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Modified Equivalent Radius Approach for Soil Damping Measurement in Torsional Testing

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Abstract

Determination of strain associated with shear modulus and damping ratio during torsional test is complicated. This is due to nonuniform stress-strain variation occurring linearly with radius in a soil specimen in torsion. A conventional equivalent radius approach proposed by Chen and Stokoe appears to be adequate for evaluating strain associated with shear modulus at low to intermediate strain levels. This approach is less accurate for damping measurement, particularly at high strain. Modified equivalent radius approach was used to account for the nonuniform stress-strain effect more precisely. The modified equivalent radius approach was applied for hyperbolic, modified hyperbolic, and Ramberg-Osgood models. The results illustrate the usefulness of the modified equivalent radius approach and suggest that using a single value of equivalent radius ratio to calculate strains is not appropriate.

1. Introduction

When a soil specimen is subjected to cyclic loading in torsional tests, hysteresis loops generated on torque-rotation relationship represent the nonlinear stress-strain behavior and energy dissipating characteristics of the soil. Several approaches have been proposed to identify a specific strain associated with shear modulus and effective damping ratio at a given rotation (Hardin and Drenevich 1972, Chen and Stokoe 1979).

An equivalent radius approach dealing with the nonuniform distribution of strain in soil specimen was developed by Chen and Stokoe (1979). The approach is based on the assumption that the representative stress and strain in soil specimen in torsion occurs at a radius called the equivalent radius. To apply the equivalent radius approach, an equivalent radius ratio, R_{eq}, which is defined as the ratio of the equivalent radius (r) and the outside radius of soil specimen (R) is used to calculate the stress and strain at a given rotation. Shear strain is calculated from:

$$\gamma = \frac{\theta r}{L} = \frac{r}{R}\gamma_{max} = R_{eq} \times \gamma_{max.} \tag{1}$$

Shear strain at the outer surface or maximum shear strain, γ_{max} can be expressed as:

$$\gamma_{max} = \frac{\theta R}{L} \tag{2}$$

where, θ = rotational angle,
R = radius of the soil specimen, and
L = length of the soil specimen.

Fig. 1 shows the longitudinal section of shearing stress in soil column for TS test

Chen and Stokoe (1979) obtained R_{eq} from a correction of the corresponded q from the effective shear modulus (G_{eff}) calculated from the total torque-rotation relationship and γ from the equal value of shear modulus (G) from the theoretical stress-strain relationship. More details on this approach can be found in Chen and Stokoe (1979) and Sasanakul (2005). According to Chen and Stokoe, R_{eq} value varies from 0.82 for strains below 10^{-3} % to 0.79 for strains at 10^{-1} % for a solid specimen. In practice, a single value of R_{eq} has been used for range of strains as shown in Fig. 1 (Hwang 1997, and Kim 1991).

The equivalent radius ratio values suggested by Chen and Stokoe (1979) is adequate for evaluating strains corresponded to shear modulus at low to intermediate strain levels but the approach does not account for soil nonlinearity and appear to be less accurate at high strains (Sasanakul 2005). In addition, the same value of R_{eq} from Fig. 2 was suggested for calculation of strain for damping. However, the equivalent radius at which a soil specimen contributes the most damping is not necessarily the same as the equivalent radius at which the representative shear modulus was contributed. Using less accurate R_{eq} will result in misinterpretation of strain corresponding

