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Revival of Phonons in High Tc Superconductors

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Abstract

We study the effects of phonon interaction on the superconducting pairing in the high T_c superconductors (HTSC). Using coupled BCS gap equations, we found that phonon interaction can induce a s-wave component to the d-wave gap, mediated by Antiferromagnetic (AFM) spin fluctuations, in the (D+iS) form. However, T_c is not enhanced compared to the pure d-wave pairing without phonon interaction. On the other hand, anisotropic phonon interaction can dramatically enhance the d-wave pairing and T_c itself, together with the AFM spin fluctuation interaction. This (D_{AFM}+ D_{ph}) type pairing exhibits strongly reduced isotope coefficient despite the large enhancement of T_c by phonon interaction.

Keywords : High Tc Superconductor, Pairing Mechanism, Phonons

I. Introduction

Phonons is usually the most dominant and important elementary excitations in solids. Therefore, physical properties of typical metals at low temperatures such as thermodynamic, transport, and optical properties, etc are mostly governed by the electron-phonon interactions. The phenomena of superconductivity (SC) in metals is not an exception and the electron-phonon interaction is known to be crucial to form "Cooper pairs" -- the electron bound states -- in conventional superconductors; we specifically use the terminology "conventional" for the phonon mediated s-wave pairing.

Let us briefly review the role of phonons for the superconducting pairing. According to the BCS theory -- which is until now the only proven fundamental mechanism for superconductivity – the electrons should form two-body bound states (Cooper pair) and then these macroscopically many Cooper pairs automatically condense at low temperature by statistics, which is the superconducting transition. Therefore, forming Cooper pairs is - in a loose sense - the necessary and sufficient condition for superconducting transition. In principle, there are various phonon modes in solids and all of them contribute to the electron-phonon interactions. However, in most of metals the dominant coupling is provided by one or two optical phonon modes and the coupling matrix between electrons and optical phonon mode(s) usually has very weak momentum dependence. Therefore, it is a good approximation to represent the coupling constant λ_{ph} induced by exchange of an optical phonon mode of ω_{ph} as

$$\lambda_{ph} \sim \frac{const}{\omega^2 - \omega_{ph}^2} \qquad (1)$$

Because the above form has a momentum independent attractive interaction for low energy $\omega < \omega_{ph}$, the s-wave orbital solution of the

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two-electron bound state (Cooper pair) becomes the lowest energy state. In combination with the singlet state of the spin part wave function, the superconductivity becomes a singlet s-wave state. During the early period of the high T_c superconductivity (HTSC) research, this phonon mediated s-wave superconducting state was a serious candidate for the HTSC state. Indeed several phonon modes of the oxygen octahedron with a copper ion at the center are likely to provide a strong coupling between the copper d-electrons and the optical phonon modes of oxygens.

In mid-90, after the pairing symmetry of the HTSC was confirmed as d-wave state by various experiments [1], the phonon scenario has quickly faded away and the problems of the HTSC have been mostly focused on the electronic correlations. Recent Angle Resolved Photoemission Spectroscopy (ARPES) experiments, however, revived the interest of the possibly important role of phonons in the high-T_c cuprates (HTC) [2]. In particular, the systematic measurements and analysis [3] of the kink structures in the quasiparticle dispersion near the Fermi surface (FS) convincingly showed that: (1) electron-phonon coupling is the strong candidate for the origin of the kink and its coupling is quite strong, (2) the typical energy of the phonon(s) is ~ 40 - 70 meV, and (3) the coupling matrix is quite anisotropic. Therefore, it is currently a pressing question what possible roles and effects, in particular, for the superconducting (SC) pairing, the phonon(s) can have on the HTSC.

Moreover, although the SC gap symmetry in the high-T_c cuprates is well established as a d-wave by most experiments, there are still continuous experimental reports such as tunneling conductance [4], penetration depth measurements [5], etc that provide rather convincing evidences for a small magnitude of a s-wave component in addition to the dominant d-wave SC gap in some of HTC compounds. Therefore, more specific question would be: is it possible to reconcile a d-wave gap and a s-wave gap together ? and in that case, what is the role of phonon(s) ? For these questions, we need first to know the mechanism of d-wave pairing. Unfortunately, there is yet no consensus on the pairing mechanism for the d-wave gap in HTC, but we believe and assume in this paper that the AFM spin fluctuations is the mediating glue for d-wave gap [6]. In this theory, simply the AFM spin fluctuations replaces the phonon interaction in the standard BCS theory. In contrast to the attractive phonon interaction, the AFM spin fluctuations provide all positive interaction between two electrons -- in momentum space and in the spin singlet channel. The crucial ingredient of the theory is then the sign changing order parameter (OP) of the d-wave gap which turns the positive interaction into an effectively attractive pairing interaction. On the other hand, the attractive phonon interaction (therefore negative in sign in momentum space) naturally leads to an isotropic s-wave pairing. With this standard pairing picture, it is clear that the AFM spin fluctuations and phonons are antagonistic and hard to cooperate for pairing because they tend to promote a different symmetry of the pairing gap each other.

