J. Appl. Math. & Informatics Vol. **26**(2008), No. 3 - 4, pp. 653 - 660
Website: http://www.kcam.biz

ON FP-FILTERS AND FPD-FILTERS OF LATTICE IMPLICATION ALGEBRA

JIAJUN LAI*,YANG XU AND ZHIYAN CHANG

ABSTRACT. In this paper, we consider the fuzzification of prime filters in Lattice Implication Algebras (briefly, LIAs), and introduce the concepts of fuzzy prime filters (briefly, FP-filters), and we also studied the properties of FP-filters. Finally, we investigate the properties of fuzzy prime dual filters (briefly, FPD-filters) in LIA, and the relations of them are investigated.

AMS Mathematics Subject Classification: 03B50; 03B52
Key words and phrases : Lattice implication algebra, FP-filters, FPD-filters.

1. Introduction

Non-classical logic has become a considerable formal tool for computer science and artificial intelligence to deal with fuzzy information and uncertain information. Many-Valued logic, a great extension and development of classical logic (see [1]), has always been a crucial direction in non-classical logic. The interest in foundation of fuzzy logic has been rapidly growing recently and several new algebras playing the role of the structures of truth values have been introduced. P. Hajek introduced the system of basic logic (BL from now on) axioms for fuzzy propositional logic and defined the class of BL-algebras (see [2]). BL is built up from two primitive connectives $\wedge$ and $\rightarrow$, and the truth constant 1. The conjunction $\wedge$ is interpreted by a continuous t-norm $*$ and the implication $\rightarrow$ by the residuum of $*$. Moreover, min-conjunction $\wedge$ and max-disjunction $\vee$ connectives are definable. The logic MTL, Monoidal t-norm based logic, was introduced be F. Esteva and L. Godo (see [3]). This logic is very interesting from many points of view. From the logic point of view, it is indeed both an strengthening of Monoidal logic (ML for short) and a weakening of BL logic. In order to research the many-valued logical system whose propositional value is given in a lattice, in 1990 Xu proposed the notion of Lattice Implication Algebras (LIAs for short) and discussed its some properties (see [4,5]). Up to now,

Received May 17, 2007. Revised November 3, 2007.

this logical algebra has been widely studied (see [6-13, 19-21, 23]). In particular, emphasis seems to have been put on the filters and ideals theory. In [14], Liu and Xu proposed the notion of prime filter and obtained some properties. Lai and Xu defined the concept of weak LI-ideal in LIA and investigated its properties (see [15]). In [16], Lai and Xu researched the properties of lattice filters in LIA. In [17], Zhao and Zhu introduced the notion of primary filter in LIA, and given three characterizations of primary filter. In [18], Lai and Xu studied FW-filters. In [20], Wang et al. defined a special kind of prime dual ideals in a LIA, and investigated its structures and properties.

In 1965, L. A. Zadeh [22] introduced the notion of fuzzy sets and studied their properties on the parallel lines to set theory. At present this concept has been applied to many mathematical branches, such as group, topology etc.. In this paper, by applying the fuzzy set concept to lattice implication algebras, the authors further study fuzzy prime filters (briefly, FP-filters) and fuzzy prime dual filters (briefly, FPD-filters) in LIA. We introduced the notions of FP-filters and FPD-filters in LIA, and give some equivalent conditions by fuzzy filters. Finally, some classes of FP-filters and FPD-filters are defined and the relations among fuzzy Boolean filters, FP-filters and FPD-filters are investigated.

2. Preliminaries

First of all, we recall some definitions and results which will be needed.

