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AN IMAGE THRESHOLDING METHOD BASED ON THE TARGET EXTRACTION

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ABSTRACT. In this paper an algorithm, based on extracting a certain target of an image, is proposed that is capable of performing bilevel thresholding of image with multimodal distribution. Each pixel in the image has a membership value which is used to denote the characteristic relationship between the pixel and its belonging region (i.e. the object or background). Using the membership values of image set, a new measurement, which simultaneously measures the measure of fuzziness and the conditional entropy of the image, is calculated. Then, thresholds are found by optimally minimizing calculated measurement. In addition, a fuzzy range is defined to improve the threshold values. The experimental results demonstrate that the proposed approach can select the thresholds automatically and effectively extract the meaningful target from the input image. The resulting image can preserve the object region we target very well.

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1. Introduction

Thresholding is a process of partitioning a digital image into mutually exclusive and distinctive regions. It is a commonly used technique for image processing, pattern recognition and computer vision, etc. A number of investigations on various thresholding techniques have been reported in the literature, such as the references [11,12,14,16,].

Generally, information may lose while we map 3-D objects into 2-D images; or there are ambiguity and vagueness in some definitions; or some ambiguity and vagueness may exist when we interpret low-level image processing results. Consequently, image processing will bear some fuzziness in nature. The nature of this ambiguity (fuzziness) in the image therefore arises from the uncertainty representation. The segments in that case may be viewed as fuzzy subsets of the

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image. The membership function in the processing is used to denote the degree of possessing a certain property by the pixels of the image. Several researchers have made much investigation on this fuzzily-based thresholding technique. The methods have been successfully applied for automatic selection of the thresholds, in which optimization of some threshold-dependent criterion function is done. The criterions are entropy measure [8] [15], index of fuzziness [10], fuzzy divergence [1,2] and the compactness [17] of the image, etc.

Another useful criterion is entropy [20], which measures probabilistic information in communications. It was introduced into thresholding by Pun [19] and then many methods based on entropy are developed [3,5,6,13,21,23]. Based on the maximum entropy theorem of information theory, Jin et al. presented a new definition of fuzzy partition entropy with a basis of condition probability and condition entropy in reference [9]. It used the relation of the probability partition and the fuzzy partition to search the optimal threshold adaptively through the maximum fuzzy entropy principle.

Traditional bilevel thresholding methods separate the pixels of an image into two regions. Usually, one is the object whose gray level values are smaller than the specific threshold, and the other is the background. However, for the image of multimodal histogram, the part concerning "object" which is meaningful to the further research doesn't necessarily consist of all the smaller pixels. That is, there is some pixel in the image actually, whose gray level value is smaller, it isn't contained by the "object", however. In that case, traditional bilevel thresholding method counts a little in classifying the pixel to the correct region. To overcome this shortcoming, a new fuzzy set based on the target is defined in our article.

Let a certain pair of values be the threshold values of the regions. The thresholds are the crossover points between the fuzzy subsets, that is, between the "object" and the "background". For simplicity, the target region in the image is denoted as "object" and the other part is as "background" in this paper. Every pixel in the image has a membership value which is used to denote the characteristic relationship between the pixel and its belonging region (i.e. the "object" or "background"). Using the membership values, a new measurement, which simultaneously measures the measure of fuzziness and the conditional entropy of the image, is calculated. Then, thresholds are found by optimally minimizing calculated measurement. In addition, a fuzzy range is defined to improve the threshold values within this range.

The design of this paper is organized as follows. Section 2 is a brief introduction to the preliminary, including the model of an image, measure of fuzziness and conditional entropy. Section 3 describes the proposed method. Section 4 presents the experimental results. Finally, a brief discussion is concluded as a conclusion in Section 5 .

