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A Far Field Solution of the Slowly Varying Drift Force on an Offshore Structure in Bichromatic Waves - Two Dimensional Problems

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ABSTRACT: *A far field solution of the slowly varying force on an offshore structure by gravity ocean waves was shown as a function of the reflection and transmission of the body disturbed waves. The solution was obtained from the conservation of the momentum flux, which simply describes various wave forces, while making it unnecessary to compute complicated integration over a control surface. The solution was based on the assumption that the frequency difference of the bichromatic incident waves is small and its second order term is negligible. The final solution is expressed in term of the reflection and transmission waves, i.e. their amplitudes and phase angles. Consequently, it shows that not only the amplitudes but also the phase differences make critical contributions to the slowly varying force. In a limiting case, the slowly varying force solution gives the one of the mean drift force, which is only dependent on the reflection wave amplitude. An approximation is also suggested in a case where only the mean drift force information is available.*

1. Introduction

The slowly varying drift force is a phenomenon where ocean structures experience a low frequency force and motion with a frequency lower than that of the incident waves. Understanding this phenomenon is of particular importance to ocean system design and operations. Offshore systems, such a guide tower, TLP, FPSO, semisubmersible, open ocean aquaculture, wave power converter, or ocean buoy, are designed to operate with a flexible mooring or positioning system to avoid direct, or first order wave frequency loads. A drawback of such a flexible system, as a bi-product, is the possible excitation of low frequency resonance by the slowly varying drift force.

Over the last thirty years, progress has been made in understanding and predicting this slowly varying force. As a near field approach, Pinkster (1979) used the bichromatic wave velocity potential to evaluate the second order force due to the wave elevation and fluid velocity on the surface of the offshore structure. Though the near field approach clearly gives the physical meaning of each force component, it has convergence problems related to the solution accuracy for the number of panels, the evaluation of velocity square terms, the singularities on sharp corners and the line integration for the wave elevation on the waterline.

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Kim and Yue (1990) showed the importance of the second order diffraction effect in evaluating the slowly varying force for an axisymmetric body. Dai et al. (2005) suggested a middle field solution in which pressure integration was performed on the fictitious surface of a middle field control volume. The advantage of the middle field solution is its superiority in solution convergence, while the near field solution is confronted with slow convergence or divergence problems.

The present paper suggests a far field formulation for the slowly varying force of a two dimensional body in an explicit form. The conservation of the momentum flux is applied in the far field fluid domain, and the time varying slowly varying terms are extracted and summed up to obtain the final solution. Theoretical computation results for a fixed finite vertical thin barrier in waves are provided. And a crude approximation form is also suggested for design purposes.

2. Mathematical Formulation

The conservation of momentum flux is expressed in term of the linear velocity potential, which describes the pressure at the far field. Bichromatic incident waves of different frequencies are then introduced to define the potential fields. Applying the bichromatic wave potential to the conservation of the momentum flux, the explicit formula for the second order slowly varying force is derived.

2.1 Momentum flux

Assume that a long cylinder body is floating at the sea surface and gravity waves progress from the left negative x-direction to the right positive x-direction perpendicular to the body, where the origin of the coordinate is located. The positive y-direction is defined as upward.

Taking the two dimensional control boundary with verticals at the infinity in the negative and positive x-directions, the momentum flux (Maruo, 1960) in the x-direction is written as

$$\frac{dM_x}{dt}=\rho\int_{-\infty}^{\infty}u_{-\infty}^2dy+\int_{-\infty}^{\infty}p_{-\infty}dy-\rho\int_{-\infty}^{\infty}u_{\infty}^2dy-\int_{-\infty}^{\infty}p_{\infty}dy-F_x \tag{1}$$

where u is the x-component velocity of the fluid, ρ is the fluid density, p is the hydrodynamic pressure and F_x is the force exerted on the body in the x-direction. The suffixes $-\infty$ and $+\infty$ denote the left and right infinite boundaries, respectively. The hydrodynamic pressure is expressed as

$$\frac{p}{\rho}=\frac{p_0}{\rho}+\frac{\partial\Phi}{\partial t}-\frac{1}{2}\left[\left(\frac{\partial\Phi}{\partial x}\right)^2+\left(\frac{\partial\Phi}{\partial y}\right)^2\right]-gy \tag{2}$$

