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자기상관관계를 갖는 수요 하에서의 재고정책

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Inventory Control under Correlated Demand

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기업이 수요에서 자기상관을 인식하지 못하게 되면, 이 기업은 수요가 서로 독립이면서 동일한 분포를 따른다는 가정 하에 개발된 전통적인 재고주문방법을 시행하게 된다. 따라서 이 경우에 안전재고량, 재고부족 확률, 재고부족량 등과 같은 성과지표의 관점에서 전통적인 재고주문방법이 정확한 재고주문방법에 비하여 얼마만큼 비효율적인가를 알아보는 것은 기업경영의 측면에서 의미 있는 일이다. 이 논문에서는 정기주문시스템 하에서의 두 가지 재고관리방법을 비교하기 위한 수리적 모형을 개발하고 이를 통하여 어떠한 상황 하에서 정확한 재고주문방법이 전통적인 재고주문방법에 비하여 더 절실한지를 살펴본다.

Keywords : Inventory Control, Periodic Review System, Correlated Demand

1. Introduction

Even though most standard inventory control models are based on the assumption that the demand is independent from one period to the next, the actual demand distribution of many consumer goods have often been found to be autocorrelated. For instance, Erkip et al. [2] observed the inventory system of a major national producer and distributor of consumer products and found high correlations between successive monthly demands. In addition, Lee et al. [5] performed the Durbin-Watson test on the weekly sales data at a supermarket to conclude that the demand distribution for most products are significantly autocorrelated at the 1% significance level.

Zinn et al. [8] conducted an extensive simulation study to evaluate the impact of autocorrelation on safety stock and found that (i) when the demand distribution is autocorrelated, stockouts are more frequent and larger than otherwise, (ii) the effect of autocorrelation on the number of stockouts is directly related to the variability of customer demand and (iii) the effect of autocorrelation on the number of stockouts is inversely related to the variability of lead time from suppliers.

Urban [7] noticed that traditional reorder point models assume that the demand distribution is independent with time. Next, the author showed that, when the assumption does not hold, while the mean demand during lead time is equal to the long run mean demand over the lead time, the mean demand in the short run may not be equal to the long run average. Then, for a continuous review system, the author examined the determination of accurate reorder levels for first order autoregressive demand processes which are updated every period, conditional to the most recently observed demand. From a simulation study, the author indicated that traditional approaches for determining reorder levels can result in excessive inventories and shortages for high levels of autocorrelation.

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Recently, Kim [4] also noticed that autocorrelated demand is a characteristic of most of consumer product industry and considered the impact of autocorrelation on the cost incurred by participants in a supply chain when the participants implement accurate approaches vs. traditional approaches. Since the author's attention was on identifying the value of accurate approaches over traditional approaches in a supply chain setting, the author did not fully consider the impact of autocorrelation at an individual firm setting, especially in terms of inventory levels to keep a certain level of customer service, which is the main focus of this paper.

Therefore, this paper addresses similar issues as those of Zinn et al. [8] and Urban [7]. However, this paper differs from Zinn et al. [8] and Urban [7] in that we use mathematical models instead of simulation models. This paper also differs from Urban [7] in that we consider a periodic review system instead of a continuous review system.

The remainder of this paper is organized as follows : Section 2 develops a simple mathematical model to compare the two approaches of determining the order-up-to level and Section 3 shows, from numerical studies, major findings about the impact of autocorrelation on the amount of safety stock, the long term mean number of stockouts and the long term mean number of excesses. Final remarks are addressed in Section 4.

2. Model Development

Demand Process The model used for the demand from customers during the period t is a first order autoregressive AR(1) process, which is described as below :

$$d_t = \mu + \rho d_{t-1} + \epsilon_t \tag{1}$$

where μ is a non-negative constant, ρ is a correlation parameter, in other words, autocorrelation, with $-1 < \rho < 1$, and the error terms, ϵ_t, are i.i.d. normal random variable with mean 0 and variance σ^2. It is assumed that μ is very large relative to σ so that the probability of negative demand is very negligible. It can be easily seen that $E[d_t] = \frac{\mu}{1-\rho}$ and $V[d_t] = \frac{\sigma^2}{1-\rho^2}$. In this paper, first order autoregressive AR(1) process is selected to model the demand from customers because it has been reported to be able to describe many actual atuocorrelated demand processes accurately (Fotopoulos et al. [3]). Notice that, from Equation (1), the demand m periods ahead, conditional to the most recent observed demand d_0, can be written as below :

$$d_m | d_0 = \frac{\mu}{1-\rho} + \rho^m \left(d_0 - \frac{\mu}{1-\rho} \right) + \sum_{i=1}^{m} \rho^{m-i} \epsilon_i \tag{2}$$

Notice also that the first term in Equation (2) is just the unconditional mean demand. However, as shown in the second term in Equation (2), the conditional mean is adjusted by an amount that is a function of the deviation of the most recent observed demand from the unconditional mean.

