

# A Review of the Expansion Behavior of Marine Pipelines

HAN-SUK CHOI\*, SEUNG-KEON LEE\* AND EUN-JEE CHUN\*

*\*Dept. of Naval Architecture and Ocean Engineering, Pusan National University, Busan, Korea*

**KEY WORDS:** Marine pipeline, Pipeline expansion, Pipe-in-pipe system, Lateral deviation

**ABSTRACT:** A comprehensive review of the expansion behavior of marine pipelines due to thermal and pressure change is presented based on research work over the last 10 years. The review is organized into five main sections, namely free expansion with uniform temperature, free expansion with temperature gradient, expansion with end restraints, expansion of pipe-in-pipe system, and lateral deviation (snaking). Based on the accumulated knowledge of the interactions between the soil and pipeline behavior, a whole pipeline system can be modeled by an accurate finite element method (FEM). This methodology requires a comprehensive understanding and engineering verification of the expansion behavior of marine pipelines.

## 1. Introduction

Many subsea pipelines are operated at a high pressure and a high temperature (HP/HT). The pipeline will be expanded at the certain distance from both ends and moving portion of the pipeline is called as a partially restrained zone. However, the expansion is constrained by friction between the pipeline and soil at the certain distance from the ends. The locations which expansion will not occur at certain distances are called as anchor points and the area between the anchor points is called as a fully restrained zone (Nes et al., 1996).

Pipelines from the subsea wells (flowline) are connected to the gathering platform and are operated with a HP/HT. These flowline require insulations to transport the production without a formation of wax or hydrate. Pipe-in-pipe (PIP) systems are often used for the HP/HT flowlines. A PIP system requires a complicated mechanical design, and expansion behavior should be incorporated with the mechanical design of the system (Choi, 2002; Choi and Do, 2006).

The sinusoidal lateral deviation or snaking behavior due to thermal expansion has been studied in recent years (Kershenbaum et al., 1996). The concept of the lateral deviation has been reflected in recent design codes.

A large amount of knowledge has been accumulated as results of recent research work. The purpose of the present paper is to review the pipeline expansion research work performed in the last 10 years. Based on the accumulated knowledge of the interactions between the soil and pipeline behavior, a whole pipeline system can be modeled by an

accurate finite element method (FEM). This methodology requires comprehensive understanding and engineering verification of the expansion behavior of marine pipelines.

## 2. Free Expansion With a Uniform Temperature

Basic expansion analysis methods for submarine pipelines have been well developed during the last two decades (AGA, 1987; Choi, 1995). The pipeline expansion was calculated with soil friction but without end restrictions. Thus, very conservative results were obtained.

The pipeline end expansion due to the temperature and pressure will create a soil friction force proportional to the length of the moving portion of the pipe. If the total friction force developed along the pipeline is sufficient to suppress expansion, no expansion will occur.

Figure 1 shows a sketch of pipeline expansion with uniform temperature. When the end restraining forces are zero, the analysis is called a free expansion problem. In Fig. 1, the two points where the movement stops are called the anchor points. Space and the lengths from the free ends to the anchor points are called the anchor lengths or moving length of the partially restrained zone. If the temperature is uniformly distributed along the pipeline, the anchor lengths at the upstream and downstream will be same. The portion of the length between the anchor points is called the fully restrained zone.

Once the anchor length,  $L_a$ , is obtained from the equilibrium of the forces in the pipeline, the total expansion and corresponding stresses can be easily obtained. The equilibrium of the forces in the pipeline is:

교신저자 최한석: 부산광역시 금정구 장전동

051-510-2343 hanchoi@pusan.ac.kr

$$F_t + F_p + F_v = F_f \quad (1)$$

where,  $F_t$  is the force due to the temperature,  $F_p$  is the force due to the pressure,  $F_v$  is the force due to Poisson contraction, and  $F_f$  is the force due to soil frictional resistance. The anchor length can be derived from the above equation:

$$L_a = \frac{1}{\mu W_s} (\alpha E A_s \Delta T + P A_i - \nu A_s \sigma_h) \quad (2)$$

where,  $\mu$  is the soil friction coefficient of pipe,  $W_s$  is the submerged weight of pipeline per unit length,  $\alpha$  is the coefficient of thermal expansion,  $E$  is the Young's modulus,  $A_s$  is the pipe steel section area,  $\Delta T$  is the temperature difference between inside and outside of pipe,  $P$  is the pressure difference between inside and outside of pipe,  $A_i$  is flow area of pipe,  $\nu$  is the Poisson's ratio, and  $\sigma_h$  is the hoop stress.

The expansion at the free end of the pipeline can be obtained using:

$$\Delta x = \int_0^{L_a} \varepsilon \cdot dx = \left\{ \alpha \Delta T + \frac{\sigma_h}{2E} (1 - 2\nu) - \frac{\mu W_s L_a}{2 A_s E} \right\} \cdot L_a \quad (3)$$

where  $\varepsilon$  is the strain of a pipeline in the partially restrained zone.

The above equations are conservatively used for the pipeline expansion. However, this approach is not valid for a very hot pipeline without good insulation system or a long pipeline whose temperature distribution has a considerable gradient along the pipeline.

