International Journal of Reliability and Applications
Vol. 9, No. 2, pp. 125-140, 2008

A Goodness of Fit Approach for Testing NBUFR (NWUFR) and NBAFR (NWAFR) Properties

M. A. W. Mahmoud
Mathematics Department, Faculty of Science
Al-Azhar University, Nasr city, 11884, Cairo, EGYPT

N. A. Abdul Alim
Mathematics Department, Faculty of Science
Al-Azhar University, Nasr city, 11884, Cairo, EGYPT

Abstract. The new better than used failure rate (NBUFR) ,Abouammoh and Ahmed (1988), and new better than average failure rate (NBAFR) Loh (1984) classes of life distributions, have been considered in the literature as natural weakenings of NBU (NWU) property. The paper considers testing exponentiality against strictly NBUFR (NBAFR) alternatives, or their duals, based on goodness of fit approach that is possible in life testing problems and that it results in simpler procedures that are asymptotically equivalent or better than standard ones. They may also have superior finite sample behavior. The asymptotic normality are proved. Powers, Pitman asymptotic efficiency and critical points are computed. Dealing with censored data case also studied. Practical applications of our tests in the medical sciences are present.

Key words : *NBUFR, NBAFR, U-statistic,hypotheses testing, life testing, exponential distribution, goodness of fit testing, efficiency, power, Mont Carlo m***1***ethods.*

1. INTRODUCTION

Aging is characterized by a non negative random variable T with distribution function $F(t) = P(T \le t)$ and a survival function $\overline{F}(t) = P(T > t)$. For practicalities, T is often assumed (but need not be) continuous with pdf $f(t) = F'(t)$. The most commonly applied concepts of positive aging are in terms of failure rate, $r(t),\ t \ge 0$, of the distribution. In this paper we provide two more criterion describing positive aging in terms of the failure (hazard) rate. Formally the NBUFR and NBAFR and their duals new worth than used failure rate (NWUFR) and new worth average failure rate (NWARFR) cf.

* Corresponding Author.
E-mail address: Nasser_anwer@yahoo.com

Loh(1984) and abouammoh and Ahmed (1988). These classes and their duals are defined as follows:

Definition 1.1 An aging r.v. T ≥ 0 is said to have NBUFR (NWUFR) if $r(0) \leq (\geq) r(t)$, for all $t \geq 0$ where $r(t) = f(t) / \overline{F}(t)$ is the failure rate at $T = t$ and $f(0) > 0$. i.e the failure rate of a new system is less (greater) than the failure rate of a used system.

Definition 2.2 An aging r.v. T ≥ 0 is said to have NBAFR (NWAFR) if

$$r(0) \leq (\geq) t^{-1} \int_0^t r(u) du, t \geq 0 .$$

Equivalently $r(0) \leq (\geq) - t^{-1} \ln \overline{F}(t)$, i.e the failure rate of a new system is less (greater) than the average failure rate of a used system.

A significant part of life testing problems is concerned with testing whether a life distribution belongs to a non parametric family of aging. See for example For testing against NBUE, NBUFR and NBAFR classes, we refer to Klefsjo (1981 and 1982), Deshpande et al. (1986), Abouammoh and Ahmed (1988), Loh (1984) and Hendi, Alnachawati, and AL-Graian (2000). Mahmoud and Abdul Alim (2002, 2003 a and 2003 b) studied testing exponentiality against new better than used renwel failure rate (NBURFR) and new better average renwel failure rate (NBARFR) based on a U-statistic for censored and non censored data.

We often encounter testing H_0 : A life distribution is exponential versus H_1 : A life distribution belongs to an aging family. In contrast to goodness of fit problems, where the test statistic is based on a measure of departure from H_0 that depends on both H_0 and H_1 , most tests in life testing settings, including those referenced above, do not use the null distribution in devising the test statistics, this resulted in test statistics that are often difficult to work with and require programming to calculated. The current work that incorporating H_0 into the measure of departure from it can lead to simpler test statistics that are easy to work with, are asymptotically equivalent in distribution to those based on another approaches and may have equal or higher efficiency than the classical procedures. They also may have better finite sample behaviors. Ahmed et al (2001) introduced the previous method with major life distributions classes which are increasing failure rate (IFR), new better than used (NBU), new better than used in convex ordering (NBUC), new better than used in expectation (NBUE) and harmonic new better than used in expectation (HNBUE). Mahmoud and Abdul Alim (2006), A used this method with testing hypothesis with new better than used renewal failure rate NBURFR and new better average renewal failure rate NBARFR.

