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AN ORDER LEVEL INVENTORY MODEL FOR PERISHABLE SEASONAL PRODUCTS WITH DEMAND FLUCTUATION

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ABSTRACT. A single item order level inventory model for perishable products is considered in which a constant fraction of on hand inventory spoils per unit time. Demand linearly depends on time. The fluctuation of demand is taken into account to determine minimum total cost of the system. Both discrete and continuous fluctuations are considered. The model is developed and solved analytically for infinite time horizon. A numerical example is presented for finite time horizon. Sensitivity analysis of the model is carried out.

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1. Introduction

This article is motivated by an actual problem presented to one of the authors by a small retailer of cold drinks of one of the metro railway stations in Kolkata. During the summer days the demand of cold drinks increases with weather significantly. But due to uncontrollable fluctuation of temperature demand fluctuates, resulting in a significant change on system running costs. The question pertains what will be the order level for the summer season to minimize the total system running cost.

Researchers have extensively discussed various types of inventory model with time varying demand after the introduction of classical inventory model in the literature by considering a uniform demand rate[16]. Donalson[3] first overruled the static demand rate, which is unrealistic in many practical situations, by introducing a time varying demand pattern in inventory modeling. Since then several researchers have studied inventory lot-sizing problems with time varying demand under a variety of modelling assumptions. Mainly four types of demand patterns have been found in the literature: (1) linear positive or negative, (2) quadratic, (3) exponentially increasing or decreasing and (4) ramp type (Hill[8],

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Wu et.al[17], Giri et.al.[5], Mondal and Pal[13]). Linear time dependent demand represents uniform change in demand per unit time. Quadratic time dependent demand indicates accelerated growth or decline in demand rate in uniform way. Khanra and Chaudhuri[10] noted that, this may be used as an alternative for the extraordinarily high rate of change for exponential demand. The ramp-type demand demonstrates a time period classified demand pattern. In different time periods the demand is either constant or its rate of change is different. These time dependent demand patterns indicate uniform rate of change and never indicate any fluctuation of demand and thus is not reflected on order level and total cost. This is unrealistic to some extent. And the present authors feel that for smooth and efficient running of business affairs proper accountability of order level and system running cost is essential under the effects of demand fluctuation. Fluctuation in demand may be simply defined as the small perturbation in demand rate caused by the impact of several marketing parameters. It is seldom found in real market that demand is smooth rather than a small perturbation in it. This perturbation is caused by the effects of several market parameters (eg. increment in unit selling price, competitors pricing strategy, advertisement, environmental effects, etc.), which may be termed as fluctuating parameters. The demand fluctuation due to the impact of fluctuating parameters may have significant effect on the order level and system running cost. The effect of demand fluctuation can be treated through the identification of all the fluctuating parameters which may have effects on a particular inventory system and then directly calculating their effects on the system by introducing them in demand. However, fluctuation in demand is uncertain, uncontrollable and unpredictable. Thus, it is random in nature and the behaviour of demand fluctuation may be analysed by considering probability distributions for fluctuating parameters.

In the literature, perishability of inventories has been discussed extensively. Nahmias[14] classified perishability in terms of fixed life time and random life time. Ghare and Scharder[4], the first proponents of deterioration, categorized it into three types: direct spoilage, physical depletion and deterioration. Yang and Wee[18] defined deterioration as decay, damage, spoilage, evaporation, obsolence, loss of utility or loss of marginal value of a commodity that results in a decreasing usefulness from the original one. Since then, several inventory modelling aspects have been examined by considering deterioration. There is a vast literature on deteriorating inventory, the outline of which can be found in research articles by Nahmias[15], Raaft[15] and Goyal and Giri[6].

In this paper, we consider an order level inventory problem for a deteriorating item having linear time dependent demand. Since the object is to determine the order level for entire summer season to minimize the system cost under fluctuating demand environment, fluctuation is taken into account by introducing a fluctuation parameter in demand. The inventory is assumed to deteriorate at a constant rate. Shortages are not allowed. Effect of fluctuation parameter is considered for both discrete and continuous case. The rest of the paper is organized as follows. In the next section assumptions and notations are provided for the development of the model. The model is developed and an algorithm to find the optimal order level is given in section 3. Section 4 deals with a numerical example. Summary and some concluding remarks are provided in section 5.

