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화이트 노이즈 음파를 이용한 막구조물의 장력 측정장치 개발

Development of Measurement Equipment of Membrane Stress Using White Noise Sound Wave

Jin, Sang-Wook* Hiroshi Ohmori* Kim, Jae-Yeol**

요 약

휨이나 압축에 저항할 수 없는 막재료에 적절한 장력을 도입함으로써 안정화 되는 막구조물은, 유지 관리면에 있어서 막면에 도입되어 있는 장력을 설계대로 유지하는 것이 매우 중요하지만, 준공후 막면에 도입되어 있는 장력을 정확하게 파악하기가 어렵다. 저자들은, 직방형의 경계를 가지는 막을 가청역의 음파를 이용해 진동시키고, 진동하는 막의 공진진동수를 측정함으로써 간접적으로 막장력을 측정하는 방법을 제안하고, 막을 진동시키는 음파로서 정현파와 화이트 노이즈를 이용해 검증실험을 해 왔다. 본 논문은 주요 막재료를 이용해 행한 막장력 측정 이론의 검증을 위한 실험 결과와, 실재하는 막구조물의 장력측정을 통해, 본 측정장치의 정확성과 폭 넓은 적용성 및 측정에 있어서의 안정성을 검증한다.

Abstract

One of the most important matters in keeping membrane structures in healthy condition is to maintain the proper tension distribution over the membrane. However, it is not easy to know the real stress level in the membrane quantitatively after completion of the structures. Authors suggested measurement method that can measure membrane stress using sound wave, and have been holding experimental tests of membrane stress measurement that used the sound external excitation with sine wave and white noise. The concept of the method is the fact that measurement of resonance frequency by vibrating membrane having rectangular boundary by audible frequency can measure membrane stress indirectly. In this paper, through the experimental tests it is proved that the equipment can be used for not only the membrane material of type A but also for types B and C. In addition, it is proved that the developed measurement equipment is available to stably measure the membrane stress which exists in the membrane material of the actual membrane structures.

키워드 : 막장력측정, 음파진동, 화이트노이즈, 현장측정

Keywords : Measurement of membrane stress, Sound excitation, White noise, Practical measurement

1. Introduction

Membrane structure is a structure that used membrane materials which cannot afford compression or bending, but is stabilized by maintaining appropriate tensile status, and thus able to endure load like snow or wind, etc. These membrane structures need accurate maintenance of tension condition but generally it is difficult. After the completion of construction, measuring stress of membrane structures is very important in the maintaining point of view, and huge expect is putted on this measuring technology.

Up to recent years, several methods on measurement of membrane stress have been proposed and some have been used in a construction site, where precise measurements of membrane stress have been tried, however, accuracy of the obtained data has been found to be far from desirable one. Such situation has not

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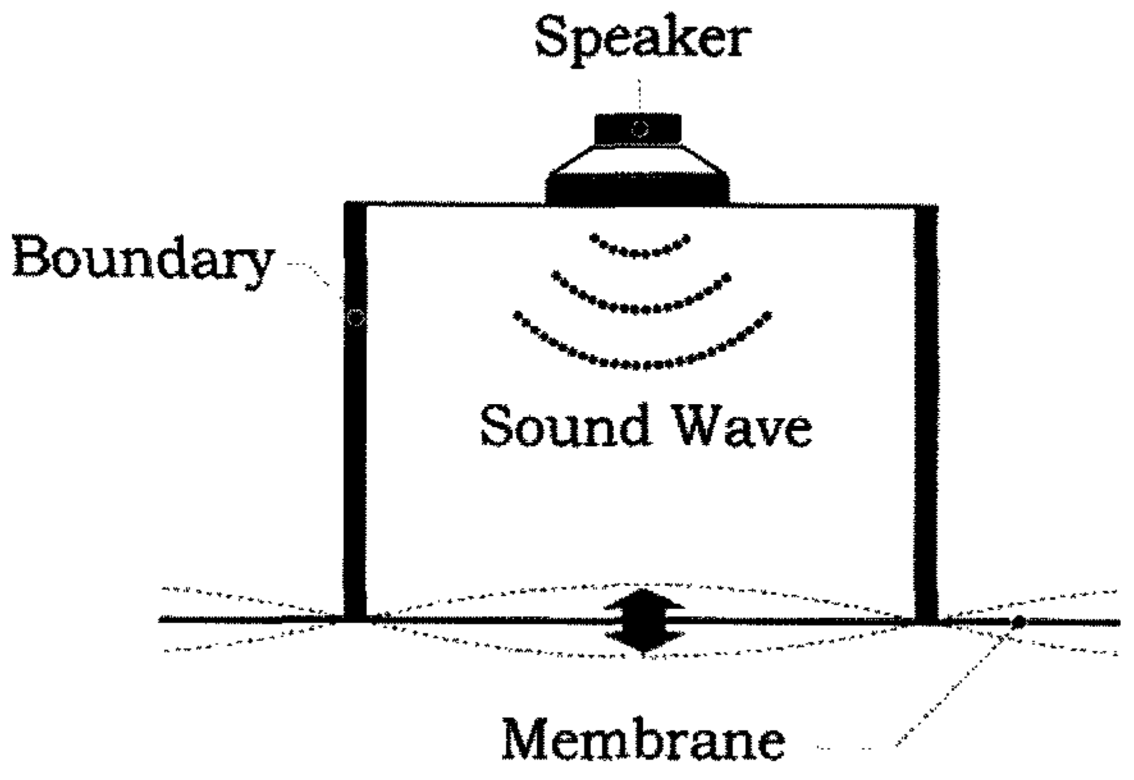
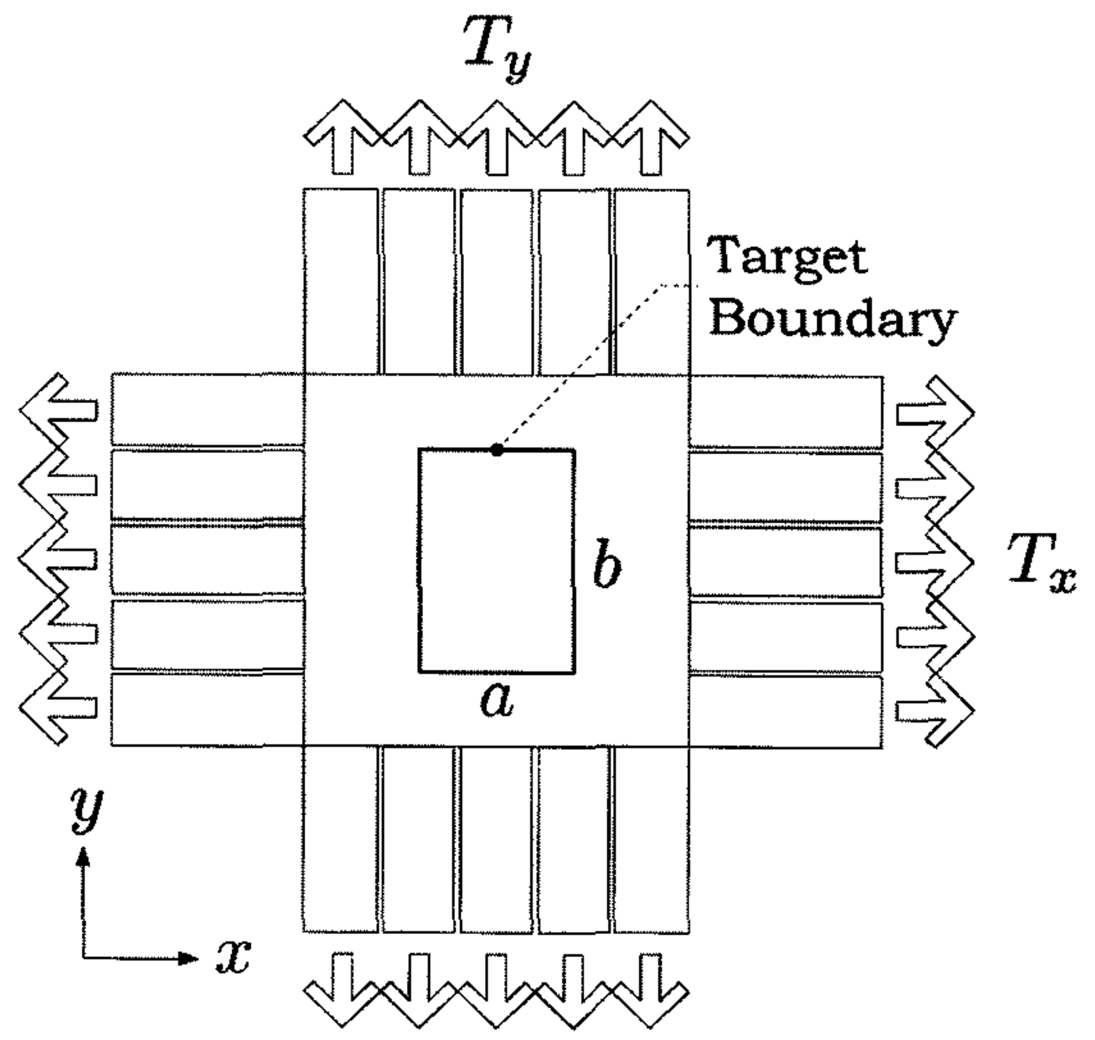


〈Fig. 1〉 Concept of measurement equipment



〈Fig. 2〉 Boundary of membrane material

been changed until recent days, and we do not have the measurement tools yet on which we can actually depend. Moreover, due to the different properties in biaxial directions for material of membrane, the stress in the warp direction is generally different from that in the fill one. We have not yet had such practicable device by which membrane stress in both directions can be measured even roughly.