In the following sections we derive four nonparametric tests for testing exponentiality against NBUFR (NWUFR) and NBAFR (NWAFR) properties respectively using goodness of fit approach.

2. TESTING AGAINST NBUFR (NWUFR) CLASS

In this Section, a new test statistic is proposed for testing exponentiality versus new better than used failure rate (NBUFR) alternatives. This test statistic, which is based on U statistic of a random sample, is readily applied in the case of small sample as well as large sample. Also, this test statistic is simpler and more efficient than the test statistic of Hendi et al (2000). Also the comparisons of NBUFR tests in the sense of asymptotic efficiency (AE) and powers are given in Section 3.

For testing the hypothesis H_0 : F is exponential against $H_1^{(1)}$: F belongs to NBUFR class and not exponential, we propose the following measure of departure

$$\delta_F^{(1)} = \int_0^\infty f(x) dF_0(x) - \int_0^\infty \overline{F}(x) f(0) dF_0(x),$$

where $F_0(t) = e^{-t}$.

Lemma 2.1.

Let X be a random variable with distribution function F. Then

$$\delta_F^{(1)} = E\left(e^{-X}\right) - f(0)\left[1 - E\left(e^{-X}\right)\right]$$

Proof.

Note that

$$\delta_F^{(1)} = \int_0^\infty f(x) e^{-x} dx - \int_0^\infty \overline{F}(x) f(0) e^{-x} dx$$

$$= E(e^{-x}) - I \text{, say.}$$

Now since by integration by parts

$$I = \int_0^\infty \overline{F}(x) e^{-x} dx = 1 - E(e^{-x}),$$

then the lemma completely proved.

Based on a random sample $X_1, X_2, X_3, ... X_n$ from a distribution F we wish to test H_0 : against $H_1^{(1)}$. Clearly, $\delta_F^{(1)}$=0 under H_0, while it is positive under $H_1^{(1)}$. Thus we may be testing on its estimate. By using the empirical form of $f(0)$ and terms of then the estimate of $\delta_F^{(1)}$ is given by

$$\hat{\delta}_{F_n}^{(1)} = \frac{1}{n} \sum_{i=1}^{n} \left(e^{-X_i} + \hat{f}_n(0)\left(1 - e^{-X_i}\right)\right)$$

$$= \frac{1}{n^2} \sum_{j=1}^{n} \sum_{i=1}^{n} \left[e^{-X_i} + \frac{1}{a_n}\left(1 - e^{-X_i}\right) K\left(\frac{-X_j}{a_n}\right)\right]. \qquad (2.1)$$

where

$$\hat{f}_n(0) = \frac{1}{na_n}\sum_{j=1}^{n} K\left(\frac{-X_j}{a_n}\right),$$

is an estimated pdf at 0 based on the kernel method with the bandwidth h and $K(u)$ be

$$h = \left(\frac{4}{3n}\right)^{1/5}\hat{\sigma} \;, \hat{\sigma} = \sqrt{\left(\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \overline{X}\right)^2\right)}$$

a known probability density function, symmetric and bounded with 0 mean and variance $\sigma_k^2 > 0$, cf. Hardle (1991). Here we set $K(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2}$, which is the standard normal density. Further it is known that, cf. Hardle (1991),

$$E\hat{f}(0) = f(0) + o(h),$$

and

$$\operatorname{var}(\hat{f}(0)) = n^{-1}(f(0) + o(h)\{h^{-1} \| K \|_2^2 - (f(0) + o(h))\}, \text{as } h \to 0.$$

By defining

$$\phi\left(X_1, X_2\right) = e^{-X_1} + \frac{1}{a_n}\left(1 - e^{-X_1}\right) K\left(\frac{-X_2}{a_n}\right),$$

and defining the symmetric kernel

$$\psi(X_1, X_2) = \frac{1}{2!}\sum_R \phi(X_{i1}, X_{i2}),$$

where, the summation over all arrangements of X_{i1}, X_{i2}, then $\hat{\delta}_{F_n}^{(1)}$ is equivalent to U-statistic

$$U_n = \frac{1}{\binom{n}{2}}\sum_R \psi(X_i, X_j).$$

The following theorem studied the large sample distribution of $\hat{\delta}_{F_n}^{(1)}$.