where g is the gravity acceleration, t the time, Φ is the fluid velocity potential and p_0 is the atmospheric pressure. We can let the atmospheric pressure p_0 be zero without losing generality. Substituting Eq. (2) to Eq. (1) yields

$$\begin{aligned}F_x=&-\frac{dM}{dt}+\frac{1}{2}\rho\int_{-\infty}^{0}\left[\left(\frac{\partial\Phi}{\partial x}\right)_{-\infty}^2-\left(\frac{\partial\Phi}{\partial y}\right)_{-\infty}^2\right]dy\\&-\frac{1}{2}\rho\int_{-\infty}^{0}\left[\left(\frac{\partial\Phi}{\partial x}\right)_{\infty}^2-\left(\frac{\partial\Phi}{\partial y}\right)_{\infty}^2\right]dy\\&+\rho\int_{-\infty}^{0}\left[\left(\frac{\partial\Phi}{\partial t}\right)_{-\infty}-\left(\frac{\partial\Phi}{\partial t}\right)_{\infty}\right]dy\\&+\rho(\zeta_{-\infty}\frac{\partial\Phi}{\partial t})_{y=0}-\rho(\zeta_{\infty}\frac{\partial\Phi}{\partial t})_{y=0}\\&-\frac{1}{2}\rho g(\zeta_{-\infty}^2-\zeta_{\infty}^2)\end{aligned} \tag{3}$$

where ζ is the wave elevation on the surface. The force expressed in Eq. (3) is an instantaneous one, including the mean drift force.

2.2 Bichromatic wave potentials

The first order wave potentials for the incoming and outgoing waves of different frequencies are defined. The incoming waves are bichromatic, or two waves of different frequencies. And each disturbed wave has its own independent frequency and amplitude.

Each velocity potential satisfies the boundary conditions. The boundary condition at the free surface is

$$\frac{\partial\Phi}{\partial t})_{y=0}-g\zeta=0 \tag{4}$$

and the kinematic condition is

$$\frac{\partial\zeta}{\partial t}=-\left(\frac{\partial\Phi}{\partial y}\right)_{y=0} \tag{5}$$

The first order velocity potential of the bichromatic waves is assumed to be a linear sum of components as

$$\Phi=\Phi_i+\Phi_j=\mathrm{Re}\phi_i e^{i\sigma_i t}+\mathrm{Re}\phi_j e^{i\sigma_j t} \tag{6}$$

where ϕ_i and ϕ_j are complex potentials associated with the wave circular frequencies σ_i and σ_j. The suffix i and j denote the wave frequencies, while i in the superscript denotes the complex quantity $i=\sqrt{-1}$. Each component of the wave potentials satisfies the dynamic boundary conditions as

$$\frac{\partial\phi_j}{\partial y}-K_j\phi_j=0 \quad \text{at } y=0 \tag{7}$$

where $K_j=\sigma_j^2/g$ is a wave number. The boundary condition in Eq. (7) also holds for the wave potential ϕ_i. The regular waves progressing in the x-direction are expressed by

$$\phi_{wj}=c_jh_je^{K_jy-iK_jx} \tag{8}$$

where the wave amplitude is h_j and the wave celerity is $c_j=\sigma_j/K_j$. At a great distance from the body, the radiation and diffraction waves caused by the cylinder take the form of regularly progressing outward. Hence, $x\to-\infty$, it can be written

$$\phi_{Bj}=c_jA_j^-e^{K_jy+iK_jx} \tag{9}$$

and at $x\to+\infty$

$$\phi_{Bj}=c_jA_j^+e^{K_jy-iK_jx} \tag{10}$$

for a wave of circular frequency σ_j, where A+ and A- are the amplitudes in complex quantity. The resultant velocity potential for the bichromatic wave system with frequencies σ_i and σ_j is written as

$$\phi = \phi_{wi} + \phi_{Bi} + \phi_{wj} + \phi_{Bj} \tag{11}$$

The wave elevation from Eq. (4) is obtained as

$$\begin{aligned}\zeta &= \frac{1}{g}\frac{\partial \Phi}{\partial t})_{y=0} \\ &= Re\left(\frac{i\sigma_i}{g}(\phi_{wi}+\phi_{Bi})_{y=0}e^{i\sigma_i t} + \frac{i\sigma_j}{g}(\phi_{wj}+\phi_{Bj})_{y=0}e^{i\sigma_j t}\right)\end{aligned} \tag{12}$$

