A Study on Linear Matrix Inequalities Robust Active Suspension Control System Design Algorithm

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Abstract—A robust optimal control system design algorithm in active suspension equipment adopting linear matrix inequalities control system design theory is presented. The validity of the linear matrix inequalities robust control system design in active suspension system through the numerical examples is also investigated.

Index Terms—Active Suspension, Bouncing Pitching, Linear Matrix Inequalities, Robust Control.

I. INTRODUCTION

This paper proposes modeling and design methods in vehicle suspension system to analyze active suspension equipment by adopting linear matrix inequalities theory to design robust h^{∞} control system. Recently, in the field of suspension system designs, it is general to adopt active control scheme for stiffness and damping. Connection with the other complicate vehicle stability control equipment is also intricate. It is required for the control system scheme to design more robust, fast response and high precision control equipment. It is known that the active suspension system is much better than passive spring-damper system in designing the suspension equipment [1]-[5].

In this paper, I deal with a design method based upon robust h^{∞} control solution which is obtained by linear matrix inequalities for improving vehicle performance and driver's ride comfort problems. In the problems to improve ride comfort, it is most important indicator to control bouncing displacement and pitching angle vibration on driving vehicle. The method to control bouncing displacement and pitching angle actively in this paper assures the robust performance and driver's ride comfort to the continuously added road disturbances under the steady speed driving condition.

The linear matrix inequalities robust h^{∞} controller is designed based on a 4our Degree of Freedom linear vehicle system model which represents the bouncing displacement and pitching angle of a vehicle concerned with front-rear parts bouncing displacements. The active suspension system with considering location of front-rear wheel and driving velocity is analyzed and the robust control system is also designed. The validity of the linear matrix inequalities robust control system design in active suspension system through the numerical examples and experiments is also investigated.

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II. SYSTEM ANALYSIS AND DESIGN

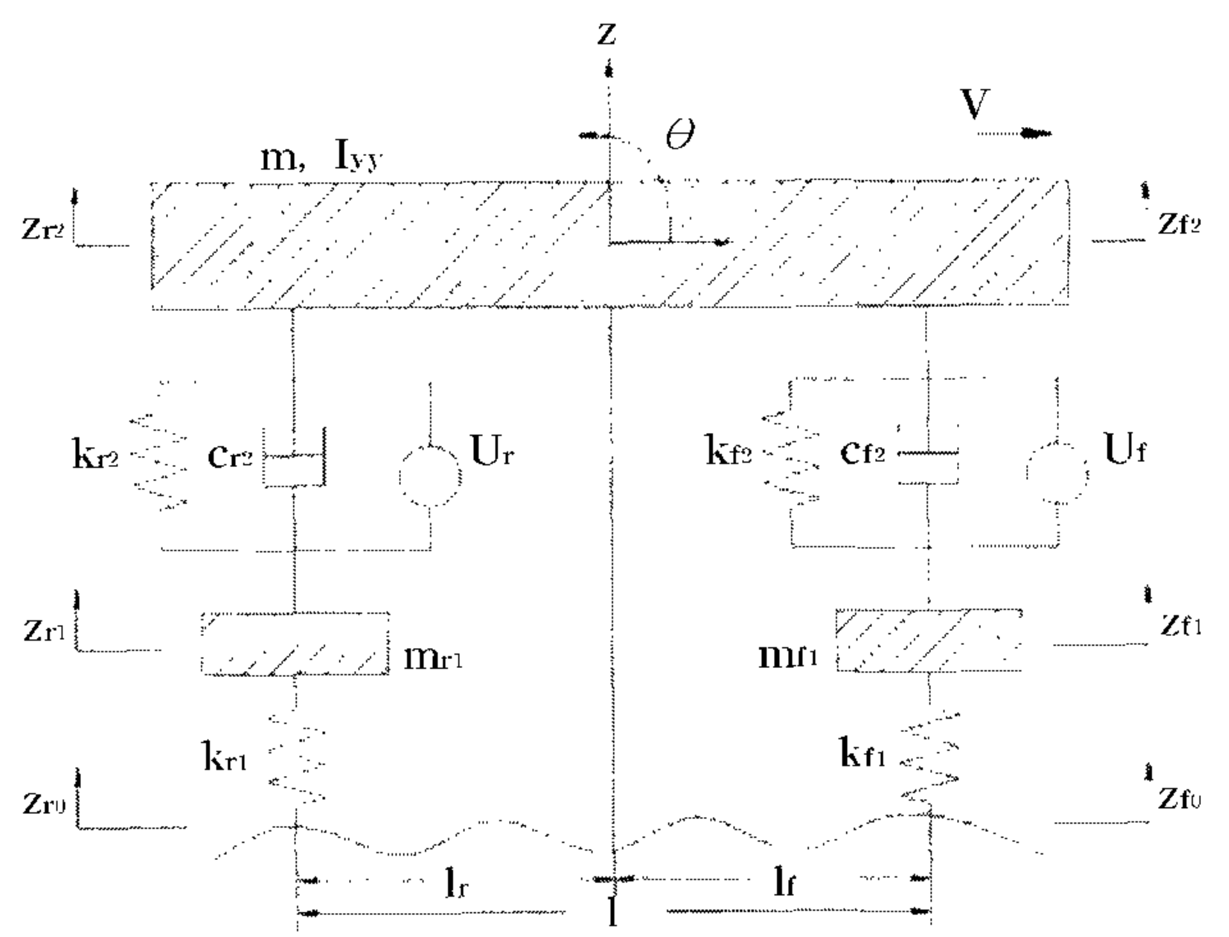

Fig. 1 4-DOF vehicle system model.

The analysis, modeling and design object used for this paper is a 4-DOF vehicle system model shown on figure 1 [6]-[7]. In Fig. 1, V denotes the velocity of the driving direction, i.e. the longitudinal velocity (x -direction); m and I_{yy} denote the mass of the vehicle and the pitch moment of inertia about its mass center in the lateral direction (y -direction); θ and z denote the pitching angle and the bouncing displacement, i.e. the upper and lower motion (z -direction); U_f and U_r denote the control inputs to the front part suspension and rear part suspension; z_{f0} and z_{r0} denote road displacement disturbances to the front part suspension and rear part suspension, respectively.