2. Assumptions and notations

The following assumptions and notations are used in developing the model.

1. The time dependent demand rate is of the form

$$D(t) = \alpha + \beta(\tau - b)t$$

where β is the rate at which demand increases or decreases. τ is the fluctuating parameter and b is it's value under standard condition. $\tau = b$ indicates that when the fluctuation has no effect with time, demand is α. If $\tau > b$, the fluctuation in demand is positive and has an effect with time, resulting in a increment in demand. $\tau < b$ indicates that fluctuation has a negative effect and leads to a decrement in demand. The fluctuating parameter τ is a random variable, which may be discrete or continuous.

2. Replenishment is instantaneous.
3. Shortages are not allowed.
4. A constant fraction θ, $0 < \theta << 1$ of on hand inventory deteriorates per unit time.
5. C_1 is the set up cost per set up. C_2 is the holding cost per unit per unit time. C_3 is the deterioration cost per unit. H is the length of the finite time horizon.

3. Model formulation and solution

Let $I_i(t)$ be the inventory level at time $(K_{i-1} + t)$ during the i-th replenishment cycle, $(0 < t < T_i), i = 1, 2, 3, \cdots$ where K_{i-1} is the total time elapsed upto and including the $(i-1)$th cycle and T_i be the length of i-th cycle. The instantaneous level of inventory during the i-th cycle is governed by the following differential equation

$$\frac{dI_i(t)}{dt} + \theta I_i(t) = -D(K_{i-1} + t), \quad 0 \le t \le T_i \tag{1}$$

with terminal condition $I_i(T_i) = 0$, i.e. inventory reaches zero level at the end of the cycle..

Solving equation (1) we have

$$I_i(t) = \frac{1}{\theta^2}\left[A\theta\{\exp[\theta(T_i - t)] - 1\} - B(\theta t - 1) + B(\theta T_i - 1)\exp[\theta(T_i - t)]\right] \tag{2}$$

where $A = \alpha + \beta(\tau - b)K_{i-1}$, $\quad B = \beta(\tau - b)$

Total cost of the system in the i-th replenishment cycle is the sum of set up cost, holding cost(HC) and deterioration cost(DC) which are given by

$$
\begin{aligned}
HC &= C_2\int_0^{T_i} I_i(t)dt = \frac{C_2}{\theta^2}\left[-A(1+\theta T_i-\exp[\theta T_i]) - B(\frac{\theta T_i^2}{2}-T_i)\right.\\
&- \left.\frac{B}{\theta}(\theta T_i - 1)(1-\exp[\theta T_i])\right]
\end{aligned}
$$

$$
\begin{aligned}
DC &= C_3\theta\int_0^{T_i} I_i(t)dt = \frac{C_3}{\theta}\left[-A(1+\theta T_i-\exp[\theta T_i]) - B(\frac{\theta T_i^2}{2}-T_i)\right.\\
&- \left.\frac{B}{\theta}(\theta T_i - 1)(1-\exp[\theta T_i])\right]
\end{aligned}
$$

Therefore, the total inventory cost per unit time in the i-th cycle is given by

$$
\begin{aligned}
\pi_i(T_i) &= \frac{1}{T_i}(C_1 + HC + DC) \qquad (3)\\
&= \frac{1}{T_i}\left[C_1 + (C_2+\theta C_3)\frac{1}{\theta^2}\left[-A(1+\theta T_i-\exp[\theta T_i]) - B\left(\frac{\theta T_i^2}{2}-T_i\right)\right.\right.\\
&- \left.\left.\frac{B}{\theta}(\theta T_i - 1)(1-\exp[\theta T_i])\right]\right]
\end{aligned}
$$

Let us now consider the effect of fluctuating parameter τ on $\pi_i(T_i)$. Since τ is a random variable, it may be discrete or continuous. We consider both cases separately.