2. Preliminary

2.1. Model of an image

Let X denote an image of size $M \times N$with L levels ranging from r_0 to r_{L-1}, and the histogram of the image is h_k, $k = 0, \cdots, L-1$, that is, the number of occurrences at the gray level r_k in X. Let $\mu_X(r_k)$ denote the membership value which represents the degree of possessing a certain property by the gray level r_k. In the notation of fuzzy set, the image set X can be written as [25]:

$$X = \frac{\mu_X(r_0)}{r_0} + \frac{\mu_X(r_1)}{r_1} + \cdots + \frac{\mu_X(r_k)}{r_k} \tag{1}$$

or

$$X = \frac{\sum_{r_k} \mu_X(r_k)}{r_k} \tag{2}$$

where "+" means union.

2.2 Measure of fuzziness

The measure of fuzziness usually indicates the degree of fuzziness of a fuzzy set. It has been used to image thresholding to partition the image space into meaningful regions. It is a function, $f : A \to R$, which gives the fuzzy set A a value to represent the degree of fuzziness of A. We will introduce the entropy measure which uses the Shannon's function defined by De Luca and Termini [15]:

The entropy measure of a fuzzy set A is:

$$E(A) = \frac{1}{nln2} \sum_i S(\mu_A(x_i)), i = 1, 2, \cdots, n \tag{3}$$

Where $S(\mu_A(x_i)) = -\mu_A(x_i)\ln[\mu_A(x_i)] - [1 - \mu_A(x_i)]\ln[1 - \mu_A(x_i)]$ is the Shannon's function. Extending to the two-dimensional image plane, the entropy measure of an image set X is expressed as:

$$E(X) = \frac{1}{MNln2} \sum_m \sum_n S(\mu_X(x_{mn})) \tag{4}$$

where $m = 1, 2, \cdots, M$, $n = 1, 2, \cdots, N$ and x_{mn} is the gray level of a (m, n) pixel in X. Using the histogram information, Eq.(4) can be further revised as:

$$E(X) = \frac{1}{MNln2} \sum_g S(\mu_X(g))h(g), g = 0, 1, \cdots, L-1 \tag{5}$$

2.3 Conditional entropy

Based on the definition of a fuzzy event [24], we can view an image as a fuzzy event modeled by a probability space (Ω, K, P), where $\Omega = \{r_0, \cdots, r_{L-1}\}$ and P is the probability measure of the occurrence of gray levels, i.e., $Pr\{r_k\} = \widehat{h}_k = h_k / \sum_{k=0}^{L-1} h_k$. Fuzzy set $A = \sum_{r_k} \mu_A(r_k)/r_k$ denote a certain fuzzy event, the probability of it can be calculated by equation (6), i.e.

$$P(A) = \sum_{r_k} \mu_A(r_k)\widehat{h}_k \tag{6}$$

The conditional probability, where the gray level r_k is classified into fuzzy set A due to A is [7]:

$$P(\{k\}|A) = \frac{\mu_A(r_k)\widehat{h}_k}{P(A)} \tag{7}$$

Definition. Let X denote an image with L levels ranging from r_0 to r_{L-1}, $\Omega = \{r_0, \cdots, r_{L-1}\}$ and A is a fuzzy event of X. $Q = \{x_1, x_2, \cdots, x_s\}$ denotes a set which has s pixels of X and $R = \{r_{x_1}, r_{x_2}, \cdots, r_{x_s}\}$ is the gray level set corresponding to the elements of Q, respectively. Then, on condition that A is given, the conditional entropy of Q is as follows:

$$H(Q|A) = -\sum_{r_k} \frac{\mu_A(r_k)\widehat{h}_k}{P(A)} log_2 \frac{\mu_A(r_k)\widehat{h}_k}{P(A)} \tag{8}$$

In communications theory, an entropy function can be used to measure how much information is contained in a data set. The present paper will measure the conditional entropy of some pixels of image to justify if the information is mostly contained after thresholding.