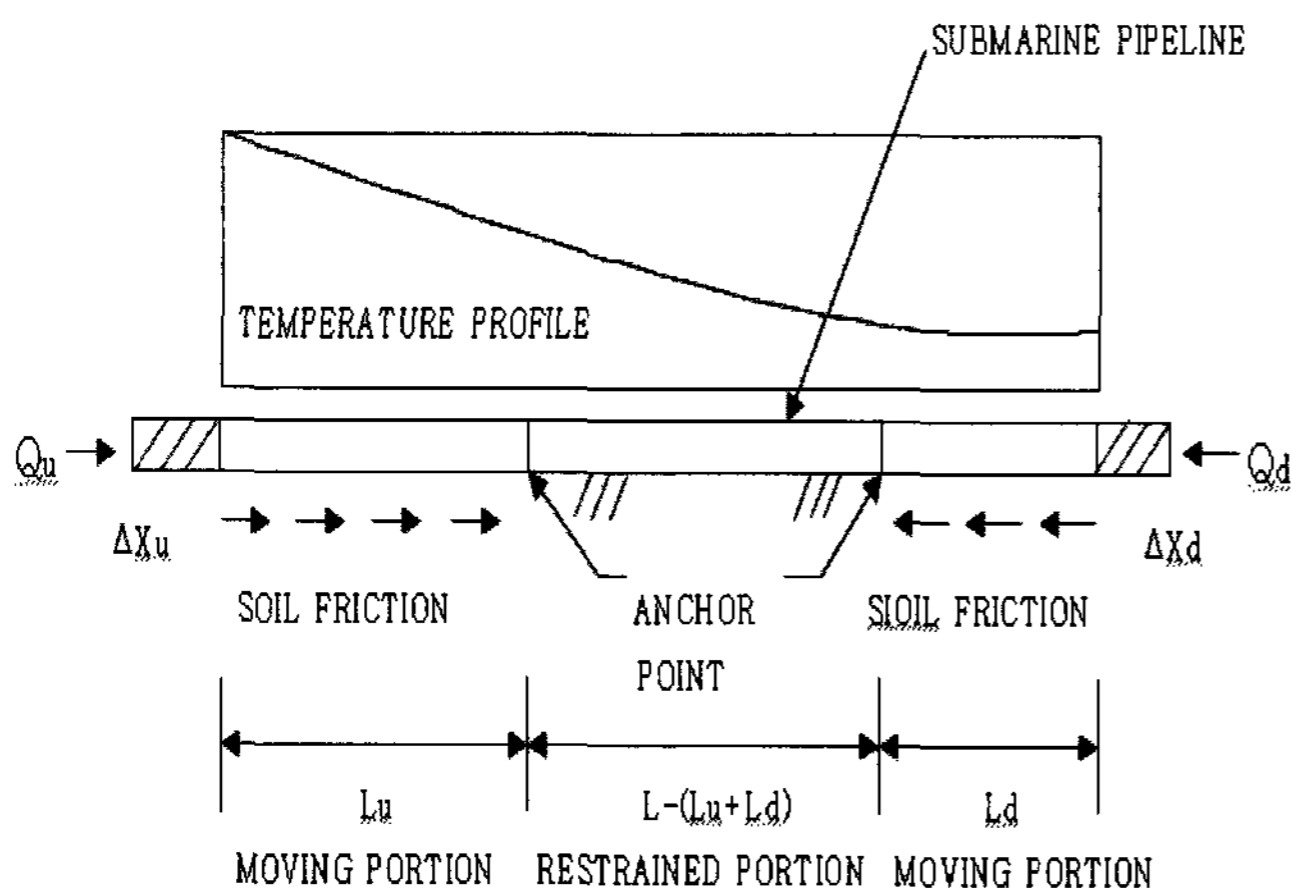


Fig. 1 A sketch of pipeline expansion

### 3. Free Expansion with a Temperature Gradient

A detailed analysis of the heat transfer can determine the temperature decay profile along the pipeline (MNET, 1991).

The temperature gradient will cause different anchor lengths at the upstream side and downstream side, i.e., the longer anchor length at the upstream side and short anchor length at the downstream side. Different anchor lengths will be resulted with temperature gradient. Both of the anchor lengths and the fully restrained portion are dependent on each other and should be obtained from the same calculation. Most of the existing software do not consider the dependence of the upstream anchor length, downstream anchor length, and restrained length. Choi (1995) has derived a closed form solution to obtain both of the anchor lengths simultaneously.

The average temperature over the moving length, at the upstream of the pipeline, can be obtained as follow:

$$DT_u = \frac{1}{L_u} \int_0^{L_u} \Delta T \cdot \exp\left(-\frac{x}{\lambda}\right) \cdot dx = -\frac{\lambda \Delta T}{L_u} \left\{ \exp\left(-\frac{L_u}{\lambda}\right) - 1 \right\} \quad (4)$$

where  $L_u$  is the upstream anchor length,  $x$  is the distance from the upstream end and  $\lambda = C_p \rho \cdot Q / U$  is the decay length of pipeline temperature,  $C_p$  is the specific heat of fluid contents,  $\rho$  is the fluid density,  $q$  is the flow rate, and  $U$  is the overall heat transfer coefficients.