An easy way to reconcile the d-wave and s-wave gap with two different pairing interactions would be to invoke two separate bands for each gap [7]. The similar idea for the two s-wave gaps was very successful for MgB_2 [8] because there two well separated bands exist. However, in HTSC, numerous experiments, in particular ARPES measurements [9], show that there is only one main band crossing FS. Then any idea of having s-wave and d-wave gaps in the single band, induced by two very different pairing interactions, seems too naive. However, if the coupling matrix of phonon interaction possesses a strong anisotropy, this combined pairing problem is not a trivial one and needs systematic investigation with a traceable formulation. Indeed, the ARPES experiments [3] provide very convincing evidence that the B_{1g} and half-breathing mode phonons are particularly strongly coupled with conduction electrons near the Fermi surface (FS) and have strongly anisotropic coupling matrices; the B_{1g} bucking mode couples mostly with antinodal quasiparticles (q.p.) and the half-breathing mode with nodal q.p.

In this paper, we took a simplest approach and studied the coupled gap equations for multiple gaps in

a single band with two pairing interactions, i.e., AFM interaction and anisotropic phonon interaction, within the BCS framework. By numerical solutions, we extensively investigated for the necessary conditions of the strength and the degree of anisotropy of phonon interaction for various multigap solutions. We found that (D+iS) type solutions are indeed possible with a proper degree of anisotropy of phonon interaction. However, T_c is not enhanced at all in this case even though an additional s-wave component is formed due to the phonon interaction. Another interesting possibility is that the anisotropic phonon interaction can induce an additional d-wave component (D_{ph}) to the AFM interaction induced d-wave component (D_{AFM}). As a result, T_c is dramatically enhanced due to the phonon interaction. We then derived an analytic T_c equation for this ($D_{AFM}+D_{ph}$) case and showed that phonon isotope effect can be strongly reduced due to the interplay between the AFM and phonon interactions despite the large increase of T_c by phonon. This result can explain a long standing puzzle of the small isotope effect in HTSC. In view of experiments [3], the best anisotropic phonon can be the B_{1g} buckling phonon mode which fulfills all necessary qualifications for our model. Similar conclusion was obtained by other authors [11] using different approaches.

II. Formalism

The interplay between phonons and electronic correlations in HTSC has been studied by numerous authors using different theoretical approaches and models [10-12] In this paper, we took a simple minded approach with a specific question for this problem. On the phenomenological basis, we consider a q.p. band and the AFM spin fluctuations as a dominant pairing interaction. In addition to that we add anisotropic phonon interactions and we study the pairing instability of the model for possible types of multi-gap solution using a generalized BCS gap equations. In this approach, still remaining interplay between spin fluctuations and phonons is ignored. The results of this paper should be taken with this caveat. The Hamiltonian is written as

$$H = \sum_{k\sigma} \epsilon(k) c^{\dagger}_{k\sigma} c_{k\sigma} + \sum_{kk'\uparrow\downarrow} V_{AFM}(k,k') c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} c_{k'\downarrow} c_{-k'\uparrow} \sum_{kk'\uparrow\downarrow} V_{ph}(k,k') c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} c_{k'\downarrow} c_{-k'\uparrow} \quad (2)$$

where $\epsilon(k)$ is the dispersion of the quasiparticles created by $c^{\dagger}_{k\sigma}$ as standard notation. $V_{AFM}(k,k')$ and $V_{ph}(k,k')$ are the effective interactions, for the singlet superconducting pairing channel, originating from the AFM spin fluctuations and phonon(s), respectively. For traceable numerical calculations, we further simplify the above Hamiltonian as follows. The real two dimensional FS is simplified as a circular FS and the interactions are also modelled accordingly as follows.

$$V_{AFM}(\Delta\phi) = V_M \frac{\phi_0^2}{(\Delta\phi \pm \phi_{AFM})^2 + \phi_0^2} \quad (3)$$

and

$$V_{ph}(\Delta\phi) = \begin{cases} -V_P & \text{for } |\Delta\phi| < \phi_{AN} \\ 0 & \text{for } |\Delta\phi| > \phi_{AN} \end{cases} \quad (4)$$

where $\Delta\phi = \phi - \phi'$ and $\phi_{AFM} = \pi/2$ representing the exchanged momentum $\mathbf{k} - \mathbf{k}'$ and the AFM ordering vector $\mathbf{Q}$ in the circular FS, respectively. V_M and V_P are chosen to be positive, so that V_{AFM} is all repulsive and V_{ph} is all attractive in momentum space.