Based on the 4-DOF vehicle system model analysis frames shown in Fig. 1, the equations of the vehicle motion can be obtained as follows. The dynamic equations of the suspension upper part mass can be defined as

$$m\ddot{z} = F_f + F_r, \quad I_{yy}\ddot{\theta} = l_f F_f - l_r F_r$$
$$z = \frac{l_r z_{f2} + l_f z_{r2}}{l}, \quad \theta = \frac{z_{f2} - z_{r2}}{l} \tag{1}$$

where $F_f = -k_{f2}(z_{f2} - z_{f1}) - c_{f2}(\dot{z}_{f2} - \dot{z}_{f1})$ denotes the force transmitted through the front part suspension, and $F_r = -k_{r2}(z_{r2} - z_{r1}) - c_{r2}(\dot{z}_{r2} - \dot{z}_{r1})$ denotes the force transmitted through the rear part suspension; k and c denote stiffness and damping coefficient. The dynamic equations of the suspension lower part mass in the front-rear parts can be also defined as follows.

$$\begin{aligned} m_{f1}\ddot{z}_{f1} &= -F_f - k_{f1}(z_{f1} - z_{f0}) + U_f \\ m_{r1}\ddot{z}_{r1} &= -F_r - k_{r1}(z_{r1} - z_{r0}) + U_r \end{aligned} \quad (2)$$

Assuming that $I_{yy} = ml_f l_r$ condition, from 2nd term of the Eq. (1) follows can be obtained.

$$\begin{aligned} m_{f2}\ddot{z}_{f2} &= F_f \quad (m_{f2} = \frac{ml_r}{l}) \\ m_{r2}\ddot{z}_{r2} &= F_r \quad (m_{r2} = \frac{ml_f}{l}) \end{aligned} \quad (3)$$

From Eqs. (2) and (3), the matrix representations of the equations of motion are expressed by

$$M_t\ddot{z}_t + C_t\dot{z}_t + K_t z_t = H_t w + F_t u \quad (4)$$

where

$$M_t = \begin{bmatrix} m_{f1} & 0 & 0 & 0 \\ 0 & m_{f2} & 0 & 0 \\ 0 & 0 & m_{r1} & 0 \\ 0 & 0 & 0 & m_{r2} \end{bmatrix}, \quad z_t = \begin{bmatrix} z_{f1} \\ z_{f2} \\ z_{r1} \\ z_{r2} \end{bmatrix},$$

$$C_t = \begin{bmatrix} c_{f2} & -c_{f2} & 0 & 0 \\ -c_{f2} & c_{f2} & 0 & 0 \\ 0 & 0 & c_{r2} & -c_{r2} \\ 0 & 0 & -c_{r2} & c_{r2} \end{bmatrix}, \quad u = \begin{bmatrix} U_f \\ U_r \end{bmatrix},$$

$$K_t = \begin{bmatrix} (k_{f1} + k_{f2}) & -k_{f2} & 0 & 0 \\ -k_{f2} & k_{f2} & 0 & 0 \\ 0 & 0 & (k_{r1} + k_{r2}) & -k_{r2} \\ 0 & 0 & -k_{r2} & k_{r2} \end{bmatrix},$$

$$H_t = \begin{bmatrix} k_{f1} & 0 \\ 0 & 0 \\ 0 & k_{r1} \\ 0 & 0 \end{bmatrix}, \quad F_t = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad w = \begin{bmatrix} z_{f0} \\ z_{r0} \end{bmatrix}.$$

In this paper, the control system is designed with the linear matrix inequalities robust h^∞ control to suppress the effect of the road disturbance. Eq. (4) can be modeled and expressed by state space equations as

$$\begin{aligned} \dot{x} &= A_t x + B_{1t} w + B_t u \\ z &= C_{1t} x + D_{12} u \\ y &= C_{2t} x + D_{21} w \end{aligned} \quad (5)$$

where x and u denote system state variables and control input; y and z denote measured output and controlled output, respectively; and w denotes the road disturbances input. System design variables and matrix parameters become as follows.

$$A_t = \begin{bmatrix} 0 & I \\ -M_t^{-1}K_t & -M_t^{-1}C_t \end{bmatrix}$$

$$B_{1t} = \begin{bmatrix} 0 \\ M_t^{-1}H_t \end{bmatrix}, \quad B_t = \begin{bmatrix} 0 \\ M_t^{-1}F_t \end{bmatrix}$$

$$C_{2t} = \begin{bmatrix} F_t^T & 0 \end{bmatrix}, \quad x = \begin{bmatrix} z_t \\ \dot{z}_t \end{bmatrix}$$

In order to design linear matrix inequalities robust h^∞ controller for controlled objective plant represented by Eq. (5), it is considered that robust controller can be expressed as follows [8].

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c y \\ u &= C_c x_c \end{aligned} \quad (6)$$

The necessary and sufficient conditions for the existence of the linear matrix inequalities robust h^∞ controller are that there exist X and Y which satisfy the follows.

$$\begin{aligned} &\begin{bmatrix} A_t X + X A_t^T + B_{1t}B_{1t}^T - B_t B_t^T & X C_{1t}^T \\ C_{1t} X & -I \end{bmatrix} < 0 \\ &\begin{bmatrix} Y A_t + A_t^T Y + C_{1t}^T C_{1t} - C_{2t}^T C_{2t} & Y B_{1t} \\ B_{1t}^T Y & -I \end{bmatrix} < 0 \\ &\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0 \end{aligned} \quad (7)$$

where

$$P \cong A_t X + X A_t^T + X C_{1t}^T C_{1t} X + B_{1t} B_{1t}^T - B_t B_t^T < 0$$

$$Q \cong Y A_t + A^T Y + Y B_{1t} B_{1t}^T Y + C_{1t}^T C_{1t} - C_{2t}^T C_{2t} < 0 \ .$$

Under the assumptions that Eq. (7) linear matrix inequalities conditions are satisfied, one of the Eq. (6) robust controllers can be obtained as follows [9]-[11].