We consider a periodic review system in which, at the end of each period, the firm reviews its inventory level, calculates its order-up-to level y from which it determines and places an order of size $y - y_0 + d_0$, where y_0 is the most recent order-up-to level and d_0 is the most recent observed demand, and it receives the shipment of this order at the beginning of $l+1$ periods ahead. We assume that the replenishment lead time from the supplier to the firm l is in constant periods. We also assume that no backorders are allowed and that the supplier has enough capacity to meet the firm's order all the time.

Accurate Order-Up-To Level From Equation (2), the total demand over the effective lead time, conditional to the most recent observed demand d_0, that is, $\sum_{i=1}^{l+1} d_i | d_0$, can be written as below :

$$\sum_{i=1}^{l+1} \left\{ \frac{\mu}{1-\rho} + \rho^i \left(d_0 - \frac{\mu}{1-\rho} \right) \right\} + \sum_{i=1}^{l+1} \sum_{j=1}^{i} \rho^{i-j} \epsilon_j \tag{3}$$

Therefore the total demand over the effective lead time, conditional to the most recent observed demand d_0, follows a normal distribution with the mean and the variance as below :

$$E\left[\sum_{i=1}^{l+1} d_i | d_0 \right] = \frac{\mu}{1-\rho}(l+1) + \left(d_0 - \frac{\mu}{1-\rho} \right) \sum_{i=1}^{l+1} \rho^i \tag{4}$$

$$V\left[\sum_{i=1}^{l+1} d_i | d_0 \right] = \sigma^2 \sum_{i=1}^{l+1} \left(\sum_{j=i}^{l+1} \rho^{l+1-j} \right)^2 \tag{5}$$

where Equation (5) follows from the fact that $\sum_{i=1}^{l+1} \sum_{j=1}^{i} \rho^{i-j} \epsilon_j = \sum_{i=1}^{l+1} \left(\sum_{j=i}^{l+1} \rho^{l+1-j} \right) \epsilon_i$. Notice also that $V\left[E\left[\sum_{i=1}^{l+1} d_i | d_0 \right] \right] = \frac{\sigma^2}{1-\rho^2} \left(\sum_{i=1}^{l+1} \rho^i \right)^2$.

Therefore, the accurate order-up-to level for the firm can be written as below :

$$y = E\left[\sum_{i=1}^{l+1} d_i | d_0\right] + z_\alpha \sqrt{V\left[\sum_{i=1}^{l+1} d_i | d_0\right]} \tag{6}$$

where z_α is a desired safety factor relating the investment of safety stock, the second term in Equation (6), to customer service level of 100(1-α)%, that is, $P(Z \geq z_\alpha) = \alpha$. For example, the value of $z_{.05}$, which is 1.645, is used to achieve a service level of 95%. Notice that, in this case, the order-up-to level y for the firm is different from period to period, depending on the most recent observed demand d_0. For details, see Nahmias [6].

Then, for a periodic review system with order-up-to level y, the probability of stockouts at the end of $l+1$ periods ahead, that is, $P\left(\sum_{i=1}^{l+1} d_i | d_0 > y\right)$, can be easily simplified as below :

$$P(Z > z_\alpha)(= \alpha) \tag{7}$$

where $z_\alpha \left(= \frac{y - E\left[\sum_{i=1}^{l+1} d_i | d_0\right]}{\sqrt{V\left[\sum_{i=1}^{l+1} d_i | d_0\right]}} \right)$ can also be interpreted as the standardized value of the order-up-to level. Therefore, from Equation (7), since the probability of stockouts every period is α, the long term probability of stockouts is α as well.