Applying Eq. (12) to Eq. (3), it is further simplified as

$$\begin{aligned}F_x &= \frac{1}{2}\rho\int_{-\infty}^{0}\left[\left(\frac{\partial \Phi}{\partial x}\right)_{-\infty}^{2} - \left(\frac{\partial \Phi}{\partial y}\right)_{-\infty}^{2}\right]dy \\ &- \frac{1}{2}\rho\int_{-\infty}^{0}\left[\left(\frac{\partial \Phi}{\partial x}\right)_{\infty}^{2} - \left(\frac{\partial \Phi}{\partial y}\right)_{\infty}^{2}\right]dy \\ &+ \rho\int_{-\infty}^{0}\left[\left(\frac{\partial \Phi}{\partial t}\right)_{-\infty} - \left(\frac{\partial \Phi}{\partial t}\right)_{\infty}\right]dy \\ &+ \frac{1}{2}\rho g\left(\zeta_{-\infty}^{2} - \zeta_{\infty}^{2}\right) - \frac{dM}{dt}\end{aligned} \tag{13}$$

We can put $dM/dt = 0$, without losing generality. The force equation, Eq. (13), accounts for all of the time variant and invariant forces acting on the cylinder. The next step is to identify and extract the slowly varying forces from Eq. (13).

2.3 Slowly varying drift force

As given in Eq. (13), the force exerted on the body can, in principle, be arranged in terms of (σ_i , σ_j , $2\sigma_i$, $2\sigma_j$ $\sigma_i - \sigma_j$, $\sigma_i + \sigma_j$) as

$$\begin{aligned}F_x = Re[&F_i^{(2)} + F_j^{(2)} + F_i^{(1)}e^{i\sigma_i t} + F_j^{(1)}e^{i\sigma_j t} \\ &+ F_i^{(2)}e^{2i\sigma_i t} + F_j^{(2)}e^{2i\sigma_j t} + F_{ijL}^{(2)}e^{i(\sigma_i-\sigma_j)t} + F_{ijH}^{(2)}e^{i(\sigma_i+\sigma_j)t}]\end{aligned} \tag{14}$$

where the superscript number denotes the order of the force with respect to the wave amplitudes. The first two terms denote the mean drift force with $F_i^{(2)} = O(h_i^2(\sigma_i))$ and $F_j^{(2)} = O(h_j^2(\sigma_j))$. The terms $F_i^{(1)} = O(h_i(\sigma_i))$ and $F_j^{(1)} = O(h_j(\sigma_j))$ are the first order forces at each frequency, $F_i^{(2)} = O(h_i^2(\sigma_i))$ and $F_j^{(2)} = O(h_j^2(\sigma_j))$ are the second order double frequency terms, $F_{ijL}^{(2)} = O(h_i(\sigma_i)h_j(\sigma_j))$ is the second order slowly varying force term coupled by two frequencies and $F_{ijH}^{(2)} = O(h_i(\sigma_i)h_j(\sigma_j))$ is the second order fast frequency term.

The mean drift force terms are derived by Maruo(1960) from Eq. (14) taking the zeroth order or time average of the momentum conservation to the monochrome frequency wave. The same equation Eq. (14) is now directly applied to the bichromatic waves to derive the time dependent slowly varying force.

Using Eq. (6), (8), (9), and (12), each partial derivative in Eq. (13) is evaluated as

$$\left(\frac{\partial \Phi}{\partial x}\right)_{-\infty} = Re\left[-iK_i c_i R_i e^{K_i y}e^{i\sigma_i t} - iK_j c_j R_j e^{K_j y}e^{i\sigma_j t}\right] \tag{15a}$$

$$\left(\frac{\partial \Phi}{\partial x}\right)_{+\infty} = Re\left[-iK_i c_i P_i e^{K_i y}e^{i\sigma_i t} - iK_j c_j P_j e^{K_j y}e^{i\sigma_j t}\right] \tag{15b}$$