$$\begin{aligned} A_c &= A_t + B_t C_c - B_c C_{2t} + Y^{-1} C_{1t}^T C_{1t} - Y^{-1} Q (I - XY)^{-1} \\ B_c &= Y^{-1} C_{2t}^T, \quad C_c = B_t^T Y (I - XY)^{-1} \end{aligned} \quad (8)$$

III. NUMERICAL SIMULATIONS

For the validities of the proposed modeling and design methods in this paper for vehicle suspension system to analyze active suspension equipment by adopting linear matrix inequalities theory to design a robust h^∞ control system, numerical simulations are carried out under the condition which continuously added a sinusoidal road disturbance. Throughout the simulations, the vehicle was driven at steady speed V=70km/h. In detail numerical simulation specifications, those were set that the vehicle mass m = 1790kg, the suspension front lower part mass m_{f1} = 134.1kg, the suspension rear lower part mass m_{r1} = 109.5kg, the pitch moment of inertia I_{yy} = 3523.6 kgm^2, the wheel and the suspension part stiffness coefficients $k_{f1} = k_{f2}$ = 1411N/m, the suspension part damping coefficients $c_{f2} = c_{r2}$ = 118Ns/m, and l the wheel base parts are l_f = 1.27m, l_r = 1.55m.

Fig. 2 shows the sinusoidal road disturbance which is continuously added to the vehicle model. On the figures of throughout the results of simulations, a solid line represents the result of controlled one, and a dotted line represents that of uncontrolled one.

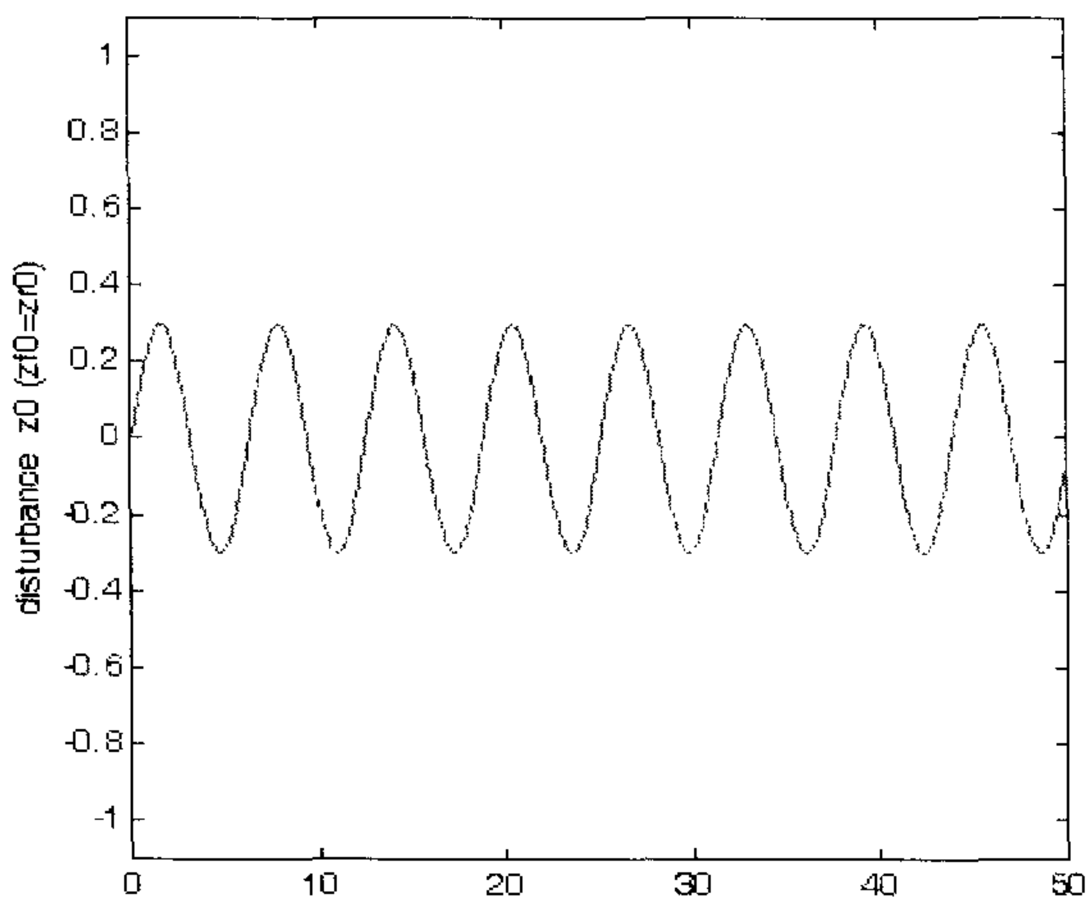

Fig. 2 Sinusoidal road disturbance.

The upper and lower motion displacement response of the suspension front lower part, the upper and lower motion displacement response of the suspension front upper part in cases of uncontrolled and controlled are shown in Fig. 3 and Fig. 4, respectively. Fig. 5 shows the front part actuator control input U_f (kgm/s^2).