Next, the mean amount of stockouts at the end of $l+1$ periods ahead, that is, $\int_y^\infty \left(\sum_{i=1}^{l+1} d_i | d_0 - y\right) dF\left(\sum_{i=1}^{l+1} d_i | d_0\right)$, where $F\left(\sum_{i=1}^{l+1} d_i | d_0\right)$ is the cumulative distribution function of the total demand over the effective lead time, conditional to the most recent observed demand d_0, using a standard transformation, can be simplified as below :

$$\sqrt{V\left[\sum_{i=1}^{l+1} d_i | d_0\right]} \int_{z_\alpha}^\infty (Z - z_\alpha)\, \phi(Z)\, dZ \tag{8}$$

where $\phi(\cdot)$ is the probability density function for the standard normal distribution and z_α is the standardized value of the order-up-to level. Equation (8) can be further simplified as below :

$$\sqrt{V\left[\sum_{i=1}^{l+1} d_i | d_0\right]} \left[\phi(z_\alpha) - z_\alpha P(Z \geq z_\alpha)\right] \tag{9}$$

For derivation of Equation (9), see Zipkin [9]. Therefore, since, from Equation (9), the mean amount of stockouts at the end of $l+1$ periods ahead is the same regardless of the specific period, Equation (9) itself represents the long term mean number of stockouts as well.

Finally, since the mean amount of excess at the end of $l+1$ periods ahead, that is, $\int_{-\infty}^y \left(y - \sum_{i=1}^{l+1} d_i | d_0\right) dF\left(\sum_{i=1}^{l+1} d_i | d_0\right)$, can be easily shown to be equal to the sum of the safety stock and the mean amount of stockouts at the end of $l+1$ periods ahead, from Equation (6), Equation (7) and Equation (9), the long term mean number of excesses can be represented as below :

$$z_\alpha \sqrt{V\left[\sum_{i=1}^{l+1} d_i | d_0\right]} + V\left[\sum_{i=1}^{l+1} d_i | d_0\right] \left[\phi(z_\alpha) - \alpha z_\alpha\right] \tag{10}$$

Traditional Order-Up-To Level It can be easily established that the unconditional total demand over the effective lead time, that is, $\sum_{i=1}^{l+1} d_i$, follows a normal distribution with the mean and the variance as below :

$$E\left[\sum_{i=1}^{l+1} d_i\right] = \frac{\mu}{1-\rho}(l+1) \tag{11}$$

$$V\left[\sum_{i=1}^{l+1} d_i\right] = \frac{\sigma^2}{1-\rho^2}\left(\sum_{i=1}^{l+1} \rho^i\right)^2 + \sigma^2 \sum_{i=1}^{l+1}\left(\sum_{j=i}^{l+1} \rho^{l+1-j}\right)^2 \tag{12}$$

Notice that Equation (12) follows from the fact that $V\left[\sum_{i=1}^{l+1} d_i\right] = V\left[E\left[\sum_{i=1}^{l+1} d_i | d_0\right]\right] + V\left[\sum_{i=1}^{l+1} d_i | d_0\right]$. Notice also that $E\left[\sum_{i=1}^{l+1} d_i\right]$ is equal to $E\left[E\left[\sum_{i=1}^{l+1} d_i | d_0\right]\right]$, where the outer expectation is taken over the most recent observed demand d_0, and that $V\left[\sum_{i=1}^{l+1} d_i\right]$ is always bigger than or equal to $V\left[\sum_{i=1}^{l+1} d_i | d_0\right]$, where the equality holds when $\rho = 0$.

Therefore, when the autocorrelation of demands goes undetected or ignored by the firm, the order-up-to level for the firm can be written as below :

$$y = E\left[\sum_{i=1}^{l+1} d_i\right] + z_\alpha \sqrt{V\left[\sum_{i=1}^{l+1} d_i\right]} \tag{13}$$

Notice that, from Equation (13), the order-up-to level y for the firm is constant from period to period, regardless of the most recent observed demand d_0.

Then, as before, the probability of stockouts at the end of $l+1$ periods ahead, that is, $P\left(\sum_{i=1}^{l+1} d_i | d_0 > y\right)$, can be simplified to $P(Z > \tilde{z})$, where $\tilde{z} = \frac{y - E\left[\sum_{i=1}^{l+1} d_i | d_0\right]}{\sqrt{V\left[\sum_{i=1}^{l+1} d_i | d_0\right]}}$. When calculating the long term probability of stockouts, the mean of $P(Z > \tilde{z})$ needs to be evaluated since $\tilde{z}$ is a random variable due to the fact that $\tilde{z}$ relies upon the most recent observed demand d_0. Notice that $\tilde{z}$ follows a normal distribution with the mean and the variance as below :

$$E[\tilde{z}] = \frac{z_\alpha \sqrt{V\left[\sum_{i=1}^{l+1} d_i\right]}}{\sqrt{V\left[\sum_{i=1}^{l+1} d_i | d_0\right]}} \quad (14)$$

$$V[\tilde{z}] = \frac{V\left[E\left[\sum_{i=1}^{l+1} d_i | d_0\right]\right]}{V\left[\sum_{i=1}^{l+1} d_i | d_0\right]} \quad (15)$$