The property of the AFM interaction with a short range correlation is simulated with the inverse correlation length parameter ϕ_0 ($\lambda = (\pi a/\sqrt{2})\phi_0$). In this paper, we chose ϕ_0=1 ($\lambda \sim 2a$) for all numerical calculations, which is quite a short range AFM correlation. The degree of anisotropy of the phonon interaction is controlled by the anisotropy angle parameter ϕ_{AN} which restricts the scattering angle between incoming and outgoing q.p momenta; for example, $\phi_{AN} = \pi$ would allow a perfectly

isotropic interaction. The phonon interaction $V_{ph}(\phi-\phi^{'})$, however, does not restrict the incoming (ϕ) and outgoing momenta ($\phi^{'}$). Now the reduced BCS Hamiltonian in the mean field theory can be written as

$$H = \sum_{\phi\xi\sigma}\epsilon(\xi)c^{\dagger}_{\phi\xi\sigma}c_{\phi\xi\sigma}+\sum_{\phi\xi}\Delta^{*}_{AFM}(\phi)c_{\phi\xi\downarrow}c_{-\phi\xi\uparrow} + \sum_{\phi\xi}\Delta^{*}_{ph}(\phi)c_{\phi\xi\downarrow}c_{-\phi\xi\uparrow} \quad (5)$$

where $\Delta_{AFM}(\phi)$ is the SC gap function induced by V_{AFM} and $\Delta_{ph}(\phi)$ is the one induced by V_{ph}. The two gap functions $\Delta_{AFM}(\phi)$ and $\Delta_{ph}(\phi)$ may or may not have the same symmetry. After diagonalizing the above Hamiltonian we obtain two self-consistent equations as

$$\Delta_{AFM}(\phi) = \sum_{\phi^{'}\xi}V_{AFM}(\phi-\phi^{'})<c_{\phi\xi\downarrow}c_{-\phi\xi\uparrow}>,$$
$$\Delta_{ph}(\phi) = \sum_{\phi^{'}\xi}V_{ph}(\phi-\phi^{'})<c_{\phi\xi\downarrow}c_{-\phi\xi\uparrow}>. \quad (6)$$

Similar coupled gap equations were studied in previous studies [7]. The key difference is that our model has only one band. Then because the same band electrons should form the multigaps, there is severe competition between different gaps and more constraint to allow multigap solutions. In the next sections, we will consider possible multigap solutions of the above coupled gap equations.

III. (D+iS) case

In this case, we assume $\Delta_{AFM}(k)=\Delta_d\cos(2\phi)$ and $\Delta_{ph}=i\Delta_s$. Accordingly, $\Delta_t(\phi)=\Delta_d\cos(2\phi)+i\Delta_s$ and $E(\phi)=\sqrt{\xi^2+\Delta_d^2(\phi)+\Delta_s^2}$. The gap equations are

$$\Delta_d(\phi) = -\sum_{\phi^{'}}V_{AFM}(\phi-\phi^{'})N(0)\int_0^{\omega_{AFM}}d\xi\frac{\Delta_t(\phi^{'})}{E(\phi^{'})}\tanh(\frac{E(\phi^{'})}{2T}),$$
$$i\Delta_s(\phi) = -\sum_{\phi^{'}}V_{ph}(\phi-\phi^{'})N(0)\int_0^{\omega_{ph}}d\xi\frac{\Delta_t(\phi^{'})}{E(\phi^{'})}\tanh(\frac{E(\phi^{'})}{2T}). \quad (7)$$

where $N(0)$ is the density of states (DOS) at FS and $\omega_{AFM,ph}$ are the BCS energy cutoffs of each pairing interactions V_{AFM} and V_{ph}, respectively. Because of the phase i between $\Delta_d(\phi)$ and Δ_s, two gaps are only indirectly coupled through the quasiparticle energy $E(\phi)$ in the pair susceptibility $\chi(\phi,\omega_{AFM,ph})=N(0)\int_0^{\omega_{AFM,ph}}d\xi\frac{\tanh(\frac{E(\phi)}{2T})}{E(\phi)}$. Therefore the coupling of two gaps is very gentle. A consequence of it is that the gap equations can allow two transition temperatures $T_{c,D}$ and $T_{c,S}$ for each gap Δ_D and Δ_S, respectively. This feature is shown in Fig. 1. The normalized d-wave gap Δ_D/ω_{AFM} and the s-wave gap Δ_S/ω_{AFM} are plotted for different values of anisotropy angle $\phi_{AN}=\pi/4,\pi/2$ and π with fixed values of coupling constants $N(0)V_M=2$ and $N(0)V_P=0.55$. The effective dimensionless coupling constants $\lambda_{AFM,ph}$ are smaller than theses values which is defined below as a projected average with a SC gap function.