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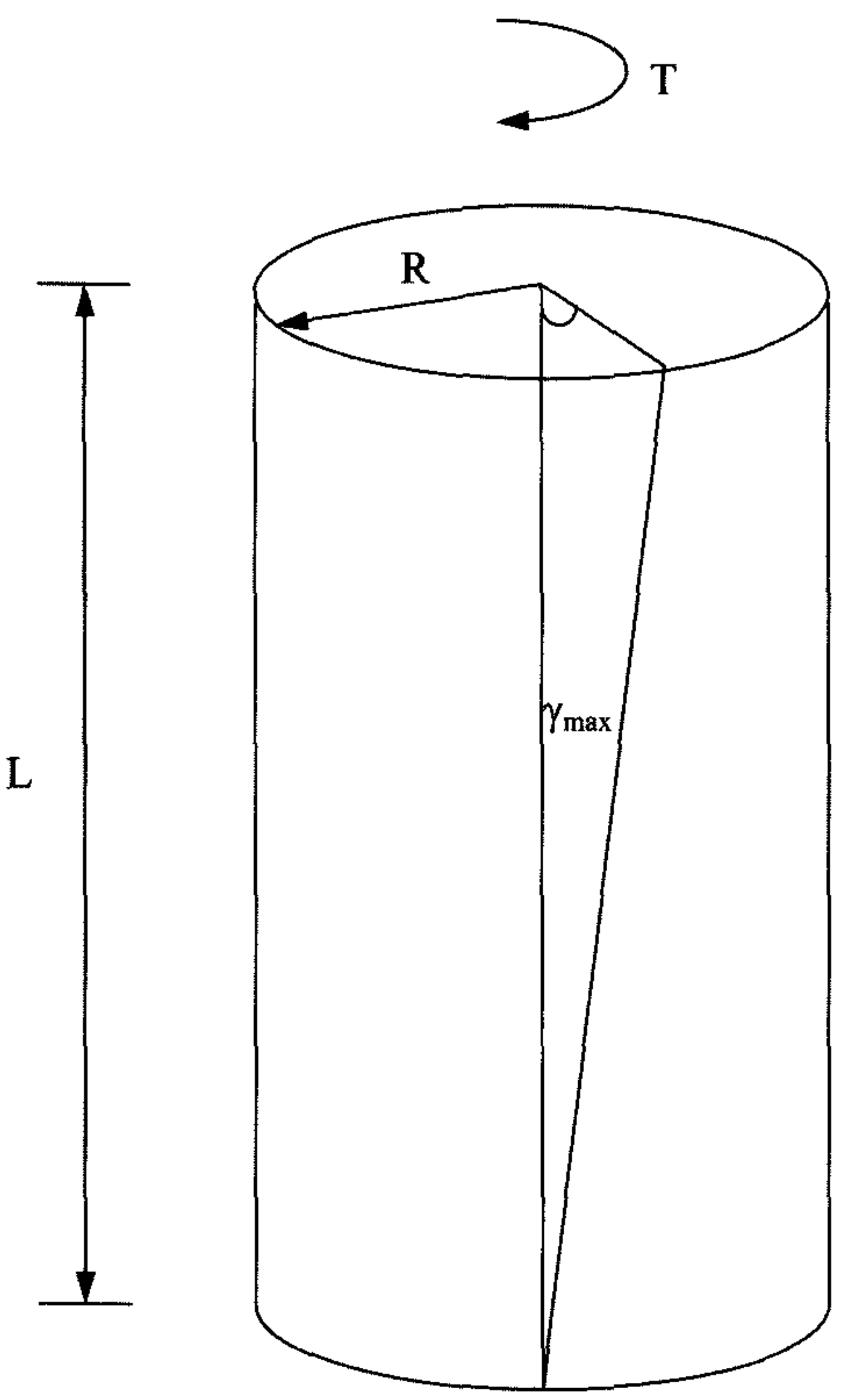


Fig. 1 Longitudinal section of shearing stress in soil column for TS test

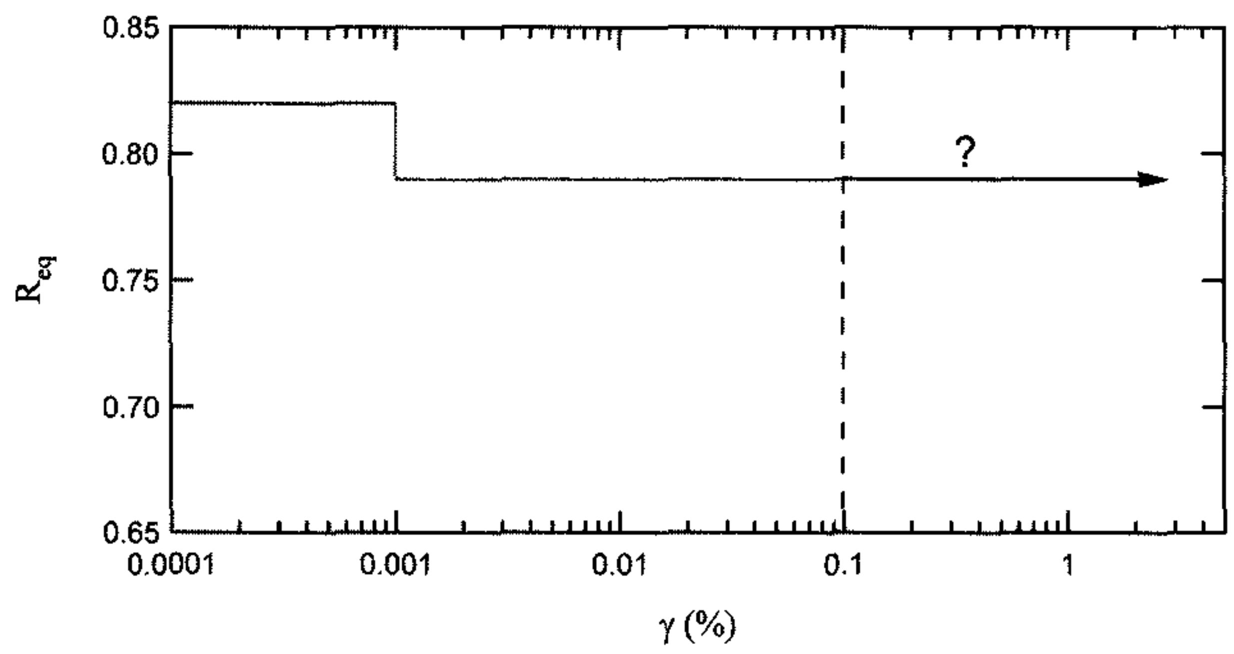


Fig. 2 R_{eq} Values Suggested by Chen and Stokoe (1979)

to a measured damping.