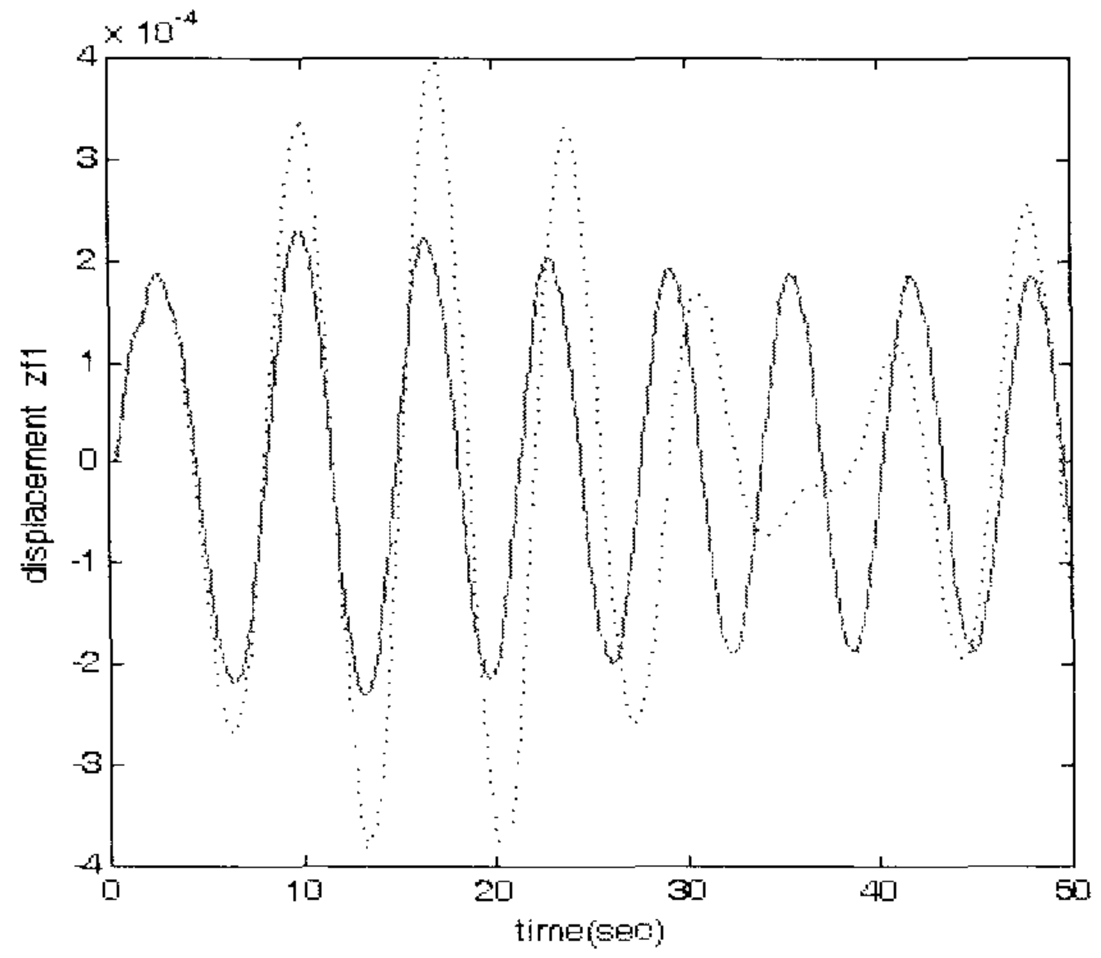

Fig. 3 Road disturbance response z_{f1}.

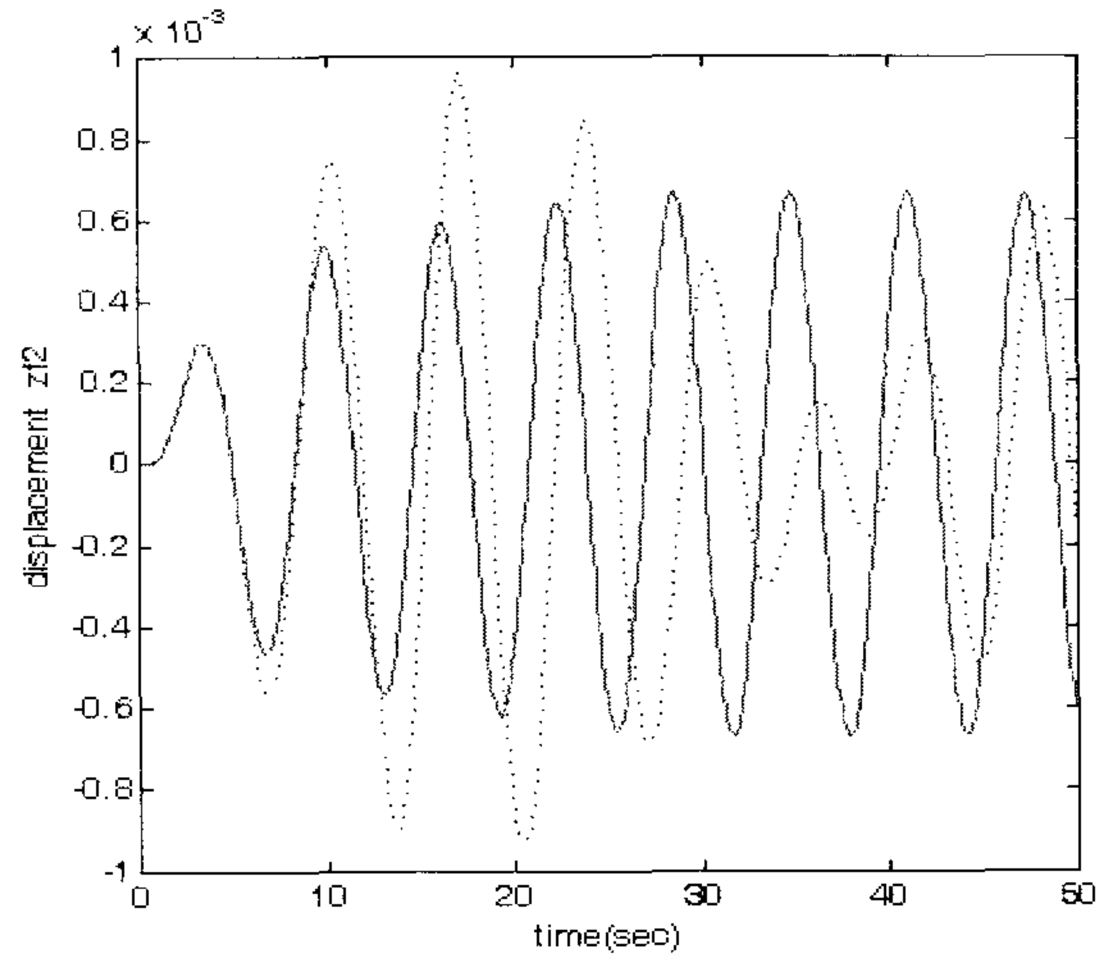

Fig. 4 Road disturbance response z_{f2}.