The average temperature over the moving length, at the downstream of the pipeline, can be obtained similarly. The average temperature over the moving length, at the downstream of the pipeline, can be obtained similarly;

$$DT_d = \frac{1}{L_d} \int_{L-L_d}^L \Delta T \cdot \exp\left(-\frac{x}{\lambda}\right) \cdot dx = -\frac{\lambda \Delta T}{L_d} \left\{ \exp\left(-\frac{L}{\lambda}\right) - \exp\left(-\frac{L-L_d}{\lambda}\right) \right\} \quad (5)$$

The anchor length of the downstream,  $L_d$ , of the pipeline was given as;

$$L_d = \frac{1}{\mu W_s} \left[ \alpha E A_s \cdot \left( \frac{\lambda \Delta T}{L_d} \right) \left\{ \exp\left(-\frac{L-L_d}{\lambda}\right) - \exp\left(-\frac{L}{\lambda}\right) \right\} + P A_i - \nu A_s \sigma_h \right] \quad (6)$$

The above equations (4) through (6) are nonlinear and can be solved by using a numerical iteration method.

Choi (1995) indicates that the anchor points shift together depending on the temperature gradient and more realistic anchor lengths and expansions can be obtained. If the temperature is uniform, the upstream and downstream

anchor lengths become same as described in the previous section.

#### 4. Expansion With End Restraints

Pipeline end movement will be less than that of free expansion, if there are restraining forces at the ends of the pipeline. The restraining force and end movement of the pipeline interact with each other and can not be obtained independently.

Choi (1995) introduced a numerical iterative method. Firstly, end moving distance can be obtained using the free expansion analysis. Secondly, a first estimate of the restraining force can be calculated using the free end movement. Thirdly, the restraining force was added to calculate a new anchor length.

$$L_a = \frac{1}{\mu W_s} (\alpha E A_s \Delta T + P A_i - \nu A_s \sigma_h - Q) \quad (7)$$

where  $q$  is the end restraining force. The reduced anchor length is used for the next step calculation of the new end expansion length. The new expansion length can be used again for an another restraining force. This iterative procedure is continued until the new anchor length and end movement varies within a specified tolerance. Choi's iterative method produced the converged solutions with a few cycles in most cases.

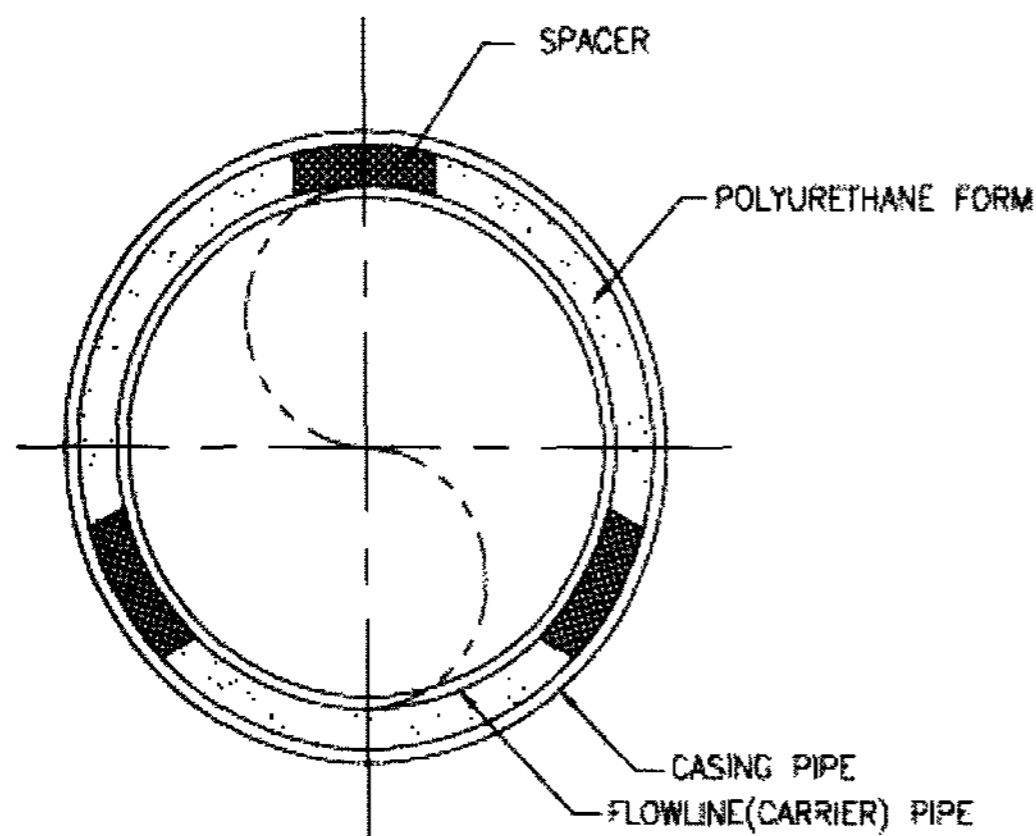


Fig. 2 Typical cross section of a pipe-in-pipe system

#### 5. Expansion of Pipe-in-pipe System

High pressure and high temperature reservoirs in subsea field are being developed as part of subsea tie-backs to existing or new platforms. There is a need to insulate the flowlines to transport the product without wax or hydrate formation (Choi, 2002; Choi and Do, 2006).