A more general approach called modified equivalent radius approach was developed by Sasanakul (2005) to account for the nonuniform stress-strain more precisely. In this paper, the approach is adopted to investigate the variation of R_{eq} versus rotation for damping measurement using three stress-strain soil models. These models are hyperbolic, modified hyperbolic, and Ramberg-Osgood. Results and discussions of each model are presented as followed.

2. Modified Equivalent Radius Approach

For more general approach to account for nonuniform stress-strain than the conventional equivalent radius approach, the modified equivalent radius approach was proposed. The nonuniform stress-strain effect was accounted for by integrating the stress over the radius of soil specimen to obtain a

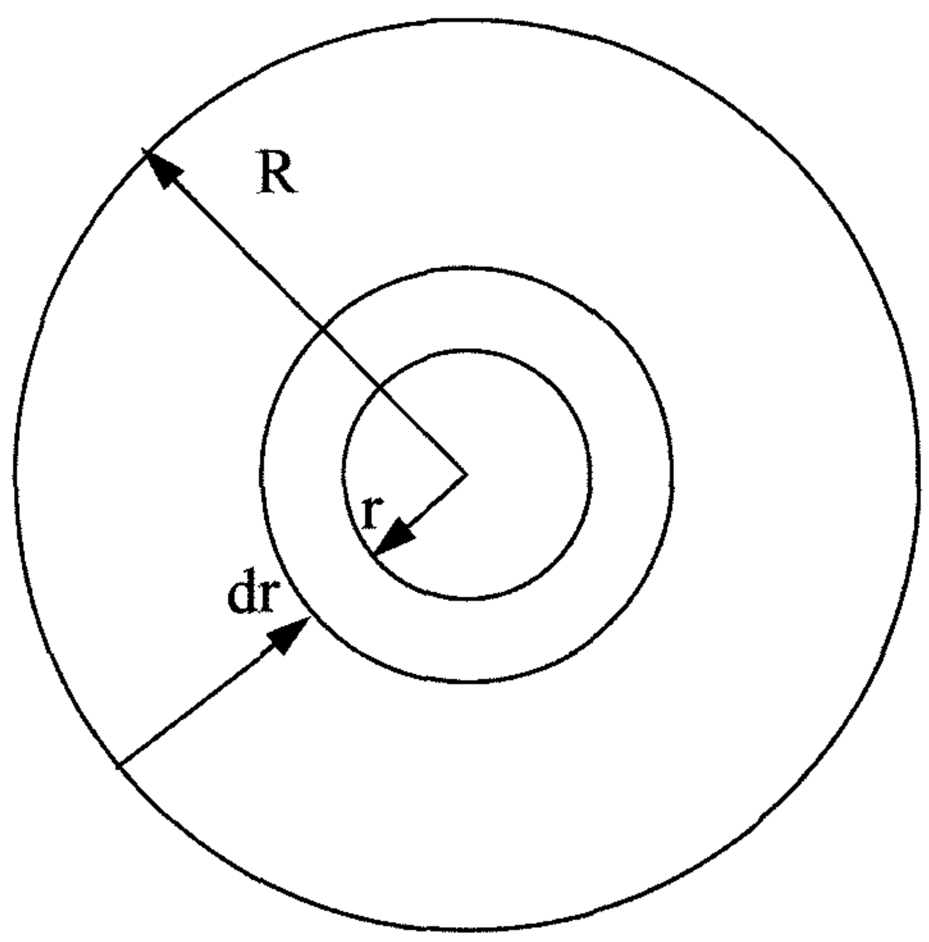


Fig. 3 Transverse section of shearing stress distribution in soil column for TS test

torque-rotation relationship.

Stress integration starts by relating the shear stress acting on a circular section to the applied torque, T, using a basic equation of mechanics shown in Fig. 3:

$$T = \int_A dM = \int \tau r dA = \int_0^R 2\pi r^2 \tau dr \tag{3}$$

where, M = resultant moment over the entire cross section area,
r = interior radius,
A = cross section area, and
t = shear stress.

If the stress-strain relationship is known, then for any given value of, the shear stress at any point in the specimen can be determined. This requires using the first part of Eq. (1) to calculate the shear strain and calculating the corresponding shear stress from the known stress-strain relationship. Thus, the distribution of shear stress and strain over the entire cross section of the soil can be evaluated. Since the shear strain varies linearly with the radius, the distribution of the shear stresses has the same shape as the stress-strain relation. Torque can be obtained from the integral of Eq. (3). As a result, the theoretical torque-rotation relationship can be developed

2.1 Developing R_{eq} Curves Based on Damping

In torsional testing, the torque-rotation relationship is measured directly and a hysteresis loop is developed in the torque-rotation plane. The effective hysteretic damping ratio, D_{eff}, can be calculated similarly to the hysteretic damping ratio, D, from the stress-strain plane except that the D_{eff} is associated with a given rotation, θ. The values of R_{eq} based on damping can be obtained by matching D-γ relationship with the D_{eff} - θ relationship similar to the conventional equivalent radius approach used to calculate R_{eq} based on shear modulus. In this study, the modified equivalent radius approach improves this procedure because the D_{eff} - θ relationship can be generated using either a close form integration or numerical integration. Procedures for developing R_{eq} based on damping using the three different soil models are described as followed.

