

ON SANDWICH THEOREMS FOR CERTAIN SUBCLASSES OF ANALYTIC FUNCTIONS INVOLVING CARLSON–SHAFFER OPERATOR

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ABSTRACT. The purpose of this present paper is to derive some subordination and superordination results involving Carlson–Shaffer operator for certain normalized analytic functions in the open unit disk. Relevant connections of the results, which are presented in the paper, with various known results are also considered.

1. Introduction

Let \mathcal{H} be the class of functions analytic in $\Delta := \{z : |z| < 1\}$ and $\mathcal{H}[a, n]$ be the subclass of \mathcal{H} consisting of functions of the form $f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \dots$. Let \mathcal{A} be the subclass of \mathcal{H} consisting of functions of the form $f(z) = z + a_2 z^2 + \dots$. With a view to recalling the principle of subordination between analytic functions, let the functions f and g be analytic in Δ . Then we say that the function f is subordinate to g if there exists a Schwarz function ω analytic in Δ with

$$\omega(0) = 0 \quad \text{and} \quad |\omega(z)| < 1 \quad (z \in \Delta),$$

such that

$$f(z) = g(\omega(z)) \quad (z \in \Delta).$$

We denote this subordination by

$$f \prec g \quad \text{or} \quad f(z) \prec g(z) \quad (z \in \Delta).$$

In particular, if the function g is univalent in Δ , the above subordination is equivalent to

$$f(0) = g(0) \quad \text{and} \quad f(\Delta) \subset g(\Delta).$$

Let $p, h \in \mathcal{H}$ and let $\phi(r, s, t; z) : \mathbb{C}^3 \times \Delta \rightarrow \mathbb{C}$. If p and $\phi(p(z), zp'(z), z^2 p''(z); z)$ are univalent and if p satisfies the second order superordination

$$(1.1) \quad h(z) \prec \phi(p(z), zp'(z), z^2 p''(z); z),$$

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then p is a solution of the differential superordination (1.1). (If f is subordinate to F , then F is superordinate to f .) An analytic function q is called a *subordinant* if $q \prec p$ for all p satisfying (1.1). A univalent subordinant \tilde{q} that satisfies $q \prec \tilde{q}$ for all subordinants q of (1.1) is said to be the best subordinant. Recently Miller and Mocanu [5] obtained conditions on h , q and ϕ for which the following implication holds:

$$h(z) \prec \phi(p(z), zp'(z), z^2p''(z); z) \Rightarrow q(z) \prec p(z).$$

Using the results of Miller and Mocanu [5], Bulboacă [2] considered certain classes of first order differential subordinations as well as superordination-preserving integral operators. Ali et al. [1] have used the results of Bulboacă [2] and obtained sufficient conditions for certain normalized analytic functions $f(z)$ to satisfy

$$q_1(z) \prec \frac{zf'(z)}{f(z)} \prec q_2(z),$$

where q_1 and q_2 are given univalent functions in Δ with $q_1(0) = 1$ and $q_2(0) = 1$. Shanmugam et al. [8] obtained sufficient conditions for a normalized analytic functions $f(z)$ to satisfy

$$q_1(z) \prec \frac{f(z)}{zf'(z)} \prec q_2(z)$$

and

$$q_1(z) \prec \frac{z^2f'(z)}{\{f(z)\}^2} \prec q_2(z),$$

where q_1 and q_2 are given univalent functions in Δ with $q_1(0) = 1$ and $q_2(0) = 1$.

Obradovic [6] introduced a class of functions $f \in \mathcal{A}$, such that, for $0 < \alpha < 1$,

$$\Re \left\{ f'(z) \left(\frac{z}{f(z)} \right)^\alpha \right\} > 0, z \in \Delta.$$

He called this class of function as “non-Bazilevič” type. Tuneski and Darus [11] obtained Fekete-Szegő inequality for the non-Bazilevič class of functions. Using this non-Bazilevič class, Wang et al. [12] studied many subordination results for the class $N(\alpha, \lambda, A, B)$ defined as

$$N(\alpha, \lambda, A, B) := \left\{ f \in \mathcal{A} : (1 + \lambda) \left(\frac{z}{f(z)} \right)^\alpha - \lambda f'(z) \left(\frac{z}{f(z)} \right)^{1+\alpha} \prec \frac{1 + Az}{1 + Bz} \right\},$$

where $\lambda \in \mathbb{C}, -1 \leq B \leq 1, A \neq B, 0 < \alpha < 1$.