Therefore, the long term probability of stockouts, that is, $E[P(Z > \tilde{z})]$, where the expectation is taken over $\tilde{z}$, can be simplified as below :

$$P(H > E[\tilde{z}])(=\alpha) \quad (16)$$

where H is a normal random variable with mean 0 and variance $V[\tilde{z}]+1$. For derivation of Equation (16), see <Appendix B>. It can be easily shown, using a standard transformation, that Equation (16) further reduces to Equation (7). Therefore, traditional order-up-to level achieves the same long term probability of stockouts as that of accurate order-up-to level, but with more amount of safety stock.

Next, since, just like the probability of stockouts at the end of $l+1$ periods ahead, the mean amount of stockouts at the end of $l+1$ periods ahead, that is, $\sqrt{V\left[\sum_{i=1}^{l+1} d_i | d_0\right]} [\phi(\tilde{z}) - \tilde{z}P(Z \geq \tilde{z})]$, is a function of a normal random variable $\tilde{z}$, the long term mean number of stockouts can be written as below :

$$\sqrt{V\left[\sum_{i=1}^{l+1} d_i | d_0\right]} E[\phi(\tilde{z}) - \tilde{z}P(Z \geq \tilde{z})] \quad (17)$$

where the expectation is taken over $\tilde{z}$. Equation (17) can be represented as below :

$$\sqrt{V\left[\sum_{i=1}^{l+1} d_i | d_0\right]} [h(E[\tilde{z}])(V[\tilde{z}]+1) - \alpha E[\tilde{z}]] \quad (18)$$

where $h(\cdot)$ is the probability density function for a normal distribution with mean 0 and variance $V[\tilde{z}]+1$. The derivation of Equation (18) can be achieved following a similar approach which was used when deriving Equation (16). Therefore, we will leave the derivation of Equation (18) as an exercise for the reader. Using Equation (14), Equation (15) and the fact that $V\left[\sum_{i=1}^{l+1} d_i\right] = V\left[E\left[\sum_{i=1}^{l+1} d_i | d_0\right]\right] + V\left[\sum_{i=1}^{l+1} d_i | d_0\right]$, Equation (18) can be further simplified as below :

$$\sqrt{V\left[\sum_{i=1}^{l+1} d_i\right]} [\phi(z_\alpha) - \alpha z_\alpha] \quad (19)$$

Finally, from Equation (13) and Equation (19), the formula for the long term mean number of excesses can be easily obtained as below :

$$z_\alpha \sqrt{V\left[\sum_{i=1}^{l+1} d_i\right]} + \sqrt{V\left[\sum_{i=1}^{l+1} d_i\right]} [\phi(z_\alpha) - \alpha z_\alpha] \quad (20)$$

By comparing Equation (6) with Equation (13), Equation (9) with Equation (18), Equation (10) with Equation (20), respectively, it can be easily understood that the amount of safety stock, the long term mean number of stockouts, and the long term mean number of excesses of traditional order-up-to level are $\frac{\sqrt{V\left[\sum_{i=1}^{l+1} d_i\right]}}{\sqrt{V\left[\sum_{i=1}^{l+1} d_i | d_0\right]}}$ times larger than those of accurate order-up-to level.

Now that we have completed our mathematical model for calculating the amount of safety stock, the long term mean number of stockouts, and the long term mean number of excesses for the two approaches, we are now ready to conduct numerical studies, which is presented in Section 3.

3. Numerical Studies

As mentioned in Section 1, the goals of this paper are to compare the two approaches of determining the order-up-to level, while maintaining a predetermined customer service level or the long term probability of stockouts, in terms of the amount of safety stock, the long term mean number of stockouts, and the long term mean number of excesses and identify the situations where the differences of the two approaches will be more consequential. For these goals, the values for the variables in the numerical studies are selected as below :

- Autocorrelation ρ : 0.0, 0.2, 0.4, 0.6, 0.8
- Lead Time l : 1, 2, 4
- Standard Deviation σ : 10, 20, 40
- Safety Factor z_α : 1.28, 2.33

Here, the value of autocorrelation ρ are confined to non-negative values since, according to Zinn et al. [8], negative autocorrelation is a rarity in practice. Notice also that the values for the safety factor z_α 1.28 and 2.33 correspond to customer service levels of 90% and 99%, respectively. The value of the constant μ is set to provide a long term mean demand of 300 each period.