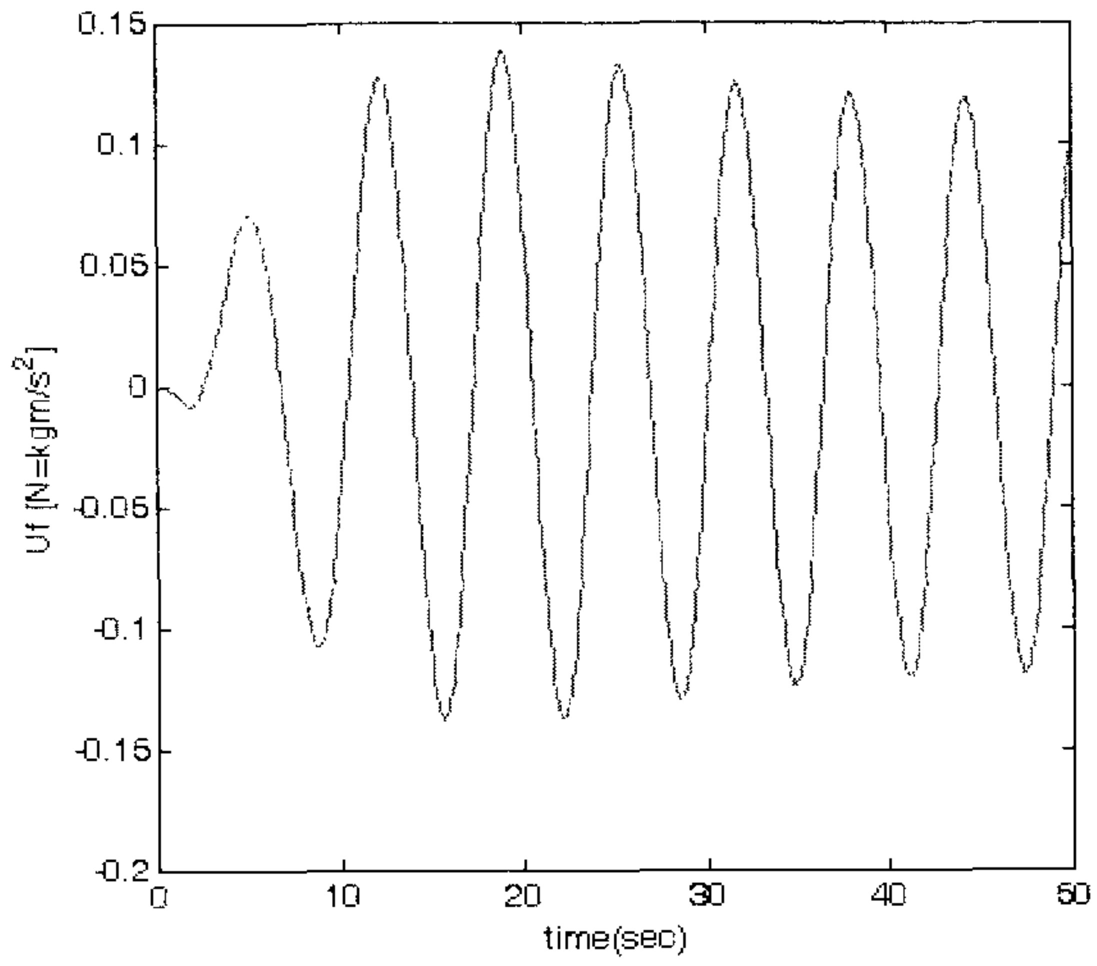

Fig. 5 Front part control input U_f.

Fig. 6, Fig. 7 and Fig. 8 also show the z-direction displacements in uncontrolled, controlled cases, and control input of the rear part, respectively.

