최소생애주기비용을 위한 PSC보 보강의 최적설계

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Optimal Design of the PSC Beam Reinforcement for Minimum Life-Cycle Cost

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Abstract : To optimize the selected reinforcing method for application to PSC Beam bridges, the reliability analysis was performed with consideration for the increase and decrease of the member section based on the standard design section, and the minimum life-cycle cost(LCC) was calculated from this analysis with consideration for the aleatory uncertainty. Moreover, the mean, 50%, 75%, and 90% distributions of the analysis results were re-evaluated quantitatively by considering the effect of the epistemic uncertainty. The reliability results gained from the application of the reinforcing method, as well as the optimal design method based on the minimum LCC, will provide more reasonable design criteria for the PSC Beam bridges.

초 록 : PSC보의 보강 시에 안전도분석을 위한 신뢰성해석과 생애주기비용을 최소화하기 위한 최적설계를 실시하였다. 신뢰성해석은 현재 공용중인 표준단면을 변형시키면서 실시하며 자연적 불확실성과 인위적 불확실성을 고려하였다. 자연적(내재적) 불확실성을 고려하여 최소생애주기비용을 구한 후에, 인위적 불확실성을 최대 90%의 분산까지 고려하여 재해석을 실시하였다. 보강방법에 대한 신뢰성해석과 최소생애주기비용해석은 보강방법을 결정하는데 매우 합리적인 기준을 제시하였다.

Key Words : reinforcing method, minimum life-cycle cost, aleatory and epistemic uncertainties, reliability analysis, optimal design

1. Introduction

Considering the traffic volume and weight of vehicles passing over the bridges currently under use, a method for rapid reconstruction and reasonable reinforcement is urgently required for PSC girder bridges, which account for the highest ratio in deterioration among concrete bridges[1]. However, since the replacement of bridges experiencing deterioration requires huge economic and social costs, it is more reasonable to reconsider the service lifetime extension of deteriorated bridges, as well as the economic efficiency, by selecting a proper reinforcing method and by performing a optimal reinforcement design that reflects the maintenance equipment and the serviceability of the bridges[2-4].

The safety assessment should be conducted using a reliability evaluation method that takes into consideration the effects of uncertainty built into structures[5,6]. However, it is critical for the reliability evaluation to reassess in greater detail the influence of the two uncertainties, aleatory and epistemic; where the former is associated with the randomness of nature and induces a probability of failure or risk, whereas the latter gives rise to a range of possible values of the calculated probability or risk[5]. In terms of economic efficiency and safety, structural design can reaso-

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nably be performed by finding the optimal design scheme based on the minimum life-cycle cost(LCC)[7]. However, the influence exerted on the minimum LCC calculation by the uncertainties included in the initial, maintenance and damage costs must also be re-evaluated[8].

In this study, the optimal design is conducted based on the minimum LCC as the following research of the previous results on the reliability analysis of To-Box reinforcing method[6,9].

2. Structural Safety and Minimum Life-Cycle Cost(LCC)

2.1. Determination of Optimal Design

The required acceptable safety level may be determined on the basis of the minimum expected LCC with appropriate adjustments for risk aversion. The expected LCC represents the overall average cost of a structure from its construction stage to the end of its life. Currently, the concept of the expected LCC has been applied to the design of buildings and bridges, as well as to the maintenance of individual bridges and bridge networks.

The minimum LCC design of a structural system may be formulated as follows:

$$E(LCC) = C_I + E(C_M) + E(C_D) \qquad (1)$$

where, $E(LCC)$ = the expected LCC

C_I = the initial cost of a structure or system

$E(C_M)$ = the expected maintenance cost over the lifetime of the structure

$E(C_D)$ = the expected damage cost over the lifetime of the structure, and $C_I = C_{ID} + C_{IC} + C_{IT}$