$$\left(\frac{\partial \Phi}{\partial y}\right)_{-\infty} = Re\left[K_i c_i R_i^+ e^{K_i y}e^{i\sigma_i t} + K_j c_j R_j^+ e^{K_j y}e^{i\sigma_j t}\right] \tag{15c}$$

$$\left(\frac{\partial \Phi}{\partial y}\right)_{+\infty} = Re\left[K_i c_i P_i e^{K_i y}e^{i\sigma_i t} + K_j c_j P_j e^{K_j y}e^{i\sigma_j t}\right] \tag{15d}$$

$$\left(\frac{\partial \Phi}{\partial t}\right)_{-\infty} = Re\left[i\sigma_i c_i R_i^+ e^{K_i y}e^{i\sigma_i t} + i\sigma_j c_j R_j^+ e^{K_j y}e^{i\sigma_j t}\right] \tag{15e}$$

$$\left(\frac{\partial \Phi}{\partial t}\right)_{+\infty} = Re\left[i\sigma_i c_i P_i e^{K_i y}e^{i\sigma_i t} + i\sigma_j c_j P_j e^{K_j y}e^{i\sigma_j t}\right] \tag{15f}$$

where

$$R_i = h_i e^{-iK_i x} - A_i^- e^{iK_i x} \tag{16a}$$

$$R_i^+ = h_i e^{-iK_i x} + A_i^- e^{iK_i x} \tag{16b}$$

$$R_j = h_j e^{-iK_j x} - A_j^- e^{iK_j x} \tag{16c}$$

$$R_j^+ = h_j e^{-iK_j x} + A_j^- e^{iK_j x} \tag{16d}$$

$$P_i = h_j e^{-iK_i x} + A_j^+ e^{-iK_i x} \tag{16e}$$

$$P_j = h_j e^{-iK_j x} + A_j^+ e^{-iK_j x} \tag{16f}$$

Expending all of the force terms in Eq. (13), the slowly varying components are collected and rearranged. To make it easy, an operator $\mathcal{L}$[ ] is defined to extract the low frequency parts as shown in the Appendix. The first term in Eq. (13) is written using Eq. (15a, b) and Eq. (16a-d) as

$$\begin{aligned}F_{ijL1}^{(2)} &= \mathcal{L}\left[\frac{1}{2}\rho\int_{-\infty}^{0}\left[\left(\frac{\partial \Phi}{\partial x}\right)_{-\infty}^{2} - \left(\frac{\partial \Phi}{\partial y}\right)_{-\infty}^{2}\right]dy\right] \\ &= \frac{1}{2}\rho\frac{\sigma_i\sigma_j}{(K_i+K_j)}Re\left(R_i R_j^* - R_i^+ R_j^{+*}\right)e^{i(\sigma_i-\sigma_j)t}\end{aligned} \tag{17}$$

In an analogous way, we can show that the slowly varying part of the second term in Eq. (13) vanishes as

$$F_{ijL2}^{(2)} = -\pounds\left[\frac{1}{2}\rho\int_{-\infty}^{0}\left\{\left(\frac{\partial\Phi}{\partial x}\right)_{\infty}^{2} - \left(\frac{\partial\Phi}{\partial y}\right)_{\infty}^{2}\right\}dy\right] \quad (8)$$
$$= -Re\frac{\rho}{2}\int_{-\infty}^{0}\left[\sigma_i\sigma_j(P_iP_j^* - P_iP_j^*)\right]e^{(K_i+K_j)y}e^{i(\sigma_i-\sigma_j)t}dy$$
$$= 0$$

and the third term in Eq. (13) contributes only the first order force, which plays an important role in the first order motion analysis as

$$F_{ijL3}^{(2)} = \pounds\left[\rho\int_{-\infty}^{0}\left[\left(\frac{\partial\Phi}{\partial t}\right)_{-\infty} - \left(\frac{\partial\Phi}{\partial t}\right)_{\infty}\right]dy\right] \quad (19)$$
$$= \rho\pounds\left[Re\int_{-\infty}^{0}\left[igR_i^+e^{i\sigma_it} - igP_j^+e^{i\sigma_jt}\right]e^{(K_i+K_j)y}dy\right]$$
$$= 0$$

