# 슬로우 다중경로 페이딩 채널환경에서 BCH 부호화기법을 이용한 FS MC-CDMA 시스템의 블록에러확률의 성능 개선

# Performance of Block Error Probabilities of FS MC-CDMA System using BCH Coding Scheme in Slow Multipath Fading Channels

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# 요 약

본 논문에서는 DS-CDMA 시스템 구현 시 복잡성을 감소시키는 다중접속간섭 등의 여러 감쇠 요인을 고려 하여 이를 개선시킬 수 있는 FS MC-CDMA 시스템의 성능을 분석하고 열악한 무선 모바일 환경에서 성능 개 선기법인 BCH 부호화기법을 채용하여 FS MC-CDMA 시스템의 성능을 개선하였다. 또한 차세대 무선 모바일 통신 채널 환경으로는 나카가미 페이딩을 적용하여 FS MC-CDMA 시스템의 성능을 분석하여 검토하였다.

#### Abstract

In this paper, we analysis the error performance improvement of the FS MC-CDMA, whose information data are spread spectrum with direct sequences(DS), and each subband signal is transmitted using a subcarrier frequency, combining BCH code in Mobile communication channels which is characterized by Nakagami fading. The effect of co-channel interference on the block error performance is also studied in this paper. As a results of study, the coding techniques provides more efficient improvement than non-coding, but the coding techniques are required more band as many coding rate. Our study demonstrates that the system is combined coding techniques, the amount of improvement is capable of efficiently exploiting over the Nakagami fading.

Key words : block error probability, FS MC-CDMA, multipath fading, wireless mobile communication

#### I. Introduction

Multicarrier code division multiple access (MC-CDMA) technique has received much attention among researchers, and it is one of the most promising techniques for the future mobile radio communication systems beyond 3G[1]. It has the properties desirable for high data-rata wireless services such as insensitivity to frequency selective channels, frequency diversity, high spectral efficiently and flexibility to generate different data rates within a fixed bandwidth[2].

Frequency selective multipath fading is common in urban and indoor environments and is a significant source of potential degradation in a wideband mobile communication system. A direct-sequence spreadspectrum (DS-SS) code-division multiple-access

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(CDMA) scheme used for high-data-rate applications is usually know as broadband CDMA (B-CDMA)[1] due to its large bandwidth. However, the high bandwidth everywhere is a key requirement of the future communicating systems[2].

The transmission of information over radio channels with multiple changing propagation paths is subject to fading. For digital transmission over a fading channel, time variation causes a changing error probability with the effect of clustering errors at the receiver output.

Broadband wireless mobile channels are typically time- varying and the received signals may experience both frequency-selective and time-selective fading [3],[4].

Previous work on error probabilities have been conducted considering mobile radio channels with Rayleigh[2]-[4], Rician[5] and m-distribution[6] fading characteristics. The m-distribution is chosen to characterize the fading channel because it takes the Rayleigh distribution as a special case, approximates the Rician distribution well, models fading conditions which are more or less severe than those of Rayleigh, and more importantly, fits experimental data better than Rayleigh or Rician distributions. [7],[8]

In this paper, we analyzed the error performance improvement of fractionally spreading MC-CDMA system (FS MC-CDMA)[5], when communicating over wireless channels exhibiting both frequency-selective and time-selective fading. We applies the BCH coding scheme to improve the performance of FS MC-CDMA system in the slow fading channel environment

# II. FS MC-CDMA System Model

#### 2-1 Transmitter Model

The transmitter diagram of the kth user is shown in Fig. 1 for the FS MC-CDMA system. In this scheme the original binary data stream having a bit duration of  $T_b$ 

is S-P converted to U parallel sub- streams, which are expressed as  $\{b_{k1}, b_{k2}, \ldots, b_{ku}\}$ . The new bit duration or symbol duration after S-P conversion is given by  $T_s$  $= UT_b$ . As shown in Fig. 1, after S-P conversion each of the substreams is spread using two time (T)-domain spreading codes, namely  $a_k(t)$  and  $c_k(t)$ . More explicitly, the first T-domain spreading code  $a_k(t)$  is applied at the fraction level and it is expressed as  $a_k(t) = \sum_{i=-\infty}^{\infty} a_{ki} P_{T_D}(t-iT_D)$  where  $T_D = T_s/N_1$ represents the fraction's time duration,  $a_{ki}$  assumes the binary values of +1 or -1 with equal probability, while  $P_{T_D}(t)$  represents the rectangular pulses of duration  $T_D$ [5].