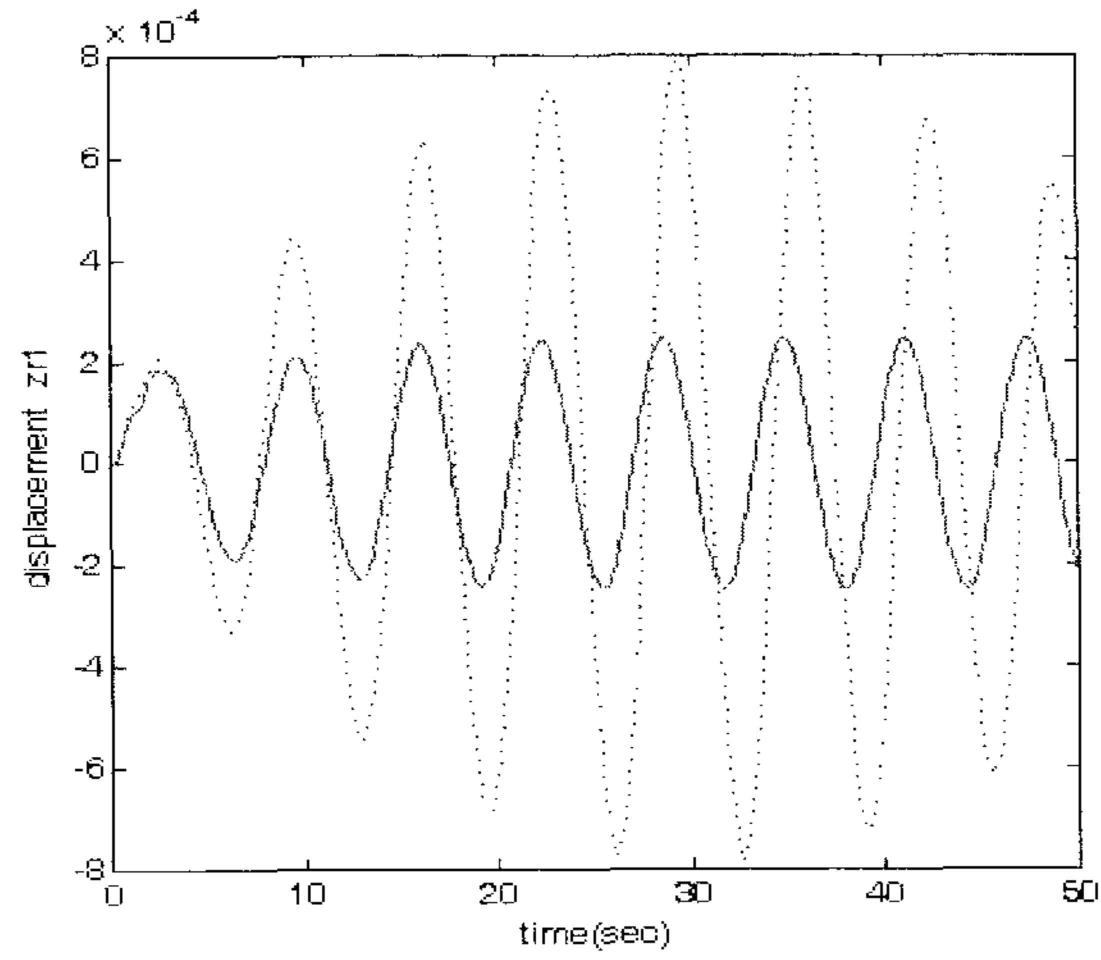

Fig. 6 Road disturbance response z_{r1}.

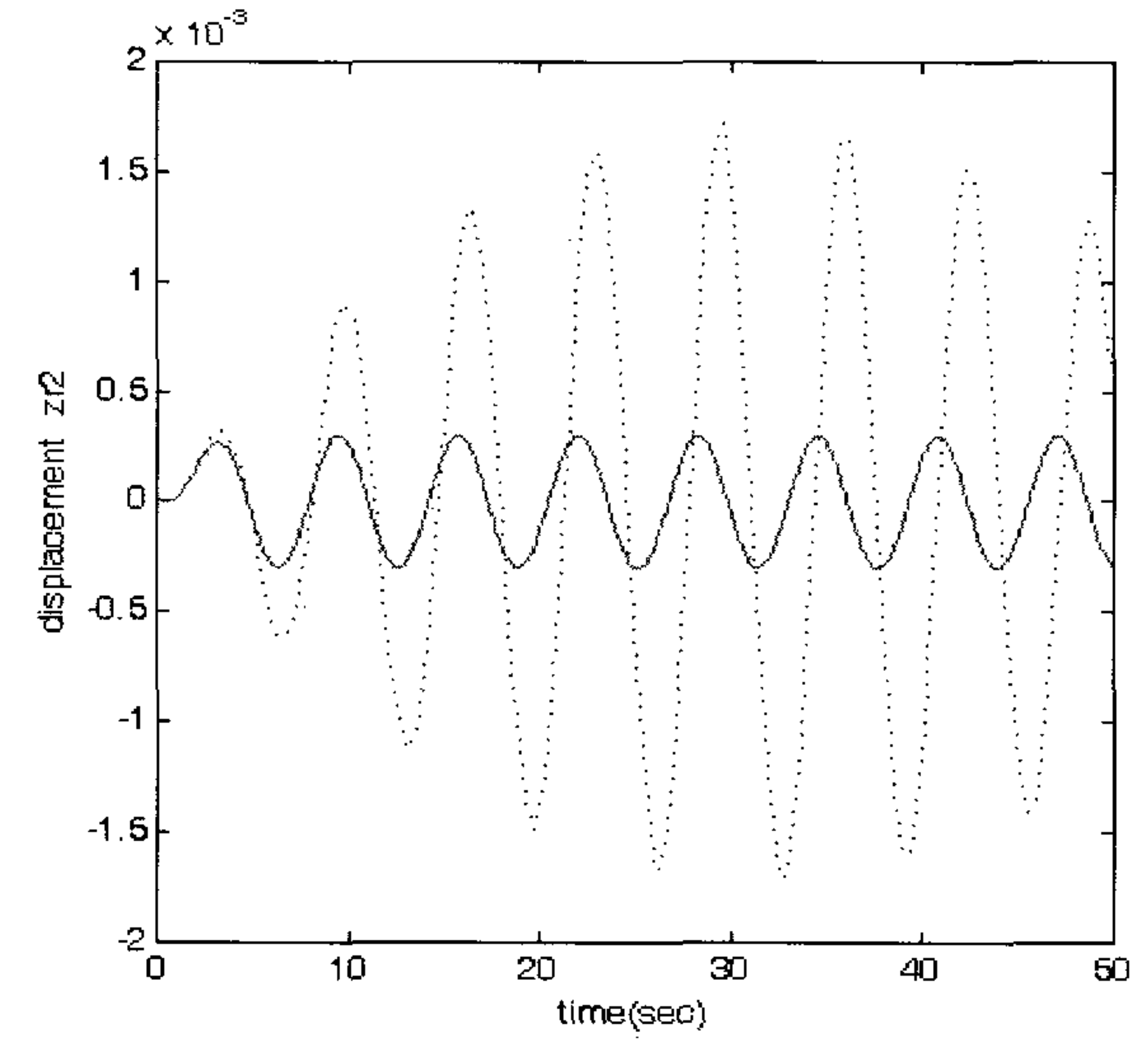

Fig. 7 Road disturbance response z_{r2}.