$$\lambda_{AFM,ph}=N(0)\frac{\sum_{\phi,\phi^{'}}V_{AFM,ph}(\phi-\phi^{'})\eta(\phi)\eta(\phi^{'})}{\sum_{\phi}\eta^2(\phi)} \quad (8)$$

where $\eta(\phi)=\cos(2\phi)$ for d-wave gap and $\eta(\phi)=1$ for s-wave gap. For the chosen potential strengths $N(0)V_M=2$ and $N(0)V_P=0.45$ in Fig.1, the effective coupling strengths are $\lambda_{AFM,D}=0.332$ and $\lambda_{ph,S}=0.55$, 0.414, 0.243 for $\phi_{AN}=\pi,\pi/2,\ \pi/4$, respectively.

Main features are summarized as: (1) when $\lambda_{ph,S}$ is too weak, the effect of phonon interaction is totally ignored and the gap and T_c is solely determined by the AFM interaction (see $\phi_{AN}=\pi/4$ case). (2) when $\lambda_{ph,S}$ is much stronger than $\lambda_{AFM,D}$, the d-wave gap is totally suppressed and the gap and T_c

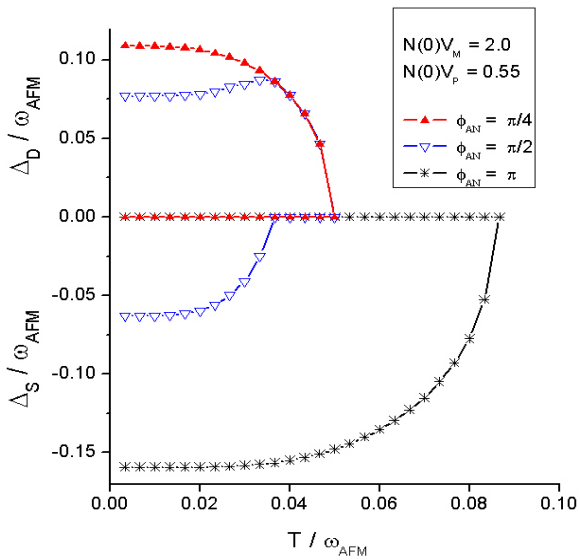

Fig. 1. (D+iS) case. Calculated magnitudes of d-wave components (Δ_D/ω_{AFM}) and s-wave components (Δ_S/ω_{AFM}) for different anisotropy angles $\phi_{AN} = \pi/4, \pi/2,$ and π. For all cases $N(0)V_M = 2.0$, $N(0)V_P = 0.5$ and ω_{ph}/ω_{AFM}=0.5.

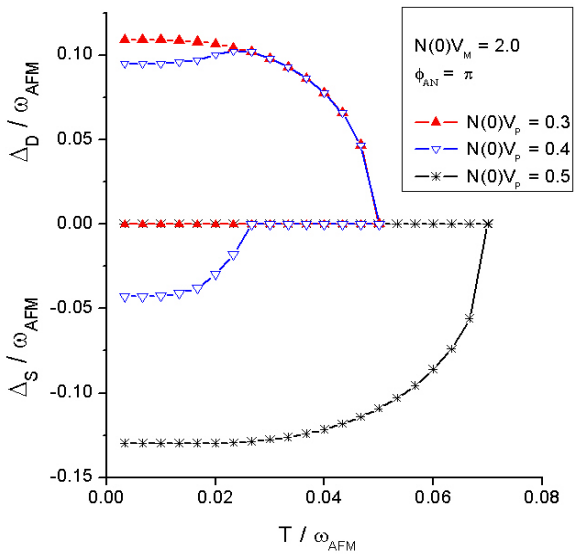

Fig. 2. (D+iS) case. Calculated magnitudes of d-wave component (Δ_d/ω_{AFM}) and s-wave component (Δ_s/ω_{AFM}) for different strength of phonon interaction $N(0)V_P = 0.3, 0.4,$ and 0.5 . For all cases $N(0)V_M = 2.0$, $\phi_{AN} = \pi$, ω_{ph}/ω_{AFM}=0.5.

is determined solely by phonon interaction (see $\phi_{AN} = \pi$ case). (3) when the effective coupling strengths are comparable, namely $\lambda_{AFM,D} \approx \lambda_{ph,S}$, both d-wave and s-wave gaps have separate transition temperatures $T_{c,D}$ and $T_{c,S}$ and two gaps can coexist at low temperatures (see $\phi_{AN} = \pi/2$ case). In this case, $T_{c,D}$ is always higher transition temperature and this transition temperature is just the same as the pure d-wave T_{c0} without phonon interaction. At the lower transition temperature $T_{c,S}$, the s-wave gap $i\Delta_S$ appears and the magnitude of d-wave gap Δ_D accordingly decreases.