Authors suggested measurement method that can measure membrane stress using sound wave, and have been holding experimental tests of membrane stress measurement that used sine wave and white noise as external excitation[1~3]. As shown in Fig. 1, the concept of the method is the fact that measurement of resonance frequency by vibrating membrane with rectangular boundaries as shown Fig. 2 by audible frequency can measure membrane stress indirectly. This is a basic concept, but from the research we realized that natural frequency of membrane is affected by air. That is because the vibrating membrane is as light as air. For estimation of theoretical natural frequency, the effect of added mass of air has to be considered. Thus by replacing vibrating surface of membrane to the circular plate having the same area, we estimated added mass by air theoretically[4,5].

In experimental test conducted by using white noise which has frequency field of 0 to 500Hz as an external excitation to measure membrane stress, verification experiments are held such as weighting upper part of equipment, improving power of speaker, sticking rubber to the base of the equipment and sharpened boundary of acrylic box[5,6]. In addition, for practical use of equipment, we have held experimental tests to verify the effectiveness of improved equipment for high accuracy[6~8].

In this paper, it is proved that the equipment can be used not only the membrane material of type A for membrane structure but also types B and C through the experimental tests. Moreover, result of the measurement of existing frame supported membrane structure covered with the membrane material of type A is reported.

2. Fundamental Theory

The equation of motion of the membrane can be expressed as the following equation,

$$-\rho_k\frac{\partial^2\omega}{\partial t^2}+\left(\frac{\partial^2\omega}{\partial x^2}T_x+\frac{\partial^2\omega}{\partial y^2}T_y\right)=p \qquad (1)$$

where $\omega=\omega(x, y, t)$ is the deflection of membrane over the surface, t is the time, p is the external pressure, ρ_k is the mass of membrane per unit area and T_x and T_y represent the existing tension per a unit length in x and y directions, respectively. Let us assume that

ω and p can be expressed as follows;

$$\omega(x,\ y,\ t)=W(x,\ y)e^{iwt},\quad p(x,\ y,\ t)=P(x,\ y)e^{iwt} \tag{2}$$

We obtain the following equation by substituting Eq. (2) into Eq. (1);

$$\rho_k w^2 W+\left(\frac{\partial^2 W}{\partial x^2}T_x+\frac{\partial^2 W}{\partial y^2}T_y\right)=P \tag{3}$$

Considering the rectangular membrane within a fixed range, the vibration form $W(x,y)$ can be assumed as

$$W=\sum_m\sum_n C_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\quad m,n=1,2,... \tag{4}$$

Substituting Eq. (4) into Eq. (3), we obtain the following equation;

$$\left\{\rho_k w^2-\pi^2\left(\frac{m^2}{a^2}T_x+\frac{n^2}{b^2}T_y\right)\right\}W_{mn}(x,\ y)=P(x,y) \tag{5}$$
$$m,\ n=1,\ 2,\ \cdots$$

For consideration of the free vibration of the membrane, where $P(x,y)=0$ can be assumed, the natural frequency w_{mn} can be expressed as follows;

$$w_{mn}=\pi\sqrt{\frac{1}{\rho_k}}\cdot\sqrt{\frac{m^2}{a^2}T_x+\frac{n^2}{b_2}T_y}\quad m,\ n=1,\ 2,\ \cdots \tag{6}$$

where m and n represent the numbers of vibration mode.

Let us consider to estimate membrane stress in two orthogonal directions from the natural frequency of membrane. By using the relationship of $2\pi f=w$, natural frequency f_{mm} becomes as shown below;

$$f_{mn}=\frac{1}{2}\sqrt{\frac{1}{\rho_k}}\cdot\sqrt{T_x\frac{m^2}{a^2}+T_y\frac{n^2}{b^2}} \tag{7}$$

Now, when the fixed membrane is resonated by two different values of the natural frequencies f_1 and f_2, the following equations are concluded by Eq. (7);

$$T_x\frac{m^2}{a_1^2}+T_y\frac{n^2}{b_1^2}=4\rho_k f_1^2 \tag{8}$$
$$T_x\frac{m^2}{a_2^2}+T_y\frac{n^2}{b_2^2}=4\rho_k f_2^2$$

where a_1, b_1 and a_2, b_2 represent the length of sides of the rectangular boundary respectively when the external excitation with the frequencies f_1 and f_2 are observed. Eq. (8) can be expressed as

$$[A]\{T\}=\{f\} \tag{9}$$

where $[A]$, $\{T\}$ and $\{f\}$ are as shown below;

$$[A]=\begin{bmatrix}\frac{m_1^2}{a_1^2} & \frac{n_1^2}{b_1^2}\\ \frac{m_2^2}{a_2^2} & \frac{n_2^2}{b_2^2}\end{bmatrix},\quad \{T\}=\begin{Bmatrix}T_x\\ T_y\end{Bmatrix},\quad \{f\}=4\rho_k\begin{Bmatrix}f_1^2\\ f_2^2\end{Bmatrix}$$

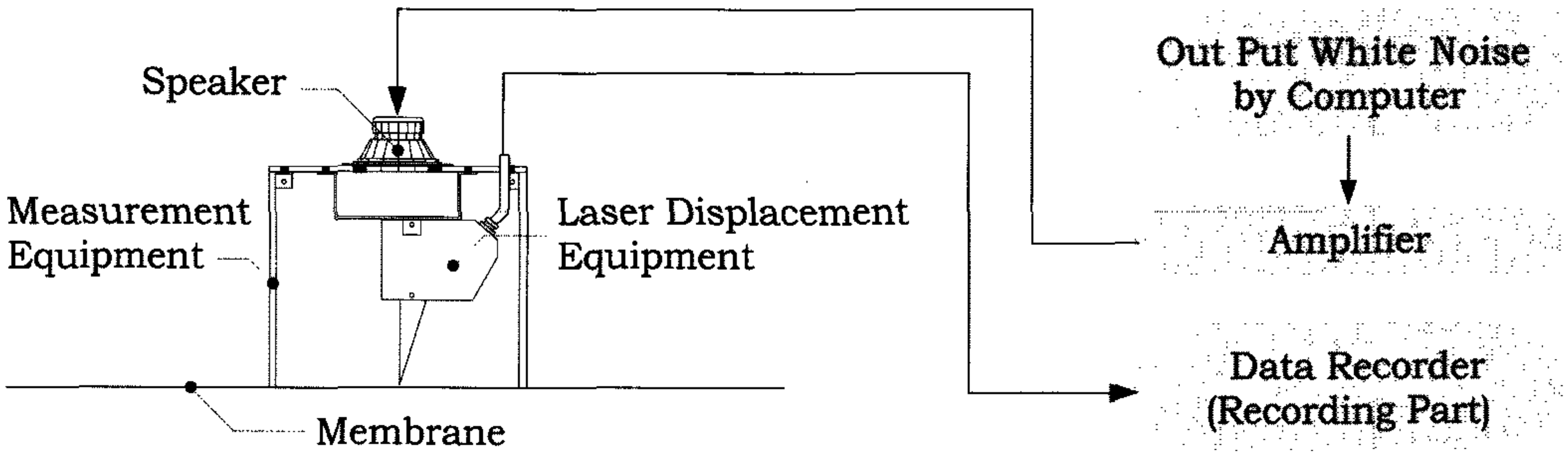
The membrane stress $\{T\}$ can be obtained by Eq. (9) as follows;