The fourth term in Eq. (13), resulting from the wave elevation at the infinity boundary, is in terms of Eq. (16e) and Eq. (16f) written as

$$F_{ijL4}^{(2)} = \pounds\left[\frac{1}{2}\rho g\left(\zeta_{-\infty}^2 - \zeta_{\infty}^2\right)\right] \quad (20)$$
$$= \frac{1}{2}\rho g Re\left[\left(R_i^+\right)\left(R_j^+\right)^* - \left(P_i\right)\left(P_j\right)^*\right]e^{i(\sigma_i-\sigma_j)t}$$

The sum of the slow varying terms Eq. (17)~Eq. (20) is then written as

$$F_{ijL}^{(2)} = \frac{1}{2}\rho\frac{\sigma_i\sigma_j}{(K_i+K_j)}Re\left(R_iR_j^* - R_i^+R_j^{+*}\right)e^{i(\sigma_i-\sigma_j)t} \quad (21)$$
$$+\frac{1}{2}\rho g Re\left[\left(R_i^+\right)\left(R_j^+\right)^* - \left(P_i\right)\left(P_j\right)^*\right]e^{i(\sigma_i-\sigma_j)t}$$

Equation (21) is then further simplified using the relationships

$$R_iR_j^* = (h_ie^{-iK_ix} - A_i^-e^{iK_ix})(h_je^{-iK_jx} - A_j^-e^{iK_jx})^* \quad (22a)$$
$$= \{h_ie^{-iK_ix}h_je^{+iK_jx}$$
$$-(h_iA_j^{-*}e^{-iK_ix}e^{-iK_jx} + h_jA_i^-e^{iK_ix}e^{iK_jx})$$
$$+A_i^-e^{iK_ix}A_j^{-*}e^{-iK_jx}\}$$

$$R_i^+R_j^{+*} = (h_ie^{-iK_ix} + A_i^-e^{iK_ix})(h_je^{-iK_jx} + A_j^-e^{iK_jx})^* \quad (22b)$$
$$= \{h_ie^{-iK_ix}h_je^{+iK_jx}$$
$$+(h_iA_j^{-*}e^{-iK_ix}e^{-iK_jx} + h_jA_i^-e^{iK_ix}e^{iK_jx})$$
$$+A_i^-e^{iK_ix}A_j^{-*}e^{-iK_jx}\}$$

$$P_iP_j^* = (h_ie^{-iK_ix} + A_i^+e^{-iK_ix})(h_je^{-iK_jx} + A_j^+e^{-iK_jx})^* \quad (22c)$$
$$= \{h_ie^{-iK_ix}h_je^{iK_jx}$$
$$+(h_iA_j^{+*}e^{-iK_ix}e^{iK_jx} + h_jA_i^+e^{iK_ix}e^{-iK_jx})$$
$$+A_i^+e^{-iK_ix}A_j^{+*}e^{iK_jx}\}$$

and considering the asymptotic behavior of

$$g - \frac{2\sigma_i\sigma_j}{K_i+K_j} = \frac{g}{\sigma_i^2+\sigma_j^2}(\sigma_i-\sigma_j)^2 = O((\sigma_i-\sigma_j)^2) \quad (23)$$

This shows that the quantity in Eq.(23) is of the square order of the frequency difference. Since we are looking for the asymptotic expression of the slowly varying force for small difference in incident wave frequencies, the term of higher order with respect to the frequency difference can be neglected. Reordering Eq. (21) with Eq. (22) and Eq. (23) the slowly varying force is written as

$$F_{ijL}^{(2)} = \frac{1}{2}\rho g Re\left[h_ie^{-iK_ix}h_je^{+iK_jx}\right. \quad (24)$$
$$\left.+A_i^-e^{iK_ix}A_j^{-*}e^{-iK_jx} - P_iP_j^*\right]_{x=0}e^{i(\sigma_i-\sigma_j)t}$$

Equation (24) is a final form of the slowly varying second order force obtained from Eq. (13). Some interpretation of Eq. (24) is necessary to understand the next step.

An examination of Eq. (24) shows that there are terms with $e^{-i((K_i-K_j)x-(\sigma_i-\sigma_j)t)}$ and $e^{i((K_i-K_j)x+(\sigma_i-\sigma_j)t)}$. The former gives the slowly varying force due to the waves propagating in the positive x-direction, i.e. the incident and transmission waves, and the later gives the force due to the waves propagating in the negative x-direction, i.e. the reflection waves. It also says that the second order slowly varying force is differently observed depending on the observation location x. Once the location x is fixed, the slowly varying force is observed as the function of just the frequency difference $\sigma_i - \sigma_j$. The observation location may vary, but the magnitude of the second order slowly varying force does not change, only the phase changes.