Let the function $\varphi(a, c; z)$ be given by

$$\varphi(a, c; z) := \sum_{n=0}^{\infty} \frac{(a)_n}{(c)_n} z^{n+1} \quad (c \neq 0, -1, -2, \dots; z \in \Delta),$$

where $(x)_n$ is the *Pochhammer symbol* defined by

$$(x)_n := \begin{cases} 1, & n = 0; \\ x(x + 1)(x + 2) \dots (x + n - 1), & n \in \mathbb{N} := \{1, 2, 3, \dots\}. \end{cases}$$

Corresponding to the function $\varphi(a, c; z)$, Carlson and Shaffer [3] introduced a linear operator $L(a, c)$, which is defined by the following Hadamard product (or convolution):

$$L(a, c)f(z) := \varphi(a, c; z) * f(z) = \sum_{n=0}^{\infty} \frac{(a)_n}{(c)_n} a_n z^{n+1}.$$

We note that

$$L(a, a)f(z) = f(z), \quad L(2, 1)f(z) = zf'(z), \quad L(3, 1)f(z) = zf'(z) + \frac{1}{2}z^2f''(z)$$

and $L(\delta + 1, 1)f(z) = D^\delta f(z)$, where $D^\delta f(z)$ is the *Ruscheweyh derivative* of f (see [7]).

The main object of the present sequel to the aforementioned works is to apply a method based on the differential subordination in order to derive several subordination results involving the Carlson-Shaffer operator. Furthermore, we obtain the previous results of Srivastava and Lashin [9] as special cases of some of the results presented here.

2. Preliminaries

In our present investigation, we shall need the following definition and results.

Definition 2.1 ([5, Definition 2, p. 817]). Denote by Q , the set of all functions f that are analytic and injective on $\bar{\Delta} - E(f)$, where

$$E(f) = \{\zeta \in \partial\Delta : \lim_{z \rightarrow \zeta} f(z) = \infty\}$$

and are such that $f'(\zeta) \neq 0$ for $\zeta \in \partial\Delta - E(f)$.

Lemma 2.2 ([4, Theorem 3.4h, p. 132]). Let $q(z)$ be univalent in the unit disk Δ and θ and ϕ be analytic in a domain D containing $q(\Delta)$ with $\phi(w) \neq 0$ when $w \in q(\Delta)$. Set $Q(z) = zq'(z)\phi(q(z))$, $h(z) = \theta(q(z)) + Q(z)$. Suppose that

- (1) $Q(z)$ is starlike univalent in Δ , and
- (2) $\Re \frac{zh'(z)}{Q(z)} > 0$ for $z \in \Delta$.

If

$$\theta(p(z)) + zp'(z)\phi(p(z)) \prec \theta(q(z)) + zq'(z)\phi(q(z)),$$

then $p(z) \prec q(z)$ and $q(z)$ is the best dominant.

Lemma 2.3 ([8]). Let q be a convex univalent function in Δ and $\psi, \gamma \in \mathbb{C}$ with $\Re \left\{ 1 + \frac{zq''(z)}{q'(z)} + \frac{\psi}{\gamma} \right\} > 0$. If $p(z)$ is analytic in Δ and

$$\psi p(z) + \gamma zp'(z) \prec \psi q(z) + \gamma zq'(z),$$

then $p(z) \prec q(z)$ and q is the best dominant.

Lemma 2.4 ([5, Theorem 8, p. 822]). *Let q be convex univalent in Δ and $\gamma \in \mathbb{C}$. Further assume that $\Re[\bar{\gamma}] > 0$. If $p(z) \in \mathcal{H}[q(0), 1] \cap Q$, $p(z) + \gamma zp'(z)$ is univalent in Δ , then*

$$q(z) + \gamma zq'(z) \prec p(z) + \gamma zp'(z)$$

implies $q(z) \prec p(z)$ and q is the best subdominant.

3. Subordination for analytic functions

By using Lemma 2.3, we first prove the following.

Theorem 3.1. *Let q be univalent in Δ , $\lambda \in \mathbb{C}$ and $0 < \alpha < 1$. Suppose q satisfies*

$$(3.1) \quad \Re \left\{ 1 + \frac{zq''(z)}{q'(z)} + \frac{\lambda}{\alpha} \right\} > 0.$$

If $f \in \mathcal{A}$ satisfies the subordination:

$$(3.2) \quad (1+a\lambda) \left(\frac{z}{L(a,c)f(z)} \right)^\alpha - a\lambda \frac{L(a+1,c)f(z)}{L(a,c)f(z)} \left(\frac{z}{L(a,c)f(z)} \right)^\alpha \prec q(z) + \frac{\lambda zq'(z)}{\alpha},$$

then

$$\left(\frac{z}{L(a,c)f(z)} \right)^\alpha \prec q(z)$$

and q is the best dominant.