$$\{T\}=[A]^{-1}\{f\} \tag{10}$$

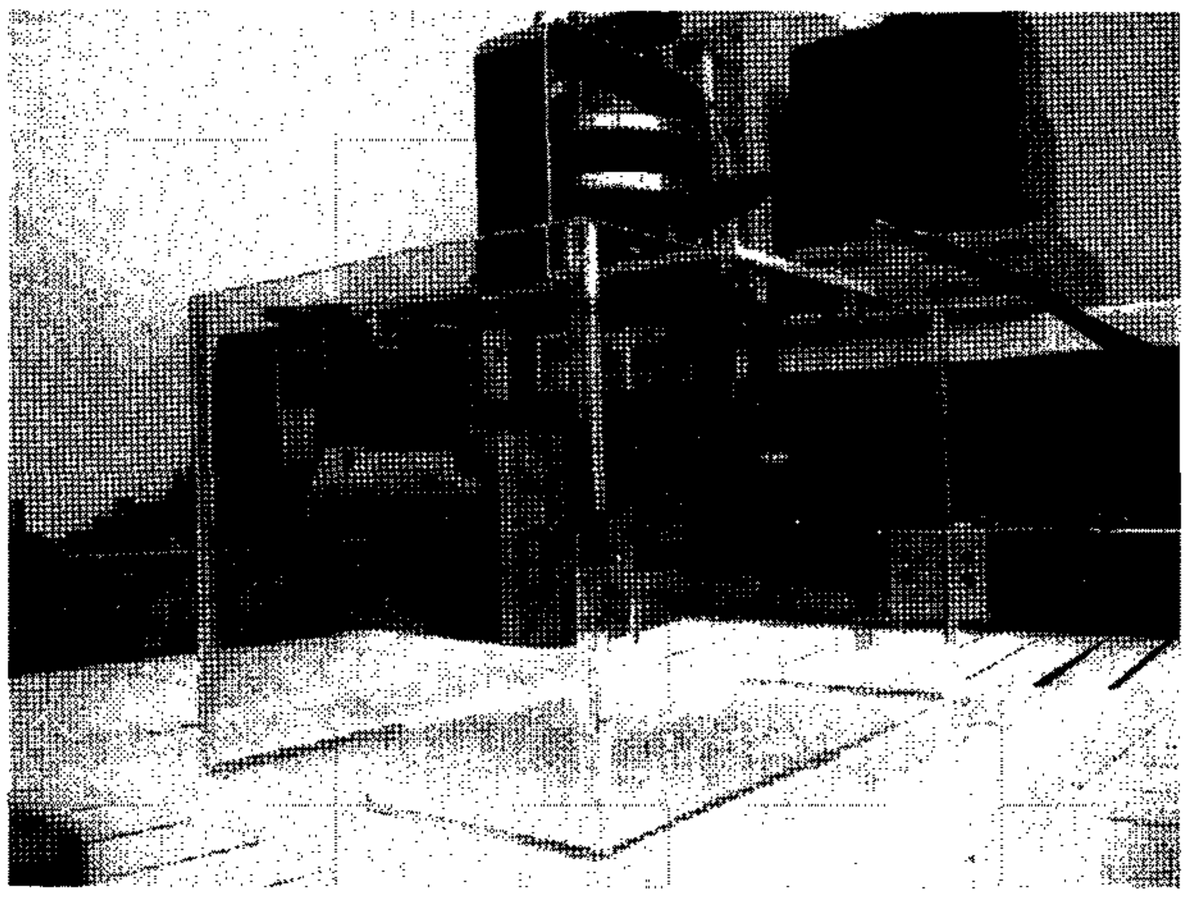
From Eq. (10), if the natural mode of vibration and natural frequency are measured in the resonance, we can obtain separately the objective values of tension stresses T_x and T_y existing in the membrane for both warp and fill directions.

3. Experimental Tests

The sweep external excitation needs rather a long measurement time because there is a possibility that the response amplitude depends on the speed and the time of sweep to secure demanded accuracy, we use white noise which has 0 to 500Hz in frequency field as external excitation in the experiment in order to improve above problems. As shown in Fig. 3, the



〈Fig. 3〉 Concept of experimental test



〈Fig. 4〉 Measurement equipment

〈Table 1〉 Types of membrane material

Name	Fiber	Coating	Density(kg/m^2)
Type A	Glass fiber	Ethylene fluoride	About 1.3
Type B	Glass fiber	Synthetic resins	About 0.9
Type C	Synthetic fiber etc.	Synthetic resins	About 0.8

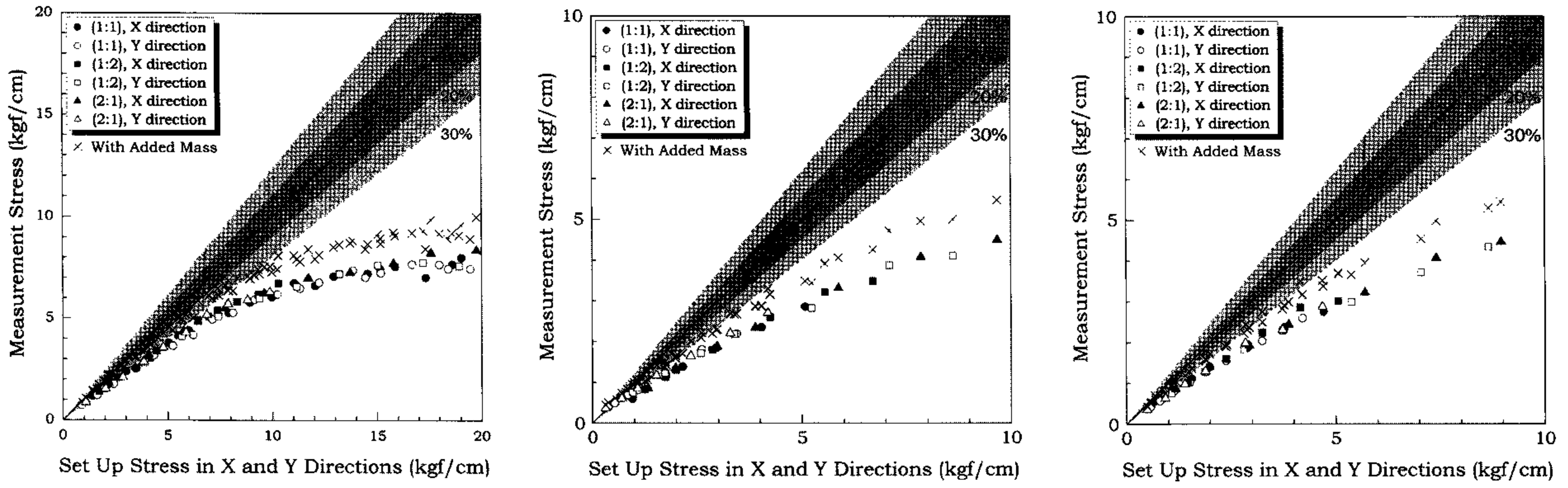
measurement equipment consists of the audio speaker for sound source, the measurement parts composed of laser displacement equipment, the box boundary and the recording part. The speaker radiates white noise sound wave having adaptable strength and frequency, and the laser displacement equipment measures displacement of membrane which is resonated by sound wave without contact. Fig. 4 shows the completed measurement equipment. By putting the equipment softly on the membrane surface, small rectangular region is made on the membrane, and the transparent acrylic box has the different lengths in its side, 200×300mm, and 200mm in height as shown in Fig. 4.

The membrane materials are classified depending on the characteristics of those materials because they are made by coating to the woven cloth as shown in Table 1. The equipment is theoretically adaptable to any types of membrane material by changing area density ρ_k of membrane in the Eq. (10). For usage in the site, we have to verify experimentally, thus experimental tests that used the membrane materials of types A, B and C were carried out.

3.1 Experimental Test of Type A Membrane Material

Membrane material of type A used in experiment is SF-2 produced by Taiyo Kogyo Corporation, having area density of 1.3 kg/m^2 which is an average value of five times measured weight of area 100×100mm. Firstly, introduced stresses on experimental test are equal in its level in the X and Y directions of which ratio in stress level is 1 to 1, the load level of which ranges from 0 to 20 kgf/cm being divided by the magnitude of 1 kgf/cm. As the second step, the tensile test with the different ratio in the stress level for two directions, such as 1 to 2 and 2 to 1 in X and Y directions was carried out. The introduced membrane stresses are read from the force sensor when the tremor of stress is observed to cease after enough time passed since the equipment had put on the membrane softly.

In Fig. 5 (a) shows the result of type A. The vertical



(a) Type A (b) Type B (c) Type C

〈Fig. 5〉 Result of each type

axis shows the measurement stress obtained by the measurement when the stress is set as the introduced stress on horizontal axis. For instance, the symbol • of 5 kgf/cm on horizontal axis shows that the stress of 3 kgf/cm was obtained by measurement in X direction when X and Y directions ratio of stress was 1 to 1. The diagonal solid line in the Fig. 5 (a) shows the objective accuracy, by which we can see the discrepancy between the measured and truly existing stress levels. Moreover, the symbols × shows the stresses which are considered the effect of added mass of air. It shows that the equipment has the accuracy of about 80% or above by considering added mass of air when the stress up to 5 kgf/cm is introduced.