To clarify the roles of the anisotropy and the strength of phonon interaction, in Fig.2 we fixed anisotropy angle as $\phi_{AN} = \pi$, which simulates a perfectly isotropic phonon interaction, and varied the interaction strength. Fig.2 shows the results of these calculations. With $N(0)V_M$ =2.0 (λ_{AFM}=0.332), we varied $N(0)V_P$ (= 0.3, 0.4, 0.5; accordingly λ_s=0.3, 0.4, 0.5). This plot demonstrates that the anisotropy of phonon interaction doesn't play a particular role in the case of the (D+iS) type gap. The mixed gap solution of the (D+ iS) type is still possible when $\lambda_{AFM,D}$ and $\lambda_{ph,S}$ are of the comparable strength regardless of the anisotropy of phonon interaction.

The summary for the (D+iS) type gap is: (1) The mixed type gap solution is possible if the phonon interaction $\lambda_{ph,S}$ is comparable but still subdominant to the AFM interaction strength $\lambda_{AFM,D}$. (2) Separate two transition temperatures $T_{c,D}$ and $T_{c,S}$ exist for each gap Δ_D and Δ_S; $T_{c,D}$ is always higher than $T_{c,S}$ and only below $T_{c,S}$ two gaps coexist. (3) T_c is not enhanced as far as the d-wave gap remains finite regardless of the presence and magnitude of the s-wave gap component. (4) Anisotropy of the phonon interaction is not particularly necessary.

IV. (D_{AFM}+D_{ph}) case

This is the case that phonon interaction also supports the d-wave pairing. Because it is clear that a

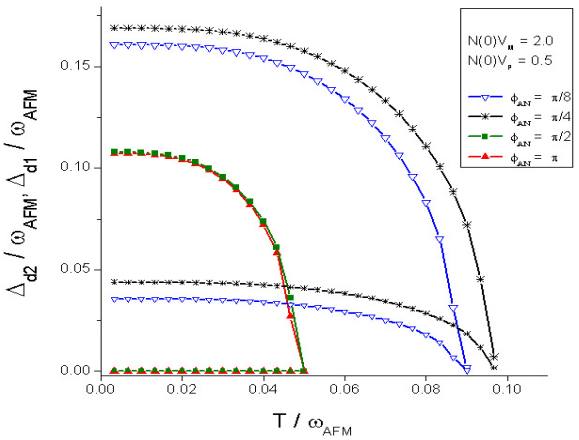

Fig. 3. (D_{AFM}+D_{ph}) case. Calculated magnitudes of D_{AFM} component (Δ_{d1}/ω_{AFM}) and D_{ph} component (Δ_{d2}/ω_{AFM}) for different anisotropy angles $\phi_{AN} = \pi/8, \pi/4, \pi/2,$ and π. For all cases $N(0)V_M = 2.0$, $N(0)V_P = 0.5$ and ω_{ph}/ω_{AFM}=0.5.

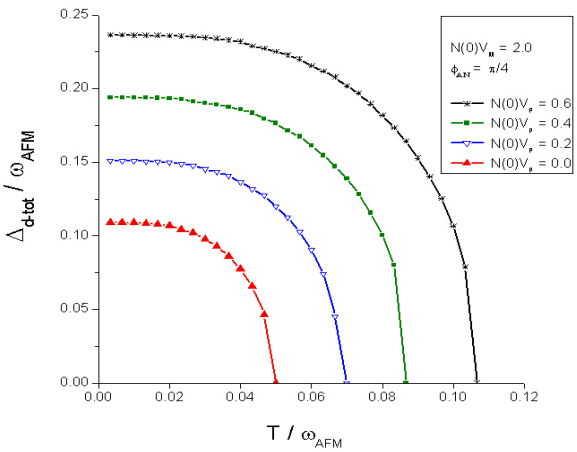

Fig. 4. (D_{AFM}+D_{ph}) case. Calculated magnitudes of the total d-wave gap ($\Delta_{d-tot}/\omega_{AFM}$) for different strength of phonon interaction $N(0)V_P = 0.0, 0.2, 0.4,$ and 0.6. For all cases $N(0)V_M = 2.0$, $\phi_{AN} = \pi/4$ and ω_{ph}/ω_{AFM}=0.5.