Proof. Define the function p by

$$p(z) := \left(\frac{z}{L(a,c)f(z)} \right)^\alpha.$$

Then

$$\frac{zp'(z)}{p(z)} = \alpha a \left[1 - \frac{L(a+1,c)f(z)}{L(a,c)f(z)} \right]$$

which, in light of hypothesis (3.2) of Theorem 3.1, yields the following subordination

$$p(z) + \frac{\lambda zp'(z)}{\alpha} \prec q(z) + \frac{\lambda zq'(z)}{\alpha}.$$

The assertion (3.2) of Theorem 3.1 now follows by an application of Lemma 2.3 with $\gamma = \frac{\lambda}{\alpha}$ and $\psi = 1$. □

Letting $a = c = 1$ in Theorem 3.1, we have the following corollary.

Corollary 3.2. *Let q be univalent in Δ , $\lambda \in \mathbb{C}$ and $0 < \alpha < 1$. Suppose q satisfies (3.1). If $f \in \mathcal{A}$, and*

$$\left(\frac{z}{f(z)} \right)^\alpha \left(1 + \lambda - \lambda \frac{zf'(z)}{f(z)} \right) \prec q(z) + \frac{\lambda zq'(z)}{\alpha},$$

then

$$\left(\frac{z}{f(z)}\right)^\alpha \prec q(z)$$

and q is the best dominant.

Letting $a = 2$ and $c = 1$ in Theorem 3.1, we have the following corollary.

Corollary 3.3. *Let q be univalent in Δ , $\lambda \in \mathbb{C}$ and $0 < \alpha < 1$. Suppose q satisfies (3.1). If $f \in \mathcal{A}$ and*

$$\left(\frac{1}{f'(z)}\right)^\alpha \left(1 + 2\lambda - \lambda \left(2 - \frac{zf''(z)}{f'(z)}\right)\right) \prec q(z) + \frac{\lambda z q'(z)}{\alpha},$$

then

$$\left(\frac{1}{f'(z)}\right)^\alpha \prec q(z)$$

and q is the best dominant.

Taking $q(z) = \frac{1+Az}{1+Bz}$ in Theorem 3.1, we have the following corollary.

Corollary 3.4. *Let $-1 \leq B < A \leq 1$ and (3.1) hold. If $f \in \mathcal{A}$ and*

$$\begin{aligned} & (1 + a\lambda) \left(\frac{z}{L(a, c)f(z)}\right)^\alpha - a\lambda \frac{L(a + 1, c)f(z)}{L(a, c)f(z)} \left(\frac{z}{L(a, c)f(z)}\right)^\alpha \\ & \prec \frac{\lambda(A - B)z}{\alpha(1 + Bz)^2} + \frac{1 + Az}{1 + Bz}, \end{aligned}$$

then

$$\left(\frac{z}{L(a, c)f(z)}\right)^\alpha \prec \frac{1 + Az}{1 + Bz}$$

and $\frac{1+Az}{1+Bz}$ is the best dominant.

Our Theorem 3.1 for the choice of $q(z) = \frac{1+z}{1-z}$, reduces to

Corollary 3.5. *Let (3.1) hold. If $f \in \mathcal{A}$ and*

$$\begin{aligned} & (1 + a\lambda) \left(\frac{z}{L(a, c)f(z)}\right)^\alpha - a\lambda \frac{L(a + 1, c)f(z)}{L(a, c)f(z)} \left(\frac{z}{L(a, c)f(z)}\right)^\alpha \\ & \prec \frac{2\lambda z}{\alpha(1 - z)^2} + \frac{1 + z}{1 - z}, \end{aligned}$$

then

$$\left(\frac{z}{L(a, c)f(z)}\right)^\alpha \prec \frac{1 + z}{1 - z}$$

and $\frac{1+z}{1-z}$ is the best dominant.