To prevent the wax and hydrate formation, the pipe-in-pipe system is the best configuration so far to insulate the flowline. In addition, the pipe-in-pipe system provides a good protection of the HP/HT flowlines. Heat loss from the flowline can be prevented by applying insulation between the flowline (carrier) pipe and the casing pipe using the PIP design concept as shown in Fig. 2.

The PIP system approach with a structurally coupled flowline pipe and a casing pipe was considered (Choi, 2002; Harrison et al., 1997; Kershenbaum et al., 1996).

For a PIP system design, the expansion analysis, the mechanical design of connection system between the two pipes and the tie-in spool design are integrated for the whole PIP system (Choi and Do, 2006).

##### 5.1 Axial forces in the pipes

Axial force in the flowline pipe ( $F_f$ ) is due to internal pressure ( $P_f$ ), external hydrostatic pressure ( $P_o$ ), and force imparted to the casing pipe ( $Q$ ) by rigid bulkheads:

$$F_f = F_{pf} - Q \quad (8)$$

$$F_{pf} = P_f A_{ff} - P_o \cdot (A_f + A_{ff}) \quad (9)$$

where,  $A_{ff}$  = Flowline flow area

$P_o$  = External hydrostatic pressure

$A_f$  = Flowline pipe steel section area

Axial force in casing pipe ( $F_c$ ) is due to internal pressure ( $P_c$ ), soil friction, and interaction force ( $Q$ ) imparted by the flowline pipe:

$$F_c = F_{pc} + Q + F_\mu \quad \text{for } x \leq Z_1 \quad \text{Region 1}$$

$$F_c = 0 \quad \text{for } Z_1 < x \leq Z_2 \quad \text{Region 3} \quad (10)$$

$$F_c = F_{pc} + Q + F_\mu \quad \text{for } Z_2 < x \leq L \quad \text{Region 2}$$

$$F_{pc} = P_c A_{an} - P_o \cdot (A_{an} + A_c) \quad (11)$$

$$F_\mu = -\mu \cdot W_{sub} \cdot x \quad (12)$$

where,  $A_{an}$  = Annular area

$A_c$  = Casing pipe steel section area

$\chi$  = Pipeline length

$L$  = Total pipeline length

##### 5.2 Strain and expansions in the pipes

Pipe strain in the flowline pipe is:

$$\varepsilon_f = \frac{1}{EA_f} \cdot (F_{pfc} + F_{yf} - Q) \quad (13)$$

$$F_{pfc} = F_{pf} - A_f \cdot \nu \cdot \sigma_f \quad (14)$$

$$F_{yf} = E \cdot A_f \cdot \alpha \cdot \Delta T \quad (15)$$

where,  $\sigma_f$  = hoop stress in flowline pipe  
Then, the expansion in the flowline pipe is:

$$\Delta L_f = \frac{1}{EA_f} \cdot (F_{pfc} + F_{yf} - Q) \cdot L \quad (16)$$

Pipe strain in the casing pipe is:

$$\varepsilon_c = \frac{F_c}{EA_c} - \nu \cdot \frac{\sigma_c}{E} \quad (17)$$

In region 1, the equation (10) becomes:

$$\varepsilon_{c1} = \frac{1}{EA_c} \cdot (F_{c1e} + F_\mu) \quad (18)$$

$$\text{where, } F_{c1e} = F_{pc} - A_c \cdot \nu \cdot \sigma_c + Q \quad (19)$$

$$Z_1 = \frac{F_{c1e}}{\mu \cdot W_{sub}} \quad (\text{Anchor point}) \quad (20)$$

In region 2, the equation (10) becomes:

$$\varepsilon_{c2} = \frac{1}{EA_c} \cdot (F_{c2e} - F_\mu) \quad (21)$$

$$\text{where, } F_{c2e} = F_{pc} - A_c \cdot \nu \cdot \sigma_c + Q - \mu \cdot W_{sub} \cdot L \quad (22)$$

$$Z_2 = -\frac{F_{c2e}}{\mu \cdot W_{sub}} \quad (\text{Anchor point}) \quad (23)$$

In region 3, the casing pipe has zero strain.