For each combination of the variables, using the mathematical model developed in Section 2, the amount of safety stock, the long term mean number of stockouts and the long term mean number of excesses are calculated, the results of which are provided in <Appendix A>. Notice that, in <Appendix A>, when the value of autocorrelation ρ is equal to 0, the two approaches provide the same results.

<Table 1> presents the results of the numerical studies concerning the amount of safety stock, the long term mean number of stockouts and the long term mean number of excesses as the autocorrelation increases. Results are provided for each of the two approaches when $z_\alpha = 1.28$, $\sigma = 10$, and $l = 1$. From <Table 1>, it is apparent that both of the two approaches experience more amount of safety stock, more long term mean number of stockouts, and more long term mean number of excesses as the autocorrelation increases and that the traditional order-up-to level experiences more amount of safety stock, more long term mean number of stockouts, and more long term mean number of excesses than the accurate order-up-to level does regardless of the specific value of autocorrelation. This is because both of the two approaches experience more uncertainty in the total demand over the effective lead time and the difference between the two approaches in the uncertainty becomes larger as the autocorrelation increases, which is reflected in Equation (5) and Equation (12). However, the rate of increase in the amount of safety stock, the long term mean number of stockouts, and the long term mean number of excesses that the traditional order-up-to level experiences, in comparison with the accurate order-up-to level, gets more consequential as the autocorrelation increases, from 1.05 times when $\rho = 0.2$ to 1.39 times when $\rho = 0.8$. This is because $\sqrt{V\left[\sum_{i=1}^{l+1} d_i\right]}$ increases at a faster rate than $\sqrt{V\left[\sum_{i=1}^{l+1} d_i | d_0\right]}$ does as the autocorrelation increases, which is also reflected in Equation (5) and Equation (12).

<Table 1> Different Levels of Autocorrelation

ρ	Accurate Order-Up-To Level			Traditional Order-Up-To Level		
	Safety Stock	Stockout	Excess	Safety Stock	Stockout	Excess
0.0	18.12	0.67	18.79	18.12	0.67	18.79
0.2	20.02	0.74	20.76	21.02	0.78	21.80
0.4	22.05	0.81	22.86	24.41	0.90	25.31
0.6	24.18	0.89	25.07	28.83	1.06	29.89
0.8	26.39	0.97	27.36	36.79	1.36	38.15

<Table 2> presents the results of the numerical studies concerning the amount of safety stock, the long term mean number of stockouts and the long term mean number of excesses as the lead time increases. Results are provided for each of the two approaches when $z_\alpha = 1.28$, $\sigma = 10$, and $\rho = 0.8$. From <Table 2>, it is apparent that both of the two approaches experience more amount of safety stock, more long term mean number of stockouts, and more long term mean number of excesses as the lead time increases and that the traditional order-up-to level experiences more amount of safety stock, more long term mean number of stockouts, and more long term mean number of excesses than the accurate order-up-to level does regardless of the specific value of lead time. This is because, as more periods are included in the total demand over the effective lead time, the two approaches experience more uncertainty in the total demand over the effective lead time and the difference between the two approaches in the uncertainty becomes larger as the

<Table 2> Different Levels of Lead Time

l	Accurate Order-Up-To Level			Traditional Order-Up-To Level		
	Safety Stock	Stockout	Excess	Safety Stock	Stockout	Excess
1	26.39	0.97	27.36	36.79	1.36	38.15
2	40.92	1.51	42.43	50.64	1.87	52.51
4	70.44	2.60	73.04	78.67	2.91	81.57

lead time increases, which is reflected in Equation (5) and Equation (12). However, the rate of increase in the amount of safety stock, the long term mean number of stockouts, and the long term mean number of excesses that the traditional order-up-to level experiences, in comparison with the accurate order-up-to level, gets less substantial as the lead time increases, from 1.39 times when $l=1$ to 1.12 times when $l=4$. This is due to the fact that, as the lead time increases, the total demand over the effective lead times of the two approaches become similar, in other words, $\sum_{i=1}^{l+1} d_i \mid d_0 \approx \sum_{i=1}^{l+1} d_i$, which, in turn, is because, as i increases, the conditional distribution of d_i, given d_0, becomes similar to the unconditional distribution of d_i.