Definition 2.1 [4, 5]. A bounded lattice $(L, \vee, \wedge, \rightarrow, ', O, I)$ with ordered-reversing involution $'$ and a binary operation $\rightarrow$ is called a lattice implication algebra if it satisfies the following axioms:

(L1) $x\rightarrow(y\rightarrow z)=y\rightarrow(x\rightarrow z)$,

(L2) $x\rightarrow x=I$,

(L3) $x\rightarrow y=y'\rightarrow x'$,

(L4) $x\rightarrow y=y\rightarrow x=I$ imply $x=y$,

(L5) $(x\rightarrow y)\rightarrow y=(y\rightarrow x)\rightarrow x$,

(L6) $(x\vee y)\rightarrow z=(x\rightarrow z)\wedge(y\rightarrow z)$,

(L7) $(x\wedge y)\rightarrow z=(x\rightarrow z)\vee(y\rightarrow z)$, for all $x, y, z \in L$.

Definition 2.2 [5, 17]. Let L be a lattice implication algebra, P a proper filter of L. P is called a prime filter if $a\vee b\in P$ implies $a\in P$ or $b\in P$ for all $a, b\in L$.

Definition 2.3 [5, 12]. Let L be a lattice implication algebra. B_L is the collections of fuzzy sets of L, $A \in B_L$ and $A \neq \emptyset$. A is called a fuzzy filter of L if it satisfies:

(1) $A(x)\leq A(I)$;

(2) $A(y) \geq \min\{A(x\rightarrow y), A(x)\}$ for any $x, y \in L$.

Definition 2.4 [18]. Let L be a lattice implication algebra, a fuzzy subset μ of L is called fuzzy weak filters (briefly, FW-filters) of L if for every $x, y \in$, $\mu((x' \oplus (x \rightarrow y)) \geq \mu(x \rightarrow y)$.

Theorem 2.5 [5, 8]. *Let L be a lattice implication algebra, P a proper filter of L. P is a prime filter if and only if $a \rightarrow b \in P$ or $b \rightarrow a \in P$ for all a, $b \in L$.*

Corollary 2.6 [5]. *If J is a proper filter and P is a prime filter such that $P \subseteq J$, then J is a prime filter.*

Theorem 2.7 [5, 17]. *Let L be a lattice implication algebra and P a filter of L. Then the following statements are equivalent:*
(1) P is a prime filter;
(2) for any filters A, B of L, $A \bigvee B \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$.

3. The FP-filters and FPD-filters of LIA

Definition 3.1. Let μ be a non-constant fuzzy filter of lattice implication algebra L. μ is called a fuzzy prime filter (FP-filter for short) if $\mu(x \vee y) = \mu(I)$ implies $\mu(x) = \mu(I)$ or $\mu(y) = \mu(I)$ for any x, $y \in L$.

Theorem 3.2. *Let μ be a non-constant fuzzy filter of LIA L. Then μ is FP-filter if and only if $\mu(x \rightarrow y) = \mu(I)$ or $\mu(y \rightarrow x) = \mu(I)$ for all x, $y \in L$.*

Proof. Assume that μ is FP-filter of LIA L and let x, $y \in$ L. Then
$\mu((x \rightarrow y) \vee (y \rightarrow x)) = \mu(I)$.
By Definition 3.1, $\mu(x \rightarrow y) = \mu(I)$ or $\mu(y \rightarrow x) = \mu(I)$.

Conversely, let μ is a non-constant fuzzy filter of LIA L. Suppose $\mu(x \bigvee y) = \mu(I)$. For x, $y \in$ L, we have $\mu(x \rightarrow y) = \mu(I)$ or $\mu(y \rightarrow x) = \mu(I)$. If $\mu(x \rightarrow y) = \mu(I)$. Since $((x \bigvee y) \rightarrow y = (x \rightarrow y) \wedge (y \rightarrow y) = I \wedge (x \rightarrow y) = x \rightarrow y$.

It follows that $\mu((x \vee y) \rightarrow y) = \mu(x \rightarrow y) = \mu(I)$. Since $\mu(I) \geq \mu(y) \geq \min\{\mu((x \vee y) \rightarrow y), \mu(x \vee y)\} = \mu(I)$.