Theorem 2.1.

The asymptotic distribution of $\sqrt{n}\left(\hat{\delta}_{F_n}^{(1)} - \delta_F^{(1)}\right)$ is normal with mean 0 and variance σ^2. Under H_0, The asymptotic distribution is normal with mean zero and variance σ_0^2. Where,

$$\sigma^2 = Var\left\{\left(e^{-X} + \int_0^\infty e^{-X} dF(x)\right)\left(1 + f(0)\right) - 2f(0)\right\} \text{ and } \sigma_0^2 = \frac{1}{3}.$$

Proof.

Using standard U-statistics theory, cf- Lee(1990), we need only the asymptotic variance which is as follows:

Recall the definition of $\phi\left(X_1, X_2\right)$, then the asymptotic variance is given by

$$\sigma^2 = Var\{E[\phi(X_1, X_2) \mid X_1] + E[\phi(X_1, X_2) \mid X_2]\},$$

but

$$E[\phi(X_1,X_2)\mid X_1]=e^{-X_1}-(1-e^{-X_1})\int_0^\infty \frac{1}{a}k(\frac{-X_2}{a})dF(X_2).$$

Similarly,

$$E[\phi(X_2,X_1)\mid X_1]=\int_0^\infty e^{-X_1}dF(x_1)-\frac{1}{a_n}K\left(\frac{-X_2}{a_n}\right)\int_0^\infty (1-e^{-X_1})dF(x_1),$$

then σ^2 obtained.

Also, under H_0

$$\sigma_0^2=E\{4e^{-2X}-4e^{-X}+1\}=1/3\,.$$

To perform the above test, calculate $\sqrt{3n}\hat{\delta}_{F_n}^{(1)}$ and rejected H_0 if this value exceed Z_α the standard normal variate.

3. ASYMPTOTIC EFFICIENCY AND POWERS FOR $\hat{\delta}_{F_n}^{(1)}$

In this section the asymptotic efficiencies of $\hat{\delta}_{F_n}^{(1)}$ for LFR, Weibull and Makeham families.

Let T_{1n}, T_{2n} be two test statistics for testing $H_0 : F_\theta \in \{F_{\theta_n}\}, \theta_n = \theta + cn^{-1/2}$ with c is an arbitrary constant, then the asymptotic efficiency of T_{1n} relative to T_{2n} is defined by

$$e(T_{1n},T_{2n})=\left[\mu_1'^2(\theta_0)/\sigma_1^2(\theta_0)\right]/\left[\mu_2'^2(\theta_0)/\sigma_2^2(\theta_0)\right].$$

where

$$\mu_i'(\theta_0)=\lim_{n\to\infty}\left(\frac{\partial}{\partial\theta}E(T_{in})\right)_{\theta\to\theta_0},$$

and

$$\sigma_i^2(\theta_0)=\lim_{n\to\infty}Var_0(T_{in}),\ \text{i=1,2},$$

is the null variance.

Carrying out the efficacy calculations for the linear failure rate Weibull , and Makeham families we get the following results.

In order to evaluate the asymptotic efficacy of $\hat{\delta}_{F_n}^{(1)}$, we shall use the definition of $\delta_F^{(1)}$ then we have

$$AE\left(\hat{\delta}_{F_n}^{(1)}\right)=\sqrt{3}\left\{\int_0^\infty f_\theta(x)e^{-x}dx-f(0)\int_0^\infty \overline{F}_\theta(x)e^{-x}dx\right\}.$$

The following results are the ARE for LFR, Weibull and Makeham families

LFR family, $\overline{F}(t)=Exp(\text{-}t\text{-}\theta\, t^2/2)$, $t>0,\theta\ge 0$

$$AE\left(\hat{\delta}^{(1)}_{F_n}\right) = \sqrt{3}\left\{\int_0^{\infty} xe^{-2x}dx\right\} = \sqrt{3}/4 .$$