3.1. Discrete fluctuation

The assumption of discrete behaviour of τ may be justified in the sense that the demand of cold drinks, winter garments, vegetables, fruits, fishes, etc do fluctuate frequently due to the fluctuation of temperature, advertisement, new fashion trends and for several other reasons. And this fluctuation occurs in discrete way. If the parameter τ fluctuates by assuming the values τ_j, $(j = 0,1,2,3,\cdots m)$ with probability $P(\tau=\tau_j)=p_j$, $(j=0,1,2,3,\cdots m)$ such that $\sum_{j=0}^m p_j = 1$ and $p_j \geq 0$ for all j. The expected total cost of the system in the i-th cycle is then given by

$$E\pi_i(T_i) = \sum_{j=0}^{m} \pi_i(T_i)p_j \qquad (4)$$

We need to determine the value of T_i to minimize (4). The necessary condition for this is $dE\pi_i(T_i)/dT_i = 0$. Since the rate of deterioration θ is very small, using the approximation $\exp[\theta T_i] = (2+\theta T_i)/(2-\theta T_i)$ and simplifying we get,

$$
\begin{aligned}
T_i^3 &+ \frac{2\sum_{j=0}^m p_j[\alpha+\beta K_{i-1}(\tau_j - b)]}{3\beta\sum_{j=0}^m(\tau_j-b)p_j}T_i^2 + \frac{2\theta C_1}{3\beta(C_2+\theta C_3)\sum_{j=0}^m(\tau_j-b)p_j}T_i \qquad (5)\\
&- \frac{4C_1}{3\beta(C_2+\theta C_3)\sum_{j=0}^m(\tau_j-b)p_j} = 0
\end{aligned}
$$

If $\tau_j - b = 1$ for all j, then the fluctuation has no effect on demand and the demand is simply linear time dependent. Thus, it has no effect on lot-size and total cost. Then, equation (5) becomes

$$T_i^3 + \frac{2(\alpha + \beta K_{i-1})}{3\beta} T_i^2 + \frac{2\theta C_1}{3\beta(C_2 + \theta C_3)} T_i - \frac{4C_1}{3\beta(C_2 + \theta C_3)} = 0 \qquad (6)$$

which agrees with the result of Chung and Ting[2] as expected. Now

$$\begin{aligned}\frac{d^2 E\pi_i(T_i)}{dT_i^2} &= \frac{C_2 + \theta C_3}{T_i}\Big[\alpha \exp[\theta T_i] + \{(T_i + K_{i-1})\exp[\theta T_i] \\ &+ \frac{1}{\theta}(\exp[\theta T_i] - 1)\}\beta \sum_{j=0}^{m} (\tau_j - b)p_j\Big] > 0 \ \forall T_i > 0\end{aligned}$$

if $\sum_{j=0}^{m}(\tau_j - b)p_j > 0$. Hence the sufficient condition for the minimization of $ETCU_i(T_i)$ holds if $\sum_{j=0}^{m}(\tau_j - b)p_j > 0$.

3.2. Continuous fluctuation

In this case we assume that the fluctuating parameter τ is a continuous random variable having probability density function $f(\tau)$. The expected total cost of the system is given by

$$E\pi_i(T_i) = \int_{-\infty}^{\infty} \pi_i(T_i, \tau_j) f(\tau) d\tau \qquad (7)$$

Which simplifies to

$$\begin{aligned}E\pi_i(T_i) &= \frac{1}{T_i}\Big[C_1 + \frac{C_2 + \theta C_3}{\theta^2}\big[(1 + \theta T_i - \exp[\theta T_i])(\alpha + \beta K_{i-1}[E(\tau) - b]) \qquad (8) \\ &-\beta[E(\tau) - b]\left(\frac{\theta T_i^2}{2} - T_i\right) - \frac{\beta[E(\tau) - b]}{\theta}(\theta T_i - 1)(1 - \exp[\theta T_i])\big]\Big]\end{aligned}$$

The necessary condition for the minimization of $E\pi_i(T_i)$ yields after simplification

$$\begin{aligned}&T_i^3 + \frac{2[\alpha + \beta K_{i-1}[E(\tau) - b]]}{3\beta[E(\tau) - b]} T_i^2 + \frac{2\theta C_1}{3\beta(C_2 + \theta C_3)[E(\tau) - b]} T_i \qquad (9) \\ &- \frac{4C_1}{3\beta(C_2 + \theta C_3)[E(\tau) - b]} = 0\end{aligned}$$

Now differentiating $E\pi_i(T_i)$ twice with respect to T_i we have

$$\begin{aligned}\frac{dE\pi_i^2(T_i)}{dT_i^2} &= \frac{C_2 + \theta C_3}{T_i}\Big[\alpha \exp[\theta T_i] + \{(T_i + K_{i-1})\exp[\theta T_i] \\ &+\frac{1}{\theta}(\exp[\theta T_i] - 1)\}\beta E(\tau - b)\Big]\end{aligned}$$

Clearly, $dE\pi_i^2(T_i)/dT_i^2 > 0$ if $E(\tau) - b > 0$ and $T_i > 0$ and the sufficient condition holds always.