<Table 3> presents the results of the numerical studies concerning the amount of safety stock, the long term mean number of stockouts and the long term mean number of excesses as the standard deviation increases. Results are provided for each of the two approaches when $z_\alpha = 1.28$, $l=1$, and $\rho = 0.8$. From <Table 3>, it is apparent that both of the two approaches experience more amount of safety stock, more long term mean number of stockouts, and more long term mean number of excesses as the standard deviation increases and that the traditional order-up-to level experiences more amount of safety stock, more long term mean number of stockouts, and more long term mean number of excesses than the accurate order-up-to level does regardless of the specific value of standard deviation. This is because the two approaches experience more uncertainty in the total demand over the effective lead time and the difference between the two approaches in the uncertainty becomes larger as the standard deviation increases, which is reflected in Equation (5) and Equation (12). However, the rate of increase in the amount of safety stock, the long term mean number of stockouts, and the long term mean number of excesses that the traditional order-up-to level experiences, in comparison with the accurate order-up-to level, remains the same, regardless of the specific value of the standard deviation, at 1.39 times. This can be easily understood by seeing that, from Equation (5) and Equation (12), both of $\sqrt{V\left[\sum_{i=1}^{l+1} d_i\right]}$ and $\sqrt{V\left[\sum_{i=1}^{l+1} d_i \mid d_0\right]}$ are directly proportional to σ.

<Table 3> Different Levels of Standard Deviation

σ	Accurate Order-Up-To Level			Traditional Order-Up-To Level		
	Safety Stock	Stockout	Excess	Safety Stock	Stockout	Excess
10	26.39	0.97	27.36	36.79	1.36	38.15
20	52.78	1.95	54.73	73.57	2.72	76.29
40	105.56	3.90	109.45	147.15	5.44	152.59

<Table 4> presents the results of the numerical studies concerning the amount of safety stock, the long term mean number of stockouts and the long term mean number of excesses as the safety factor increases. Results are provided for each of the two approaches when $\sigma = 10$, $l=1$, and $\rho = 0.8$. From <Table 4>, it is apparent that both of the two approaches experience more amount of safety stock, less long term mean number of stockouts, and more long term mean number of excesses as the safety factor increases and that the traditional order-up-to level experiences more amount of safety stock, more long term mean number of stockouts, and more long term mean number of excesses than the accurate order-up-to level does regardless of the specific value of safety factor. This is because more safety stock naturally leads to less long term mean number of shortages and more long term mean number of excesses and the uncertainty in the total demand over the effective lead time that the traditional order-up-to level experiences is always larger than that of the accurate order-up-to level. However, the rate of increase in the amount of safety stock, the long term mean number of stockouts, and the long term mean number of excesses that the traditional order-up-to level experiences, in comparison with the accurate order-up-to level, remains the same, regardless of the specific value of the safety factor, at 1.39 times, which can be easily understood by seeing that, from Equation (5) and Equation (12), both of $\sqrt{V\left[\sum_{i=1}^{l+1} d_i\right]}$ and $\sqrt{V\left[\sum_{i=1}^{l+1} d_i \mid d_0\right]}$ are not related to z_α at all.

<Table 4> Different Levels of Safety Factor

z_α	Accurate Order-Up-To Level			Traditional Order-Up-To Level		
	Safety Stock	Stockout	Excess	Safety Stock	Stockout	Excess
1.28	26.39	0.97	27.36	36.79	1.36	38.15
2.33	47.90	0.07	47.97	66.78	0.10	66.88

4. Final Remarks

The mathematical model developed in this paper has been used for numerical studies to evaluate the value of accurate order-up-to level over traditional order-up-to level. The observations from these numerical studies comparing the two approaches of determining the order-up-to level can be summarized as below :

- Using accurate order-up-to level over traditional order-up-to level always benefits the firm in terms of inventory levels to maintain a certain level of customer service.
- The benefits of using accurate order-up-to level over traditional order-up-to level increases when autocorrelation, lead time, standard deviation or safety factor increases.
- The relative advantage of accurate order-up-to level over traditional order-up-to level becomes more consequential when autocorrelation increases, becomes less consequential when lead time increases, remains the same when standard deviation or safety factor increases.

These findings imply that the firm must detect items displaying high autocorrelation with shorter lead time first and then responding to them accordingly. Since most of today's consumer products exhibit high autocorrelation and the lead time is steadily being reduced through recent technological advances such as electronic data interchange and Just-In-Time (Buzzel and Ortmeyer [1]), the need for using accurate order-up-to level will be increasing in the future.