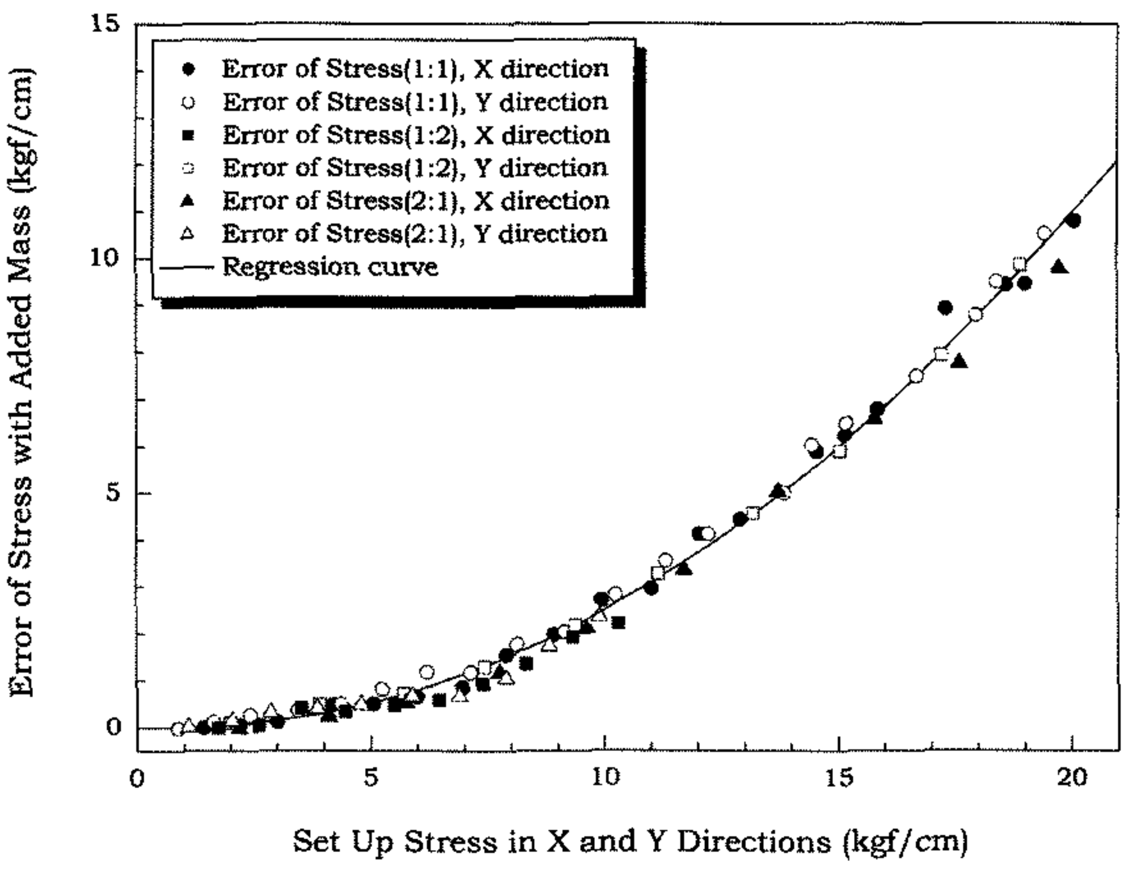
3.2 Experimental Tests of Membrane Materials of Type B and C

Membrane materials of type B and C used in the experimental tests are CMX270NM and SCC200, respectively made by Taiyo Kogyo Corporation. The area density of membrane materials are 0.8114 and 0.8692 kg/m^2 which are measured by the same method as that for type A. Introduced stresses are established in three patterns that are even stresses of ratio 1 to 1 with types B and C where the stress levels of 0.5, 1, 1.5, 2, 3, 4 and 5 kgf/cm are introduced and the different stresses of ratio 1 to 2, 2 to 1 in X and Y directions, respectively. The introduced membrane stresses are read from the force sensor when the tremor change of stress stops in the membrane after enough time passed since the equipment had put on the membrane softly like the experimental test that used for type A.

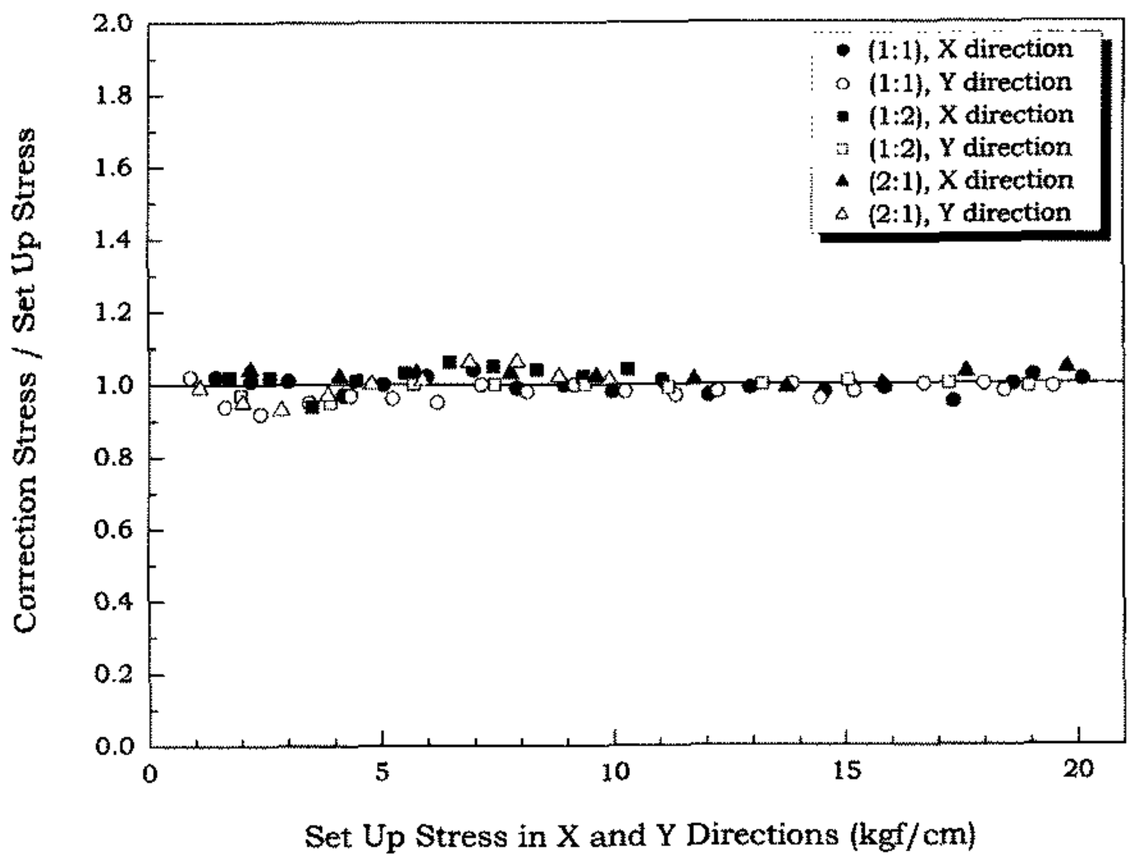
Fig. 5 (b) and (c) are showing the results for the type B and C respectively. Though accuracy falls compared with the result of type A, these results present that the equipment has the accuracy of about 70% or above by considering added mass of air when the stress up to 5 kgf/cm is introduced. It is clearly confirmed that the equipment is adaptable to any types of membrane material by changing area density ρ_k of membrane in Eq. (10).

4. Error Correction

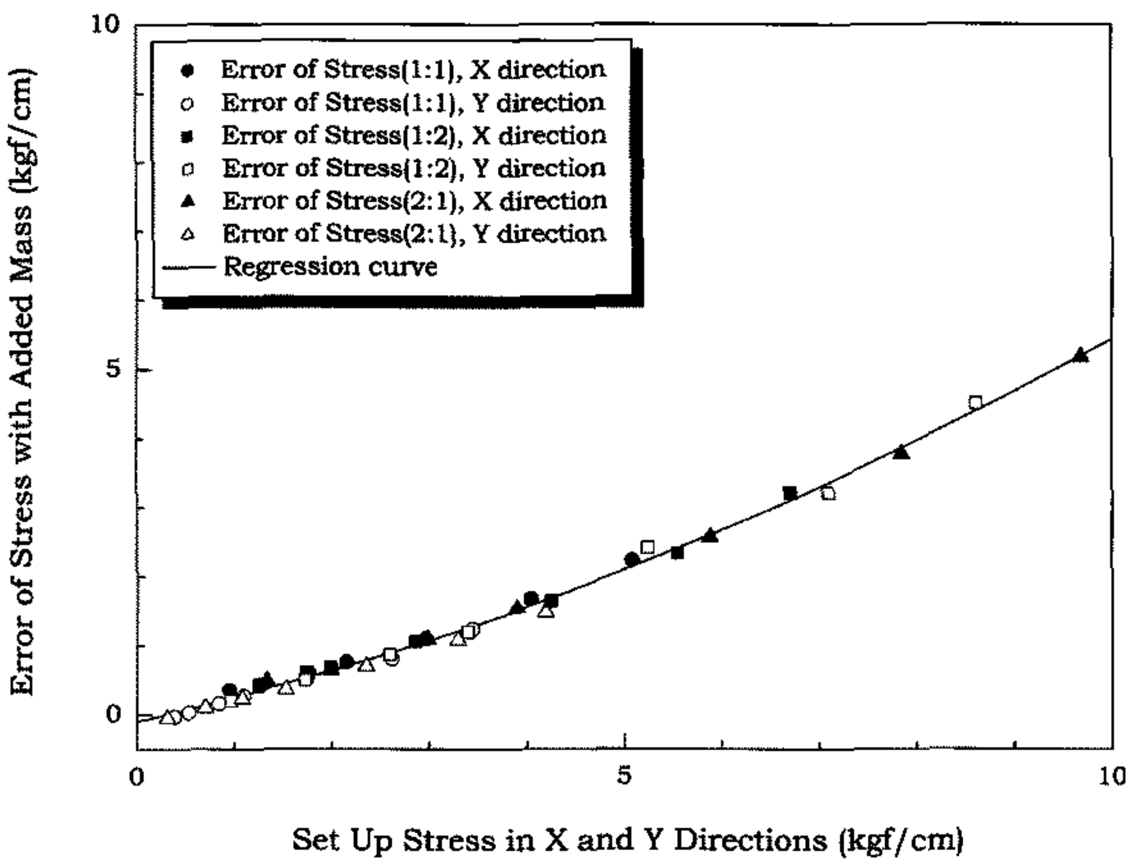
As shown in Fig. 6, errors with added mass of air increase almost in the way of those described by quadric function, and to examine the tendency of the errors, the regression analysis was carried out by using a polynomial expression. Symbols in Fig. 6 present the error level in all stress level of X and Y directions that are introduced in experimental test and a curved line presents result of regression curve obtained by using quadric expression. Table 2 shows the coefficients of the regression curve. Determination coefficient means square of correlation coefficient that is obtained in regression analysis and it shows how it is available