3.3. An algorithm for solution in finite time horizon

Though the above model is developed for infinite time horizon, it can also be applicable for finite time horizon. In this section we present an algorithm to apply it in general case using goal programming. The algorithm is as follows,

Step-1: $K_{i-1} = 0$, *number of orders* (n) = *0, i=1,* $E\pi F_n = 0$
Step-2: *Repeat Step 3 and Step 4 until* $(K_{i-1} \le H \le K_i)$
Step-3: *Find* T_i *using any iterative method*
Step-4: *Compute* $E\pi_i(T_i)$ *and* $I_i(0)$, $K_i = K_{i-1} + T_i$, $i = i + 1$
Step-5: *If* $(H - K_{i-1} \ge K_i - H)$ *then*
$n = i$, $E\pi F_n = E\pi_i(T_i)$
solve the goal programming problem
$\max(d_1^+ + d_2^+)$
subject to call Procedure $P(n)$
else
$n = i - 1$, $E\pi F_n = E\pi_{i-1}(T_{i-1})$
solve the goal programming problem
$\max(d_1^+ + d_2^-)$
subject to call Procedure $P(n)$
Step-6: *write* n, T_i, $E\pi_i(T_i)$, $I_i(0)$ *for* $i = 1, 2, \cdots, n$

Procedure $P(n)$

$$E\pi_n(T_n) + d_1^- - d_1^+ = E\pi F_n$$
$$K_{n-1} + T_n + d_2^- - d_2^+ = H$$
$$d_1^+ d_1^- = 0$$
$$d_2^+ d_2^- = 0$$
$$T_n, d_1^+, d_1^-, d_2^+, d_2^- \ge 0$$

$E\pi F_n$ is the system cost per unit time in the n-th cycle, obtained due to dissatisfaction of the time bound H. d_1^-, d_1^+ are the under and over deviational variables for the expected unit cost. And d_2^-, d_2^+ are the under and over deviational variables for the last cycle time. The optimization problems in Step-5 are represented in goal programming formulation. From the decision maker's point of view if $H - K_{i-1} \le K_i - H$, there are two objectives to be satisfied. Firstly, the final order cycle will not be considered and *(i-1)*th cycle will be continued up to H. Secondly $E\pi F_n$ should be minimized. A way to achieve these two conflicting objects is to apply goal programming. Since, the number of order cycles remains same and $H - K_{i-1} \le K_i - H$, the objectives of maximization of the over deviational variable d_1^+ in cost constraint and under deviational variable d_2^- in time constraint will lead to achieve both goals simultaneously. Similarly, if $H - K_{i-1} \ge K_i - H$ then i-th cycle will be considered and d_1^+ and d_2^+ should be maximized simultaneously. The constraints $d_1^+ d_1^- = 0$ and $d_2^+ d_2^- = 0$ indicate the positiveness of one deviational variable at a time and preclude the positiveness of the other. Goal programming was developed by Charnes and Cooper[1] and extended by Lee[12], Ignizio[9].

However, several algorithms are availabe in the literature (Goyal and Giri[7], Khouja and Goyal[11], Khanra and Chaudhuri[10], etc.), in which the adjusted optimal replenishment interval $T_i^{/}$ has been calculated . This adjustment is done to equalize the over or under deviation for the length of finite time horizon, arises for the inequality of $\sum_{j=1}^{n-1} T_j$ or $\sum_{j=1}^{n} T_j$ with H. This adjustment is computed through $T_i^{/} = T_i(H/\sum_{j=1}^{n-1} T_j)$ if $H - \sum_{j=1}^{n-1} T_j \leq \sum_{j=1}^{n} T_j - H$, otherwise, $T_i^{/} = T_i(H/\sum_{j=1}^{n} T_j)$. Using the adjusted optimal replenishment interval $T_i^{/}$ one may calculate $TCU_i(T_i^{/})$ and $I_i(0), i = 1, 2 \cdots$. The main limitation of the algorithm addresses the number of decision variable. If it is more than one then adjustment of any one may lead to the non-achievement of the optimal values of the remaining variables and hence the optimal system cost. But, instead of adjusting each cycle length over the finite time horizon, if the last cycle length is determined by minimizing the over and under deviation of $H - K_{i-1}$ or $K_i - H$ from H and at the same time over or under deviation of the cost for last order cycle then we may get better result than the algorithms presented in the literature. Simultaneously, the crisis for non-achievement of optimal values of other decision variables due to adjustment of one variable may be overcome. From this point of view, the algorithm is general in nature to solve these types of problem.