Hence, $\mu(y) = \mu(I)$. Similarly, from $\mu(y \rightarrow x) = \mu(I)$ we can obtain $\mu(x) = \mu(I)$. This means that is FP-filter. □

It is easy to prove that as following conclusion:

Theorem 3.3. *Let be a non-constant fuzzy filter of LIA L. Then is FP-filter if and only if for all $t \in [0, 1]$, $\mu_t = \{x \mid x \in L, \mu(X) \geq t = \emptyset$ or μ_t is a prime filter of L if it is proper, if and only if for all x, $y \in L$.*

Definition 3.4. A fuzzy set v in LIA L is called a fuzzy ultra-filter of L if it is a fuzzy filter of L that satisfies the following condition: $v(x) = v(I)$ or $v(x') = v(I)$ for all $x \in L$.

Example 3.1. Let L= { O, a, b, I } in which $\vee, \wedge, \rightarrow, '$ and the Hasse Diagram of L are defined by table 2.4 of Page 31 in reference [5]. Then (L, $\vee, \wedge, \rightarrow, '$, O, I) is a LIA.

Let ϑ be a fuzzy set in L given by ϑ (x) = s for x $\in$ { a, I }, otherise ϑ (x) = t for x $\in$ { O, b }, where s > t in closed internal [0, 1]. Let L be a LIA, then ϑ is a fuzzy ultra-filter of L.

Theorem 3.5. *A non-constant fuzzy set ϑ in L is fuzzy ultra-filter of L if and only if is a fuzzy Boolean filter and FP-filter of L.*

Proof. Assume that ϑ is a fuzzy Boolean filter and FP-filter of L. For all x $\in$ L, we can obtain $\vartheta (x \vee x') = \vartheta(I)=\vartheta (x) \vee \vartheta (x')$. Suppose $\vartheta (x) \neq \vartheta (I)$, then $\vartheta (x) < \vartheta (I)$. Since $\vartheta (x) \leq \vartheta (I)$ and $\vartheta (x') \leq \vartheta (I)$. From this and $\vartheta(I)=\vartheta (x) \vee \vartheta (x')$. We get $\vartheta (x') = \vartheta (I)$. Thus ϑ is fuzzy ultra-filter of L.

Conversely, let ϑ be a fuzzy ultra-filter of L. Since $x \leq x \vee x'$ and $x' \leq x \vee x'$ for all x $\in$ L, hence $\vartheta (x) \leq \vartheta (x) \vee \vartheta (x)'$ and $\vartheta (x)' \leq \vartheta (x) \vee \vartheta (x)'$. According to the Definition 3.4, we have $\vartheta (x) = \vartheta (I)$ or $\vartheta (x') = \vartheta (I)$. Hence, $\vartheta(I) \leq \vartheta (x \vee x')$. Form this and Definition 2.4, we can obtain $\vartheta (I) = \vartheta (x \bigvee x')$. This means that ϑ is fuzzy Boolean filter of L. Moreover, we have $\vartheta (x \vee y) =\vartheta ((x \to y) \to y) =\vartheta ((y \to x) \to x)$. From $y \geq O$ we get $x \to y \leq x \to O$, $(x \to y) \to y \leq x' \to y$. Therefore, for all x, y $\in$ L, i.e. $\vartheta((x \to y) \to y) \leq \vartheta(x' \to y)$. Suppose $\vartheta(x') = \vartheta(I)$, then $\vartheta(x \vee y) \leq \vartheta (I) = \vartheta (x) \leq \vartheta(x \vee y)$. If $\vartheta(x) \neq \vartheta (I)$, then $\vartheta(x')= \vartheta (I)$. Hence, $\vartheta(y) \geq \vartheta(x') \bigwedge \vartheta(x' \to y) = \vartheta (I) \wedge \vartheta(x' \to y) = \vartheta(x' \to y)$. Moreover, $\vartheta(x \vee y) \leq \vartheta(y) \leq \vartheta(x \vee y)$. This means that is a FP-filter of L. □

In what follows we denote by F (X) the set of all fuzzy subsets of X. For A $\in F$ (X), let $A_t = \{ x \mid x \in X, A(x) \geq (t \}$.