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<Appendix A> Results of Numerical Studies

When $z_\alpha = 1.28$ (90% customer service level)

σ	l	ρ	Accurate Order-Up-To Level			Traditional Order-Up-To Level		
			Safety Stock	Stockout	Excess	Safety Stock	Stockout	Excess
		0.0	18.12	0.67	18.79	18.12	0.67	18.79
		0.2	20.02	0.74	20.76	21.02	0.78	21.80
	1	0.4	22.05	0.81	22.86	24.41	0.90	25.31
		0.6	24.18	0.89	25.07	28.83	1.06	29.89
		0.8	26.39	0.97	27.36	36.79	1.36	38.15
		0.0	22.20	0.82	23.02	22.20	0.82	23.02
		0.2	25.56	0.94	26.50	26.38	0.97	27.35
10	2	0.4	29.76	1.10	30.86	31.75	1.17	32.92
		0.6	34.87	1.29	36.15	38.95	1.44	40.39
		0.8	40.92	1.51	42.43	50.64	1.87	52.51
		0.0	28.66	1.06	29.71	28.66	1.06	29.71
		0.2	34.14	1.26	35.40	34.76	1.28	36.04
	4	0.4	42.02	1.55	43.58	43.53	1.61	45.14
		0.6	53.54	1.98	55.52	56.76	2.10	58.85
		0.8	70.44	2.60	73.04	78.67	2.91	81.57
		0.0	36.25	1.34	37.59	36.25	1.34	37.59
		0.2	40.04	1.48	41.52	42.04	1.55	43.59
	1	0.4	44.10	1.63	45.73	48.81	1.80	50.61
		0.6	48.36	1.79	50.15	57.66	2.13	59.79
		0.8	52.78	1.95	54.73	73.57	2.72	76.29
		0.0	44.39	1.64	46.03	44.39	1.64	46.03
		0.2	51.12	1.89	53.01	52.75	1.95	54.70
20	2	0.4	59.53	2.20	61.72	63.49	2.35	65.84
		0.6	69.73	2.58	72.31	77.91	2.88	80.79
		0.8	81.83	3.02	84.86	101.29	3.74	105.03
		0.0	57.31	2.12	59.43	57.31	2.12	59.43
		0.2	68.28	2.52	70.80	69.52	2.57	72.09
	4	0.4	84.05	3.10	87.15	87.06	3.22	90.28
		0.6	107.08	3.96	111.03	113.51	4.19	117.71
		0.8	140.87	5.20	146.08	157.33	5.81	163.14
		0.0	72.50	2.68	75.17	72.50	2.68	75.17
		0.2	80.07	2.96	83.03	84.08	3.11	87.18
	1	0.4	88.19	3.26	91.45	97.62	3.61	101.23
		0.6	96.72	3.57	100.29	115.31	4.26	119.57
		0.8	105.56	3.90	109.45	147.15	5.44	152.59
		0.0	88.79	3.28	92.07	88.79	3.28	92.07
		0.2	102.24	3.78	106.01	105.50	3.90	109.40
40	2	0.4	119.05	4.40	123.45	126.99	4.69	131.68
		0.6	139.46	5.15	144.62	155.82	5.76	161.57
		0.8	163.67	6.05	169.71	202.57	7.48	210.05
		0.0	114.63	4.23	118.86	114.63	4.23	118.86
		0.2	136.56	5.04	141.60	139.04	5.14	144.18
	4	0.4	168.09	6.21	174.30	174.12	6.43	180.56
		0.6	214.15	7.91	222.06	227.03	8.39	235.41
		0.8	281.75	10.41	292.16	314.66	11.62	326.29

When $z_\alpha = 2.33$ (99% customer service level)