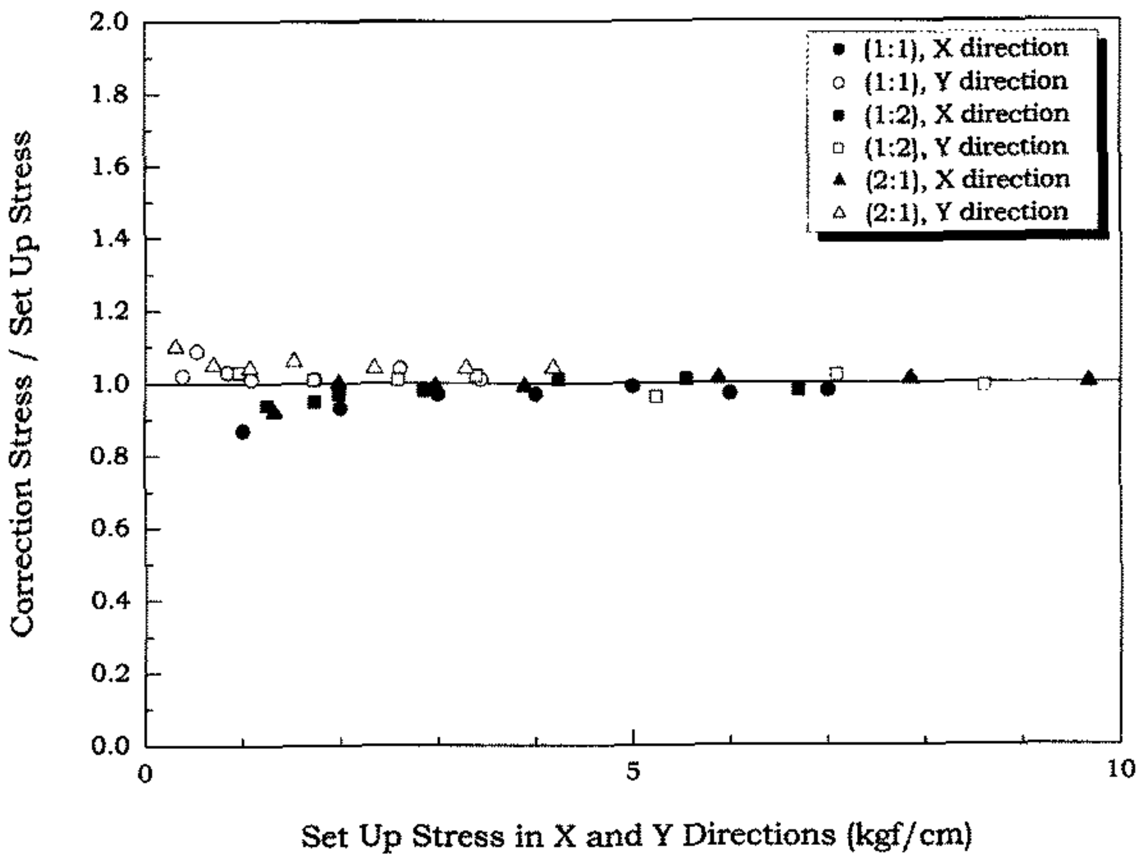
〈Fig. 6〉 Regression analysis (Type A)



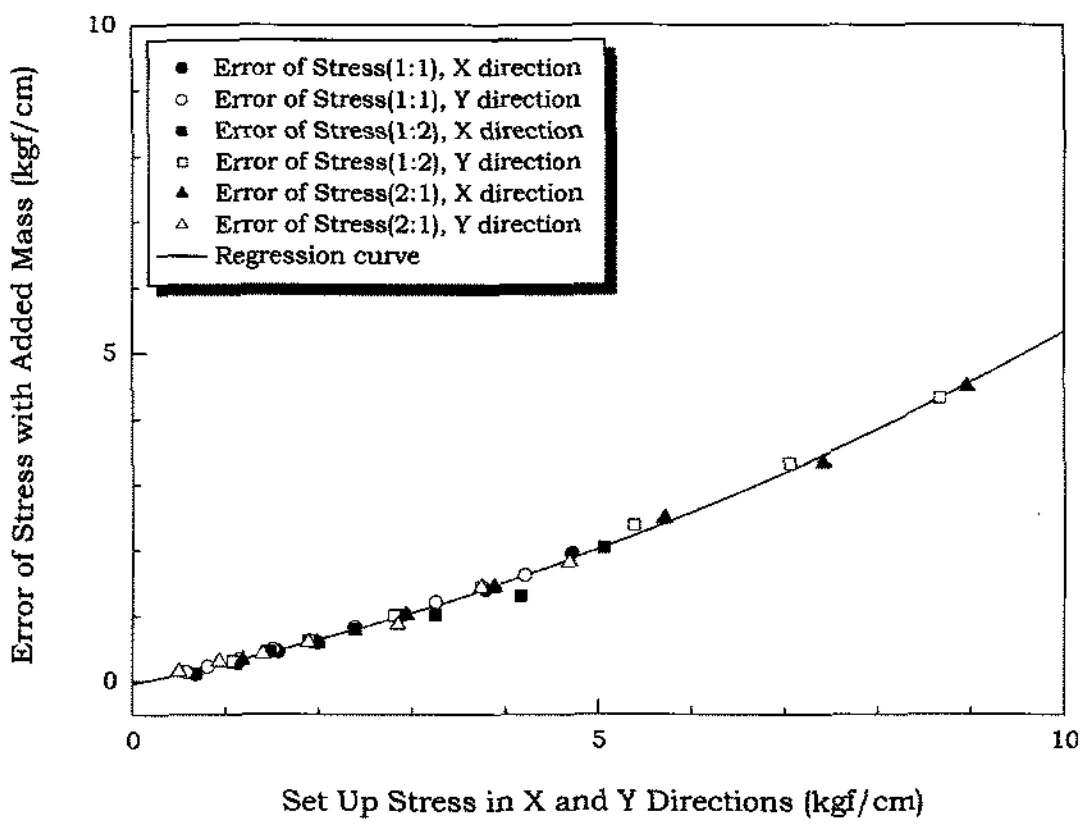
〈Fig. 7〉 Error correction (Type A)