Theorem 3.6. *Let L be a lattice implication algebra, $A \in F (L)$ and $A \neq \emptyset$. A is FP-filter of L if and only if for all $x \in L$, A_t is a prime filter of L when $A_t \neq \emptyset$.*

Proof. Suppose that A is a FP-filter of L. Then there exists $x \in A_t$, and hence A(I) = A(x), i.e. $I \in A_t$. If $x\vee y \in A_t$, then A(x $\vee$ y)= A(I), it follows that A(x) $\vee$ A(y)= A(I), i.e. , A(x) = A(I) or A(y)= A(I). Thus, A_t is a prime filter of L.

Conversely, suppose that for any x, y $\in$ L, A_t is a prime filter of L when $A_t \neq \emptyset$. Therefore, $x \vee y \in A_t$ implies $x \in A_t$or $y \in A_t$, i.e. , A(x $\vee$ y) = A (I) implies A(x) = A (I) or A(y) = A (I). By Definition 3.1, it follows that A is a FP-filter of L. □

Theorem 3.7. *Let L be a LIA. Every filter of L is a FP-filter.*

Proof. Let μ be a non-empty fuzzy subset of L, for any x, y $\in$ L. Assume μ is a FW-filter, then $\mu (x' \bigoplus (x \to y)) \geq \mu (x \to y)$ by Definition 2.4. Also, $\mu (x' \bigoplus (x \to y)) \vee = \mu (x' \bigoplus (x \to y))$. Since, $(x' \bigoplus (x \to y)) \vee (x \to y)=((x' \bigoplus (x \to y)) \to (x \to y) \to)=((x \to (x \to y)) \to (x \to y))$

$\rightarrow$(x $\rightarrow$ y) = (x $\vee$ (x $\rightarrow$ y)) $\rightarrow$ (x $\rightarrow$ y)=(x $\vee$ (x $\rightarrow$ y)) $\wedge$ I =x$'$ $\bigoplus$ (x $\rightarrow$ y). Hence, μ ((x$'$ $\bigoplus$ (x $\rightarrow$ y)) $\vee$ (x $\rightarrow$ y))=μ (x$'$ $\bigoplus$ (x $\rightarrow$ y)). It follows that μ ((x$'$ $\bigoplus$ (x $\rightarrow$ y)) $\vee$ (x $\rightarrow$ y))=μ ((x$'$ $\bigoplus$ (x $\rightarrow$ y))) $\vee$ μ (x $\rightarrow$ y), this means that μ is a FP-filter of L by Theorem 3.3. This completes the proof. □

Corollary 3.8. *Let L is a LIA. The following are hold. (1) Every fuzzy filter of L is a FP-filter; (2) Every fuzzy implicative filter is a FP-filter.*

Proof. It is easy to prove by Theorem 3.7 and reference [18]. □

Let L be a LIA, A is a fuzzy subset of L. We denote S= $A_I \bigcup \{ \emptyset , L\}$, $A_I = \{ x \mid x \in L, A(x) = A(I) \}$, Where A_I is a prime filter of L.

Theorem 3.9. *Let L is a LIA, $A_I \in S$, $t \in L$, $A_I \subseteq L$ and P_t be a set in L given by $P_t = \{ t \in L \mid x \rightarrow t$ is not $\in A_I \}$. Then the following propositions hold:*

(1) *if $t \in A_I$ then ($P_t \neq \emptyset$) $\overset{\rightrightarrows}{\exists}$ x such that A (x) =A (I);*
(2) *if A is a FP-filter of L and t is not $\in A_I$, then P_t is a prime filter;*
(3) *$P_t \in S$;*
(4) *if $s \in L$ and $t \leq s$ then $P_s \subseteq P_t$.*

Proof. (1) Let t $\in A_I$, then A (x) =A (I). So, we get $P_t = \emptyset$.