2.1.1 Hyperbolic Model

For the hyperbolic model, the theoretical torque-rotation relationship can be calculated using the closed form solution presented by Sasanakul (2005). The hyperbolic stress-strain soil model and the closed form solution for torque-rotation relationship are presented in Eqs. (4) and (5), respectively.

$$\tau = \frac{G_{max}\gamma}{1+\left(\frac{\gamma}{\gamma_r}\right)} \qquad (4)$$

Where, G_{max} = shear modulus at small strain, and
γ_r = reference strain

$$T=\frac{1}{3}\pi G_{max}\gamma_r R\left[2R^2-3R\left(\frac{\gamma_r L}{\theta}\right)+6\left(\frac{\gamma_r L}{\theta}\right)^2\right]$$
$$+2\pi G_{max}\gamma_r^4\left(\frac{L}{\theta}\right)^3\left[\ln(\gamma_r)-\ln\left(\gamma_r+\frac{\theta R}{L}\right)\right] \qquad (5)$$

To generate the hysteresis loop, it was assumed that the soil behaves according to Masing behavior. The hysteretic damping ratio in the stress-strain plane is obtained as (Ishihara, 1996):

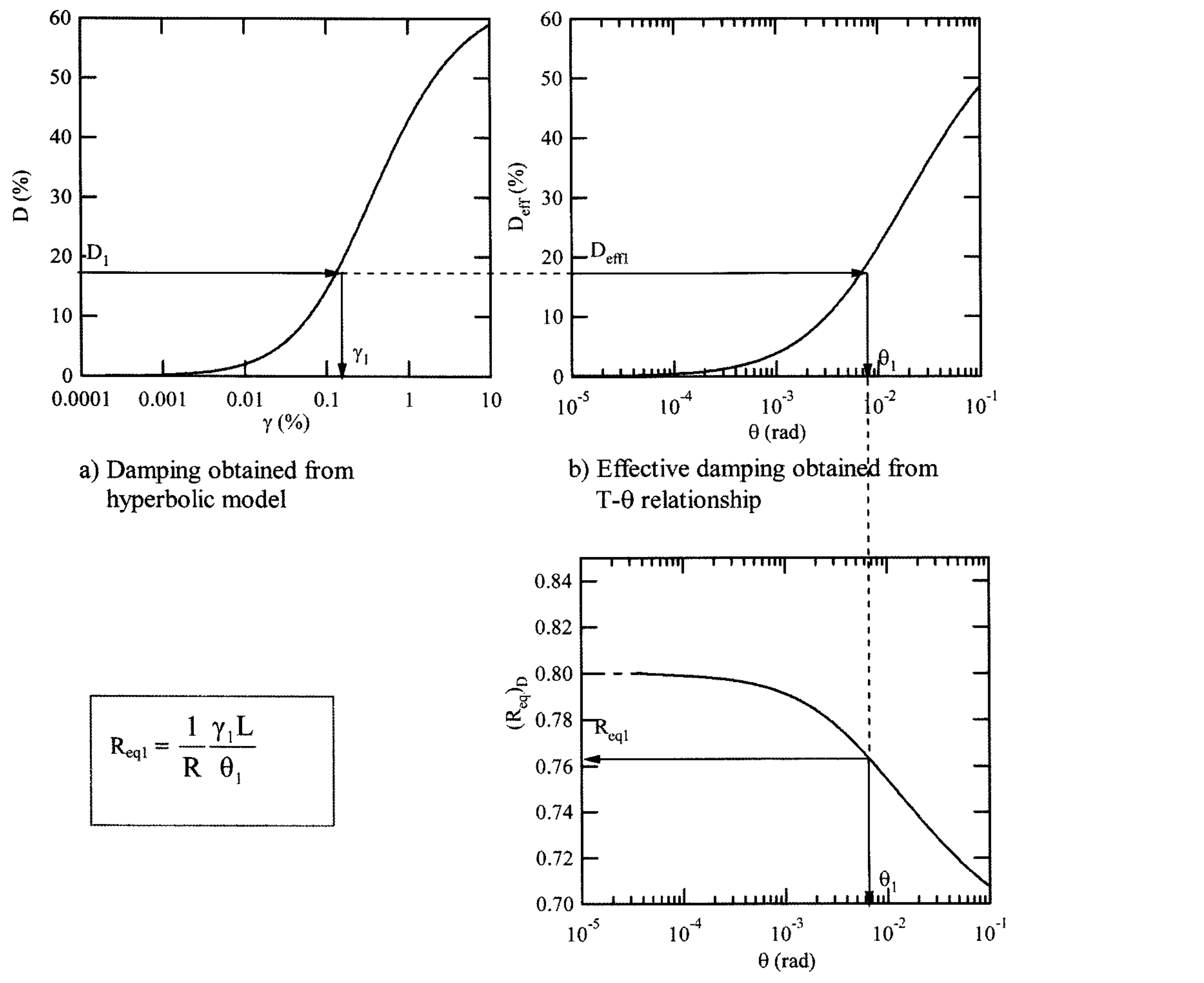
$$D=\frac{2}{\pi}\left[\frac{2\int_0^{\gamma}\tau(\gamma)d\gamma}{\gamma\tau(\gamma)}-1\right] \qquad (6)$$

By introducing Eq. (4) into Eq. (6), the damping ratio, D can be obtained from:

$$D=\frac{4}{\pi}\left[1+\frac{1}{\gamma/\gamma_r}\right]\left[1-\frac{\ln(1+\gamma/\gamma_r)}{\gamma/\gamma_r}\right]-\frac{2}{\pi} \qquad (7)$$

Eq. (7) is used to generate the D-γ relationship for the hyperbolic model shown in Fig. 4(a).