Though the formulation starts in the far field from the cylinder body, the final solution for the slowly varying force acting on the cylinder can be observed at any location, even at the origin. We can put $x=0$ for observation at the origin.

For simplicity, we can define the reflection and transmission coefficients as

$$A_i^- = r_ih_ie^{i\epsilon_{ri}} \quad (25a)$$

$$A_j^- = r_jh_je^{i\epsilon_{rj}} \quad (25b)$$

$$h_i + A_i^+ = t_ih_ie^{i\epsilon_{ti}} \quad (25c)$$

$$h_j + A_j^+ = t_jh_je^{i\epsilon_{tj}} \quad (25d)$$

where r, t, ϵ stands for the reflection coefficients, the transmission coefficients and the phase, respectively. And r, t, and ϵ are real. The energy relation between the transmission and reflection coefficients holds as

$$r_i^2+t_i^2=1\ ,\quad r_j^2+t_j^2=1 \tag{26}$$

Equation (25) is rewritten as

$$\frac{F_{ijL}^{(2)}}{\frac{1}{2}\rho g h_i h_j} = Re\left[(1+r_i r_j e^{i(\epsilon_{ri}-\epsilon_{rj})} - t_i t_j e^{i(\epsilon_{ti}-\epsilon_{tj})})e^{i(\sigma_i-\sigma_j)t}\right] \tag{27}$$

The magnitude of the slowly varying force is then

$$\frac{|F_{ijL}^{(2)}|}{\frac{1}{2}\rho g h_i h_j} = \sqrt{}\{(1+r_i r_j\cos(\epsilon_{ri}-\epsilon_{rj}) - t_i t_j\cos(\epsilon_{ti}-\epsilon_{tj}))^2 + (r_i r_j\sin(\epsilon_{ri}-\epsilon_{rj}) - t_i t_j\sin(\epsilon_{ti}-\epsilon_{tj}))^2\} \tag{28}$$

Equation (28) shows the nature of the slowly varying force, which is the sum of the interactions between the reflection waves and transmission waves. The effect of disturbances by the floating or fixed body, including the radiation and diffraction potentials, are carried in the reflection and transmission waves at the far field.

In conclusion, the slowly varying force in 2-dimension can be computed if the reflection and transmission waves including their phases are known. There is no need to evaluate the velocity square terms in this far field approach.

3. Discussion

The validity of Eq. (28) is discussed, taking the limit cases. One is an asymptote of the frequency difference to the mean drift force, the other is the analytical results of the slowly varying force for a simple two dimensional structure, used for comparison. It gives clear physical insight into the nature of the slowly varying force.

3.1 Asymptote to the mean drift force

When the two waves of the bichromatic wave train are identical, the slowly varying force in Eq. (27) formally becomes

$$\frac{F_{iiL}^{(2)}}{\frac{1}{2}\rho g h_i^2} = \lim_{j\to i}\frac{|F_{ijL}^{(2)}|}{\frac{1}{2}\rho g h_i h_j} = 2r_i^2 \tag{29}$$

In the limit, the slowly varying force approaches the mean drift force which is already known (Maruo 1960). It appears to be twice the mean drift force of the monochrome wave. Actually it is the limiting value of two waves, not one. The quadratic transfer function is then half of Eq. (28).

3.2 A solution example for a fixed vertical surface barrier

The analytic solution for the transmission and reflection waves is known for a fixed thin vertical surface barrier. Using the solution the slowly varying force can easily be calculated by applying Eq. (28).

The reflection and transmission coefficients for a thin barrier with draft D are given in terms of the second kind Bessel function (Wehausen and Laitone, 1960) as

$$r(\sigma') = \frac{\pi I_1(\sigma')}{\sqrt{\pi^2 I_1^2(\sigma') + K_1^2(\sigma')}} \tag{30a}$$

$$t(\sigma') = \frac{K_1(\sigma')}{\sqrt{\pi^2 I_1^2(\sigma') + K_1^2(\sigma')}} \tag{30b}$$

$$\epsilon_r = \tan^{-1}(K_1/\pi I_1) \tag{30c}$$

$$\epsilon_t = \frac{\pi}{2} - \epsilon_r \tag{30d}$$

where $\sigma' = \sigma\sqrt{D/g}$ is the nondimensionalized frequency.