Theorem 3.6. Let q be univalent in Δ , $\gamma, \mu \neq 0 \in \mathbb{C}$ and $0 \leq \beta \leq 1$. Let $f \in \mathcal{A}$. Suppose q satisfies

$$(3.3) \quad \Re \left\{ 1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)} \right\} > 0.$$

If

$$1 + \gamma\mu \left\{ \frac{(a+1)\beta L(a+2,c)f(z) + a(1-2\beta)L(a+1,c)f(z) - (1-\beta)(a-1)L(a,c)f(z)}{(1-\beta)L(a,c)f(z) + \beta L(a+1,c)f(z)} - 1 \right\} < 1 + \gamma \frac{zq'(z)}{q(z)},$$

then

$$\left[\frac{(1-\beta)L(a,c)f(z) + \beta L(a+1,c)f(z)}{z} \right]^\mu < q(z)$$

and q is the best dominant.

Proof. Define the function p by

$$p(z) := \left[\frac{(1-\beta)L(a,c)f(z) + \beta L(a+1,c)f(z)}{z} \right]^\mu.$$

Then a computation shows that

$$\mu \left\{ \frac{(a+1)\beta L(a+2,c)f(z) + a(1-2\beta)L(a+1,c)f(z) - (1-\beta)(a-1)L(a,c)f(z)}{(1-\beta)L(a,c)f(z) + \beta L(a+1,c)f(z)} - 1 \right\} = \frac{zp'(z)}{p(z)}.$$

By setting

$$\theta(\omega) := 1 \quad \text{and} \quad \phi(\omega) := \frac{\gamma}{\omega},$$

it can be easily observed that θ is analytic in \mathbb{C} , ϕ is analytic in $\mathbb{C} \setminus \{0\}$ and that

$$\phi(\omega) \neq 0 \quad (\omega \in \mathbb{C} \setminus \{0\}).$$

Also, we let

$$Q(z) = zq'(z)\phi(q(z)) = \gamma \frac{zq'(z)}{q(z)}$$

and

$$h(z) = \theta\{q(z)\} + Q(z) = 1 + \gamma \frac{zq'(z)}{q(z)}.$$

From (3.3), we find that $Q(z)$ is starlike univalent in Δ and that $\Re \left(\frac{zh'(z)}{Q(z)} \right) = \Re \left\{ 1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)} \right\} > 0$, by the hypothesis (3.3) of Theorem 3.6. Thus, by applying Lemma 2.2, our proof of Theorem 3.6 is completed. \square

For a special case when $a = 1$, $c = 1$, $\beta = 0$, $q(z) = \frac{1}{(1-z)^{2b}}$ ($b \in \mathbb{C} \setminus \{0\}$), $\gamma = \frac{1}{b}$ and $\mu = 1$, Theorem 3.6 reduces at once to the following known result obtained by Srivastava and Lashin [9].

Corollary 3.7. *Let b be a non zero complex number. If $f \in \mathcal{A}$ and*

$$1 + \frac{1}{b} \left[\frac{zf'(z)}{f(z)} - 1 \right] \prec \frac{1+z}{1-z},$$

then

$$\frac{f(z)}{z} \prec \frac{1}{(1-z)^{2b}}$$

and $\frac{1}{(1-z)^{2b}}$ is the best dominant.

For a special case when $a = c = 1, \beta = 1, q(z) = \frac{1}{(1-z)^{2b}}$ ($b \in \mathbb{C} \setminus \{0\}$), $\gamma = \frac{1}{b}$ and $\mu = 1$. Theorem 3.6 reduces at once to the following known result obtained by Srivastava and Lashin [9].

Corollary 3.8. *Let b be a non zero complex number. If $f \in \mathcal{A}$ and*

$$1 + \frac{1}{b} \frac{zf''(z)}{f'(z)} \prec \frac{1+z}{1-z},$$

then

$$f'(z) \prec \frac{1}{(1-z)^{2b}}$$

and $\frac{1}{(1-z)^{2b}}$ is the best dominant.

4. Superordination for analytic functions

Next, applying Lemma 2.4, we obtain the following two theorems.

Theorem 4.1. *Let q be convex univalent in Δ , $\lambda \in \mathbb{C}$ and $0 < \alpha < 1$. Suppose $f \in \mathcal{A}$ satisfies*

$$(4.1) \quad \Re \{ \lambda \} > 0$$

and $\left(\frac{z}{L(a,c)f(z)} \right)^\alpha \in \mathcal{H}[q(0), 1] \cap \mathcal{Q}$. Let

$$(1 + a\lambda) \left(\frac{z}{L(a,c)f(z)} \right)^\alpha - a\lambda \frac{L(a+1,c)f(z)}{L(a,c)f(z)} \left(\frac{z}{L(a,c)f(z)} \right)^\alpha$$

be univalent in Δ . If $f \in \mathcal{A}$ satisfies the superordination:

$$(4.2) \quad q(z) + \frac{\lambda z q'(z)}{\alpha} \prec (1 + a\lambda) \left(\frac{z}{L(a,c)f(z)} \right)^\alpha - a\lambda \frac{L(a+1,c)f(z)}{L(a,c)f(z)} \left(\frac{z}{L(a,c)f(z)} \right)^\alpha,$$

then

$$q(z) \prec \left(\frac{z}{L(a,c)f(z)} \right)^\alpha$$

and q is the best subdominant.