σ	l	ρ	Accurate Order-Up-To Level			Traditional Order-Up-To Level		
			Safety Stock	Stockout	Excess	Safety Stock	Stockout	Excess
		0.0	32.90	0.05	32.95	32.90	0.05	32.95
		0.2	36.34	0.05	36.39	38.15	0.06	38.21
	1	0.4	40.02	0.06	40.08	44.30	0.06	44.37
		0.6	43.89	0.06	43.96	52.33	0.08	52.41
		0.8	47.90	0.07	47.97	66.78	0.10	66.88
		0.0	40.29	0.06	40.35	40.29	0.06	40.35
		0.2	46.40	0.07	46.46	47.88	0.07	47.95
10	2	0.4	54.03	0.08	54.11	57.63	0.08	57.71
		0.6	63.29	0.09	63.38	70.71	0.10	70.81
		0.8	74.27	0.11	74.38	91.93	0.13	92.06
		0.0	52.02	0.08	52.09	52.02	0.08	52.09
		0.2	61.97	0.09	62.06	63.10	0.09	63.19
	4	0.4	76.28	0.11	76.39	79.02	0.12	79.13
		0.6	97.19	0.14	97.33	103.03	0.15	103.18
		0.8	127.86	0.19	128.05	142.80	0.21	143.01
		0.0	65.80	0.10	65.89	65.80	0.10	65.89
		0.2	72.68	0.11	72.78	76.31	0.11	76.42
	1	0.4	80.05	0.12	80.16	88.61	0.13	88.73
		0.6	87.79	0.13	87.91	104.66	0.15	104.81
		0.8	95.80	0.14	95.94	133.56	0.19	133.75
		0.0	80.59	0.12	80.70	80.59	0.12	80.70
		0.2	92.79	0.14	92.93	95.76	0.14	95.90
20	2	0.4	108.05	0.16	108.21	115.26	0.17	115.42
		0.6	126.58	0.18	126.77	141.42	0.21	141.63
		0.8	148.55	0.22	148.76	183.86	0.27	184.13
		0.0	104.04	0.15	104.19	104.04	0.15	104.19
		0.2	123.95	0.18	124.13	126.20	0.18	126.38
	4	0.4	152.56	0.22	152.79	158.04	0.23	158.27
		0.6	194.37	0.28	194.65	206.06	0.30	206.36
		0.8	255.72	0.37	256.09	285.60	0.42	286.01
		0.0	131.60	0.19	131.79	131.60	0.19	131.79
		0.2	145.35	0.21	145.57	152.62	0.22	152.84
	1	0.4	160.10	0.23	160.33	177.21	0.26	177.47
		0.6	175.57	0.26	175.83	209.32	0.30	209.62
		0.8	191.61	0.28	191.89	267.11	0.39	267.50
		0.0	161.17	0.23	161.41	161.17	0.23	161.41
		0.2	185.59	0.27	185.86	191.52	0.28	191.80
40	2	0.4	216.11	0.31	216.42	230.51	0.34	230.85
		0.6	253.16	0.37	253.53	282.85	0.41	283.26
		0.8	297.10	0.43	297.53	367.72	0.54	368.26
		0.0	208.07	0.30	208.38	208.07	0.30	208.38
		0.2	247.89	0.36	248.25	252.40	0.37	252.76
	4	0.4	305.13	0.44	305.57	316.08	0.46	316.54
		0.6	388.74	0.57	389.31	412.11	0.60	412.71
		0.8	511.44	0.74	512.19	571.19	0.83	572.02

<Appendix B> Derivation of Equation (15)

We need to evaluate $E[P(Z>\tilde{z})]$. Frist, notice that we can write as below :

$$E[P(Z>\tilde{z})]=\int_{-\infty}^{+\infty}\left(\int_{\tilde{z}}^{+\infty}\phi(Z)\,dZ\right)f(\tilde{z})d\tilde{z}$$
$$=\int_{-\infty}^{+\infty}\left(\int_{-\infty}^{Z}f(\tilde{z})\,d\tilde{z}\right)\phi(Z)\,dZ$$
$$=\int_{-\infty}^{+\infty}P(\tilde{z}\leq Z)\phi(Z)dZ,$$

where

$$f(\tilde{z})=\frac{1}{\sqrt{2\pi V[\tilde{z}]}}e^{-\frac{1}{2}\frac{(\tilde{z}-E[\tilde{z}])^2}{V[\tilde{z}]}}$$

is the probability density function of $\tilde{z}$. Notice that $\int_{-\infty}^{+\infty}P(\tilde{z}\leq Z)\phi(Z)\,dZ$ is a convolution of $\tilde{z}$ and $-Z$. In other words, $\int_{-\infty}^{+\infty}P(\tilde{z}\leq Z)\phi(Z)\,dZ$ is just $P(\tilde{z}-Z\leq 0)$, where $\tilde{z}-Z$ is a normal random variable with mean $E[\tilde{z}]$ and variance $V[\tilde{z}]+1$. Therefore, we can write $\tilde{z}-Z=E[\tilde{z}]-H$, where H is a normal random variable with mean 0 and variance $V[\tilde{z}]+1$ and then we have as below :

$$P(\tilde{z}-Z\leq 0)=P(E[\tilde{z}]-H\leq 0)=P(H\geq E[\tilde{z}]),$$

which is the desired result.