Pipe expansion in casing pipe in region 1 is:

$$\Delta L_{c1} = \int_0^{Z_1} \varepsilon_{c1} \cdot dx = \int_0^{Z_1} \frac{1}{E \cdot A_c} (F_{c1e} + F_\mu) dx \quad (24)$$

Pipe expansion in casing pipe in region 2 is:

$$\Delta L_{c2} = \int_{Z_2}^L \varepsilon_{c2} \cdot dx = \int_{Z_2}^L \frac{1}{E \cdot A_c} (F_{c2e} - F_\mu) dx \quad (25)$$

Total expansion of the casing pipeline is:

$$\Delta L_c = \Delta L_{c1} + \Delta L_{c2} \quad (26)$$

### 5.3 Interaction force between the pipes

Interaction force ( $Q$ ) can be obtained from:

$$\Delta L_f = \Delta L_c \quad (27)$$

$$(\varepsilon_{yf} + \varepsilon_{pf} + \varepsilon_{cf}) \cdot \frac{L_f}{2} - \frac{Q \cdot \frac{L_f}{2}}{A_f \cdot E} = (\varepsilon_{pc} + \varepsilon_{cc}) \cdot L_{anch} + \frac{Q \cdot L_{anch}}{A_c \cdot E} \quad (28)$$

$$\text{where, } L_{anch} = \frac{A_c \cdot E (\varepsilon_{pc} + \varepsilon_{cc} + \frac{Q}{A_c \cdot E})}{Fr_c} \quad (29)$$

$$\varepsilon_{pc} = -\frac{\nu \cdot OD_c \cdot \Delta P_c}{2 \cdot t_c \cdot E} \quad (30)$$

$$\varepsilon_{cc} = \frac{1}{A_c \cdot E} (P_c \cdot A_{an} - P_o \cdot \frac{\pi \cdot OD_c^2}{4}) \quad (31)$$

### 5.4 Mechanical Design

The PIP system, i.e., flowline pipe and casing pipe have mechanical connections through the water stops and the bulkheads. The former is evenly distributed along the pipeline and the latter is located at both ends of the pipeline.

The finite element (FE) analysis program has been used in the analysis of the bulkheads and water stops. Dimensions were established and a detailed FE analysis was conducted to capture local stresses within the bulkhead and water stop under the design loads.

The models and applied loads considered in the analyses are presented in Fig. 3 for the bulkheads and Fig. 4 for the water stops.

## 6. Lateral Deviation

Several studies were undertaken to investigate pipeline expansion behavior with consideration of lateral deviation. Apparently, the first study was dedicated to a similar problem: thermal expansion and buckling of steel railway tracks by Kerr (1974; 1978). Expanded pipeline upheaval buckling was studied by Hobbs (1984), Hobbs and Liang (1989), Ballet and Hobbs (1992), Brennodden and Stokkeland (1992) and Raof and Maschner (1993). Subsea thermal expanded pipeline upheaval buckling effects have been investigated analytically and by experimentally. Typically, initial pipeline imperfection was assumed on the sea bottom

and considered as the initiator of buckling. A similar approach undertaken by Sævik and Levold (1995) for pipeline lateral displacement or snaking behavior was formulated with the assumption of pipeline initial imperfections. Kershenbaum et al. (1996) have presented the basic formula of the lateral deviation with different soil friction model.

Pipelines with potential upheaval buckling were prevented by providing an overburden through dumping of large amount of rocks on the pipeline. Ellinas et al. (1990) presented this rock dumping technology for hot submarine pipelines. The rock dumping method is costly in most cases. Vermeulen (1995) presented a method of preventing upheaval buckling by means of the pre-snaking method. A pre-calculated bend is applied in a pipeline at regular intervals while it is being installed on the seabed from a layvessel. The pre-bent reduced the axial stiffness and prevented large axial forces from the thermal expansion. However, this improved method has a limit with pipe size and an additional bending machine is required on the lay vessel. Kershenbaum et al. (1996) have developed a more realistic model of the lateral deviation as described in the next subsection.

### 6.1 Buckling with plastic lateral resistance

The pipeline longitudinal axis was assumed to be unstable due to buckling and forms a sinusoidal lateral deviation (snaking):

$$z = A(x) \cdot \sin \frac{\pi x}{l(x)} \quad (32)$$

where,  $A(x)$  is the amplitude of pipeline lateral deviation,  $l(x)$  is a half of the snaking wave length, and  $x$  is coordinates from the restrained end of pipeline. The temperature distribution along the pipeline has been calculated and soil resistance creates and assumed frictional (plastic) force against the lateral deviation. Then the total energy in the pipeline segment of lateral deviation (length of  $L$ ) is as follows:

$$W = W_e - W_l + W_f \quad (33)$$

It is known that pipe bending strain energy can be represented:

$$W_e = \frac{EI}{2} \cdot \int_0^L \left( \frac{d^2 z}{dx^2} \right)^2 \cdot dx \quad (34)$$