In this case, a theoretical soil is used hence the model parameters; G_{max} and γ_r are known, thus the torque-rotation relationship is obtained directly from the closed form solution presented in Eq. (5). Eq. (6) can be transformed to obtain the effective damping, D_{eff} obtained from the torque-rotation



a) Damping obtained from hyperbolic model

b) Effective damping obtained from T-θ relationship

c) $(R_{eq})_D$ ratio curve based on damping

Fig 4. Determination of R_{eq} Based on Damping (from Sasanakul, 2005) plane thus the D_{eff} is calculated from:

$$D_{eff} = \frac{2}{\pi}\left[\frac{2\int_0^{\theta} T(\theta)d\theta}{\theta T(\theta)} - 1\right] \quad (8)$$

Procedure to obtain values of R_{eq} is as follows. The damping ratio, D_1 corresponding to a given shear strain, γ_1 can be obtained from Fig. 4(a). A value θ_1 that is associated with the D_{eff} value equal to D_1 is determined in Fig. 4(b). Then using Eq. (9), a value of R_{eq} is calculated from:

$$R_{eq} = \frac{1}{R}\frac{\gamma L}{\theta} \quad (9)$$

The R_{eq} values can be obtained and plotted for wide range of strains as shown in Fig. 4(c).

It is observed that the R_{eq} curves for soils with different G_{max} and γ_r always merge to the same value of 0.8 at low strain and the R_{eq} value decreases as the strain increases. The effect of soil nonlinearity can be accounted for by plotting the R_{eq} versus normalized rotation as presented in Fig. 5. The term θ_r is a reference rotation defined as:

$$\theta_r = \frac{\gamma_r L}{R} \quad (10)$$

The R_{eq} curve based on damping is also compared with the R_{eq} curves based on shear modulus. The procedure to determine R_{eq} curves based on shear modulus can be found in Sasanakul (2005). As shown in Fig. 5, the R_{eq} curve based on damping is significantly lower than the R_{eq} curves based on shear modulus, especially at high strains. The R_{eq} value suggested by Chen and Stokoe (1979) appears to be adequate for shear modulus but not suitable for damping. This result also suggests that using a singe value of R_{eq} for a wide range of strains is not appropriate for damping.

2.1.2 Modified Hyperbolic Model

Modified equivalent radius approach is extended to generate R_{eq} values for damping using modified hyperbolic model. The modified hyperbolic model proposed by Darendeli and Stokoe (1997) is presented as;

$$\tau = \frac{G_{max}\gamma}{1+\left(\frac{\gamma}{\gamma_r}\right)^a} \quad (11)$$

where, a = curvature coefficient.

Theoretical soils with the different curvature coefficients are used to generate the D-γ relationship. The D-γ relationship is developed by introducing Eq. (11) into Eq. (6) and performing integration. Fig. 6 shows the D-γ curve for modified hyperbolic model.

The torque-rotation relationship can be established by numerically integration of Eq. (3) using the stress-strain relationship from Eq. (11). The method of numerical integration relative to strain can be employed for the modified hyperbolic model. The D_{eq}-θ curve can be obtained from Eq. (8). Same procedure as the hyperbolic model is applied to determine the R_{eq} value presented in Fig.4. Fig. 7 shows the R_{eq} based on damping for the modified hyperbolic model using different curvature coefficients. The values of R_{eq} merge to approximately 0.8 at low strain similar to the conventional hyperbolic model. More variation is observed for different curvature coefficients at high strain level.