3. Proposed method

It is different from the early literature on the bilevel thresholding, the proposed method can extract a certain meaningful target from an image with multimodal distribution effectively. In other words, object as considered by traditional bilevel thresholding method-based intensity values is only a part which contains pixels with gray values smaller than the threshold value. However, for the image of multimodal histogram, the part concerning "object" which is meaningful to the further research usually doesn't necessarily consist of all the smaller pixels. That is, there is some pixel in the image, whose gray level is smaller actually, it isn't contained by the "object", however. In that case, traditional bilevel thresholding method counts a little in classifying the pixel to the correct region. Our algorithm is to devote to this problem.

Given an $M \times N$ image X with L levels. Let g_{max} and g_{min} respectively represent the maximum and minimum gray levels. Given a certain pair of threshold values t_1 and t_2 $(t_2 > t_1)$. It is expected that the thresholds are the crossover points between the fuzzy subsets, i.e., the "object" we want to extract and the "background" of the image. For the specific threshold values t_1 and t_2, the average gray levels of the "object" can be obtained from the following equations:

$$\mu_0 = \sum_{k=t_1}^{t_2} r_k h_k / \sum_{k=t_1}^{t_2} h_k \tag{9}$$

It can be considered as the target value of the region we expect for the given threshold values t_1 and t_2. The relationship between a pixel in X and its belonging region should intuitively depend on the difference of its gray level and the target value of its belonging region. Thus, the membership function of the "object" is defined by the absolute difference between the gray level ($\in [t_1, t_2]$) and the target value of the region (the "object" region we target). In order to emphasize the "object", we let the membership function of the "ackground" be equal to 0, that is, the membership function for the pixels of the X will be defined as:

$$\mu_X(x_{mn}) = \mu_{object}(x_{mn}) = \begin{cases} \dfrac{1}{1 + |x_{mn} - int(\mu_0)|/C}, & t_1 \le x_{mn} \le t_2 \\ 0 & otherwise \end{cases} \tag{10}$$

Where C is a constant value such that $1/2 \le \mu_{object}(x_{mn}) \le 1$. Here, let $C = g_{max} - g_{min}$. In this case, the membership function in (10) really reflects the relationship of a pixel with its belonging region. μ_0 in Eq.(10) is taken as integer value so that the membership value and the measure of fuzziness of each gray level can be evaluated in advance. Thus, we obtain a fuzzy set that characterizes the original image X.

Generally, there is more than one gray level in the target. Therefore, we select five pixels randomly, let $Q = \{r_{l_1}, r_{l_2}, \cdots, r_{l_5}\}$, which can contain the basic characteristics of the target region. Find a gray level r_{l_b} from the pixels we select such that h_{l_b} is larger than the others, $b = 1, 2, \cdots, 5$. It is believed that the more the extractive region contains r_{l_b}, the nearer the region extracted by our method is to the real appointed target (i.e. "object") in the image. Put r_{l_b} in the closed interval $[t_1, t_2]$ to assure that the appointed target can appear in the extracted region. It is the aim of the proposed method.

For a given fuzzy set, on one hand, it is expected that the measure of fuzziness should be as small as possible. Hence, we select the appropriate threshold values so that the measure of fuzziness of the fuzzy set is minimal. On the other hand, in the communications theory, an entropy function can be used to measure how much information is contained in a data set. The conditional entropy function in Eq.(8) can also be used to measure how much information is contained when the fuzzy event A (here, A is the "object" of X) is given. Thus, it is believed that the conditional entropy should be as large as possible. Based on the discussion above, a new measurement, which measures both the measure of fuzziness and the conditional entropy of the pixels we select (i.e., the set Q) on condition that

A is given, is described as:

$$d = E(A) - P(Q|A) \tag{11}$$

In order to minimize the measure of fuzziness and maximize the conditional entropy of Q simultaneously, we use minimum operator for optimization. It is obviously that when the measurement in Eq.(11) has a minimum d, the measure of fuzziness is as small as possible while on condition that A is given, the conditional entropy of Q is as large as possible. The values of t_1 and t_2 for which the criterion function Eq.(11) is minimal, are the optimum thresholds. The computation method of the proposed approach will be presented in the following. (The target region is taken as "object" and the other region in the image is as "background" during the processing. X represents the image set and A is the fuzzy event which stands for "object".)