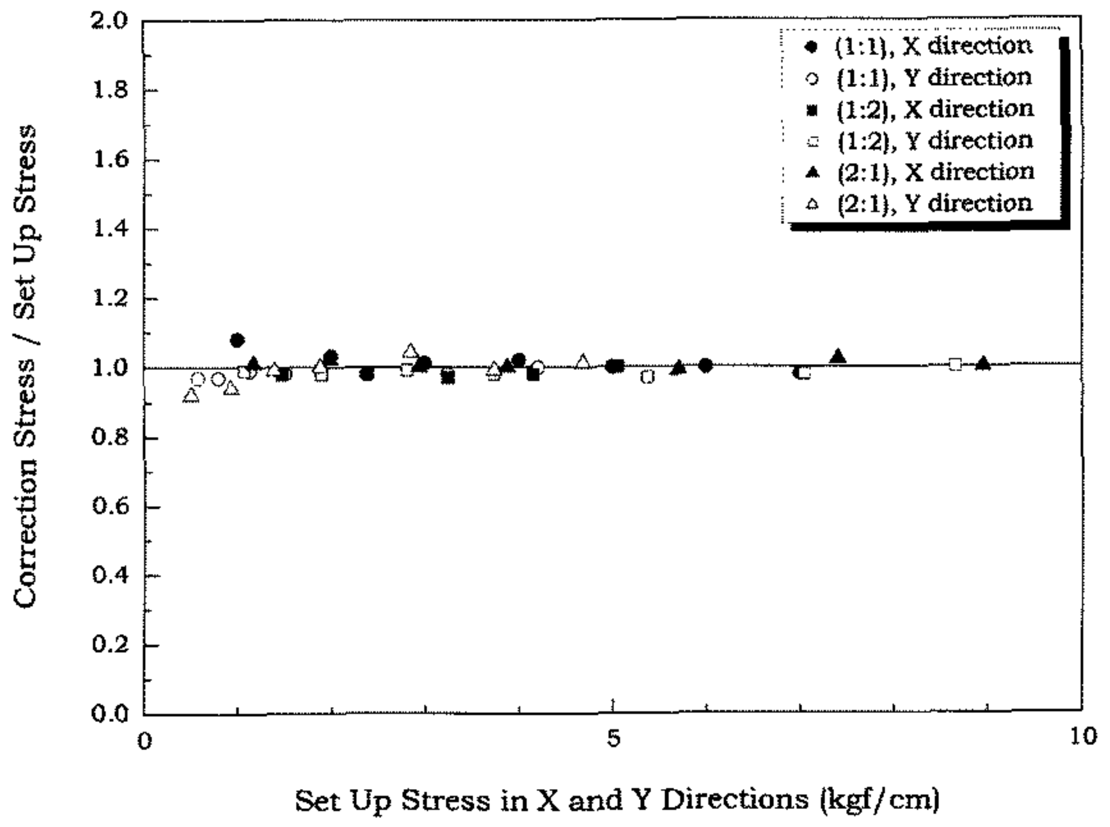
〈Fig. 8〉 Regression analysis (Type B)



〈Fig. 9〉 Error correction (Type B)



〈Fig. 10〉 Regression analysis (Type C)



〈Fig. 11〉 Error correction (Type C)

to explain by regression curve from total amount of fluctuation. Therefore determination coefficient of 0.99662 explains all of fluctuation of object function by regression curve is 99.662%. Fig. 7 shows the result that is obtained by correcting errors. Accuracy of error below 10% is obtained in total introduced stress level.

Figs. 8 and 9 show the results of regression analysis for type B by using the error of measurement and

〈Table 2〉 Coefficients of the regression curve (Type A)

$y = A + B_1 x + B_2 x^2$		
Parameter	Value	Standard Deviation
A	0.03845	0.07636
B_1	0.12371	0.01766
B_2	0.02475	0.87855E-4
Determination coefficient	Standard Deviation	N (Number of Data)
0.99662	0.21089	80

〈Table 4〉 Coefficients of the regression curve (Type C)

$y = A + B_1 x + B_2 x^2$		
Parameter	Value	Standard Deviation
A	-0.03711	0.03198
B_1	0.28478	0.01847
B_2	0.02511	0.00209
Determination coefficient	Standard Deviation	N (Number of Data)
0.99565	0.07354	42

measurement accuracy by correction respectively. Table 3 also shows the results of the coefficients of the regression curve. As in the case of type A, errors in the stress level can be prescribed by the increasing quadric function. It can be known that accuracy of error below 10% is obtained in total introduced stress level by correcting error with regression curve that has determination coefficient 0.99579 in all data.

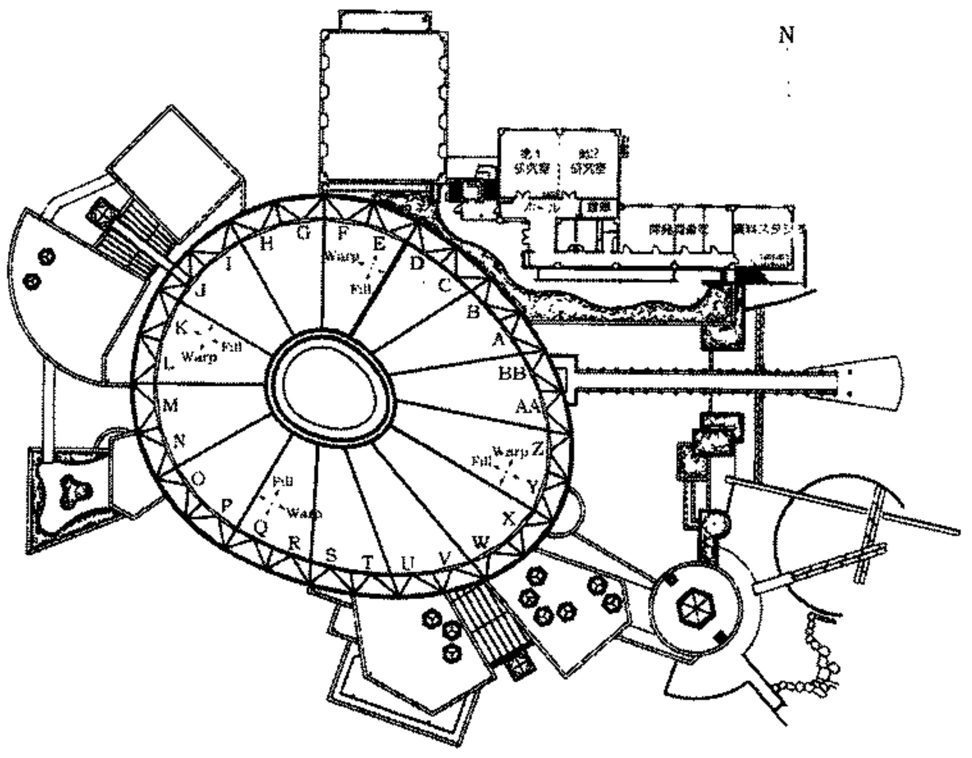
Figs. 10 and 11 show regression analysis of type C by using the error of measurement and the measurement accuracy by correction respectively. Table 4 also shows the results of the coefficients of regression curve. As can be seen in the case of type A and B, the accuracy of error bellow 10% is obtained in total introduced stress level by correcting error with regression curve that has determination coefficient 0.99565 in all data.

5. Practical Measurement of Actual Frame Supported Membrane Structure

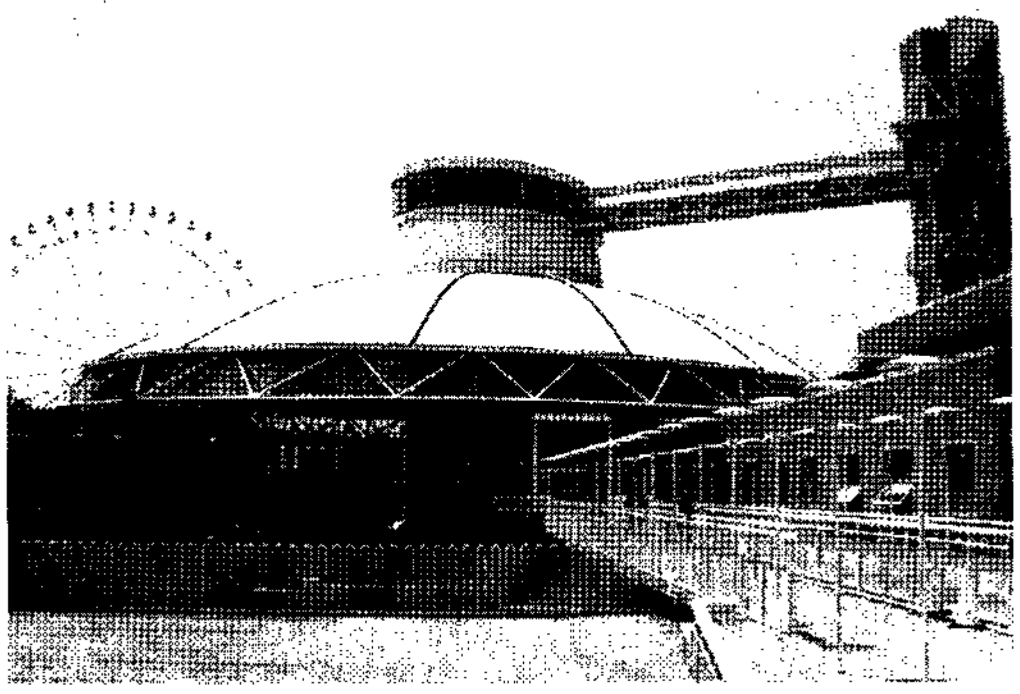
The stress of existing frame supported membrane

〈Table 3〉 Coefficients of the regression curve (Type B)