perfectly isotropic phonon interaction has null effect on the d-wave gap, anisotropy of the phonon interaction is crucial for this case. The total gap is, therefore, $\Delta_t(\phi) = (\Delta_{d1} + \Delta_{d2})\cos(2\phi)$ and $E(\phi) = \sqrt{\xi^2 + \Delta_t^2(\phi)}$. The gap equations are written as

$$\Delta_{d1}(\phi) = -\sum_{\phi'} V_{AFM}(\phi - \phi')N(0)\int_0^{\omega_s} d\xi \frac{\Delta_t(\phi')}{E(\phi')}\tanh(\frac{E(\phi')}{2T}),$$
$$\Delta_{d2}(\phi) = -\sum_{\phi'} V_{ph}(\phi - \phi')N(0)\int_0^{\omega_D} d\xi \frac{\Delta_t(\phi')}{E(\phi')}\tanh(\frac{E(\phi')}{2T}). \quad (9)$$

The above two gap equations can be combined to a single gap equation but we keep the separate form in order to trace the effect of the phonon interaction. In contrast to the previous cases, *T_c can be dramatically enhanced* when the phonon interaction $V_{ph}(\Delta\phi)$ has a proper degree of anisotropy. Other authors [11] also obtained a similar result using different approaches. In Fig.3, we plot Δ_{d1} and Δ_{d2} separately; between the same symbols the smaller value is the phonon induced d-wave gap Δ_{d2} and the larger one is the AFM induced d-wave gap Δ_{d1}. We change the anisotropy angle of $V_{ph}(\Delta\phi)$ with the fixed interaction strengths of $N(0)V_M = 2.0$ ($\lambda_{AFM,D} = 0.332$) and $N(0)V_p = 0.5$. For the anisotropy angle $\phi_{AN} = \pi/2$ and π, the phonon interaction has absolutely no effect on the d-wave pairing; this is simply because the d-wave projected average Eq.(8) becomes zero for these two commensurate angles with our model potential Eq.(4). With a stronger anisotropy ($\phi_{AN} = \pi/4$; $\lambda_{ph,D} = 0.12$), the phonon scattering sees only a limited part of the d-wave gap, therefore the d-wave gap can be seen effectively as a s-wave gap for the anisotropic phonon. For a even stronger anisotropy ($\phi_{AN} = \pi/8$; $\lambda_{ph,D} = 0.098$), too narrow scattering angle reduces the effective coupling strength. In our model potential, $\phi_{AN} = \pi/4$ is the optimal anisotropy. In fact, even weaker anisotropy of $\pi/4 < \phi_{AN} < \pi$ – except the special commensurate angles $\phi_{AN} = \pi/2$ and π – produces some uncompensated effective interaction $\lambda_{ph,D}$ and boosts the d-wave pairing. This is an artifact of the model potential Eq.(8) which always emphasizes the small angle scattering. Therefore, for some real phonons, it is possible to have a repulsive (destructive for d-wave pairing) $\lambda_{ph,D}$ if large angle (around $\Delta\phi = \pi/2$) scattering overweighs small angle scattering.

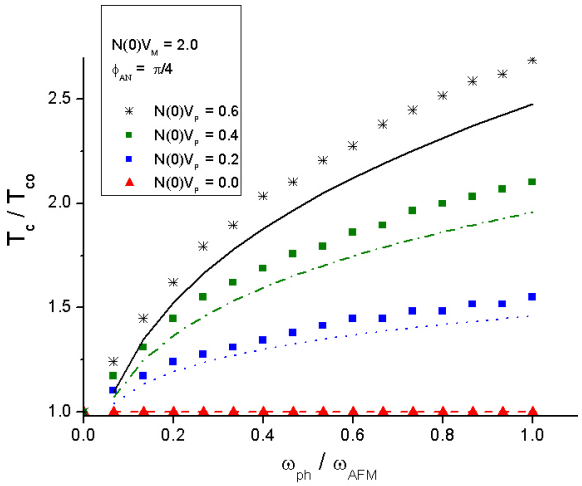

Fig. 5. (D_{AFM}+D_{ph}) gap. T_c (symbols) calculated as a function of ω_{ph}/ω_{AFM} for different strength of phonon interaction $N(0)V_P = 0.0, 0.2, 0.4$, and 0.6, normalized by T_{c0} the transition temperature with $N(0)V_P = 0.0$. For all cases, $N(0)V_M = 2.0$ and $\phi_{AN} = \pi/4$. Lines are the results of the analytic formula of T_c Eq.(10). The symbols and lines of the same colors have the same parameters set.