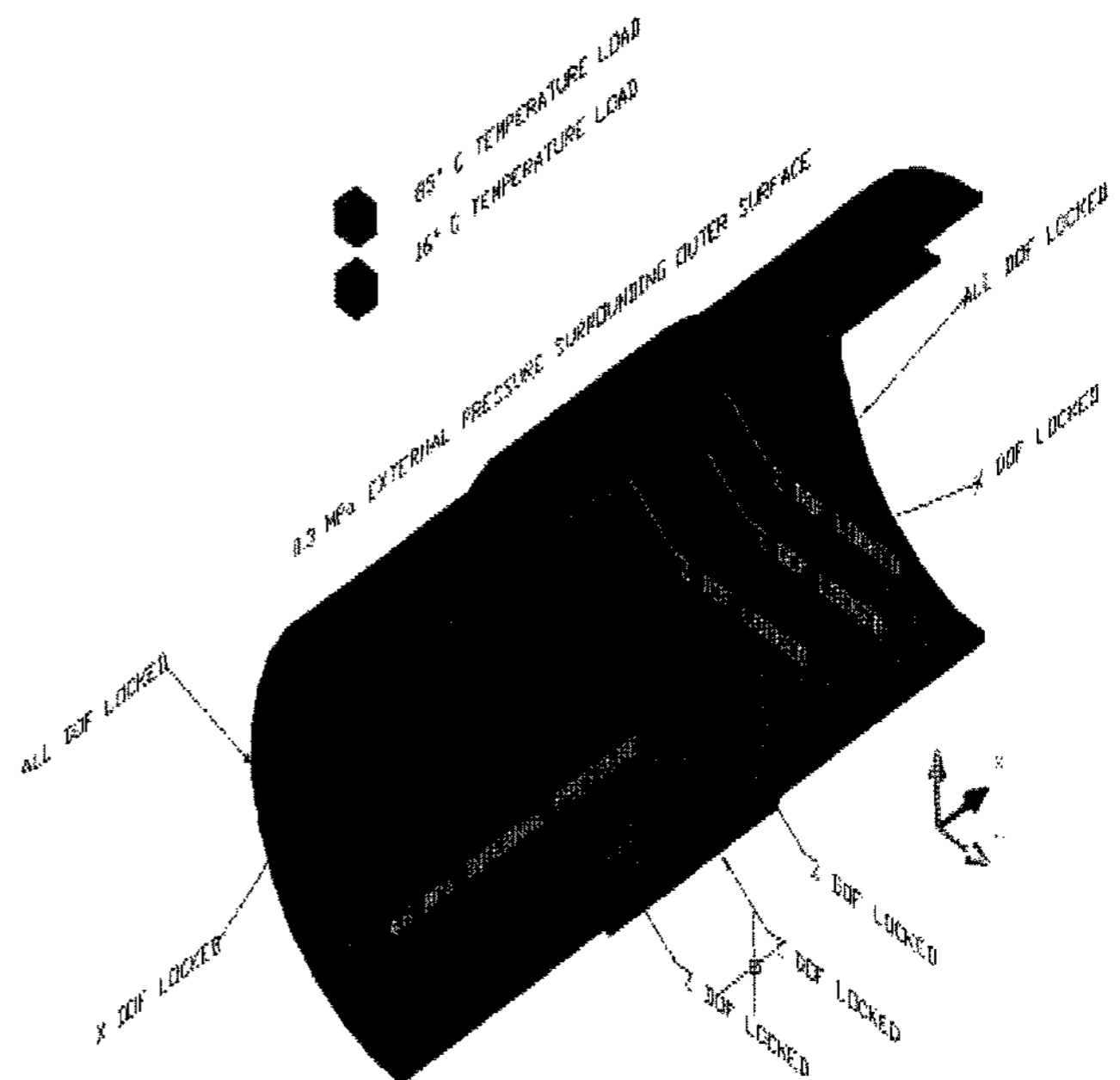


Fig. 3 Bulkhead model and loading

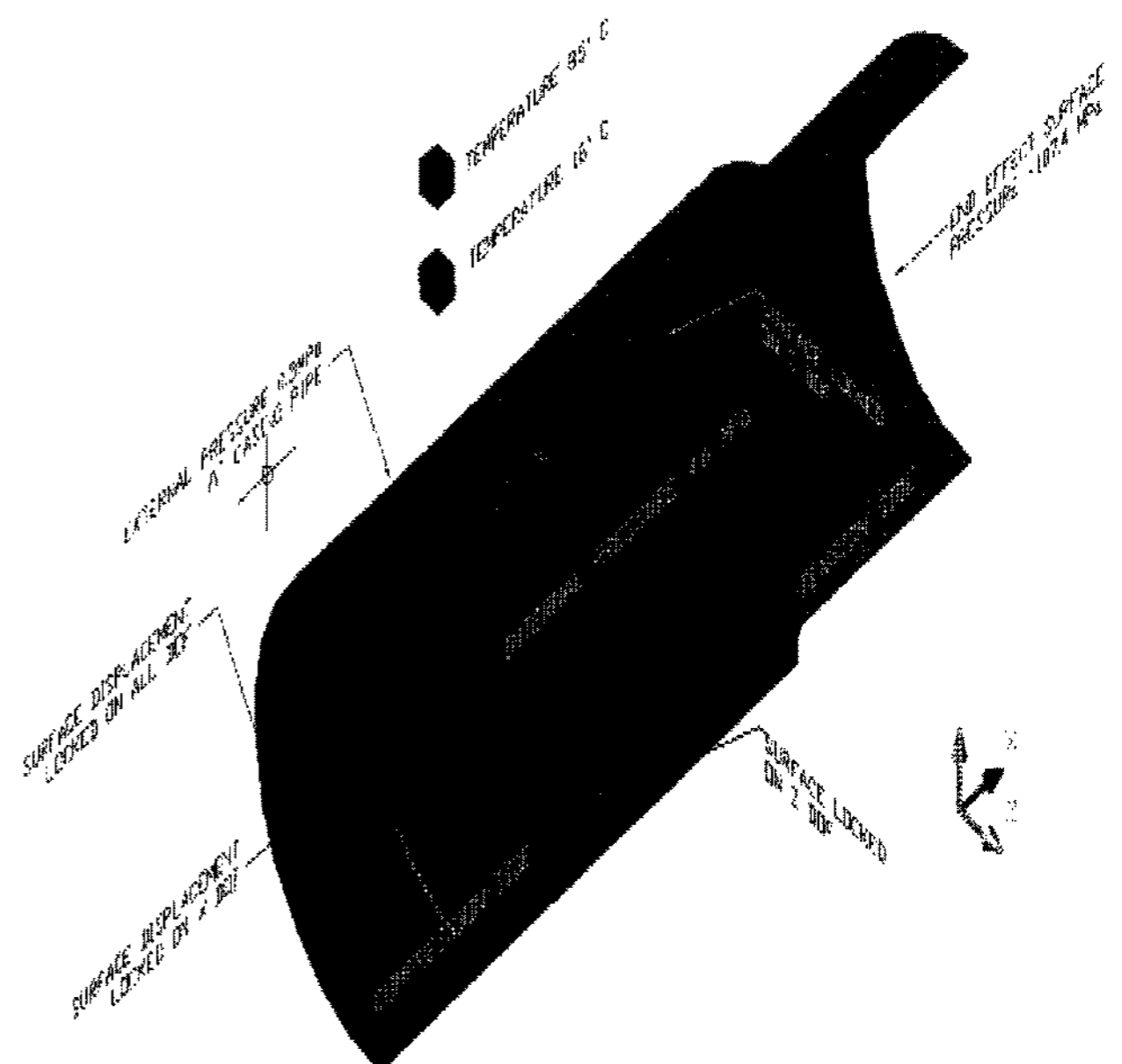


Fig. 4 Water stop model and loading

Where,  $\bar{l}$  is the average half of the snaking wave length and  $\bar{A}$  is the average amplitude of pipeline lateral deviation.

The direction of the soil frictional resistance to pipe lateral deviation is against pipe movement; hence, frictional work in lateral direction is:

$$W_f = \int_0^L |z| \cdot wf \cdot dx \quad (35)$$

Longitudinal work due to lateral deviation of pipeline can be expressed:

$$W_l = \int_0^L F_c \frac{d\Delta L}{dn} \cdot dl \quad (36)$$

Where the axial compressive force due to Euler's instability is as follows:

$$F_c = \frac{\pi^2 EI}{l^2(x)} \equiv \frac{\pi^2 EI}{\bar{l}^2} \quad (37)$$

Then

$$\Delta L = \int_0^L \sqrt{1+z'^2} \cdot dx - L \equiv \frac{\bar{A}^2 \cdot L \cdot \pi^2}{4\bar{l}^2} \quad (38)$$

For the particular case, when  $\bar{l} = L$  there is an expression:

$$\bar{A} \equiv \frac{2\bar{l}}{\pi} \sqrt{\frac{\Delta L}{L}} \quad (39)$$

Then the equation (32) can be rewritten as follow:

$$W = \frac{\bar{A}^2 \cdot EI \cdot \pi^4 \cdot L}{4\bar{l}^4} - \int_0^L F_c \frac{d\Delta L}{dl} \cdot dl + \frac{2\bar{A}L \cdot wf}{\pi} \quad (40)$$