The second T-domain spreading code  $c_k(t)$  at the chip level and is defined as  $c_k(t) = \sum_{i=-\infty}^{\infty} c_{ki} P_{T_c}(t-iT_c)$  where  $c_{ki}$  is again a random sequence with  $c_{ki} \in \{+1, -1\}$  and  $P_{T_c}(t)$  is the rectangular chip waveform defined over the time interval  $[0, T_c]$ .

Let the total number of subcarrier frequencies, namely US, where S is defined as the length of the F-domain spreading codes to be invoked addi- tionally. As shown in Fig. 1, after T-domain spreading the uth substream, where u = 1, 2, ..., U, is further spread in the frequency(F)-domain using an S-chip F-domain spreading code  $\{F_{ki}, F_{k2}, ..., F_{kS}\}$  associated with the S number of subcarrier frequencies of  $\{f_{1u}, f_{2u}, ..., f_{Su}\}$ . Finally, the US number of subcarrier-modulated substreams are superimposed on each other in order to form the transmitted signal, which can be expressed as

$$S_{k}(t) = \sqrt{\frac{2E_{b}}{T_{s}S}} \sum_{u=1}^{U} \sum_{s=1}^{S} b_{ku}(t) \qquad (1)$$
$$\times a_{k}(t)c_{k}(t)F_{ks}$$
$$\times \cos\left(2\pi f_{su}t + \varphi_{su}^{(k)}\right)$$

where  $E_b$  represents the energy per bit,  $b_{ku}(t)$ denotes the *u*th binary data's waveform after the S-P conversion, while  $\phi_{su}^{(k)}$  represents a random phase due to carrier modulation. Assuming  $N_2 = T_D/T_c$  being an integer, then the total T-domain spreading factor is  $N = T_s/T_c = T_s/T_D \times T_D/T_c = N_1N_2$ 



Fig.2. Receiver schematic diagram of fractionally spread MC-CDMA.

# 2-2. Receiver Model

Since we assume that  $T_m < T_c$  the number of resolvable paths associated with each subcarrier is therefore one, i.e., each subcarrier signal experiences flat fading. As shown in Fig. 1, the FS MC-CDMA transmitter usually employs S-P conversion and U data bits are transmitted in parallel within each symbol duration[5]. Hence, the symbol duration is  $T_s = UT_b$ . Based on the above assumptions, the asynchronous signal received by the base station can be expressed as

$$r(t) = \sqrt{\frac{2E_b}{T_s S}} \sum_{k=1}^{K} \sum_{n=-\infty}^{\infty} \sum_{u=1s=1}^{U} \sum_{s=1}^{S} b_{ku}(t-\tau_k)$$
(2)  
  $\times a_{kn} P_{T_D}(t-nT_D-\tau_k)c_k(t-\tau_k)F_{ks}$   
  $\times \cos(2\pi f_{su}t+\varphi_{su}^{(k)}) + n(t)$ 

where n(t) represents the AWGN noise having zero mean and a double-sided power spectrum density of  $N_0/2$ . Furthermore, in (2)  $\alpha_{uns}^{(k)}$  is an amplitude fading parameter associated with the *k*th user, with the *n*th fraction as well as with the subcarrier indexed by the values of, *u* and *s*. Note that,  $\alpha_{uns}^{(k)}=1$ , when non-fading AWGN channels are considered, while  $\alpha_{uns}^{(k)}=\alpha_{us}^{(k)}$ , i.e. it is independent of the fraction index of *n*, when considering frequency-selective slow fading channels.

