cl}\mu). \end{aligned}$$

Conversely, let μ be a $(\tau_i,\tau_j)-f\gamma c$ set on X. Then

$$\begin{aligned} (\tau_i^*,\tau_j^*)-\gamma\mathrm{Cl}(f(\mu)) &\le f\Big((\tau_i,\tau_j)-\gamma\mathrm{Cl}\mu\Big) \\ &= f(\mu). \end{aligned}$$

Consequently, $f(\mu)$ is a $(\tau_i^*,\tau_j^*)-f\gamma c$ set on Y and therefore f is a $fp\gamma$-irresolute closed mapping. □

Theorem 3.5. *Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a bijection. Then the following statements are equivalent:*

(1) *f is $fp\gamma$-irresolute closed.*

(2) *$f^{-1}\Big((\tau_i^*, \tau_j^*) - \gamma Cl\nu\Big) \le (\tau_i, \tau_j) - \gamma Cl(f^{-1}(\nu))$ for each fuzzy set ν on Y.*

(3) *f is $fp\gamma$-irresolute open.*

(4) *f^{-1} is $fp\gamma$-irresolute continuous.*

Proof. (1) implies (2): Let ν be a fuzzy set on Y. Then, by Proposition 3.4,

$$(\tau_i^*, \tau_j^*) - \gamma \mathrm{Cl}\Big(f(f^{-1}(\nu))\Big) \le f\Big((\tau_i, \tau_j) - \gamma \mathrm{Cl}(f^{-1}(\nu))\Big).$$

Hence

$$f^{-1}\Big((\tau_i^*, \tau_j^*) - \gamma \mathrm{Cl}(f(f^{-1}(\nu)))\Big) \le f^{-1}\Big(f((\tau_i, \tau_j) - \gamma \mathrm{Cl}(\nu))\Big).$$

Since f is a bijection,

$$f^{-1}\Big((\tau_i^*, \tau_j^*) - \gamma \mathrm{Cl}\nu\Big) \le (\tau_i, \tau_j) - \gamma \mathrm{Cl}(f^{-1}(\nu)).$$

(2) implies (1): Let μ be a fuzzy set on X. Then

$$f^{-1}\Big((\tau_i^*, \tau_j^*) - \gamma \mathrm{Cl}(f(\mu))\Big) \le (\tau_i, \tau_j) - \gamma \mathrm{Cl}(f^{-1}(f(\mu))).$$

Hence

$$f(f^{-1}\Big((\tau_i^*, \tau_j^*) - \gamma \mathrm{Cl}(f(\mu)))\Big) \le f\Big((\tau_i, \tau_j) - \gamma \mathrm{Cl}(f^{-1}(f(\mu)))\Big).$$

Since f is a bijection,

$$(\tau_i^*, \tau_j^*) - \gamma \mathrm{Cl}(f(\mu)) \le f\Big((\tau_i, \tau_j) - \gamma \mathrm{Cl}\mu\Big).$$

Therefore, by Proposition 3.4, f is $fp\gamma$-irresolute closed.

(2) implies (3): Let ν be a fuzzy set on Y. Then

$$f^{-1}\Big((\tau_i^*, \tau_j^*) - \gamma \mathrm{Cl}(\nu^c)\Big) \le (\tau_i, \tau_j) - \gamma \mathrm{Cl}(f^{-1}(\nu^c)).$$

By Lemma 2.5,

$$\begin{aligned}(\tau_i, \tau_j) - \gamma \mathrm{Int}(f^{-1}(\nu)) &= \Big((\tau_i, \tau_j) - \gamma \mathrm{Cl}(f^{-1}(\nu^c))\Big)^c \\ &\le f^{-1}\Big(((\tau_i^*, \tau_j^*) - \gamma \mathrm{Cl}(\nu^c))^c\Big) \\ &= f^{-1}\Big((\tau_i^*, \tau_j^*) - \gamma \mathrm{Int}\nu\Big).\end{aligned}$$

Hence f is $fp\gamma$-irresolute open from Theorem 3.3.

(3) implies (4): Let ν be a fuzzy set on Y. Then

$$(\tau_i, \tau_j) - \gamma\text{Int}(f^{-1}(\nu)) \leq f^{-1}\Big((\tau_i^*, \tau_j^*) - \gamma\text{Int}\nu\Big).$$

Since f is a bijection, by Proposition 2.10, f^{-1} is $fp\gamma$-irresolute continuous.

(4) implies (2): It is clear from Proposition 2.9. □

We have the following corollaries from Proposition 2.9, Proposition 3,4 and Theorem 3.3.