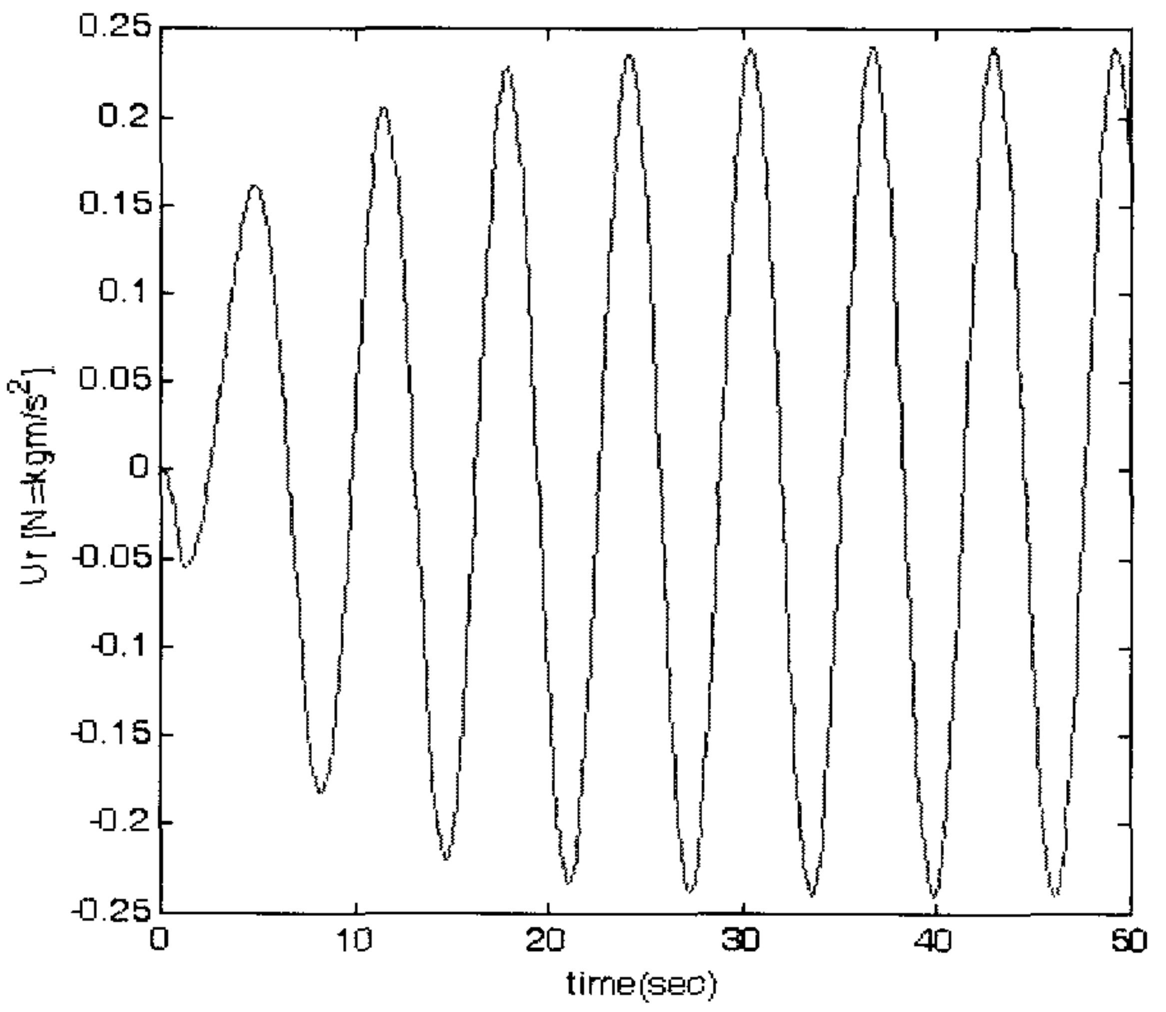

Fig. 8 Rear part control input U_r.

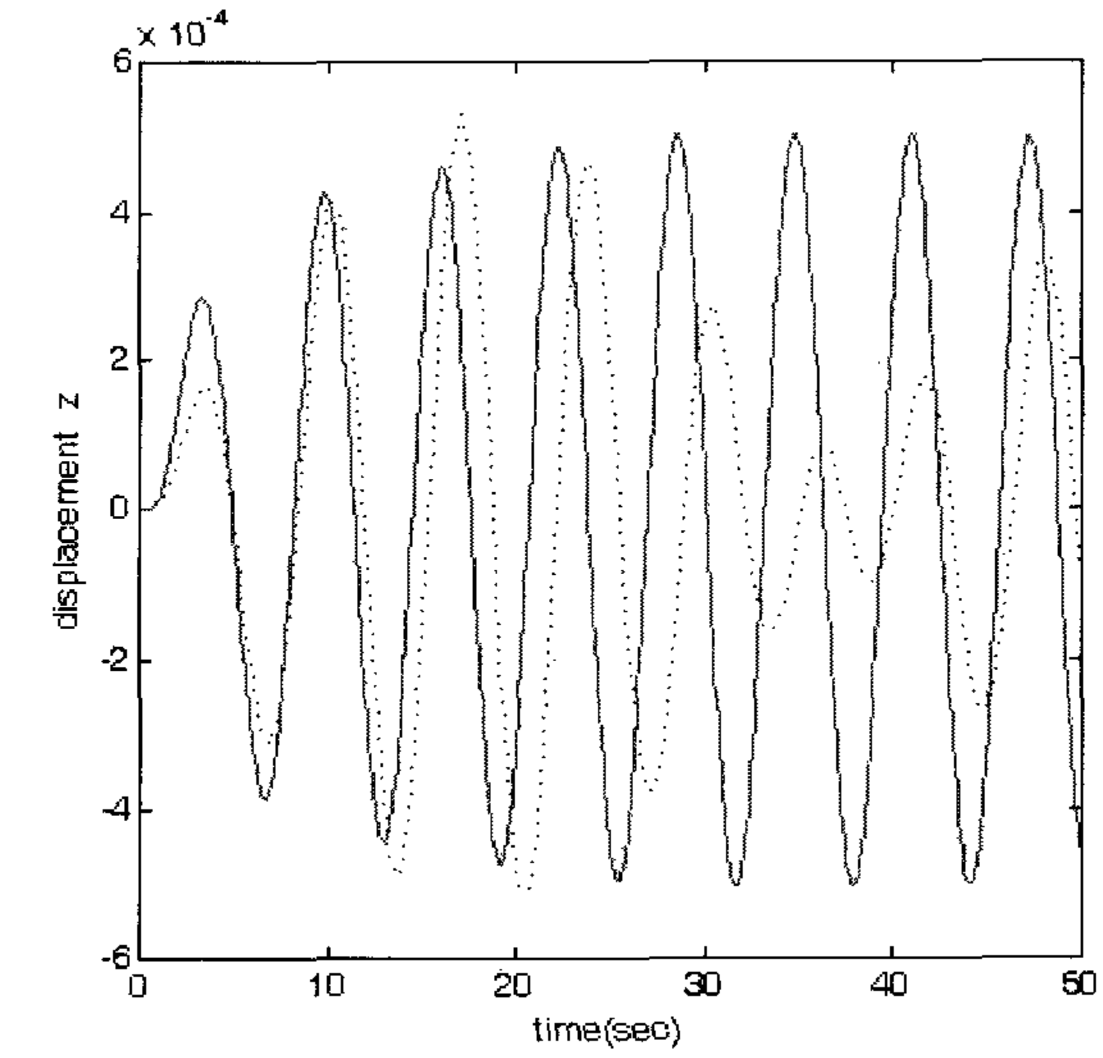

Fig. 9 Road disturbance bouncing response z.