The FS MC-CDMA receiver's schematic diagram is shown in Fig. 2, which is suitable for receiving the FS MC-CDMA signals. In Fig. 2 each subcarrier signal is first despread using the T-domain spreading code c(t) of the reference user associated with each fraction. Then, the subcarrier signals conveying the same data bit are despread using the F-domain spreading code  $\{F_1, F_2, \dots$ . ,  $F_{S}$  and combined using a MRC scheme with the aid the channel's fading envelope estimates  $\{\alpha_{un1},\alpha_{un2},...,\alpha_{unS}\}_{u\,=\,1}^{U}.$  Finally, the  $N_{1}$  number of signals corresponding to  $N_1$  fractions of the same symbol are despread using the T-domain spreading code a(t), yielding the decision variable  $Z_{\!u}, u = 1, ..., U$ acquired for the uth binary bit. The process of generating the decision variable  $Z_{\mu}$  for the first symbol can be summarized using the following equations.

$$Z_{u} = \sum_{n=0}^{N_{1}-1} a_{n} Z_{un} \qquad U = 1, ..., U$$
(3)

$$Z_{uns} = a_{uns} \times \int_{nT_D}^{(n+1)T_D} r(t)c(t)\cos(2\pi f_{su}t)dt$$
(4)

where we assumed that  $\tau = 0$  and  $\psi_{uns} = 0$ , representing perfect synchronization with the subcarrier signal of the fraction that is being considered.

# III. Block Error Probability

# 3-1. Nakagami Fading Model

A Nakagami distribution characterizes channels with different fading depth through a parameter called amount of fading AF. The AF of a signal is assumed to be 1/m. The signal envelope,  $\alpha$  is a random variable with a Nakagami probability density function (pdf), i.e.,

$$P_{\alpha}(\alpha) \frac{2}{\Gamma(m)} \left(\frac{m}{2X}\right) \alpha^{2m-1} \exp\left(\frac{m}{2X}\alpha^2\right)$$
 (5)

where  $\Gamma(\ {f \circ}\ )$  is the Gamma function, X is the mean signal power, and m  $\geq 1/2.$ 

# 3-2 BER Model

In this section we summarize the BER expressions for the proposed FS MC-CDMA system, when communicating over the AWGN, and over the slow frequency-selective Nakagami-m fading channels.

The BER of the FS MC-CDMA system communicating over AWGN channels can be expressed as

$$P_{b} = Q(\sqrt{2.SINR})$$

$$= Q\left(\left[\frac{K-1}{3N_{1}N_{2}S} + \left(\frac{2E_{b}}{N_{0}}\right)^{-1}\right]^{-1/2}\right)$$
(6)

In the context of communicating over frequency-selective slow Nakagami-m fading channels, the BER of the FS MC-CDMA systems can be expressed as

$$P_b = \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{m \sin^2 \theta}{\gamma_c + m \sin^2 \theta} \right)^{mS} d\theta \tag{7}$$

where *m* is the Nakagami-*m* fading parameter,  $\gamma_c$  represents the average signal-to-noise ratio (SNR) received and which shows the diversity order achieved over the frequency-selective slow Nakagami-*m* fading

channels is S. Furthermore, the SNR in (7) is given by

$$\gamma_{c} = \left[\frac{2(K-1)}{3N_{1}N_{2}} + \left(\frac{\Omega E_{b}}{SN_{0}}\right)^{-1}\right]^{-1}$$
(8)

where  $\Omega = E[(\alpha_{uns}^{(k)})^2]$ . Explicitly, (7) shows that the diversity order achieved is *S*. It can be shown [6] that the limit of (7) with respect to  $m \rightarrow \infty$  will converge to (6), which quantifies the BER in the context of AWGN channels. This characteristic implies that when the channel quality improves and the fading envelope becomes near-constant, the FS MC-CDMA will, automa- tically leverage the diversity gain into spreading gain.

In order to gain the achievable BER performance of the considered FS MC-CDMA system, in this contribution a kind of coding technique is applied, namely BCH code.

BCH (Bose, Ray-Chaudhuri, Hocquenghem) code is a much studied code within the study of coding theory and more specifically error-correcting codes. In technical terms a BCH code is a multi-level, cyclic, error-correcting, variable-length digital code used to correct multiple random error patterns.

In the context of communicating over frequencyselective slow fading channels, the BER of the BCH code can be expressed as

$$P_{BCH} = \frac{1}{n} \sum_{i=t+1}^{n} \cdot P_b^i \cdot (1 - P_b)^{n-i}$$
(9)

where  $P_b$  is the bit error rate of modulated signal  $(E_b/N_0=r \cdot E_b/N_0)$ .

#### 3-3 Block Error Probability Model

In deriving block error probabilities, we need to calculate the probability of more than M bit errors in a block of N bit, P(M, N). The block error probability P(M, N) of FS MC-CDMA with signal-to-noise  $\gamma_c$  in

a Nakagami fading channel is

$$p(r_{c},M,N) = \sum_{i=M+1}^{N} \binom{N}{i} p_{b}^{i}(r_{c}) [1 - p_{b}(r_{c})]^{N-i} \quad (10)$$

### IV. Performance Analysis and Discussion

In this paper, we now consider the performance of FS MC-CDMA systems in Nakagami fading channel. As a technique for the performance improvement, BCH coding have been used, and the performance have been compared and analyzed.

In Fig. 3 we show the corresponding comparison of the block error probabilities performance of the FS MC-CDMA system, when communicating over both the AWGN, well non-fading as as over the frequency-selective non-coding and coding slow fading, assuming Rayleigh (m = 1) and Rician (m = 3) fading models. The curves in the figure were plotted against the average SNR per bit of  $E_b/N_0$  for the parameters of  $N_1 = 4$ ,  $N_1 = 31$ , S = 4 and K = 30. From the results of Fig. 3 we observe that for a given SNR perbit value, the block error performance of the frequency-selective slow fading channel is significantly worse than that over AWGN channels, regardless of m = 1 and m = 3 at the block error probabilities of  $10^{-6}$ . Form = 1 we observe the formation of error for an floor the frequency-selective slow fading channel model at the SNR per bit values about 20dB or 22dB, respectively.

Fig. 6 demonstrates the comparison of the block error probabilities performance versus the number of users K for the FS MC-CDMA system, when communicating over both non-fading AWGN, as well as over frequency-selective with the BCH code and non-BCH code slow Rayleigh (m=1) fading. The curves in Fig. 6 were plotted versus the number of users K for the parameters of  $N_1 = 4$ ,  $N_1 = 31$ , S = 4 and  $E_b/N_0 =$ 15dB. The results of Fig. 6 also show that the achievable block error probabilities of the performance is better in frequency-selective slow fading with coding, than non-coding environments. However, the coding effect is minimal since the channel tends to fading.

Finally, Fig. 5 we illustrate the effect of the frequency-selectivity of the slow fading channels on the block error probabilities performance of the FS MC-CDMA system. The results of the Fig. 5 were computed against the block error probabilities per bit value of  $E_b/N_0$ . Note that the block error probabilities in the coding slow fading channels increases slow with increasing values m.

# V. Conclusion

The block error probabilities of FS MC-CDMA under slow Nakagami fading are derived in this paper. The application of BCH code to improve block error performance is investigated and co-channel interference effect is also studied. The effect of fading amount on block error performance is observed. It is shown that BCH code improves block error performance under slow fading. The FS MC-CDMA scheme with BCH code is beneficent for employment over wireless channels. However, some techniques can yield better results than others, but can be more costly. Therefore, the decision for one or another technique depends on the analysis of cost versus effectiveness. In the following we shall be going to investigate this trade-off.

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Fig. 3. block error probabilities versus  $E_b/N_0$ performance of the using non-code & BCH code of the FS-MC-CDMA, when communicating over non-fading AWGN, as well as slow frequencyselective Rayleigh (m = 1) or Rician (m = 3) fading channels.



Fig. 4. block error probabilities performance versus the number of users K for the FS MC-CDMA, when communicating over both using AWGN as well as over slow frequency-selective and slow frequencyselective with the aid of BCH code Rayleigh(m = 1) fading channels



Fig. 5. block error probabilities versus  $E_b/N_0$ performance of the FS MC-CDMA, when communicating over slow frequency-selective (M = 0) fading channels.

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