Corollary 3.6. *Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then, f is a $fp\gamma$-irresolute closed and $fp\gamma$-irresolute continuous if and only if $f\Big((\tau_i, \tau_j) - \gamma Cl\mu\Big) = (\tau_i^*, \tau_j^*) - \gamma Cl(f(\mu))$ for each fuzzy set μ on X.*

Corollary 3.7. *Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then, f is $fp\gamma$-irresolute open and $fp\gamma$-irresolute continuous if and only if $f^{-1}\Big((\tau_i^*, \tau_j^*) - \gamma Cl\nu\Big) = (\tau_i, \tau_j) - \gamma Cl(f^{-1}(\mu))$ for each fuzzy set ν on Y.*

A bijection $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ is called *a fuzzy pairwise γ-irresolute homeomorphism* if f and f^{-1} are fuzzy pairwise γ-irresolute continuous mappings.

Theorem 3.8. *Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a bijection. Then the following statements are equivalent:*

(1) f is a fuzzy pairwise γ-irresolute homeomorphism.

(2) f^{-1} is a fuzzy pairwise γ-irresolute homeomorphism.

(3) f and f^{-1} are $fp\gamma$-irresolute open ($fp\gamma$-irresolute closed).

(4) f is $fp\gamma$-irresolute continuous and $fp\gamma$-irresolute open ($fp\gamma$-irresolute closed).

(5) $f\Big((\tau_i, \tau_j) - \gamma Cl\mu\Big) = (\tau_i^*, \tau_j^*) - \gamma Cl(f(\mu))$ for each fuzzy set μ on X.

(6) $f\Big((\tau_i, \tau_j) - \gamma\text{Int}\mu\Big) = (\tau_i^*, \tau_j^*) - \gamma Int(f(\mu))$ for each fuzzy set μ on X.

(7) $f^{-1}\Big((\tau_i^*, \tau_j^*) - \gamma\text{Int}\nu\Big) = (\tau_i, \tau_j) - \gamma Int(f^{-1}(\nu))$ for each fuzzy set ν on Y.

(8) $(\tau_i, \tau_j) - \gamma Cl(f^{-1}(\nu)) = f^{-1}\Big((\tau_i^*, \tau_j^*) - \gamma Cl\nu\Big)$ for each fuzzy set ν on Y.

Proof. (1) implies (2): It follows immediately from the definition of a fuzzy pairwise γ-irresolute homeomorphism.

(2) implies (3) and (3) implies (4): It follows from Theorem 3.5.

(4) implies (5): It follows from Theorem 3.5 and Corollary 3.6.

(5) implies (6): Let μ be a fuzzy set on X. Then, by Lemma 2.5,

$$\begin{aligned} f((\tau_i,\tau_j)-\gamma\mathrm{Int}\mu) &= \Big(f((\tau_i,\tau_j)-\gamma\mathrm{Cl}(\mu^c))\Big)^c \\ &= \Big((\tau_i^*,\tau_j^*)-\gamma\mathrm{Cl}(f(\mu^c))\Big)^c \\ &= (\tau_i^*,\tau_j^*)-\gamma\mathrm{Int}f(\mu). \end{aligned}$$

(6) implies (7): Let ν be a fuzzy set on Y. Then

$$\begin{aligned} f((\tau_i,\tau_j)-\gamma\mathrm{Int}(f^{-1}(\nu)) &= (\tau_i^*,\tau_j^*)-\gamma\mathrm{Int}(f(f^{-1}(\nu))) \\ &= (\tau_i^*,\tau_j^*)-\gamma\mathrm{Int}\nu. \end{aligned}$$

Hence

$$f^{-1}\Big(f((\tau_i,\tau_j)-\gamma\mathrm{Int}(f^{-1}(\nu))\Big) = f^{-1}((\tau_i^*,\tau_j^*)-\gamma\mathrm{Int}\nu).$$

Therefore,

$$(\tau_i,\tau_j)-\gamma\mathrm{Int}(f^{-1}(\nu)) = f^{-1}((\tau_i^*,\tau_j^*)-\gamma\mathrm{Int}\nu).$$

(7) implies (8): Let ν be a fuzzy set on Y. Then, by Lemma 2.5,

$$\begin{aligned} (\tau_i,\tau_j)-\gamma\mathrm{Cl}(f^{-1}(\nu)) &= \Big(f^{-1}((\tau_i^*,\tau_j^*)-\gamma\mathrm{Int}(\nu^c))\Big)^c \\ &= \Big((\tau_i,\tau_j)-\gamma\mathrm{Int}(f^{-1}(\nu^c))\Big)^c \\ &= f^{-1}((\tau_i^*,\tau_j^*)-\gamma\mathrm{Cl}\nu). \end{aligned}$$

(8) implies (1): It follows from Theorem 3.5 and Corollary 3.7. □

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