For equilibrium condition of pipeline, the variational principle of energy must be satisfied:

$$\frac{dW}{d\bar{l}} = 0 \quad (41)$$

The solution is as follows:

$$\bar{A} \equiv \frac{1}{\frac{EI \cdot \pi^5}{16 \cdot \bar{l}^4 \cdot wf} + \frac{\pi}{2 \sqrt{\Delta L \cdot L}}} \quad (42)$$

But,  $\Delta L/L = \bar{\epsilon}$  is an average elongation of the pipeline. The local elongation of the pipeline. The local elongation is as follows:

$$\epsilon(x) = \frac{\Delta l}{l(x)} = \alpha \cdot \Delta T(x) + \frac{\Delta P \cdot A_i}{A_s E} - \frac{\mu \cdot \Delta P \cdot d}{2tE} + \frac{\pi^2 I}{A_s \cdot l^2(x)} \quad (43)$$

## 6.2. Buckling with elastic lateral resistance

If the pipe has embedded into the bottom soil with more than a half of the diameter, then elastic type of soil resistance to lateral movement must be considered. At the same time, the pipe axial movement is accompanied by sliding along the soil surface where frictional type of resistance predominate.

The longitudinal critical force associated with pipeline lateral buckling resulting from the energy variational approach can be used. Once the bulking force and the mode number are obtained for the elastic soil resistance, the rest of the solution can be obtained with the same procedure described in the previous section.