Proposed method:

Step 1: Input the image, compute the histogram, h_k, and the probability of the occurrence of the gray levels, $\widehat{h}_k$, $k = 0, 1, \cdots, L-1$.

Step 2: Select five pixels randomly from the target region of the input image which can contain the basic characteristics of the "object" we target, let $Q = \{r_{l_1}, r_{l_2}, \cdots, r_{l_5}\}$. Find a gray level r_{l_b} from Q such that h_{l_b} is larger than the others, $b = 1, 2, \cdots, 5$.

Step 3: Based on the membership function (10), the probability of this fuzzy event, $P(A)$, and the entropy measure, $E(A)$, respectively obtained as (6) and (4) or (5). Compute the conditional entropy by using the Equation (8):

$$\text{for } t_1 = g_{min} \ to \ r_{l_b}$$
$$\text{for } t_2 = r_{l_b} + 1 \ to \ g_{max} - 1$$

Step 4: Use the exhausted search method to find the threshold values t_1 and t_2 such that the fuzzy event has a minimum measurement d defined as Eq.(11).

Step 5: The resulted image $B = (b_{mn})_{M \times N}$ is:

$$b_{mn} = \begin{cases} 0 & t_1 \leq x_{mn} \leq t_2 \\ 1 & otherwise \end{cases} \tag{12}$$

It is obviously that the resulted image is a binary image, the pixels of the "object" we will extract are black, and the others are white.

In general, the threshold is located at the obvious and deep valley of the histogram [18]. In order to locate the threshold values near the real valley, a fuzzy range is selected before Step 5. That is, we will select t_1^* and t_2^* within the range as follows:

$$t_1^* = \min_i(h_{t_1 - i}), i \in \{1, \cdots, 10\} \tag{13}$$

$$t_2^* = \min_i(h_{t_2 + i}), i \in \{1, \cdots, 10\} \tag{14}$$

Experimental results can testify that t_1^* and t_2^* have a good chance of being located at the real valley.

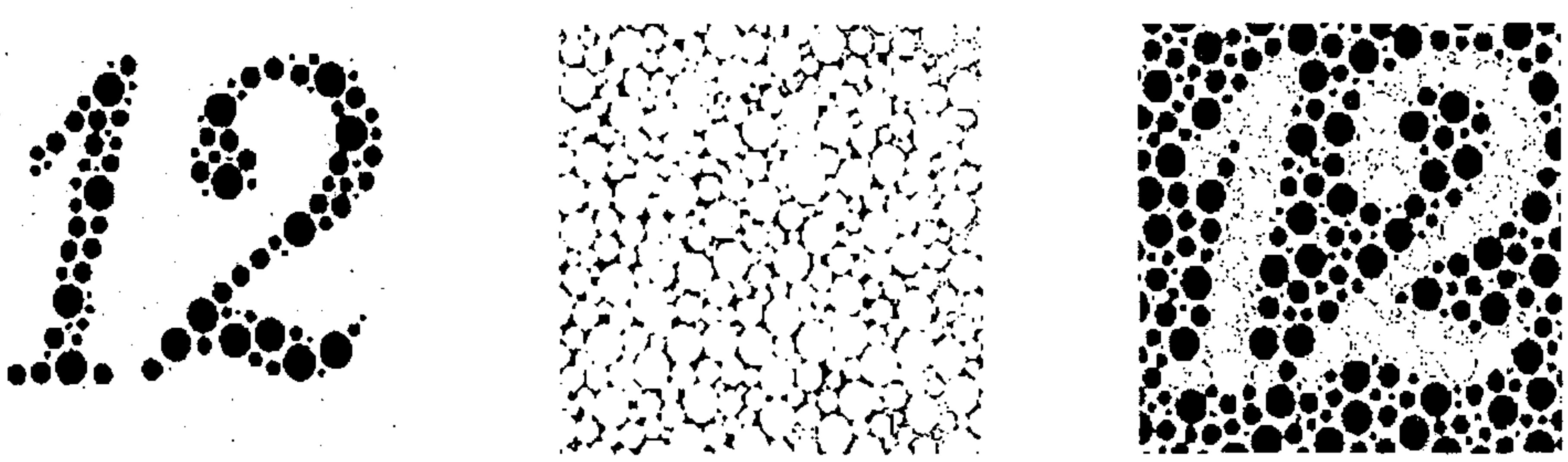
4. Experimental results

To evaluate the effectiveness of the proposed method, several images are tested in this section. These images are all multimodal with gray level $L = 256$. Each peak of histogram is a meaningful object region in the image.

Fig. 1 (a) shows the "color blindness test" image with size 199×198. Its histogram is shown in Fig. 1 (b).

(a) (b)

FIGURE 1. (a) Input image "color blindness test",(b) the histogram for the image

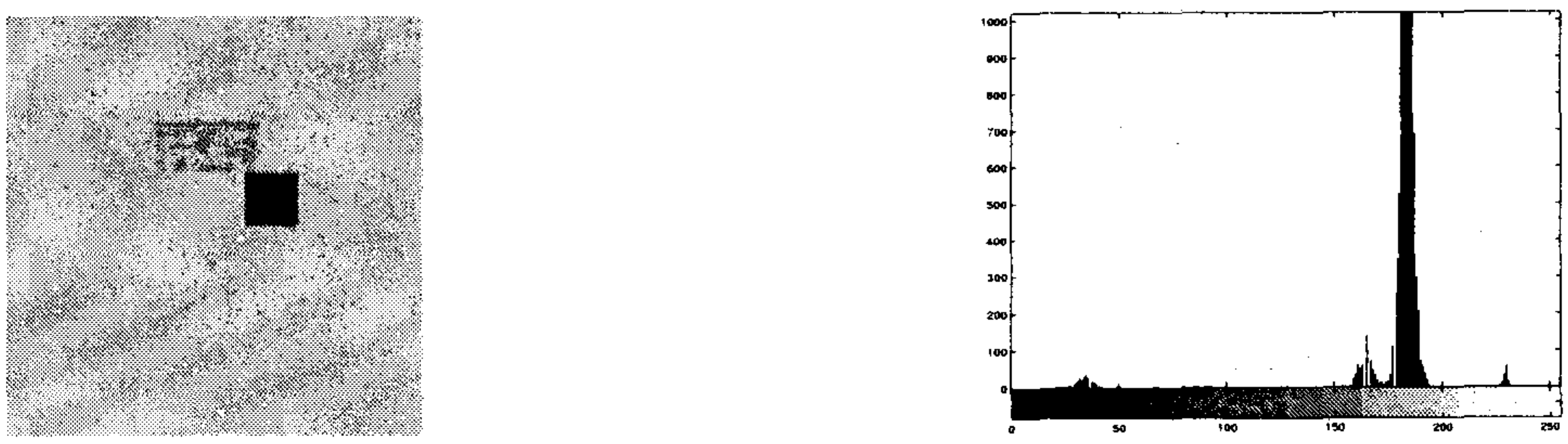


(a) (b) (c)

FIGURE 2. The thresholded image for "color blindness test" image. (a) the "object" is the number 12, (b) the whiter parts of the original image, (c) the part consists of the rest gray points in the original image

It is obviously that there are three peaks in Fig. 1 (b). Thus, there are three target regions in the original image corresponding to the peaks in the histogram i.e., the number 12, the region which is whiter than other parts in Fig. 1 (a)

and the part consists of the rest gray points. We can extract each part from the original image through the proposed bilevel thresholding method respectively. The thresholded images corresponding to the "objects" (i.e., the number 12, the part which is whiter than other parts in the image, and the part consists of the rest gray points) are presented in Fig. 2 (a) – (c), respectively. Using the proposed approach, the threshold values are respectively $(t_1^* = 0; t_2^* = 52)$, $(t_1^* = 59; t_2^* = 83)$ and $(t_1^* = 204; t_2^* = 229)$. They are located near the real valley in the histogram.



(a) (b)

FIGURE 3. (a) Input image "synthetic", (b) the histogram for the image

(a) (b) (c)

FIGURE 4. The thresholded image for "synthetic" (a) the "object" is the black square, (b) the rectangle of "synthetic", (c) the rest parts besides (a) and (b) in "synthetic"

Fig. 3 (a) is the original image of "synthetic" with 135×137 and its histogram is shown in Fig. 3 (b). The meaningful object regions in the image are (1) a black square, (2) a rectangle, and (3) the rest parts besides (1) and (2) of the image. An "object" region - the black square is shown in Fig. 4 (a) with the threshold values $(t_1^* = 29; t_2^* = 43)$. The rectangle is another "object" in Fig. 3 (a).

For the threshold values ($t_1^* = 152; t_2^* = 173$), its resulted image in Fig. 4 (b) presents the basic components of it. Fig. 4 (c) is the thresholded image for the parts besides the black square and the rectangle of the image, with the threshold values ($t_1^* = 178; t_2^* = 195$). Obviously, it is the surplus by removing the black square and the rectangle from the original image.

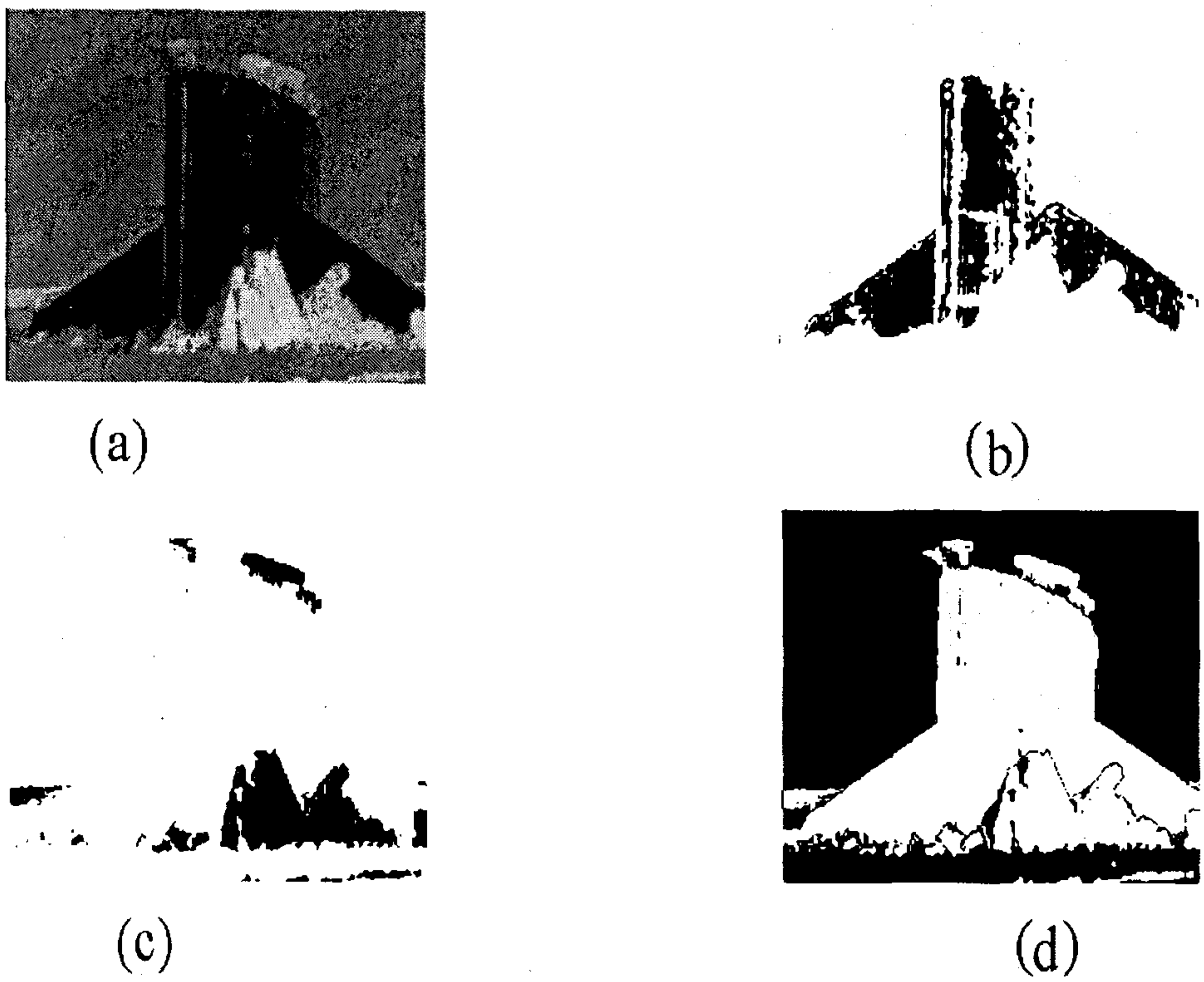


FIGURE 5. (a) the original image "submarine", (b) the body– submarine of the image, (c) the snow and ice around the submarine, (d) the sky and sea in the original image

Fig. 5 (a) is the original image of "submarine" with size 224×276, and Fig. 5 (b) is the body of the submarine with threshold values ($t_1^* = 29; t_2^* = 47$). The snow and ice around the submarine are shown in Fig. 5 (c) with the threshold values ($t_1^* = 166; t_2^* = 206$). For the threshold values ($t_1^* = 138; t_2^* = 164$), we extract the "objects" – the sky and sea. The resulted image is shown in Fig. 5 (d).

In addition, although using the traditional bilevel thresholding method can separate the pixels of an image into two regions, it cannot assure that the pixels can be classified into its real belonging region. In other words, using the traditional method, some pixel belonging to "background" may be viewed as the pixel in "object" for its gray-level is smaller. The superiority of our method can be seen in Fig. 6 (a) – (d). The minimum measure of fuzziness approach [8] is used for comparison. The reason for choosing this approach is that it is a traditional bilevel thresholding method.