(2) Suppose A is a FP-filter of L, then we have A_I is a prime filter of L by Theorem 3.6. Since I=t $\rightarrow$ t $\in A_I$, i.e., t is not $\in P_t$. Thus $P_t \neq$ L, P_t is a proper subset of L. For any x, y $\in P_t$, we get x $\rightarrow$ t is not $\in A_I$ or y $\rightarrow$ t is not $\in A_I$. That is, (x $\wedge$ y) $\rightarrow$ t = (x $\rightarrow$ t) $\vee$ (y $\rightarrow$ t) is not $\in A_I$, i. e., x $\wedge$ y $\in P_t$. For any x $\in$ L, y $\in A_I$, if x $\geq$ y then x $\rightarrow$ t $\leq$ y $\rightarrow$ t and y $\rightarrow$ t is not $\in$ A_I. Hence, we can obtain x $\rightarrow$ t is not $\in A_I$, i. e., y $\in P_t$. Let x $\vee$ y $\in P_t$ for any x, y $\in$ L, then (x $\vee$ y) $\rightarrow$ t is not $\in P_t$, i. e., (x $\rightarrow$ t) $\vee$ (y $\rightarrow$ t) is not P_t. So x $\rightarrow$ t is not A_I or y $\rightarrow$ t is not A_I, i. e., x $\in P_t$ or y $\in P_t$. This means that P_t is a prime filter of L.

(3) if $A_I = \emptyset$ or A_I =L, then P_t = L or $P_t = \emptyset$, i. e., $P_t \in$ S. Assume that A is FP-filter of L, then A_I is a prime filter. Let t $\in A_I$, then $P_t = \emptyset \in$ S. If t is not $\in A_I$, we get a prime filter P_t of L by (2).

(4) For any x $\in P_s$, then x $\rightarrow$ s $\in A_I$. Suppose t $\leq$ s, we have x$\rightarrow$t $\leq$ x$\rightarrow$s. Therefore x$\rightarrow$t is not $\in A_I$, i. e., x$\in P_t$. Hence we have $P_s \subseteq P_t$. □

Definition 3.10. A non-constant fuzzy ideal μ of a LIA L is said to be a fuzzy prime dual filter (briefly, FPD-filter) of L, if μ (x $\wedge$ y) = μ (O) implies μ (x) = μ (O) or μ (y) = μ (O) for any x, y $\in$ L.

Theorem 3.11. *A non-constant fuzzy dual filter μ of a LIA L. Then μ is prime if and only if μ (($x \rightarrow y$) $'$) = μ (O) or μ (($y \rightarrow x$) $'$) = μ (O) for any x, y $\in$ L.*

Proof. Suppose μ is FPD-filter of L and let x, y $\in$ L. Then $\mu((x \to y)') \wedge \mu((y \to x)') = \mu(((x \to y) \vee (y \to x))') = \mu(I') = \mu(O)$. By Definition 3.10, $\mu((x \to y)') = \mu(O)$ or $\mu((y \to x)') = \mu(O)$.

Conversely, let μ be a non-constant fuzzy dual filter of L. Suppose $\mu(x \wedge y) = \mu(O)$. For x, y $\in$ L, we have $\mu((x \to y)') = \mu(O)$ or $\mu((y \to x)') = \mu(O)$. If $\mu((x \to y)') = \mu(O)$, since $((x \wedge y)' \to x')' = ((x' \vee y')' = ((x' \to x') \wedge (y' \to x'))' = O \vee (y' \to x')' = (x \to y)'$. So, $\mu(((x \wedge y)' \to x')') = \mu(O)$. By the definition of fuzzy dual ideal we have $\mu(O) \geq \mu(x) \geq \min\{\mu(x \wedge y), \mu(((x \wedge y)' \to x')')\} = \min\{\mu(O), \mu(O)\} = \mu(O)$. Hence, we get $\mu(x) = \mu(O)$. Similarly, from $\mu((y \to x)') = \mu(O)$ we can obtain $\mu(y) = \mu(O)$. This μ means that is a FPD-filter of L. □