where, C_{ID} = the design costs of a structure

C_{IC} = the construction costs of the structure

C_{IT} = the load testing costs before use of the structure

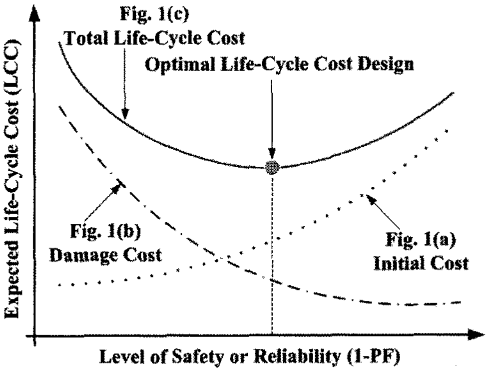

Fig. 1. Minimum Life-Cycle Cost(LCC) Design.

The above concept can be portrayed graphically as shown in Fig. 1. Clearly, the initial cost of a structure will increase with increasing safety or reliability, $1\text{-}P_F$, as shown in Fig. 1(a), whereas, conversely, the expected life-time damage cost of the same structure will decrease with $1\text{-}P_F$, as illustrated in Fig. 1(b). The minimum $E(LCC)$ is determined by adding the ordinates of Fig. 1(a) and 1(b), as indicated in Fig. 1(c). Furthermore, in Eq. (1), the damage cost C_D is generally composed of several cost items, C_i.

$$CD = \sum_{1}^{k} Ci \qquad (2)$$

It is reasonable to assume that there will be epistemic uncertainties in estimating each of the expected cost items in Eq. (1). In estimating the LCC, there is seldom sufficient data or information to evaluate the expected maintenance or damage cost. Therefore, estimating the expected maintenance cost may require reliance on past experiences and judgments gained from similar structures. In this study, the expected maintenance cost of a PSC Beam bridge is estimated from information of economic reports and maintenance ordinances for bridge safety in Korea.

2.2. Significance and Treatment of Uncertainties

The formulation outlined above contained unavoidable uncertainties in the structural capacity, loadings, member stiffness and cost item estimation. It is therefore important to distinguish between two broad types of uncertainty that have different respective significances: aleatory and epistemic. The former is associated with natural randomness and its effect leads to an underlying probability of failure, while the latter is associated with imperfections in modeling the real world and thus its effect leads to a range of possible values of the calculated failure probability(i.e., an uncertainty in the calculated failure probability). The probability of failure, P_F, of the structure, therefore, should reflect only the effects of the randomness in the applied loads and the structural capacities. However, because of the epistemic uncertainties in estimating the mean or median values of the applied loads and structural capacities, the true failure probability may be represented as a random variable.

The optimal LCC design may be determined based on the expected LCC, as outlined above, and will remain the same using other percentile LCC in the optimization process. However, the true failure probability(or reliability) of the optimal design will be a random variable because of the epistemic uncertainties in the structural loading and capacity. For design purposes, a conservative value of the calculated failure probability, or safety index, may be preferred. For example, the 75% or 90% safety index of the minimum *E*(*LCC*) design may be adopted in the formulation of criteria to insure sufficient risk aversion.

3. Optimal Design on the Reinforcement of PSC Beam Bridges

3.1. Determination of Initial Cost

In order to determine the optimal design based on minimum expected LCC, seven alternative designs were considered, including the standard one based on the reinforcing scheme, by increasing and decreasing the member sections relative to the scheme[6,9]. The To-Box reinforcement cost for the PSC beam bridge was assumed by logically considering the area of the reinforcement panel[10]. In the reinforcing scheme, the initial cost was assumed based on information from construction reports of the bridge and related data, to be 114% of the initial cost for the non-reinforcement design. All of the initial costs for the scheme and those of the different alternative designs are shown in Table 1(in US dollars).

Table 1. Initial Costs of Alternative Designs[in million USD]

Design	Initial Cost, C_I
80% of Standard Design	67.341
90% of Standard Design	71.136
95% of Standard Design	72.248
Standard Design of Scheme 4	74.100
105% of Standard Design	77.805
110% of Standard Design	81.510
120% of Standard Design	92.625

3.2. Determination of Expected Damage Cost

The expected damage cost includes all the tangible and intangible values resulting from a structural damage or failure of PSC beam bridges(including the cost associated with the closing of the bridge to traffic). Even though bridge collapse is highly unlikely under normal circumstances, the damage cost must include this possibility, as well as the insurance cost[11]. Therefore, the expected damage cost, C_D, may consist of the following several components C_i:

$$C_D = C_{FR} + C_{FL} + C_{FH} + C_{FD} + C_{FEN} \tag{3}$$

where, C_{FR} = bridge replacement cost
C_{FL} = loss of lives and equipment costs
C_{FH} = culture and historical costs
C_{FD} = functional disruption costs
C_{FEN} = environmental and social costs

Specifically, in estimating the LCC of the PSC beam bridge, the initial cost items, plus the maintenance cost and damage cost items as percentage of the initial cost, can be summarized as shown in Table 2.

Table 2. Total Expected Life-Cycle Cost(LCC) Items C_D

Cost Items	Classification of Cost Items	% of Initial Cost
Initial Cost (C_I)	Design Costs	7% C_I
	Construction costs	90% C_I
	Load Testing Costs	3% C_I
Maintenance Cost (C_M)	Inspections Costs(every 1 year) Detailed Inspections Costs(every 5 year) Repair Costs	10% C_I
Damage Cost (C_D)	Structural Failure Costs	
	- Bridge Replacement Costs	150% C_I
	- Loss of Lives and Cost of Injuries	500% C_I
	- Cultural and Historical Costs	10% C_I
	Functional Disruption Costs	50% C_I
	- Traffic Delayed Costs	
	- Traffic Detour Costs	
	- Heavy Traffic Costs	
	Environmental and Social Costs	15% C_I

All the above damage cost items must be expressed in present value. For this purpose, each potential future damage cost item must be multiplied by the Present Value Factor, *PVF*, as follows[12],

$$PVF = [1 - \exp(= \alpha L)] / (\alpha L) \tag{4}$$

where, $\alpha = \ln(1+q)$; q = annual discount rate; and L = lifetime of structure

This study assumes a lifetime of the PSC beam bridge of 50 years and an annual discount rate of 4.0%. The expected LCC for the PSC beam bridge, therefore, becomes:

$$E(C_T) = \overline{C}_I + \overline{C}_M + PVF(\overline{C}_{FR} + \overline{C}_{FL} + \overline{C}_{FH} + \overline{C}_{FD} + \overline{C}_{FEN}) \tag{5}$$

in which the individual damage cost items are the respective mean values of those described in Eq. 3.

3.3. Significance of Epistemic Uncertainties

The assessment of the initial and maintenance costs, C_I, C_M, respectively, for each of the alternative designs may contain some uncertainty(epistemic type). The actual initial and maintenance costs may vary by ±20% and may be expressed in terms of the COVs, $\Delta C_I = \Delta C_M = 0.20$, which represent the epistemic uncertainty in C_I and C_M.

Moreover, the COVs representing the respective epistemic uncertainties for each of the damage cost components may be assumed to be the values shown in Table 3.

Table 3. Epistemic Uncertainties in Damage Cost Items

Damage Cost Items	C.O.V. ΔC_i
C_{FR}	0.20
C_{FL}	0.40
C_{FH}	0.40
C_{FD}	0.40
C_{FEN}	0.80

Based on the information assumed in Tables 2 and 3, the expected damage cost is:

$$\overline{C}_D = \overline{C}_{FR} + \overline{C}_{FL} + \overline{C}_{FH} + \overline{C}_{FD} + \overline{C}_{FEN} = 7.25\overline{C}_I$$

and the variance of C_D will be,

$$Var(C_D) = [0.2(1.5\overline{C}_I)]^2 + [0.4(5.0\overline{C}_I)]^2 + [0.4(0.1\overline{C}_I)]^2$$
$$+ [0.4(0.5\overline{C}_I)]^2 + [0.8(0.15\overline{C}_I)]^2 = 4.16\overline{C}_I{}^2$$

Therefore, $\sigma C_D = 2.04\overline{C}_I$ and hence the COV of C_D will be,

$$\Delta C_D = \frac{2.04\overline{C}_I}{7.25\overline{C}_I} = 0.28$$

from which the mean and variance of the expected LCC become,

$$E(C_T) = \overline{C}_I + \overline{C}_M + \overline{C}_D,$$
$$Var(C_T) = (0.2\overline{C}_I)^2 + 0.2\overline{C}_M)^2 + (0.28\overline{C}_D)^2 \tag{6}$$

3.4. Minimum Life-Cycle Cost (LCC) Designs

With the information summarized above, the LCC for all nine alternative designs was evaluated. The

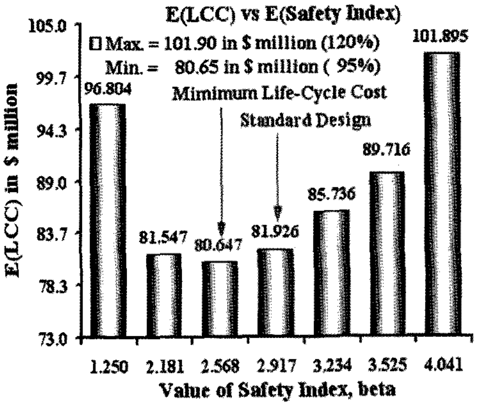

Fig. 2. E(LCC) versus E(β) due to Aleatory Uncertainties.

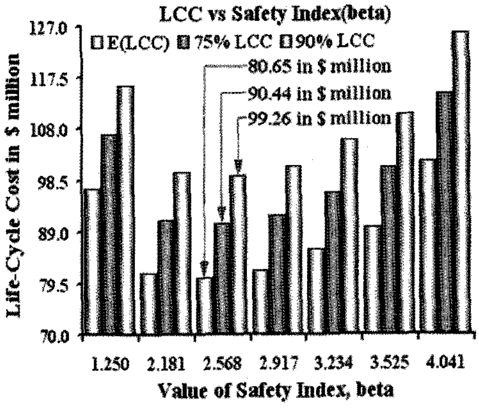

Fig. 3. Mean or Percentile LCC versus E(β) with Epistemic Uncertainties $\Delta C_I = \Delta C_M = 0.2$ and $\Delta C_D = 0.28$.

results were plotted between the mean β and the expected LCC, while considering only the aleatory uncertainties, as shown in Fig. 2. Similarly, the mean safety index was plotted versus the 75% and 90% LCC values. These results are summarized graphically in Fig. 3, which shows that, irrespective of the percentile LCC, the same optimal design was obtained at a mean safety index of $E(\beta) = 2.5679$.

Finally, because of the epistemic uncertainties indicated above, the true failure probabilities and the corresponding safety indices will be random variables. In particular, the histograms of the system failure probability and safety index generated through the Monte Carlo Simulation with a sample size of 10,000 are portrayed respectively in Figs. 4 and 5.

Figs. 4 and 5 show the histograms of the calculated failure probability, P_F, and the safety index, β, for the optimal design, respectively. From these two figures, the mean, 75%, and 90% P_F values and corresponding β values were determined and are tabulated in Table 4 for the optimal design of the PSC beam bridge.

The main results for the PSC beam bridge can be summarized as follows:

Table 4. Failure Probabilities and Safety Indices

Percentile	P_F	β
Mean	5.1027E-3	2.5649
Median	5.1053E-3	2.5669
25% P_F; 75%β	4.0269E-3	3.1013
10% P_F; 90%β	3.1006E-3	3.5753

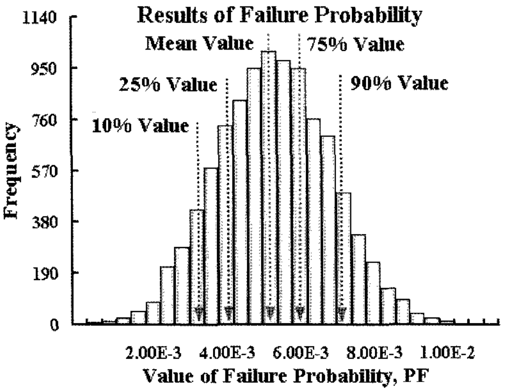

Fig. 4. Frequency Histogram of Failure Probability for Optimal Design by Epistemic Uncertainties.

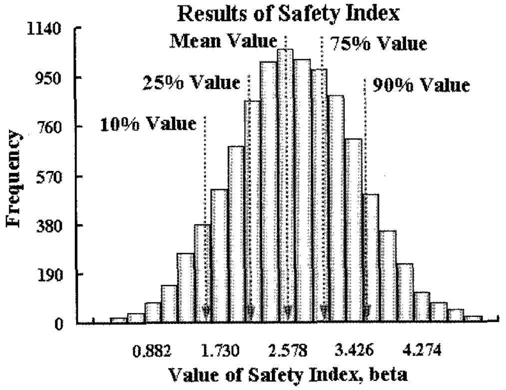

Fig. 5. Frequency Histogram of Safety Index for Optimal Design by Epistemic Uncertainties.

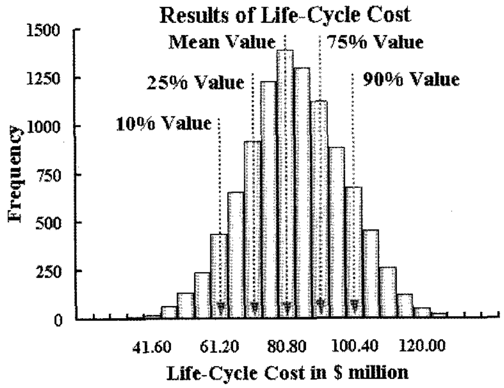

Fig. 6. Frequency Histogram of LCC of Optimal Design with $\Delta C_I = \Delta C_M = 0.2$ and $\Delta C_D = 0.28$.

Failure probabilities: Mean(P_F) = 5.103E-3, 50% P_F = 5.105E-3, 75% P_F = 6.156E-3, 90% P_F = 7.099E-2.

Corresponding safety indices: Mean β = 2.565, 50% β = 2.567, 75% β = 3.101, 90% β = 3.575.

The frequency diagram of the expected LCC for the optimal design is shown in Fig. 6. As expected, LCC increased increasing percentile level, which represents the extra cost of additional confidence. From Fig. 6, the following mean, and 50%, 75% and 90% LCC values for the optimal design were obtained (in million USD):

Mean LCC = 80.757, Median LCC = 80.549,
75% LCC = 90.538, 90% LCC = 99.884

4. Summary and Conclusions

In applying the selected reinforced method for the optimal design of the PSC beam bridge, reliability analysis was conducted by increasing and decreasing the member sections and the minimum LCC design of a reinforced PSC beam bridge was determined with separate consideration for the two types of uncertainties. This allowed the specification of prescribed percentile values of the pertinent results. For the PSC beam bridge, the results indicated that the 90% safety index value for the structural optimal design was 3.5753 and the corresponding 90% LCC value was assessed to be US$99.884. The study results confirmed that the current design of PSC beam bridges in Korea is close to optimal in terms of minimizing LCC.

In risk-based engineering, it is important to distinguish between two broad types of uncertainty: aleatory, which is part of the randomness of natural phenomena whose significance can be expressed in terms of the probability of occurrence, and epistemic, which is associated with imperfections in modeling and estimation of reality and thus leads to uncertainty (lack of complete confidence) in the calculated probability or risk. The epistemic uncertainty may be limited to the imperfections in the evaluation or predication of the mean (or median) value of a variable or parameter. Because of these epistemic uncertainties, the calculated results, such as failure probability, safety index, and expected LCC, become random variables with respective distributions (or histograms). The distributions of the calculated results allow high percentile values of the essential design parameters (such as the safety index) to be specified in order to insure sufficient risk aversion. For example, the 90% or 75% value of the safety index may be appropriate, leading to conservative designs, which are particularly important for reinforced PSC beam bridges.

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