The magnitude of the slowly varying force coefficients, as obtained from the right side of Eq. (28), is plotted in Fig. 1.

3.2 An approximation with minimal information

Often it is necessary to approximate the slowly varying forces without knowing details about the hydrodynamics during the design stage. If the reflection and transmission, as well as their phases, are not known, a crude approximation can help. For this purpose, Newman(1967) suggested an approximation formula,

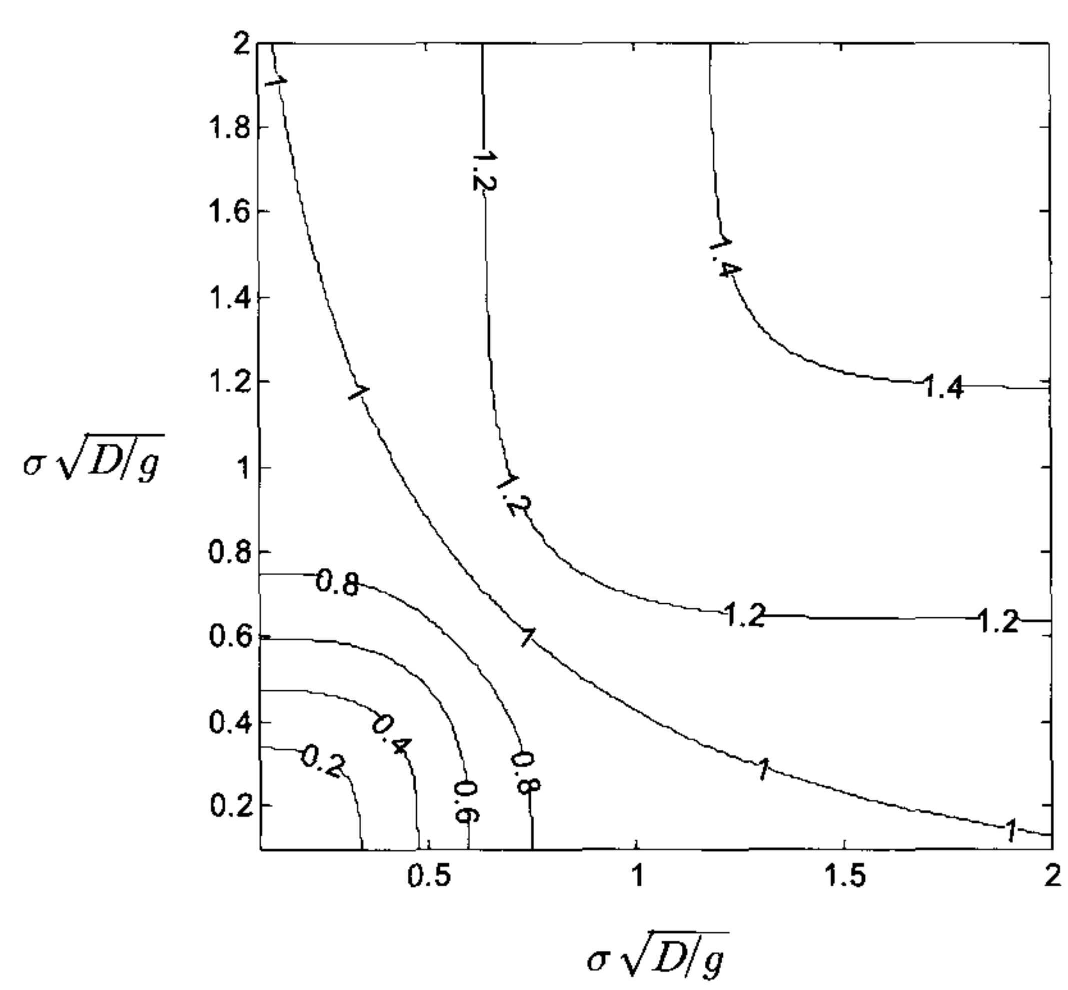

Fig. 1 The slowly varying drift force coefficients of the vertical barrier

validity of which was discussed by the authors. A new approximation can also be suggested using Eq. (27).