Fig.4 shows the results of total gap $\Delta_{d-tot} = \Delta_{d1} + \Delta_{d2}$ with the optimal anisotropy angle $\phi_{AN} = \pi/4$ and varying phonon coupling strength $N(0)V_P$ =0.0, 0.2, 0.4, and 0.6 ($\lambda_{ph,D} = 0.0, 0.047, 0.095,$ and 0.142, respectively). The results demonstrate that the relatively weak phonon interaction $\lambda_{ph,D}$ can significantly increase T_c from the pure AFM interaction induced T_{c0}.

Considering the experimental findings about phonons in the high-T_c cuprates, the best candidate for the anisotropic pairing phonon is B_{1g} buckling mode of the plane oxygen motion [3]. Devereaux and coworkers extensively analyzed the behaviors of B_{1g} mode and found : (1) B_{1g} mode has a strong anisotropic coupling matrix element concentrating around antinodal points; (2) the maximum coupling strength can reach as high as $\lambda \sim 3$. Compared to our model calculations, the optimal anisotropy for the d-wave pairing $\phi_{AN} = \pi/4$ is just about the same degree of anisotropy of the B_{1g} mode. Regarding the coupling strength, for example, in Fig.4 the maximum coupling strength λ_{max} is $N(0)V_p$ =0.6 and the corresponding average strength is $\lambda_{ph,D}$= 0.142. These values are much smaller than the ones extracted by Devereaux and coworkers [3].

Since we found that an anisotropic phonon can dramatically enhance T_c of d-wave pairing, we have to consider the isotope effect of the phonon, which is reported to be anomalously small, in particular, near optimal doping region by many experiments [13] . For this purpose, Fig.5 shows the calculated T_c's (symbols) from Eq.(9) as a function of the phonon energy cutoff ω_{ph}/ω_{AFM} for various phonon coupling strength $N(0)V_P = 0.0, 0.2, 0.4$ and 0.6, respectively. The BCS theory predicts $T_c \sim \omega_{ph}$ ($\alpha = 0.5$), which should show up as a linear lines in Fig.5. Our numerical results show much weaker power than the linear one indicating $\alpha < 0.5$. For more analytic investigation for the isotope effect and the origin of the T_c enhancement by phonon in the (D_{AFM}+D_{ph}) case, we derived an analytic T_c equation. The gap equations Eq.(9) can reduce to a single T_c equation by adding two equations and taking a limit of $\Delta_{d1}, \Delta_{d2} \to 0$. The pair susceptibility $\chi(\omega_{AFM,ph})$ is integrated out using BCS approximation which is $\int_0^{\omega_D} d\xi[\tanh \frac{\xi}{2T}]/\xi \approx \ln[2C\ \omega_D/\pi T]$ (C = $e^{\gamma} \approx 1.7807$), valid when $\omega_D/2T_c \gg O(1)$. Finally we obtained the T_c formula of the (D_{AFM}+D_{ph}) case as

$$T_c \simeq 1.13\ \omega_{AFM}^{\tilde{\lambda}_{AFM}} \cdot \omega_{ph}^{\tilde{\lambda}_{ph}}\ e^{-1/\lambda_t}. \qquad (10)$$

where $\lambda_t = (\lambda_{AFM} + \lambda_{ph})$, $\tilde{\lambda}_{AFM} = \lambda_{AFM}/\lambda_t$ and $\tilde{\lambda}_{ph} = \lambda_{ph}/\lambda_t$, and λ_{AFM} and λ_{ph} are the dimensionless effective coupling constants obtained from Eq.(8) with d-wave gap average for both couplings. ω_{AFM} and ω_{ph} are the energy cutoffs of the AFM interaction and phonon interaction, respectively. As mentioned above, this T_c formula is derived assuming $\omega_{AFM,ph}/2T_c \gg O(1)$ and this condition is well satisfied when $\lambda_t < O(1)$. In Fig.5, we also plot the results of T_c's from Eq.(10) (lines)

with the same parameters and compare them with the numerical calculations (symbols) from Eq.(9). It shows a reasonably good agreement between two results although the deviations increase with increasing the coupling constant $N(0)V_p$ (or $\lambda_{ph,D}$) as expected. However, the overall and quantitative behavior of T_c is well captured by Eq.(10).

Now we are in position to read the phonon isotope coefficient α from Eq.(10), which is

$$\alpha = \frac{1}{2}\tilde{\lambda}_{ph} = \frac{1}{2}\frac{\lambda_{ph}}{\lambda_{AFM} + \lambda_{ph}}. \qquad (11)$$

For example, with representative values of $\lambda_{AFM} = 0.33$, $\lambda_{ph} = 0.1$ and $\omega_{ph}/\omega_{AFM} = 0.5$, we obtain $\alpha \approx 0.116$, which is pretty small value compared to the standard BCS value of 0.5 while T_c is enhanced by about 100% from T_{c0} the transition temperature without phonon interaction (see Fig.5; $N(0)V_P = 0.6$ is $\lambda_{ph} = 0.142$). This isotope coefficient equation Eq.(14) provides a very plausible resolution why the isotope coefficient is so low near optimal doping region where T_c is the highest and increases toward underdoped regime (decreasing T_c). The phonon coupling strength (λ_{ph}) is likely unchanged with doping but the effective coupling strength of the AFM mediated interaction (λ_{AFM}) will change sensitively with doping. If we assume that λ_{AFM} increases with increasing doping but λ_{ph} remains constant, Eq.(14) describes the general trend of the experimentally observed oxygen isotope effect α_O [13].

V. Conclusions

In this paper we studied the effects of phonon interaction on the superconducting pairing in the background of d-wave gap already formed by the AFM interaction. In particular, we studied the role of anisotropy of the phonon interaction and possible multigap type solutions within a generalized BCS theory. For many cases the anisotropy of the phonon interaction is a crucial condition for interesting interplay with the AFM interaction but not always; the (D+iS) type gap is rather insensitive with the anisotropy of phonon interaction because two gaps are more gently coupled in the (D+iS) case. Contrary to an expectation, T_c is not enhanced at all by the additional s-wave component induced by phonon interaction as far as the d-wave gap component remains finite.

Anisotropic phonon can boost T_c together with the AFM interaction in the (D_{AFM}+D_{ph}) type solution. With numerical calculations and analytic T_c equation, we showed that T_c is enhanced dramatically following the modified BCS exponential form $T_c \sim \exp(-1/\lambda_t)$ with the total coupling strength $\lambda_t = \lambda_{AFM} + \lambda_{ph}$. This type of solution can also explain how and why the phonon isotope effect is strongly reduced despite the large enhancement of T_c by phonon. Our isotope coefficient formula Eq.(11) provides a good description of the overall trend of oxygen isotope coefficient α_O [13]. Although we didn't show in this paper, the combined type gap solution, namely (D_{AFM}+D_{ph} +iS$_{ph}$) gap, is possible and this solution displays the combined features: T_c enhancement and a minor s-wave component to the main d-wave component gap. This work will be reported elsewhere.

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References

[1] D. J. Van Harlingen, Rev. Mod. Phys. **67**, 515 (1995).

[2] X.J.Zhou et al., cond-mat/0604284 (unpublished); W. Meevasana et al., Phys. Rev. B **75**, 174506 (2007).

[3] T.P. Devereaux et al., Phys. Rev. Lett. **93**, 117004 (2004); T. Cuk et al., Phys. Rev. Lett. **93**, 117003 (2004).

[4] Kohen, G. Leibovitch, G. Deutscher, Phys. Rev. Lett. **90**, 207005 (2003).

[5] R. Khasanov et al., Phys. Rev. Lett. **98**, 057007 (2007); R. Khasanov et al., Phys. Rev. Lett. **99**, 237601 (2007).

[6] V. Chubukov, D. Pines, J. Schmalian, in 'The Physics of Conventional and Unconventional Superconductors' edited by K.H. Bennemann and J.B. Ketterson (2002); P. Monthoux, A. V. Balatsky, and D. Pines, Phys. Rev. Lett. **67**, 3448 (1991).

[7] Bussmann-Holder et al., Euro. Phys. Lett., **77**, 27002 (2007).

[8] Y. Liu, I. I. Mazin, and J. Kortus, Phys. Rev. Lett. **87**, 087005 (2001); H. Suhl, B. T. Matthias, and L. R. Walker, Phys. Rev. Lett. **3**, 552 (1959).

[9] Shen RMP A. Damascelli, Z. Hussain, and Z.X. Shen Rev. Mod. Phys. **75**, 473 (2003).

[10] W. Meevasana et al., Phys. Rev. B **75**, 174506 (2007).

[11] Honerkamp, H.C. Fu, D.H. Lee, Phys. Rev. B **75**, 014503 (2007); T.S. Nunner,J. Schmalian, and K.H. Bennemann, Phys. Rev. B **59**, 8859 (1999).

[12] N. Bulut and D.J. Scalapino, Phys. Rev. B **54**, 14971 (1996); A. Nazarenko and E. Dagotto, Phys. Rev. B **53**, 2987 (1996); R. Zeyher and G. Zwicknagl, Z. Phys. B **78**, 175 (1990).

[13] R. Khasanov et al., Phys. Rev. B **74**, 064504 (2006); H. J. Bornemann and D. E. Morris, Phys. Rev. B **44**, 5322 (1991); J.P. Franck et al., Phys. Rev. B **44**, 5318 (1991).