4. An Illustrative example

To expose the development of the model, we consider the case reported above. In which the parameter values are taken as, $\alpha = 1000/\text{month}$, $\beta = 150$, $\theta = 0.03$, $C_1 = \$200.0$, $C_2 = \$3.0$, $C_3 = \$0.4$. According to the meteorological department the summer season lasts for about 5 months and the temperature lies between 35 to 41 degree Celsius. Past records of meteorological department also suggest that the temperature follows the following probability distribution,

Temperature(^0C):	35	36	37	38	39	40	41
Probability:	0.11	0.12	0.18	0.2	0.13	0.15	0.11

Since, the maximum probability corresponds to 38 degree Celsius, we assume that $b = 38$. The results are represented in Table-1. In Table-2, the optimal cycle length and associated inventory cost and optimal order quantity is depicted when the temperature fluctuation has no effect. In the case of continuous fluctuation, it is assumed that the temperature lies between 35 to 42 degree Celsius and it follows the uniform distribution

$$f(\tau) = \begin{cases} \frac{1}{7}, & if \quad 35 \leq \tau \leq 42 \\ 0, & otherwise \end{cases}$$

Note that there is a discrepancy for the maximum value of τ in discrete and continuous case. This is allowed deliberately in order to demonstrate Step-5 of the algorithm. The results are given in Table-3. From Table-1, it is found that

Table-1: Optimal cycle length total cost and order quantity in the i-th order cycle under discrete fluctuation

n	K_{i-1}	T_i	ETC	I_i^*
1	0	0.896509	398.869	909.287
2	0.896509	0.895912	398.870	909.894
3	1.792421	0.895312	398.869	910.496
4	2.687733	0.894724	398.872	911.107
5	3.582457	0.894133	398.873	911.713
6	4.47659	0.523418	267.963	531.3

when the fluctuation in demand is taken into account and it is discrete in nature, 6 orders should be placed, the total cycle length being 4.47695+0.523418 = 5.000008, almost equal to 5, ETC^* = 2262.316 and total amount of order quantity is 5083.797. Note that when fluctuation has no effect on demand, number of order cycles is 7, the total cycle length is 4.487051+0.512919 = 4.99997, TC^* = 2628.848, which is 13.943% higher than that under discrete fluctuation, and

Table-2: Optimal cycle length total cost and order quantity in the i-th order cycle without fluctuation

n	K_{i-1}	T_i	TC	I_i^*
1	0	0.82465	381.862	846.788
2	0.82465	0.787469	384.142	942.659
3	1.612119	0.756047	385.91	993.125
4	2.368166	0.729004	387.319	1039.29
5	3.09717	0.705382	388.47	1081.94
6	3.802552	0.684499	389.426	1121.66
7	4.487051	0.512919	311.719	884.714

I^* = 6950.176 which is also 26.854% higher than that of discrete fluctuation. From Table-3 it is found that if the fluctuation is continuous then number of orders to be placed is 6, ETC^* = 2624.537, -0.164% higher than TC^*, I^* = 6040.366, 13.09% less than I^* of no fluctuation case and total cycle length is 4.077185+0.922819 = 5.000004. For discrete fluctuation and when the fluctuation has no effect $H-\sum_{j=1}^{5} T_j > \sum_{j=1}^{6} T_j - H$ and $H-\sum_{j=1}^{6} T_j > \sum_{j=1}^{7} T_j - H$ respectively. Thus we have to consider a 6th and a 7th order cycle in the case of discrete fluctuation and no fluctuation respectively. And in both the cases d_1^+ and d_2^+ are maximized. Whereas for continuous fluctuation $H - \sum_{j=1}^{6} T_j < \sum_{j=1}^{7} T_j - H$, so the same order cycle is modified by the maximization of d_1^+ and d_2^-. The number of orders is less by one in both the discrete and the continuous case in comparison to that of the no fluctuation case. Therefore, the fluctuation (discrete or continuous) in demand has a significant effect on the number orders, lot-size as well as on total system cost and suggests to consider the fluctuation in demand in inventory accounting.