2.1.3 Ramberg-Osgood Model

Modified equivalent radius approach was also extended to

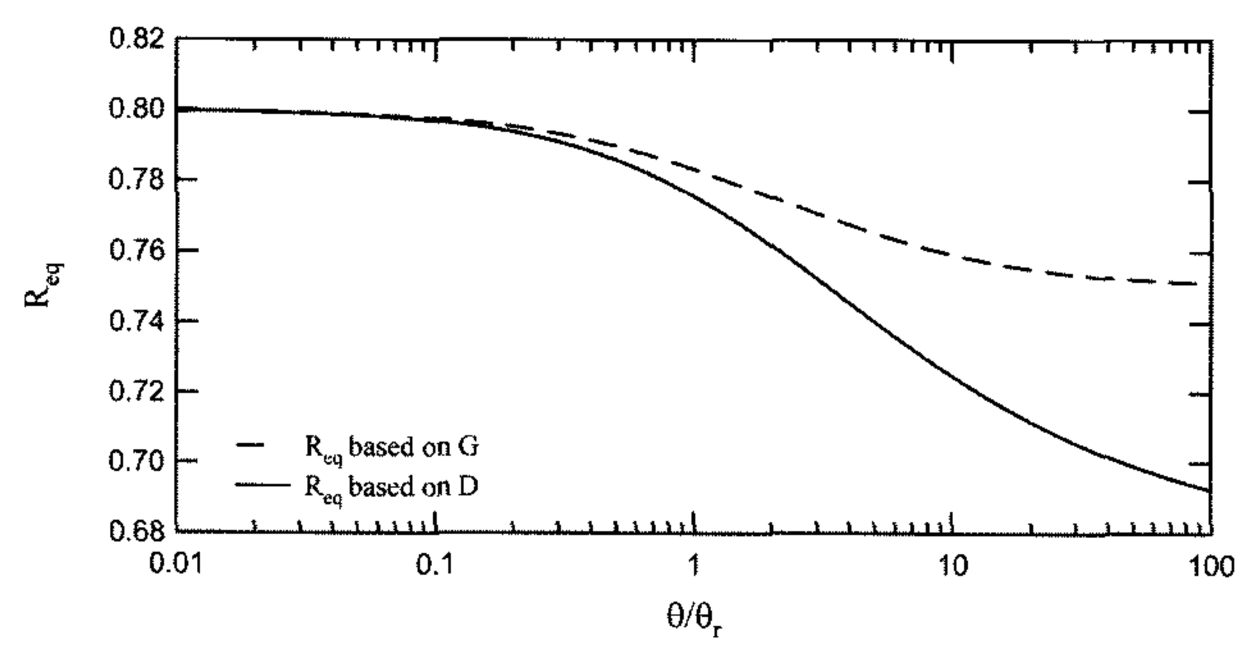


Fig 5. Normalized R_{eq} Curves Based on both Shear Modulus and Damping Obtained from the Hyperbolic Model (from Sasanakul, 2005)

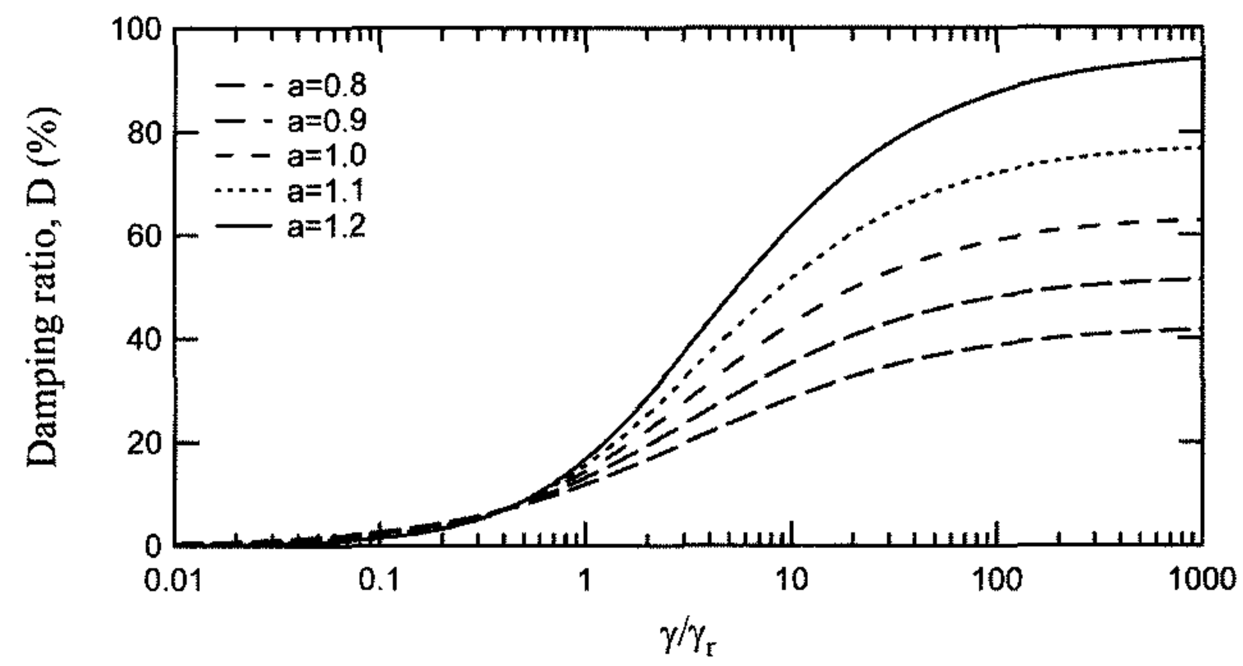


Fig. 6. Damping Ratio for the Modified Hyperbolic Model (from Bae, 2007)