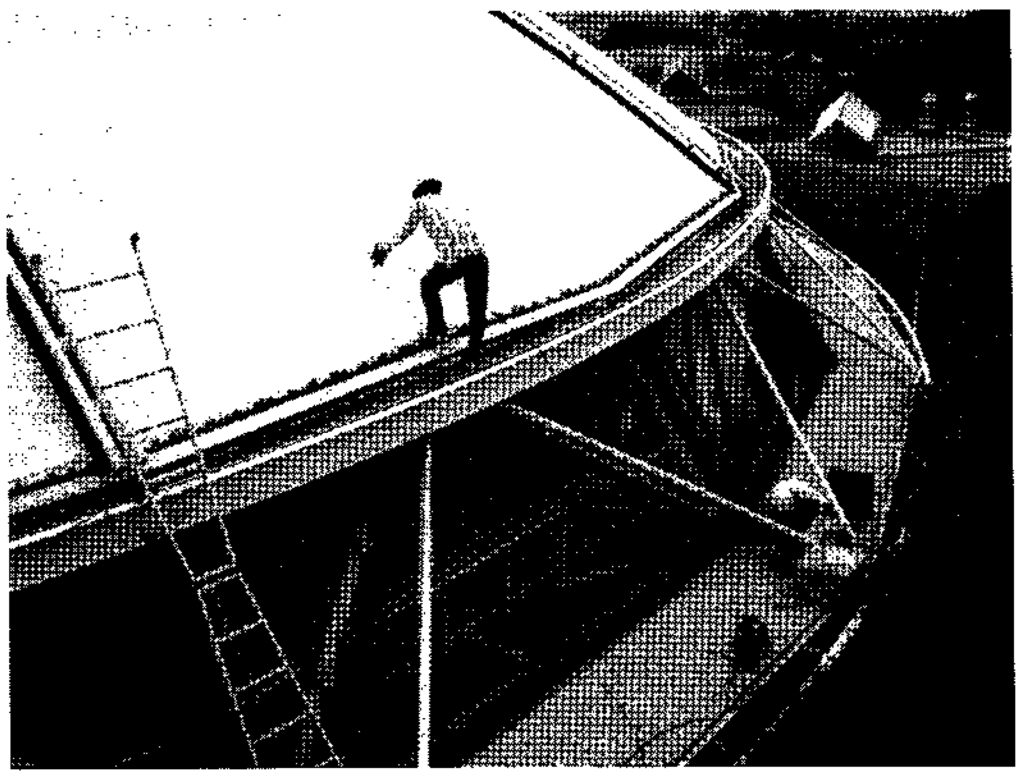
$y = A + B_1 x + B_2 x^2$		
Parameter	Value	Standard Deviation
A	-0.10006	0.03312
B_1	0.32128	0.01918
B_2	0.233	0.0021
Determination coefficient	Standard Deviation	N (Number of Data)
0.99579	0.08267	42



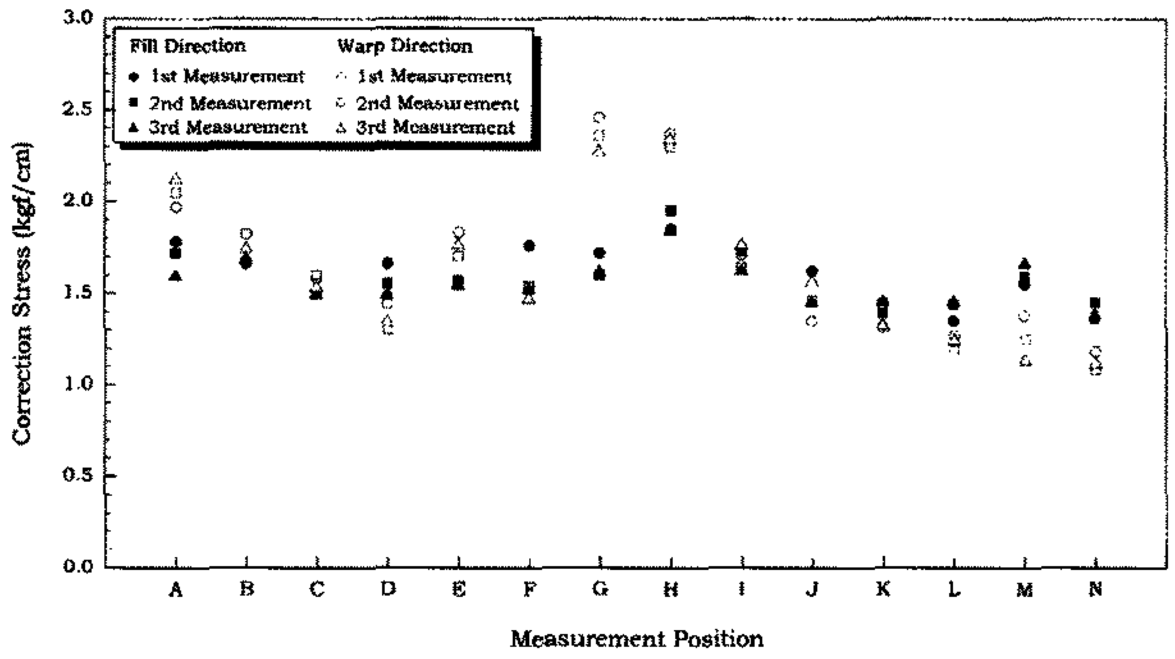
〈Fig. 12〉 Measurement position of Aichi Children's Center



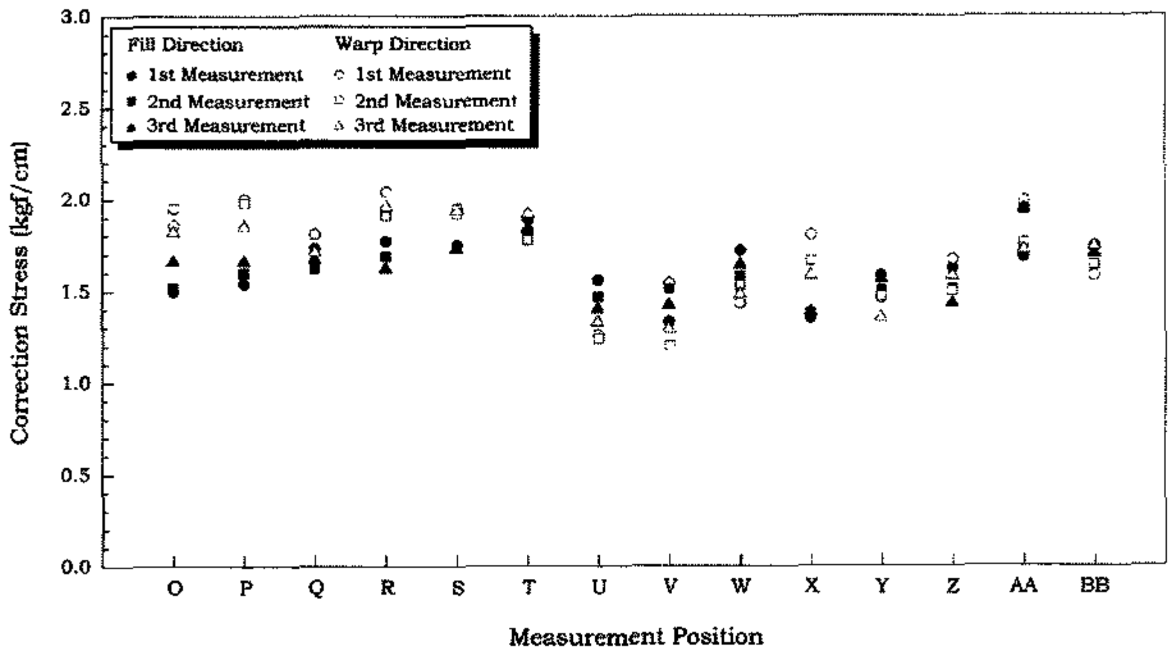
〈Fig. 13〉 Front view



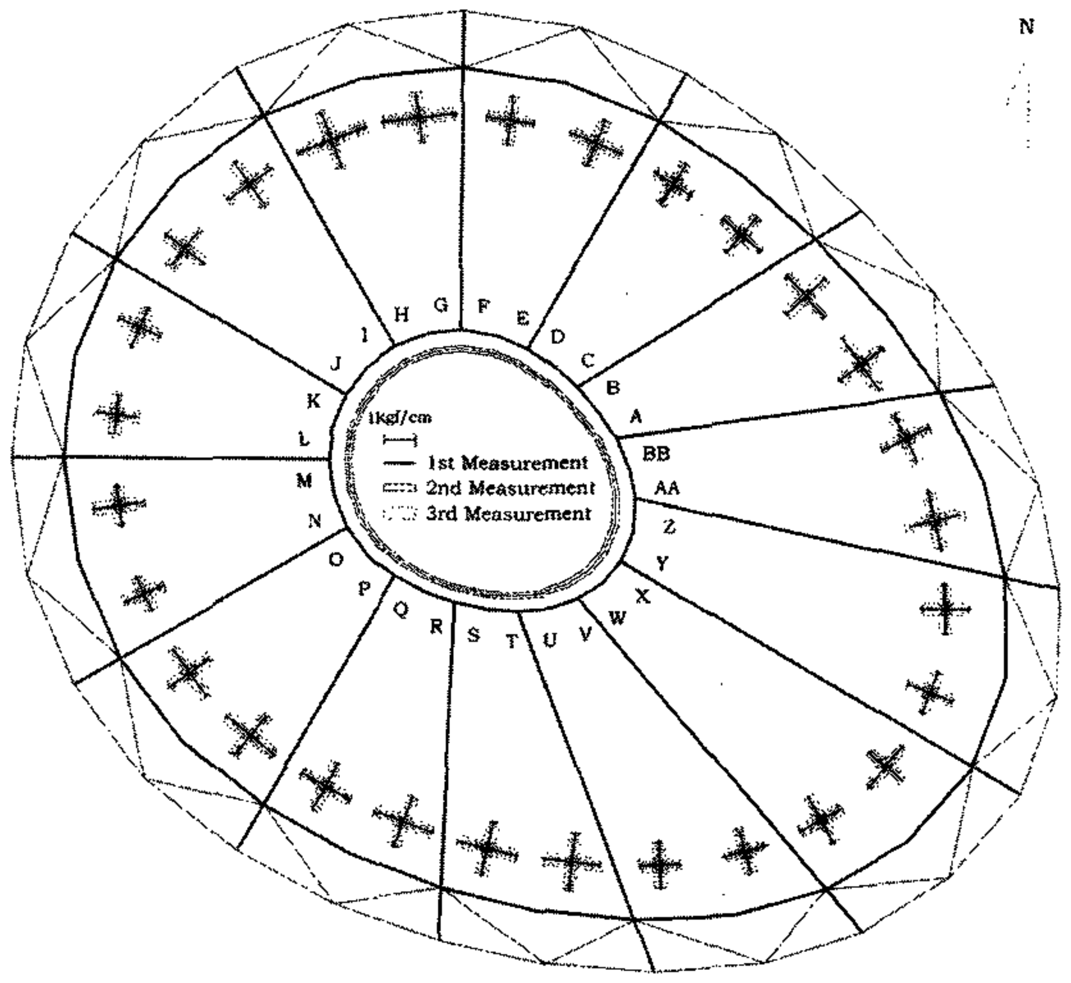
〈Fig. 14〉 Appearance of measurement



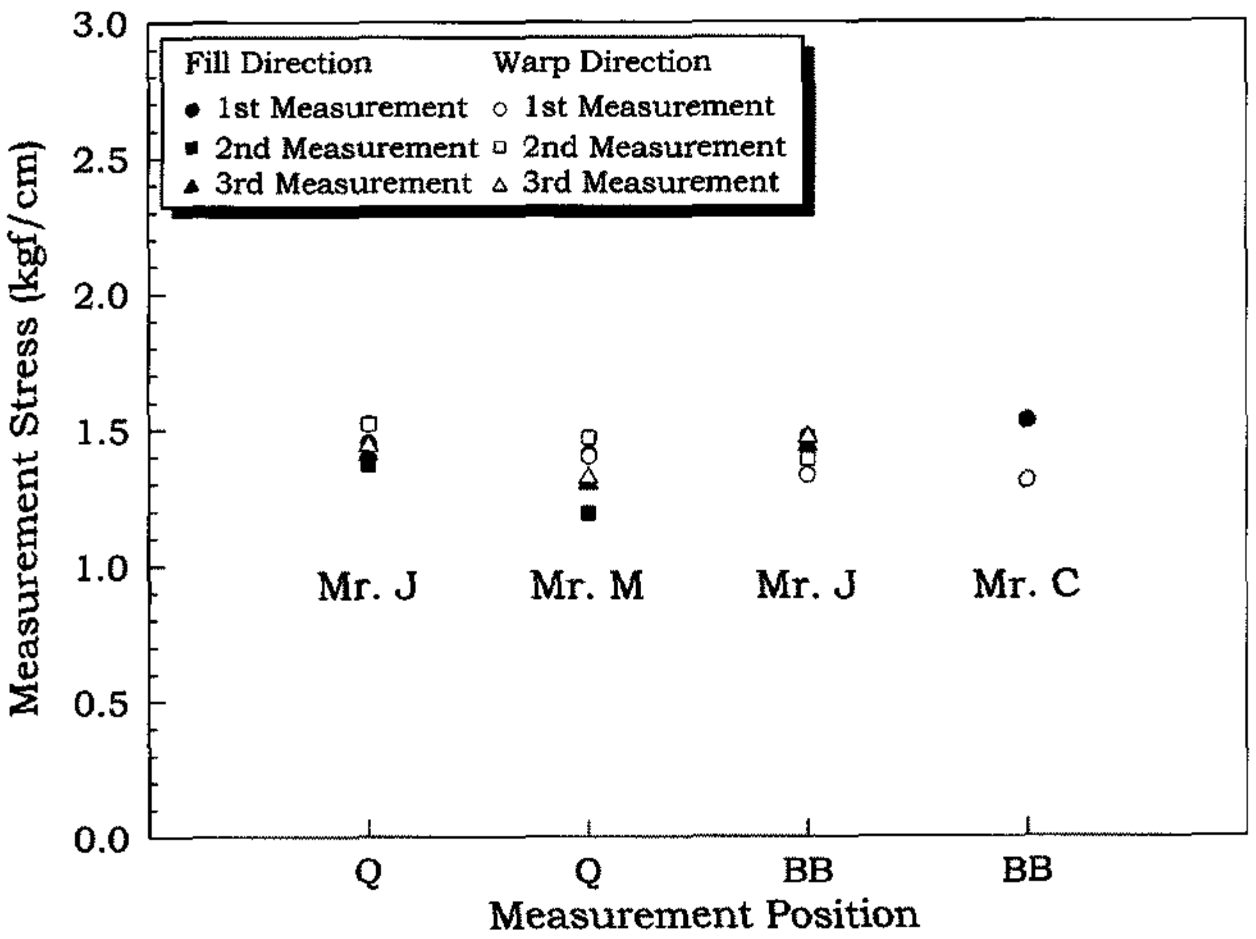
〈Fig. 15〉 Result of measurement (A to N)



〈Fig. 16〉 Result of measurement (O to BB)



〈Fig. 17〉 Measurement stress



〈Fig. 18〉 Effect of difference of measurer

structure is practically measured based on verified results mentioned above. The target membrane structure for measurement is the Aichi Children's Center located in Expo 2005 Aichi Commemorative Park, Japan. As shown in Figs. 12 and 13, it is the frame supported membrane structure which completed its construction in 1996 and have a form of an egg shape, of which size is 51m×41m. The membrane material is type A of SF-2 produced by Taiyo Kogyo Corporation which is the same one used in the preceding experimental test.

As shown in Fig. 12, the positions of measurement are composed of twenty eight parts. A panel is made up of 6 pieces of membrane material and the measurements are conducted at the peripheral area where measurer's hand was able to reach as shown in Fig. 14. The measurement was carried out by pushing the device onto the membrane with minimum power so that the device should not slip on the sloped membrane. The measurements are done three times on the same measurement position to prevent mistakes.

Figs. 15 and 16 show the results obtained by error correction as shown in Table 2. The vertical axis represents measured stress and the horizontal axis is for measured position, where the alphabets of the horizontal axis show the positions shown in Fig. 12. In the measurement, deviation among the measured values through the plural trials can be regarded as small, and the stable results are obtained. The stresses in position G and H are measured higher than those of other positions. The measurement was carried out over three days and G panel is the first one measured on the second day. When the measurement of position G was done, it was raining before dawn and the membrane was not completely dry and a little bit damp. Therefore, there is a possibility that moisture influenced the area density of membrane. Moreover, Fig. 17 shows magnitude of stress in diagram, where the symbol of ▬ means the results of the 1st measurement, ▭ is for the 2nd measurement and ⬚ represents the results of the 3rd measurement.

Fig. 18 shows the result of verification of how much influence can be given to the result of measurement on the position Q and BB by three different measurers. The results of measurement of Mr. M and C are a little low and high compared with the result of Mr. J on the position of Q and BB respectively. However, it is shown that it is almost installed within the range of error and the difference of measurer does not have a large influence on the measurement result. Consequently, it was confirmed that the present equipment did not need a special skill in the measurement.

The stress of membrane of Aichi Children's Center is designed to have 2 kgf/cm in both warp and fill directions and the measurement stresses are obtained around the value. It is judged that the designed tension has been introduced into the membrane though 11 years have passed since the completion.

6. Conclusions

In this paper, a basic concept of a new method for measuring the membrane stress in high accuracy, which is based on resonance phenomenon induced by the sound wave is presented.

In the verification experiments in which white noise is used, it is confirmed that the accuracy up to 80% or above at a low stress region up to 5 kgf/cm is assured. It is clearly shown that the white noise excitation is quite effective to obtain the resonance frequency of the membrane and suitable for the measurement equipment of membrane stress. In addition through experimental tests, it is proved that this equipment can be used not only for the membrane material of type A but also for type B and C.

In verification experiments of type A, B and C, the error to stress level behaves in such a way of the increasing quadric function and accuracy of error bellow 10% is obtained by correcting error where the regression analysis is used. It is proved that the equipment is adaptable to any types of membrane by changing area density ρ_k of membrane materials.

In the measurement of frame supported membrane structure, deviation can be regarded as small and the stable results are obtained. In addition it was confirmed that the equipment did not need a special skill in the measurement because the measurer's difference does not have a large influence on the measurement result. It is shown that this device can be used enough on the site.

Acknowledgments

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