Table-3: Optimal cycle length total cost and order quantity in the i-th order cycle under continuous fluctuation

n	K_{i-1}	T_i	ETC	I_i^*
1	0	0.85729	389.409	896.447
2	0.85729	0.834285	390.796	925.673
3	1.691575	0.813528	390.871	953.243
4	2.505103	0.794666	391.456	979.373
5	3.299769	0.777416	391.968	1004.23
6	4.077185	0.922819	670.007	1254.37

In Table-4 some sensitivity analysis on the number of orders and on the expected total cost in discrete case is depicted by changing the parameter values −40%, −20%, 20% and 40%, taking one at a time and keeping the remaining unchanged. Similarly the sensitivity analysis for other two cases can be performed. It is found from expected total cost that the model is highly sensitive to the error in the estimation of the parameter values a, C_1 and C_2 and moderately sensitive to the error in the estimation of the parameters θ and C_3. While low sensitivity is found for the change in the parameter value b. Thus, proper attention should be given for the estimation of the values of the parameters a, C_1 and C_2.

5. Summary and concluding remarks

In this paper, an order level inventory model for perishable product is discussed under the assumption that the linear trend is a stochastic random variable. Conditional on the trend taking a particular value the total cost of meeting demand over a given time period is determined. The mean total cost is then derived by taking the expectation over the stochastic random variable. The model is then solved analytically for infinite time horizon. Using the concept of goal programming an algorithm is presented to find the solution for finite time horizon. A numerical example is presented. Which indicates that randomness of fluctuating parameters may have a significant effect on system running costs as well as on order level. Therefore, the variability in time dependent demand pattern, in which the increment with respect to time is uniform, must be accounted for smooth and efficient running of business. However, the model is developed by considering a parameter whose fluctuation is random and, thus, proper determination of fluctuation parameters and their effect on the smooth demand is essential for the successful evaluation of order level and system running costs. When it is difficult to identify the fluctuating parameter one may treat the problem by keeping a fluctuating stock level at the beginning of each order cycle from which inventories will be supplied to fulfill the fluctuated amount if the demand fluctuation results in a higher demand. And the order cycle ends with keeping the same fluctuating stock as the beginning. The elegance of this strategy is that if the demand fluctuation leads to the depletion of lower inventory volume than the normal demand then the surplussed inventories will be depleted through the increment of system running time. But the main limitation of fluctuating stock level is that it requires proper guessing about its amount. If the amount is higher than the fluctuating amount the system running cost increases due to the increment of holding cost and the lower volume of it may lead to the introduction of shortage cost and hence increment of system running cost in both the cases.

Table-4: Sensitivity analysis of expected total cost and lot-size with respect to the change in parameter values

Parameter	% change	n	% change in I^*	ETCU	% change in ETC
a	-40	4	-39.47	1794.732	-20.668
	-20	6	-19.752	1994.827	-11.824
	20	6	19.886	2441.47	7.919
	40	7	39.583	2665.319	17.814
b	-40	6	-.193	2261.415	-.04
	-20	6	-.073	2261.992	-.0174
	20	6	.074	2262.739	.019
	40	6	.134	2263.074	.015
θ	-40	5	-.367	2167.75	-4.18
	-20	5	-.047	2230.962	-1.386
	20	6	.247	2289.184	1.188
	40	6	.479	2319.827	2.54
C_1	-40	7	-.209	1728.971	-23.577
	-20	6	-.025	2001.27	-11.539
	20	5	.219	2444.595	8.057
	40	5	.238	2666.178	17.852
C_2	-40	5	.266	1863.622	-17.622
	-20	5	.219	2041.542	-9.795
	20	6	-.045	2402.075	6.178
	40	6	-.006	2601.788	15.005
C_3	-40	5	.247	2174.311	-3.89
	-20	5	.243	2234.608	-1.225
	20	6	-.015	2287.962	1.134
	40	6	-.023	2317.128	2.423

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