Weibull family, $\overline{F}(t) = e^{-t^{\theta}}, t > 0, \theta \geq 0$

$$AE\left(\hat{\delta}^{(3)}_{F_n}\right) = \sqrt{3}\left\{\int_0^{\infty} e^{-2x}(1+\ln(x))dx\right\} = 0.23414\sqrt{3} .$$

Weibull family, for Hendi et al (2000)

$$AE\left(\hat{\delta}^{(3)}_{F_n}\right) = \sqrt{3}\left\{\int_0^{\infty} \left[(e^{-x} - e^{-2x} - xe^{-x})\ln(x) + e^{-x} - e^{-2x}\right]dx\right\} = 0.13518\sqrt{3} .$$

Similarly ,

Makeham family $\overline{F}(t) =$ Exp(-t- θ (t+ $e^{-\theta}$ -1)

$$AE\left(\hat{\delta}^{(1)}_{F_n}\right) = \sqrt{3}\left\{\int_0^{\infty} \left(e^{-2x} - e^{-3x}\right)dx\right\} = \sqrt{3}/6 .$$

It is also easy to see that the above test is consistent and unbiased. For samples 5(1)50 and using 5000 replications, Mont Carlo null distribution critical values for $\hat{\delta}^{(1)}_{F_n}$ test and powers for the mentioned three alternatives are given as in Tables A.1 , appendix, and Table A.2.

Now, we consider here dealing with Censored Data

Let $X_{(1)}, X_{(2)}, X_{(3)},..., X_{(n)}$ be the order statistics of $X_1, X_2, X_3,...X_n$. Let $Y_1, Y_2, Y_3..., Y_n$ be independent and identical distributed each with distribution function *G*. Y_i is the censoring time associated with T_i. We can only observe (Z_1,δ_1), (Z_2,δ_2), (Z_3,δ_3), …, (Z_n,δ_n) where $Z_i = \min(X_i, Y_i)$,

and

$$\delta_i = I(X_i \leq Y_i) = \begin{cases} 1 & if \quad X_i \leq Y_{i,} \ \ that\ is\ \ X_i\ is\ not\ censored \\ 0 & if \quad X_i \geq Y_{i,} \ \ that\ \ is\ \ X_i\ \ is\ \ censored \end{cases}$$

Notice that $Z_1, Z_2, Z_3,..., Z_n$ are independent and identical distributed with some distribution function *F*. Also $\delta_1, \delta_2, \delta_3,..., \delta_n$ contain the censoring information.

Here, a test statistic proposed to test H_0 versus $H_1^{(1)}$ with randomly right censored samples. In the censoring model, if we use the pairs (Z_i,δ_i), i=1,2,3,…,n, in the Kaplan and Meier (1958) estimator and also by using the following variable kernel estimator of the hazard rate in the censored case

$$\hat{r}(t) = \frac{1}{2R_k}\sum_{i=1}^{n}\left[\delta_{(i)}/(n-i+1)K\left((t - Z_{(i)})/2R_k\right)\right], \text{ Tanner(1983)}$$

where

R_k : distance between time t and its k^{th} nearst failure time

k : gilb (n^b), $1/2 < b < 1$

$k(.)$: a function of bounded variation with compact support on the interval [-1,1]

$n-i+1$ is the number of items at risk at $t = Z_{(i)}$, and is $n.\overline{F}_n(Z_{(i-1)})$.

Tanner (1983) proved the strong consistency of $r(t)$.

Then the proposed test statistic is

$$\hat{\delta}_{F_n}^{c(1)} = \int_0^\infty (\hat{r}_n(t) - \hat{r}_n(0)) d\overline{F}_{0n}(t)$$

For computation use, $\hat{\delta}_{F_n}^{c(1)}$ can be written in the following form

$$\hat{\delta}_{F_n}^{c(1)} = \sum_{i=1}^{n} (\hat{r}_n(Z_{(i)}) - \hat{r}_n(0)) e^{-Z_{(i)}} (Z_{(i)} - Z_{(i-1)}) ,$$

then the final emperical form of $\hat{\delta}_{F_n}^{c(1)}$ is given by

$$\hat{\delta}_{F_n}^{c(1)} = \frac{1}{2R_k} \sum_{i=1}^{n} \sum_{j=1}^{n} \left\{ \frac{\delta_{(i)}}{n-i+1} K\left(\frac{Z_{(i)} - Z_{(j)}}{2R_k} \right) - K\left(\frac{-Z_{(j)}}{2R_k} \right) \right\} e^{-Z_{(i)}} \left(\left(Z_{(i)} - Z_{(i-1)} \right) \right), \qquad (3.1)$$

where $dx = Z_i - Z_{i-1}$, and $C_k = (n-k)/(n-k+1)$.