Proof. Define the function p by

$$p(z) := \left(\frac{z}{L(a, c)f(z)} \right)^\alpha.$$

Then a computation shows that

$$p(z) + \frac{\lambda}{\alpha} zp'(z) = (1+a\lambda) \left(\frac{z}{L(a, c)f(z)} \right)^\alpha - a\lambda \frac{L(a+1, c)f(z)}{L(a, c)f(z)} \left(\frac{z}{L(a, c)f(z)} \right)^\alpha.$$

Theorem 4.1 follows as an application of Lemma 2.4. □

Taking $q(z) = \frac{1+Az}{1+Bz}$ in Theorem 4.1, we get the following corollary.

Corollary 4.2. *Let $-1 \leq B < A \leq 1$. Let q be convex univalent in Δ . Suppose $f \in \mathcal{A}$ satisfies $\Re\{\lambda\} > 0$ and $\left(\frac{z}{L(a, c)f(z)}\right)^\alpha \in \mathcal{H}[q(0), 1] \cap Q$. Let*

$$(1+a\lambda) \left(\frac{z}{L(a, c)f(z)} \right)^\alpha - a\lambda \frac{L(a+1, c)f(z)}{L(a, c)f(z)} \left(\frac{z}{L(a, c)f(z)} \right)^\alpha$$

be univalent in Δ . If

$$\frac{\lambda(A-B)z}{\alpha(1+Bz)^2} + \frac{1+Az}{1+Bz} \prec (1+a\lambda) \left(\frac{z}{L(a, c)f(z)} \right)^\alpha - a\lambda \frac{L(a+1, c)f(z)}{L(a, c)f(z)} \left(\frac{z}{L(a, c)f(z)} \right)^\alpha,$$

then

$$\frac{1+Az}{1+Bz} \prec \left(\frac{z}{L(a, c)f(z)} \right)^\alpha$$

and $\frac{1+Az}{1+Bz}$ is the best subdominant.

Theorem 4.3. *Let q be convex univalent in Δ , $\gamma \in \mathbb{C}$, $0 \leq \beta \leq 1$ and $f \in \mathcal{A}$. Suppose $0 \neq \left[\frac{(1-\beta)L(a, c)f(z) + \beta L(a+1, c)f(z)}{z}\right]^\mu \in \mathcal{H}[q(0), 1] \cap Q$, and let*

$$(4.3) \quad \begin{aligned} &\Psi(a, c, \beta, z) \\ &:= 1 + \gamma\mu \left\{ \frac{(a+1)\beta L(a+2, c)f(z) + a(1-2\beta)L(a+1, c)f(z) - (1-\beta)(a-1)L(a, c)f(z)}{(1-\beta)L(a, c)f(z) + \beta L(a+1, c)f(z)} - 1 \right\}. \end{aligned}$$

If $\Psi(a, c, \beta, z)$ is univalent in Δ and

$$1 + \gamma \frac{zq'(z)}{q(z)} \prec \Psi(a, c, \beta, z),$$

then

$$q(z) \prec \left[\frac{(1-\beta)L(a, c)f(z) + \beta L(a+1, c)f(z)}{z} \right]^\mu$$

and q is the best subdominant.

5. Sandwich results

Combining the results of differential subordination and superordination, we state the following “sandwich results”:

Theorem 5.1. *Let q_1 and q_2 be convex univalent in Δ , $\lambda \in \mathbb{C}$, and $0 < \alpha < 1$. Suppose q satisfies (3.1) and (4.1). If $\left(\frac{z}{L(a,c)f(z)}\right)^\alpha \in \mathcal{H}[q(0), 1] \cap Q$, $(1 + a\lambda) \left(\frac{z}{L(a,c)f(z)}\right)^\alpha - a\lambda \frac{L(a+1,c)f(z)}{L(a,c)f(z)} \left(\frac{z}{L(a,c)f(z)}\right)^\alpha$ is univalent in Δ .*

And if $f \in \mathcal{A}$ satisfies

$$(5.1) \quad \begin{aligned} q_1(z) + \frac{\lambda z q_1'(z)}{\alpha} &< (1 + a\lambda) \left(\frac{z}{L(a,c)f(z)}\right)^\alpha - a\lambda \frac{L(a+1,c)f(z)}{L(a,c)f(z)} \left(\frac{z}{L(a,c)f(z)}\right)^\alpha \\ &< q_2(z) + \frac{\lambda}{\alpha} z q_2'(z), \end{aligned}$$

then

$$q_1(z) < \left(\frac{z}{L(a,c)f(z)}\right)^\alpha < q_2(z)$$

and q_1 and q_2 are respectively the best subordinant and best dominant.

Theorem 5.2. *Let q_1 and q_2 be convex univalent in Δ , $\gamma \neq 0 \in \mathbb{C}$, $\mu \neq 0 \in \mathbb{C}$ and $0 \leq \beta \leq 1$. Let $f \in \mathcal{A}$ and (3.3) hold. Suppose*

$$0 \neq \left[\frac{(1-\beta)L(a,c)f(z) + \beta L(a+1,c)f(z)}{z} \right]^\mu \in \mathcal{H}[q(0), 1] \cap Q$$

and

$$(5.2) \quad \begin{aligned} &\Psi(a, c, \beta, z) \\ &:= 1 + \gamma\mu \left\{ \frac{(a+1)\beta L(a+2,c)f(z) + a(1-2\beta)L(a+1,c)f(z) - (1-\beta)(a-1)L(a,c)f(z)}{(1-\beta)L(a,c)f(z) + \beta L(a+1,c)f(z)} - 1 \right\}. \end{aligned}$$

If

$$\Psi(a, c, \beta, z)$$

is univalent in Δ and

$$1 + \gamma \frac{z q_1'(z)}{q_1(z)} < \Psi(a, c, \beta, z) < 1 + \gamma \frac{z q_2'(z)}{q_2(z)},$$

then

$$q_1(z) < \left[\frac{(1-\beta)L(a,c)f(z) + \beta L(a+1,c)f(z)}{z} \right]^\mu < q_2(z)$$

and q_1 and q_2 are respectively the best subordinant and best dominant respectively.

References

- [1] R. M. Ali, V. Ravichandran, M. Hussain Khan, and K. G. Subramanian, *Differential sandwich theorems for certain analytic functions*, Far East J. Math. Sci. (FJMS) **15** (2005), no. 1, 87-94.
- [2] T. Bulboacă, *A class of superordination-preserving integral operators*, Indag. Math., New Ser. **13** (2002), no. 3, 301-311.

- [3] B. C. Carlson and D. B. Shaffer, *Starlike and prestarlike hypergeometric functions*, SIAM J. Math. Anal. **15** (1984), no. 4, 737–745.
- [4] S. S. Miller and P. T. Mocanu, *Differential Subordinations: Theory and Applications*, Pure and Applied Mathematics No. 225, Marcel Dekker, New York, 2000.
- [5] ———, *Subordinants of differential superordinations*, Complex Variables **48** (2003), no. 10, 815–826.
- [6] M. Obradovic, *A class of univalent functions*, Hokkaido Math. J. **27** (1998), no. 2, 329–335.
- [7] S. Ruscheweyh, *New criteria for univalent functions*, Proc. Amer. Math. Soc. **49** (1975), 109–115.
- [8] T. N. Shanmugam, V. Ravichandran, and S. Sivasubramanian, *Differential sandwich theorems for some subclasses of analytic functions*, to appear in Aust. J. Math. Anal. Appl.
- [9] H. M. Srivastava and A. Y. Lashin, *Some applications of the Briot-Bouquet differential subordination*, J. Inequal. Pure Appl. Math. **6** (2005), no. 2, Article 41, 7 pp.
- [10] N. Tuneski, *On certain sufficient conditions for starlikeness*, Internat J. Math. Math. Sci. **23** (2000), no. 8, 521–527.
- [11] N. Tuneski and M. Darus, *Fekete-Szegő functional for non-Bazilevic functions*, Acta Mathematica Academia Paedagogicae Nyiregyhaziensis **18** (2002), 63–65.
- [12] Z. Wang, C. Gao, and M. Liao, *On certain generalized class of non-Bazilevič functions*, Acta Mathematica Academia Paedagogicae Nyiregyhaziensis **21** (2005), 147–154.

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