Theorem 3.12. *Let L be a LIA and μ be a FP-filter of L. Then (1) μ is a FW-filter; (2) μ is a FL-filter of L.*

Proof. (1) Let μ is a FP-filter of LIA L. Then $\mu(x' \vee y) = \mu(I)$ implies $\mu(x') = \mu(I)$ or $\mu(y) = \mu(I)$ for any x, y $\in$ L. Since $\mu(x \to y) \geq \mu(x' \vee y)$ by $x \to y \geq x' \vee y$. So $\mu(x' \bigoplus (x \to y) \geq \mu(x' \vee (x \to y)) = \mu(x') \vee \mu(x \to y) = \mu(I) \geq \mu(x \to y)$.

Hence, $\mu(x' \bigoplus (x \to y) \geq \mu(x \to y)$ holds for any x, y $\in$ L. This means that μ is a FW-filter of L. (2) Clearly, μ is a FL-filter by (1) and Theorem 2.9. □

In example 3.1, the fuzzy dual filter μ satisfied: $\mu(x) = \mu(O)$ or $\mu(x') = \mu(O)$ for any x $\in$ L. From this, we introduced the following concept.

Definition 3.13. A fuzzy dual filter μ of a LIA L is called Ultra if for every x $\in$ L, $\mu(x) = \mu(O)$ or $\mu(x') = \mu(O)$.

Similarly as for Theorem 3.5, we can prove the following theorems about FPD-filters.

Theorem 3.14. *Let μ be a non-constant fuzzy dual filter of a LIA L. Then μ is both a fuzzy primr dual filter (FPD-filter) and a fuzzy dual Boolean filter of L if and only if μ is fuzzy dual ultra filters.*

Theorem 3.15. *Let L be a LIA and μ be a non-constant fuzzy ideal of L. Then the following are equivalent:*

(1) *μ is a fuzzy dual ultra filter;*

(2) *μ is a FPD-filter and a fuzzy dual Boolean filter;*

(3) *μ is a FPD-filter and a fuzzy implicative ideal.*

4. Conclusion

We introduce the concepts of FP-filters and FPD-filters in LIA, and investigated its properties. As a logical foundation of uncertain information processing theory, it is well known that the filters with special properties play an important role in researching the structure of logical system. Therefore, we hope above work would serve as a foundation for further on study the structure of LIA and develop corresponding many-valued logical system.

5. Acknowledgment

The authors acknowledge partial support by the National Science Foundation of P.R. China under Grant No. 60474022, and the Specialized Research Fund for the Doctoral Program of Higher Education of China under Grant No. 20060613007

References

1. D.W. Borns, J.M. Mack, *An algebraic Introduction on Mathematical logic*, Springer, Berlin, 1975.
2. P. Hajek, *Metamathematics of Fuzzy Logic*, Kluwer Academic Publishers, 1998.
3. F. Esteva, L. Godo,*Monoidal t-norm based logic: towards a logic for left-continuous t-norms* ,J. Fuzzy Sets and Systems**124** (2001), 271-288.
4. Xu Yang, *Lattice implication algebras*, J. Southwest Jiaotong Univ. **28(1)** (1993), 20-27(in Chinese).
5. Y. Xu, D. Ruan, K. Y. Qin, J. Liu, *Lattice-Valued Logic*, Springer, Berlin,(2003).
6. Y. Xu, K. Y. Qin, *Lattice H implication algebras and lattice implication algebra classes*, J. Hebei Mining Civil Engineering Institute **3** (1992), 139-143(in Chinese).
7. Liu Jun, Xu Yang, *On filters and structures of lattice implication algebras*, J. Chinese Science Bulletin **42(10)**(1997), 1049-1052(in Chinese).
8. Qin Keyun, Xu Yang, *The Ultra-filter of filter of lattice implication algebras*, J. Southwest Jiaotong Univ. **34(1)**(1999), 51-54(in Chinese).
9. X. F. Wang, Y. Xu, Z. M. Song, *Some of properties of filters in lattice implication algebras*, J. Southwest Jiaotong University **36(5)**(2001), 536-539(in Chinese).
10. Y. B. Jun, Y. Xu, K. Y. Qin, *Positive implication and associative filters of lattice implication algebras*, J. Bull. Korean Math. Soc. **35(1)**(1998), 53-61.
11. Y. Xu, K. Y. Qin, *On filters of lattice implication algebras*, J. Fuzzy Math.**1(2)**(1993), 251-260.
12. Y. Xu, K. Y. Qin, *Fuzzy lattice implication algebras*, J. Southwest Jiaotong University **30(2)**(1995), 121-127(in Chinese).
13. W. Wang, Y. Xu, K. Y. Qin, *The topological structure of lattice implication algebras*, J. Southwest Jiaotong University **37(3)**(2002), 341-346(in Chinese).
14. J. Liu and Y. Xu, *On prime filters and decomposition theorem of lattice implication algebras*, J. Fuzzy Math.(Los Angeles) **6(4)** (1998), 1000-1008.
15. Lai Jiajun, Xu Yang, Zeng Zhaoyou et al, *Weak LI-ideals in lattice implication algebras*, Inter. J. of Computer Science and Network Security **6(9)** (2006), 28-32.
16. Lai Jiajun, Xu Yang, Song Zhenming, *The logic properties of lattice filters of lattice implication algebra*, J.Southwest Jiaotong University(E. E.) **15(4)** (2007), 353-356.
17. B. L. Meng, *The prime filters of lattice implication algebras*, J. Northwest University **28(3)** (1998), 189-192 (in Chinese).
18. J. J. Lai and Y. Xu, *Fuzzy Weak Filters in Lattice Implication Algebras*, (Submitted for publication).
19. J. J. Lai and S. W. Chen and Y. Xu and K. Y. Qin et al,*On relationships of fuzzy filters in lattice implication algebra*, The Proceeding of the Second International Conference of Fuzzy Information and Engineering (ICFIE): Fuzzy Information and Engineering (ICFIE

2007). Bing-Yuan Cao Editor,Springer-Verlag Berlin Heidelberg 2007 (May 13-16, 2007, Guangzhou, P.R.China)(2007), 1000-1008.

20. Lai Jiajun and Xu Yang, *Lattice implication algebra inequality*, J. Jiangnan University(Natural Science Edition) **6(3)** (2007), 366-370.
21. J. B. Zhao and H. Zhu, *Primary filter of residuated lattice implication algebras*, J. Nanyang Teacher s College **5(3)** (2006), 14-16 (in Chinese).
22. L. A. Zadeh, *Fuzzy sets*, Inform. Control. 8(1965), 338-353.
23. J. J. Lai and Y. Xu and J. Ma, *On Extension of LI-ideal in Lattice Implication Algebra*, The Proceeding of IFSA2007:Theoretical Advances and Appliications of Fuzzy Logic and Soft Computing (IFSA 2007) Editors: Oscar Castillo, Patricia Melin, Oscar Montiel Ross, Roberto Sepulveda Cruz, itold Pedrycz, Janusz Kacprzyk (Eds.) (June 18-21, 2007, Cancun, Mexico)(2007), Springer-Verlag Berlin Heidelberg 2007, 337-348.

Jiajun Lai, Intelligent Control Development Center, Southwest Jiaotong University, Chengdu 610031, P.R. China.
E-mail address: `eagles_laijiajun7309@126.com`

Yang Xu, Intelligent Control Development Center, Southwest Jiaotong University, Chengdu 610031, P.R. China.
E-mail address: `xuyang@home.swjtu.edu.cn`

Zhiyan Chang, Intelligent Control Development Center, Southwest Jiaotong University, Chengdu 610031, P.R. China.
E-mail address: `changzhiyanyantai@163.com`