Table A.2 in appendix gives the critical values of statistic $\hat{\delta}_{F_n}^{c()}$ for sample sizes 5(1),50 and 81. Table 3.1, 3.2 and 3.3 show that the test has very good powers for all alternatives.

Table 3.1. Power calculations for samples from LFR

Sample size	θ					
	0.25	0.5	0.75	1	1.5	2,0
10	0.984	0.997	0.999	1.000	1.000	1.000
20	0.989	0.998	1.000	1.000	1.000	1.000
30	0.991	1.000	1.000	1.000	1.000	1.000

Table 3.2. Power calculations for samples from Pareto

Sample size	θ					
	0.2 5	0 .5	0 .75	1	1. 5	2,0
10	0.9 99	1 .000	1 .000	1 .000	1. 000	1.000
20	1.0 00	1 .000	1 .000	1 .000	1. 000	1.000
30	1.0 00	1 .000	1 .000	1 .000	1. 000	1.000

Table 3.3. Power calculations for samples from weibull

Sample size	θ					
	0.25	0.5	0.75	1	1.5	2.0
10	0.994	0.981	0.961	0.947	0.984	1

20	1.00	0.994	0.965	0.944	0.978	1
30	1.00	0.998	0.985	0.953	0.989	0.999

4. TESTING AGAINST NBAFR CLASS

For testing the hypothesis H_0 : F is Exponential against $H_1^{(2)}$: F is NBAFR and not exponential , we propose the following measure departure

$$\delta_F^{(2)} = \int_0^\infty e^{-x\, f(0)} dF_0(x) - \int_0^\infty \overline{F}(u) dF_0(x).$$

Lemma 4.1. Let X be a random variable has NBAFR property with distribution function F. Then

$$\delta_F^{(2)} = E\left(e^{-Xf(0)}\right) - \left(1 - E\left(e^{-X}\right)\right).$$

Proof.

Since

$$\delta_F^{(2)} = \int_0^\infty \overline{F}(x) e^{-x} du = 1 - E(e^{-x})$$

So as in lemma 2.1. the result follows.

Based on a random sample $X_1, X_2, X_3, \ldots X_n$ from a distribution F, the empirical form of $\delta_F^{(2)}$ is given by

$$\hat{\delta}_{F_n}^{(2)} = \frac{1}{n} \sum_{i=1}^{n} \left(e^{-X_i \hat{f}_n(0)} + e^{-X_i} - 1\right), \tag{4.1}$$

By taking

$$\phi(X) = e^{-X \hat{f}_n(0)} + e^{-X} - 1,$$

we prove the following theorem to obtain the asymptotic properties of $\delta_F^{(2)}$.

Theorem 4.1.

As $n \to \infty$, $\sqrt{n}\left(\hat{\delta}_{F_n}^{(2)} - \delta_F^{(2)}\right)$ is asymptotically normal with mean 0 and variance σ^2 that is as in (5). Under H_0, $\sigma_0^2 = 1/3$.

Proof.

It is straightforward by noting that $\hat{\delta}_{F_n}^{(2)}$ is just an average, thus using the central limit theorem the result follows. For the variance σ^2, it can be shown by

$$\sigma^2 = Var\left[e^{-Xf(0)} - 1 + e^{-X}\right], \tag{4.2}$$

which under H_0, becomes

$$\sigma_0^2 = E\left(\left[2e^{-X} - 1\right]^2\right) = 1/3.$$

To perform the above test, calculate $\sqrt{3n}\,\hat{\delta}_{F_n}^{(2)}$ and reject H_0 if this value exceeds Z_α the standard normal variate. Table A.3 in appendix gives the critical values of $\hat{\delta}_{F_n}^{(2)}$ test for sample sizes 5(1),50. The powers of the statistic $\hat{\delta}_{F_n}^{(2)}$ for LFR, Weibull and Pareto families as alternatives can be shown in Table A.3 for sample sizes 10,20,30 and θ values .25,.5,.75,1.0,1.5,2.0. