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Activities and Programs to Cultivate Mathematical Interest and Ability[1]

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Young children have manifold potentialities. As any teacher or parent knows, a child's most obvious strengths contribute to their development in unexpected ways. A sporting or musical *forte* may provide an invaluable youthful opportunity to experience "the pursuit of excellence," but may then be laid aside. It is exceedingly rare for a strength which informed observers might "identify" at school level to develop in a predictable way. Most strengths blossom and are then laid aside, whilst some evolve sideways (for example, when the inner muse shies away from the required level of commitment, or takes fright at the miniscule prospects of success in the given field). In their place other strengths — which one may have noticed, but which were never "diagnosed" in the same way — take over and flourish.

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INTRODUCTION

My starting point may seem unpromising — though the subsequent details may be of more interest. After 40 years of working with students and teachers it seems clear that passive, *a priori* notions such as "mathematical giftedness" are an attempt to give substance to something which we think we experience, but which we cannot quite put our finger on. However, I would like to suggest that they either do not exist, or are irrelevant.

What is clear is that in fields such as mathematics and music, a tiny number of

[1] This article is an extended version of the paper (Gardiner, 2008) presented at Topic Study Group 6 (Activities and Programs for Gifted Students) of the 11th International Congress on Mathematical Education (ICME-11) held at the Universidad Autonoma de Nuevo Leon (UANL), Monterrey, Mexico, July 6–13, 2008.

children "stand out" in the sense that experienced practitioners interact with them as if they were mature *colleagues*, and it seems natural to interpret this as indicating great potential.

However, it is important first to note that this level of recognition is rare; and that even then it is easy for the observer to be misled — especially when diagnosing future potential: an observed combination of inner musicality and technique may later encounter unanticipated technical obstructions, or personal limitations. So it would be rash to elevate the undoubted phenomenon of impressive early performance to the status of a dogma, in which we import the "medical model" (first diagnose, then treat) into classrooms to subject otherwise healthy children to spurious "identification" rituals.

Secondly, even in those cases where it is hard to resist the conclusion that one is confronted with something extraordinary, this observation is in no way predictive of future success: one could even argue that the prospect of long-term survival within a community which is inevitably highly competitive is often markedly *reduced* by such early "identification"! Early "identification" can impose strains of expectation which adolescents may be ill-equipped to handle, so that when the going gets tough they lack the maturity and tenacity to persevere. In contrast, "identification" which occurs quietly and at a suitably late stage (say taking the form of implicit or explicit encouragement from a recognized expert), can serve as a confirmation (or "lying on of hands") which helps develop determination and commitment.

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Our job as humane educators is to develop opportunities and approach which might contribute to this rich flux — so that all who engage with what is on offer emerge the richer for it, without having to run the gauntlet of "identification" and subsequent disappointment.

Hence we concentrate on the question:

- What opportunities or activities give youngsters the best chance to allow their mathematical potential to blossom in a robust, lasting way?

- What follows constitutes one answer — there can be no definitive answer.

EXTENSION MATHEMATICS

Every answer depends crucially on what happens in the mathematics classroom, week-in, week-out. Hence it is important to engage with official curricula, and to develop materials that enrich students' daily mathematical diet as well as to supplement that diet in ways that appeal to different tastes. In particular, in seeking to open students' eyes to a wider mathematical world, we should take care not to alienate them from the mundane everyday world of school mathematics.

Extension mathematics Gardiner (2007) is a series designed in the above spirit for classroom use with around 25% of students aged 11–15. This is no resource for those who like hard problems in the Olympiad tradition! Rather, the series focuses on those topics from the standard curriculum which are most important for students' future mathematical development, and seeks to establish them on firmer foundations than has been generally attempted in recent decades.

In recent years mathematics teaching and its assessment in English schools has degenerated. The increase in central assessment and the emphasis on ever-improving test scores has led to the neglect of what should be the central challenge of all mathematics teaching - namely to not only master "atomic skills" (the easy bit), but to master them robustly and *to integrate these atomic skills into extended chains to solve simple multi-step problems*. The result of this neglect has been to produce a nation of students trained only to complete "one piece jigsaws," whose skills prove worthless as soon as they are faced with the simplest genuine problem. In short, the mathematical "baby" has been discarded, leaving only the worthless "bathwater." The collection (Gardiner, 2007) serves as a reminder of what has "gone missing," and why it matters.

The approach is intended to be *inclusive* and *enabling* for a large group of students, to give those who may be in a position to enjoy elementary mathematics the necessary basic tools and experiences to develop whatever potential may lie lurking within.

These are not typical textbooks. Each book contains 70–80 sections — each between 1 and 4 pages long. Each section takes it for granted that the relevant topic has already been "introduced," and seeks to develop this basic material to a higher level. Each section includes a short introductory text, which summarises the relevant background and alerts the teacher to likely prerequisites. But the bulk of every section consists of a graded sequence of rich problems.

The sections are on three "levels": *Tasters*, *Core* and *Extension*. *Tasters* are intended for all those in the target group; *Core* is likely to be slightly more demanding; *Extension* sections are likely to appeal only to those with an unusual appetite.

The "inclusive" philosophy implies that the books consciously avoid "acceleration": they lean over backwards to avoid introducing material earlier than necessary. Hence the first two books contain almost no symbolic algebra! Instead they concentrate on establishing a more profound grasp of *integer arithmetic* (emphasizing flexible use of the *base 10* system and exploiting algebraic structure in a numerical context — starting on the lowest level with "16×75 = ?", "375×48 = ?", "$(14 \times 7) - (2 \times 14)$ = ?", and so on), then applying this to achieve a stronger mastery of the *simplification* and *arithmetic of fractions*, and the arithmetic of *surds*.

It would be nice to think that large numbers of students will have their appetite for serious mathematics whetted as a result of struggling with this rich, but relatively mundane material. However, it remains unclear whether schools and teachers in England retain sufficient professional independence of spirit to take what they often see as the "risk" of teaching good *mathematics*, rather than dulling their students' judgment by concentrating on narrow preparation for the central assessments which now dominate the local educational scene.

NATIONAL COMPETITIONS

English schools vary enormously: the best are very good, and the worst are very poor (often for reasons partly beyond their control). In practice, potential young mathematicians are only likely to become visible in the better schools. But any event which seeks to encourage those who lurk in the secondary undergrowth needs to include as many schools as possible, and to be accessible to their students — no matter how little they may know.

This might explain the character of the UK competition scene UKMT[2] (2008) which was developed in the late 1980s.

On the most basic level, all events involve just one round, last around 1 hour, and

[2] The UK Mathematics Trust (UKMT) is a registered charity whose aim is to advance the education of children and young people in mathematics. The UKMT organises national mathematics competitions and other mathematical enrichment activities for 11–18 year old UK school pupils. It was established in 1996 and last academic year over 600,000 pupils from 4000 schools took part in the three individual challenges, the UK"s biggest national maths competitions. Each challenge leads into a follow-on Olympiad round and it run mentoring schemes and summer schools for high performing students as well as training the team of six to represent the UK in the International Mathematical Olympiad (IMO). It also run team maths competitions for two age ranges, publish books and organise enrichment seminars for teachers. See http://www.ukmt.org.uk

include 25 multiple choice questions. None of the events is called a "competition." Results are provided to individual schools, but no national results are published which might allow one to compare schools with each other. And the problems set are traditionally "lighter" in spirit than in many other countries — with much effort going into devising problems that might cause a smile, or lead to subsequent discussion. 40% of participants receive a certificate (6% Gold, 13% Silver, and 21% Bronze). Separate papers are set for ages 9–11, 11–13, 13–16, and 16–18 with current entry numbers around 100K, 260K, 210K, and 80K.

In each age group the top 1000 or so scorers are invited to take a second round, at which stage the level of difficulty increases — with the events for ages 11–18 shifting from multiple choice to written "Olympiad-Style." For those aged 14–18 the pyramid extends with limited opportunities to attend Summer School or more serious Olympiad training camps.

The last few years have seen the development of hugely popular national team competitions for those aged 12–14 and aged 16–17. There are also many interesting local events.

PROBLEM SOLVING JOURNAL

Forty years ago, the top 25% or so of English secondary students were often taught by teachers who had a relatively good mathematical background. (I know of no reliable documentation, but this would seem to have been due in part to the situation facing young soldiers returning from the Second World War These young men had to find some kind of a job in difficult circumstances, and often drew in part on the general sense of social vision and responsibility which accompanied the new Labour government, the reforms associated with William Beveridge's vision of a Welfare State, and the 1944 Education Act. These teachers slipped unnoticed into retirement during the 1970s and 1980s and were never replaced.)

There was at that time no national curriculum; textbooks (often written by a succession of exceptionally gifted textbook writers — such as Durell, Godfrey, Hall, and Knight) laid the foundations needed for elementary and higher mathematics; and examinations for those leaving school at age 18 were controlled by leading universities, and designed as university-entrance exams. Each of these examinations had a harder version — a "scholarship" paper — which the better students could work towards; and Oxford and Cambridge had additional entrance papers for mathematics, which provided an even stiffer hurdle for those who needed something more than this. In such a climate, there seems to have been little perceived need for additional enrichment or challenge.

Hence "problem solving" came to be linked with "preparation for entry to Oxford and Cambridge at age 18"; there was no perceived need to develop a tradition of the kind illustrated by *Kömal* (in Hungary) or *Kvant* (in Russia).

The current position is very different: the curriculum is dominated by centrally controlled "National Strategies", a national curriculum (introduced in 1989), and annual centrally controlled assessments — which are obliged to indicate increased "success", and which therefore lead to the systematic (if unintentional) neglect of genuine problems.

A number of voluntary activities have emerged in response to this unfortunate vacuum. For example,

1. The national challenges run by UKMT (2008) involve large numbers of schools and students, but they do so via timed papers, with almost all students taking part on just one day each year.
2. NRICH[3] (2008) is a fantastic website with fresh sets of problems for every conceivable group renewed on a monthly basis; but its very extent can be off-putting for many teachers.

The Problem Solving Journal (for secondary students)[4] is a more modest affair, which sits somewhere between these two examples, and which seeks to make mathematical problem solving a natural and regular activity for a larger number of adolescents. It is currently in its fifth year and is produced three times a year as a cheap 12-page, A5 booklet — giving students roughly 2 months to work on sets of problems at various levels ("easy" and "hard", for ages 11–13, 13–16, and 16–18). Each issue gives complete solutions and lists successful solvers to problems in the previous issue — a commitment which makes the turn-round time for marking, producing, printing, packing, and posting each issue very tight!

The obstacles to widespread adoption appear to be similar to those facing *Extension mathematics*: teachers are often tightly focused on meeting narrower goals; they may be put off by the fact that they themselves find the problems "hard"; they may also be unaware of the formative potential of such independent activity on the part of their students. Current subscription levels mean around 5500 copies of each issue are sent to around 300 schools.

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