## 7. Concluding Remarks

A comprehensive review on expansion behavior of marine pipelines due to thermal and pressure change was conducted based on the last 10 years research work. Five different subjects were discussed and the characteristic of the problems were identified.

Based on the accumulated knowledge of the interaction between the soil and pipeline behavior, a whole pipeline system can be modeled by an accurate finite element method (Bai, 2001; Ose et al., 1999). Am FEM modeling with a whole pipeline system requires huge capacity of computers. However, modeling a whole pipeline with a FEM has been continuously developed. This methodology requires a comprehensive understanding and engineering verification of the expansion behavior of marine pipelines described in the paper.

## Acknowledgements

This work is supported for 10 months by Brain Korea 21 (BK 21) of Pusan National University research grant.

## References

- AGA (1987). Pipeline Riser System Design and Application Guide, AGA Project Number 178-622, Reported by Brown and Root.
- Bai, Y. (2001). Pipelines and Risers, Elsevier Ocean Engineering Book Series, Vol 3.
- Ballet, J.P. and Hobbs, R.E. (1992). "Asymmetric Effect of Prop Imperfections on the Upheaval Buckling of Pipelines", Thin-Walled Structures, Vol 13, pp 355-373.
- Brennodden, H. and Stokkeland, A. (1992). "Time-Dependent

- Pipe-Soil Resistance for Soft Clay", Proc. of 24th Offshore Technology Conference, OTC 6846.
- Choi, H.S. (1995). "Expansion Analysis of Offshore Pipelines Close to Restraints", Proc. of 5th Int. Offshore and Polar Engineering Conference, Vol 2, pp 81-88.
- Choi, H.S. (2002). "Expansion Analysis of Subsea Pipe-in-Pipe Due to High Temperature and High Pressure Product", Journal of Ocean Engineering and Technology, Vol 16, No 5, pp 56-60.
- Choi, H.S. and Do, C.H. (2006). "Integarated Expansion Analysis of Pipe-In-Pipe System", Journal of Ocean Engineering and Technology, Vol 20, No 5, pp 9-14.
- Ellinas, C.P., Supple, R.P. and Morris, D.V. (1990). "Prevention of Upheaval Buckling of Hot Submarine Pipelines by Means of Intermittent Rock Dumping", Proc. 22nd offshore Tech Conf., pp 519-528.
- Harrison, G.E., Kershenbaum, N.Y. and Choi, H.S. (1997). "Expansion Analysis of Subsea Pipe-In-Pipe Flowline", Proc. of 7th Int. Offshore and Polar Engineering Conference, Vol 2, pp 293-298.
- Hobbs, R.E. (1984). "In-Service Buckling of Heated Pipelines", Journal Transport Engineering, pp 178-179.
- Hobbs, R.E. and Liang, F. (1989). "Thermal Buckling of Pipelines Close To Restraints", Proc. of 8th Int. Conf. on Offshores Mechanics and Arctic Engineering, Vol 4, pp 121-127.
- Kerr, A.D. (1974). "On the Stability of Railroad Track in the Vertical Plane", Rail International, Vol 5, pp 132-142.
- Kerr, A.D. (1978). "Analysis of Thermal Track Buckling in the Lateral Plane", Acta Mechanica, Vol 30, pp 17-50.
- Kershenbaum, N.Y., Harrison, G.E. and Choi, H.S. (1996). "Subsea Pipeline Lateral Deviation Due to High Temperature Product", Proc. of 6th Int. Offshore and Polar Engineering Conf., Vol 2, pp 74-79.
- MNET (1991). Multiphase Network Simulation Model User's Guide, Ver 1.0, Scientific Software-Intercomp, Houston.
- Nes, H., Sævik, S., Levold, E. and Johannesen, A., (1996). "Expansion Control Design of Large Diameter Pipelines", Proceedings 15th Int. Conf. on Offshore Mechanics and Arctic Engineering, Vol 5, pp 279-285.
- Ose, B.A., Bai, Y., Nystrøm, P.R. and Damsleth, P.A. (1999). "A Finite Element Model for In-situ Behavior of Offshore Pipelines on Uneven Seabed and Its Application to On-Bottom Stability", Proc. of 9th Int. Offshore and Polar Engineering Conf., Vol 2, pp 132-140.
- Raof, M. and Maschner, E. (1993). "Thermal Buckling of Subsea Pipelines", Proc. of 12th Int. Conf. on Offshore Mechanics and Arctic Engineering, Vol 5, pp 21-29.
- Saevik, S. and Levold, E. (1995). "High Temperature Snaking Behavior of Pipelines", Proc. of 5th Int. Offshore and Polar Engineering Conf., Vol 2, pp 63-71.
- Vermeulen, H.R. (1995). "Theory and Practice of Installing Pipelines by the Pre-Snaking Method", Proc. of 5th Int. Offshore and Polar Engineering Conf., Vol 2, pp 47-52.

---

2007년 10월 16일 원고 접수

2008년 4월 1일 최종 수정본 채택