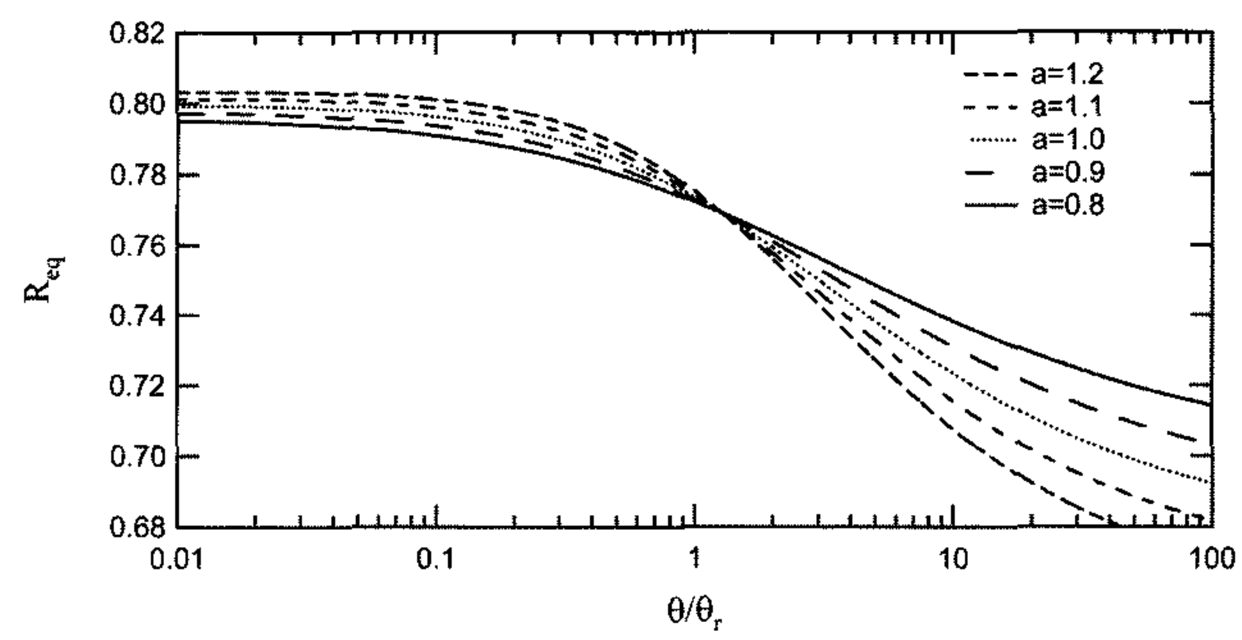


Fig. 7 Req curves Based on Damping for the Modified Hyperbolic Model (from Bae, 2007)

generate R_{eq} values for damping using the Ramberg-Osgood model. The Ramberg-Osgood model has been adopted and applied to soils by Idriss, et al. (1978). The stress-strain relationship described by the Ramberg-Osgood model is presented as;

$$\gamma = \frac{\tau}{G_{max}}\left(1+\alpha\left|\frac{\tau}{G_{max}\gamma_r}\right|^{b-1}\right) \tag{12}$$

Theoretical soils with different model parameters: a and b are used to develop the D-γ relationships presented in Fig. 8. It is noted that the value of damping approaches lower value at high strain when comparing with the other two soil models. Numerical integration is performed to generate the torque-rotation relationship and similar procedure as the other two soil models is used to obtain the R_{eq} value for the Ramberg-Osgood model.

Fig. 9 presents the R_{eq} based on damping for the Ramberg-Osgood model using different model parameters (a and b). Overall the value of R_{eq} for the Ramberg-Osgood model is in the same range as the other two models. Less variation of R_{eq} at high strain is observed in comparison with the modified hyperbolic models.

3. Conclusions

The modified equivalent radius approach provides improvement from the conventional equivalent radius approach for damping measurement in torsional test. It is clear that using a single value of R_{eq} to calculate strains corresponding to shear modulus and damping for a wide range of strains is not appropriate. In this study, the modified equivalent radius approach accounts for the variation of R_{eq} over the range of strains and the soil nonlinearity. Three different stress-strain soil models provide differences in values of R_{eq}. The modified equivalent radius approach provides flexibility to select the best model that shows the best match to the experimental data to represent the stress-strain relationship for a soil specimen.

There are two limitations of the modified equivalent radius approach that should be taken into consideration. First, the approach assumes only the hysteretic damping to represent the damping of soil but it is believed that the damping in soil consists of both hysteretic and viscous damping. There is no available damping model that can fully describe soil damping behavior. Second, the hysteresis loop is developed by assuming Masing behavior applies according the Masing rule. This assumption results in the asymptote of hysteretic damping has a value of $2/\pi$ or 63.7 percent. This value may not be realistic for the actual soil behavior.