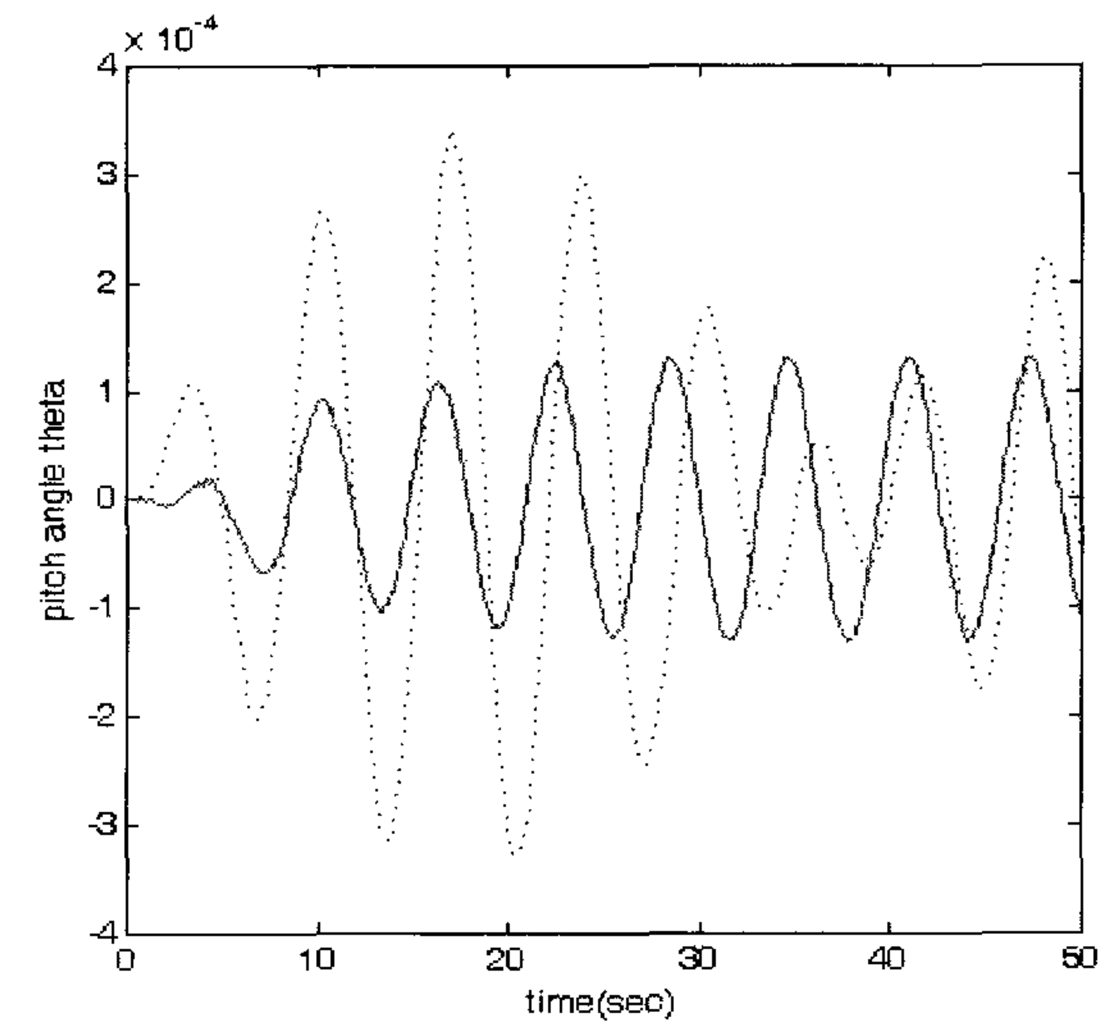

Fig. 10 Road disturbance pitch response θ.

Fig. 3 to Fig. 8 show that the active suspension 4-DOF vehicle model with the linear matrix inequalities robust h^{∞} controller exhibit better performances in upper and lower motion displacement vibration control than those of the vehicle with uncontrolled passive spring-damper suspension system. An appropriate robust h^{∞} control input solutions designed by the method based upon linear matrix inequalities are also shown in Fig. 5 and Fig. 8.

Fig. 9 and Fig. 10 show that the control method in this paper is prominent for improving vehicle performance and driver's ride comfort problems. In Fig. 9, Road disturbance bouncing response z about its mass center becomes smaller after 20 seconds in case of uncontrolled suspension than response of controlled one. However from a point of view which is concerned with ride comfort problem, steady vibration bouncing response (a solid line in Fig. 9; controlled) to the continuous road disturbances means better performance than irregular bouncing response to the regularly added sinusoidal road disturbance (a dotted line in Fig. 9; uncontrolled). Pitch angle response in Fig. 10 shows the control method in this paper is prominent for improving ride comfort.

IV. CONCLUSIONS

In this paper, I dealt with a design method based upon robust h^{∞} control solution which is obtained by linear matrix inequalities for improving vehicle performance and driver's ride comfort problems. The linear matrix inequalities robust h^{∞} controller was designed based on a 4our Degree of Freedom linear vehicle system model which represents the bouncing displacement and pitching angle of a vehicle concerned with front-rear parts bouncing displacements. In order to design linear matrix inequalities robust controller, the necessary and sufficient conditions for the existence of the linear matrix inequalities to solve robust h^{∞} control problem was investigated. The active suspension system with considering location of front-rear wheel and driving velocity was analyzed and the robust control system was also designed. The validity of the linear matrix inequalities robust control system design in active suspension system through the numerical examples and experiments was investigated.

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