Assuming that the reflections are small and the phases are all equal, we can expand Eq. (27) using Eq. (26) in a series form as

$$\frac{F_{ijL}^{(2)}}{\frac{1}{2}\rho g h_i h_j} \approx Re\left[\{(1+r_ir_j-t_it_j)\}e^{i(\sigma_i-\sigma_j)t}\right] \tag{31}$$

$$= Re(1+r_ir_j-\sqrt{1-r_i^2-r_j^2+r_i^2r_j^2})e^{i(\sigma_i-\sigma_j)t}$$

$$\approx (r_ir_j+\frac{1}{2}(r_i^2+r_j^2))e^{i(\sigma_i-\sigma_j)t}$$

$$= \frac{1}{2}(r_i+r_j)^2\cos(\sigma_i-\sigma_j)\,t$$

This approximation needs only the reflection coefficients of the component, r_i and r_j. And the approximation is neither a geometric or arithmetic mean. We can easily show the inequality with the geometric and arithmetic mean as

$$\sqrt{r_i^2r_j^2} \le \frac{1}{4}(\sqrt{r_i^2}+\sqrt{r_j^2})^2 \le \frac{1}{2}(r_i^2+r_j^2) \tag{32}$$

Readers can see that the first term in Eq. (32) was used by Newman(1976) as an approximation, and that there is a slight difference between Newman's equation and Eq. (31). The latter approximation gives a safer prediction.

5. Conclusions

A far field solution of the slowly varying force acting on a long cylinder in bichromatic waves was presented. The explicit form solution says that once the transmission and reflection waves, along with their magnitude and phases, are known, the slowly varying force can be predicted.

The validity of the solution was proved by showing that the slowly varying force approaches the mean drift force in the limit. For further comparison the numerical result of the slowly varying drift force on a thin vertical fixed surface barrier was presented for which the analytical solution of the transmission and reflection coefficients was known.

A simple approximation based on the far field solution was also suggested for practical application.

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Appendix

A. An operator for extracting the low frequency part

Let $\Psi = Re(A_m e^{i\sigma_m t} + A_n e^{i\sigma_n t})$ with complex quantities A_m and A_n. By substituting $A_m = \alpha_m e^{i\beta_m}$ and $A_n = \alpha_n e^{ib_n}$ with real α_n and β_n, the square of Ψ is then given as

$$\Psi^2 = \{\alpha_m\cos(\beta_m+\sigma_m t)+\alpha_n\cos(\beta_n+\sigma_n t)\}^2 \tag{A.1}$$

$$= \frac{1}{2}\alpha_m^2+\frac{1}{2}\alpha_m^2\cos 2(\beta_m+\sigma_m t)$$

$$+\frac{1}{2}\alpha_n^2+\frac{1}{2}\alpha_n^2\cos 2(\beta_n+\sigma_n t)$$

$$+\alpha_m\alpha_n\cos(\beta_m+\beta_n+(\sigma_m+\sigma_n)t)$$

$$+\alpha_m\alpha_n\cos(\beta_m-\beta_n+(\sigma_m-\sigma_n)t)$$

The last term denotes the slowly varying frequency difference term. Let us define a new operator, so that the low frequency second order, or frequency difference part, is collected as

$$\mathcal{L}[\Psi^2] = \mathcal{L}\left[\{Re(A_m e^{i\sigma_m t}+A_n e^{i\sigma_n t})\}^2\right] \tag{A.2}$$

$$= \alpha_m\alpha_n\cos(\beta_m-\beta_n+(\sigma_m-\sigma_n)t)$$

$$= Re\,\alpha_m\alpha_n e^{i(\beta_m-\beta_n+(\sigma_m-\sigma_n)t)}$$

$$= Re\,\alpha_m e^{i(\beta_m+\sigma_m t)}\alpha_n e^{-i(\beta_n+\sigma_n t)}$$

$$= Re\,A_m e^{i\sigma_m t}A_n^* e^{-i\sigma_n t})$$

$$= \frac{1}{2}Re\{A_mA_n^*+(A_mA_n^*)^*\}e^{i(\sigma_m-\sigma_n)t}$$

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