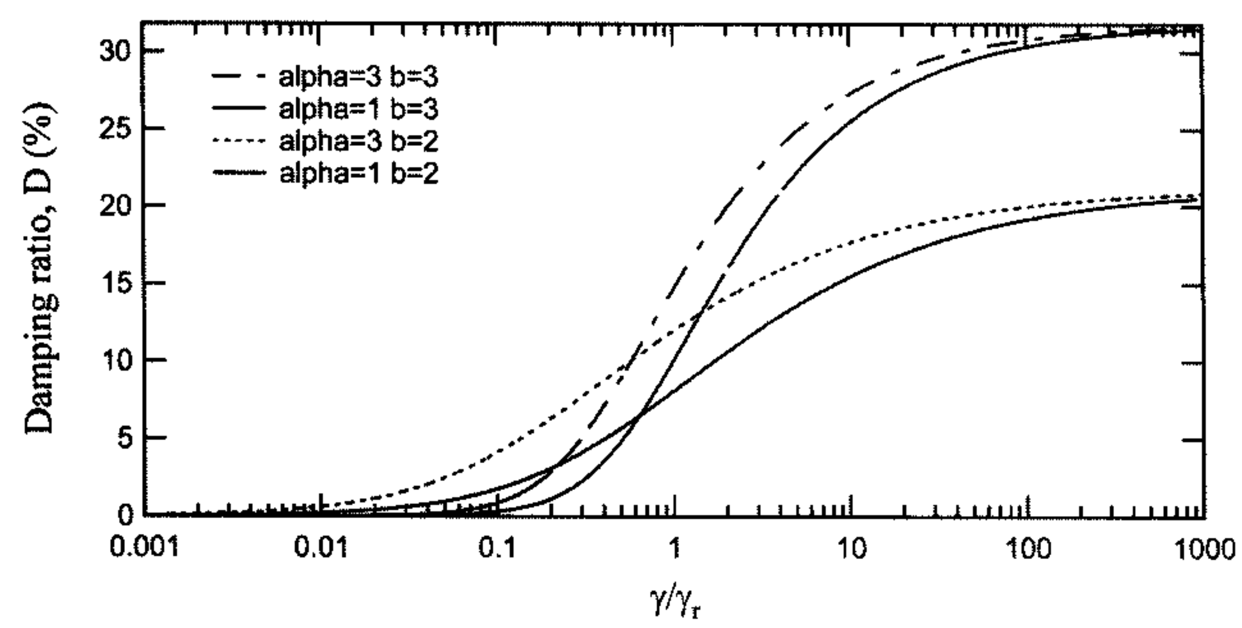


Fig. 8. Damping Ratio for Ramberg-Osgood Model (from Bae, 2007)

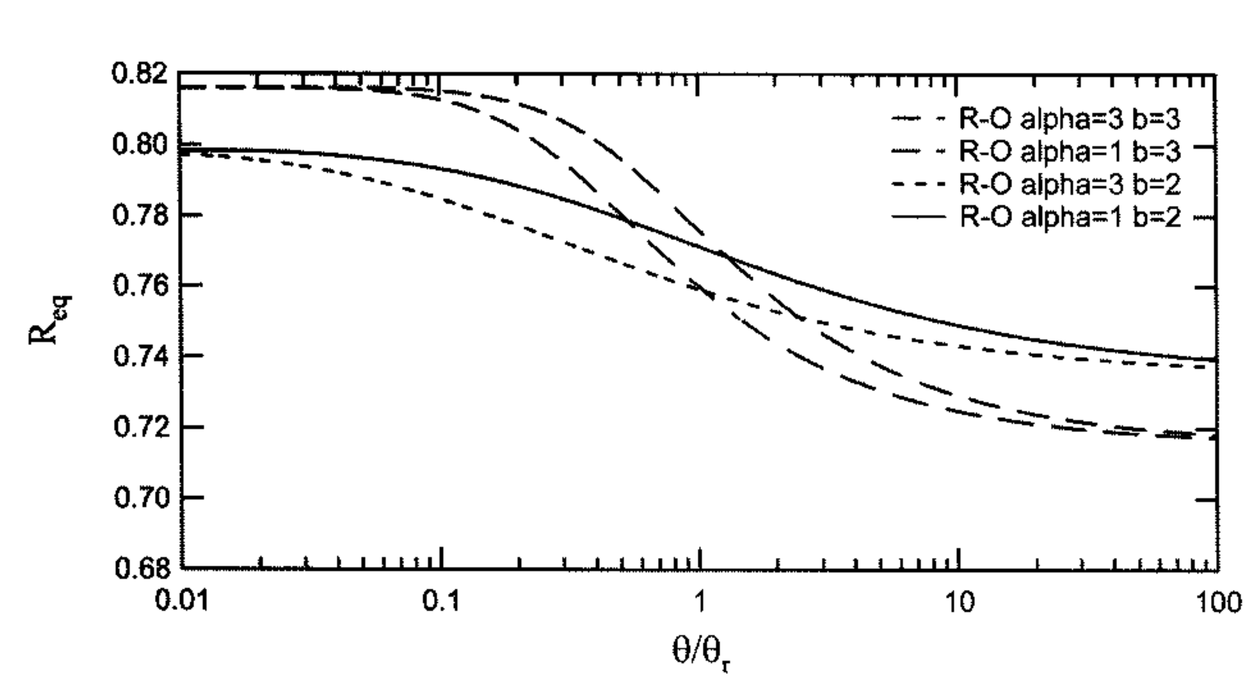


Fig. 9. R_{eq} Curves Based on Damping for the Ramberg-Osgood Model (from Bae, 2007)

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