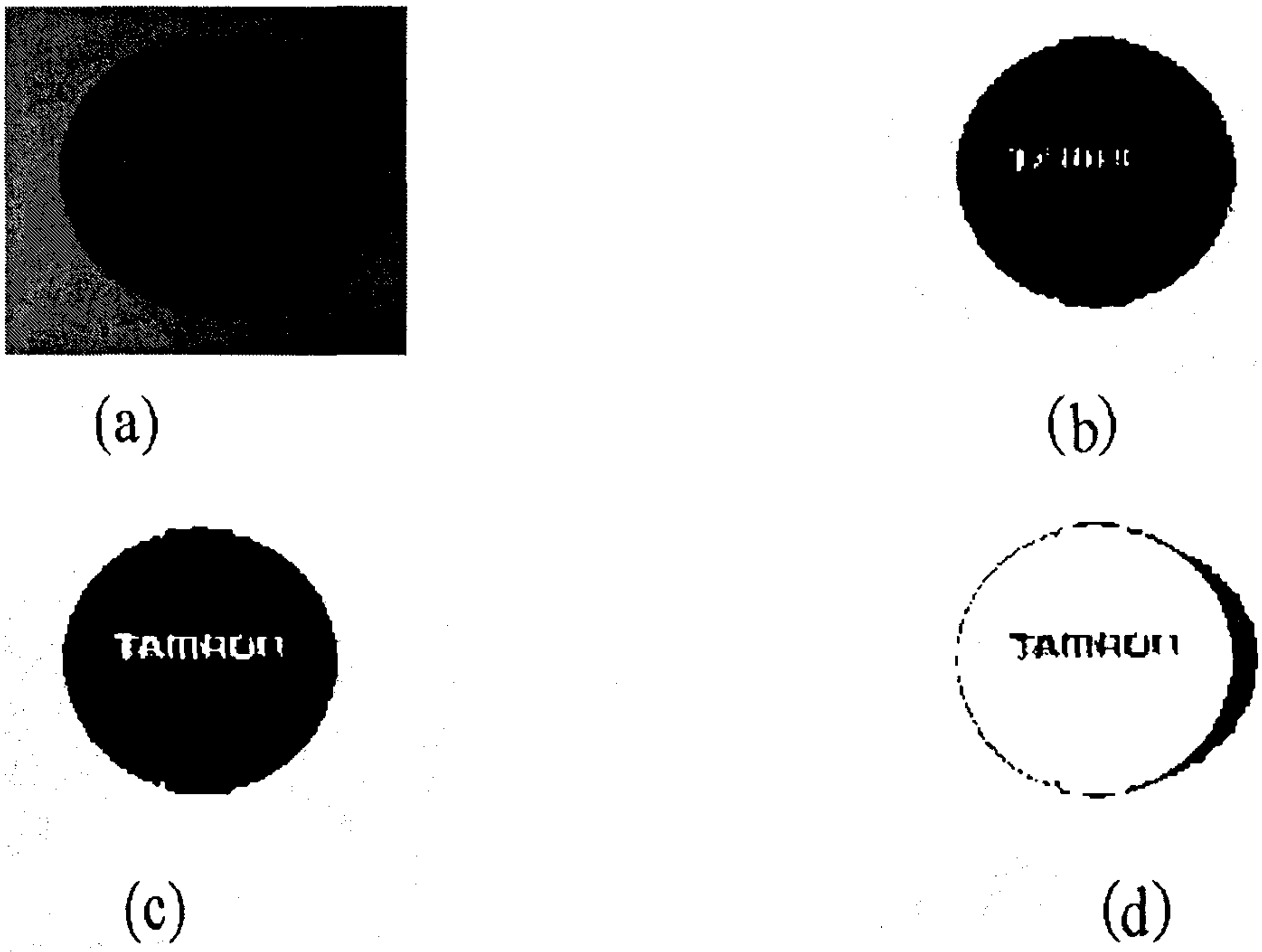


FIGURE 6. (a) the original image "ball", (b) bilevel thresholding image using the minimum measure of fuzziness approach, (c) using the proposed method, take the ball as "object", (d) sing the proposed method, the alphabets is the "object"

Fig. 6 (a) is the original image "ball". Using the minimum measure of fuzziness approach, bilevel thresholded image is shown in Fig. 6 (b) with the threshold value 34. The extractive image using the proposed method are shown in Fig. 6 (c) and (d), with the threshold values $(t_1^* = 1; t_2^* = 21)$ and $(t_1^* = 22; t_2^* = 48)$ respectively. The experiment results in Fig. 6 show that our method achieves good results for the further task-oriented study. The proposed bilevel thresholding method can extract either the ball or the alphabets on the ball if it is necessary. However, the extraction result is rough by using the minimum measure of fuzziness approach.

5. Conclusion

A new bilevel thresholding method based on the target extraction is proposed. It utilizes the measure of fuzziness of an input image and the conditional entropy to identify the appropriate threshold values. In conclusion, the experimental results indicate that the proposed method can find the appropriate threshold values effectively. However, we should select some pixels from the image during the performance, it contains some subjective factors. It is an open question.

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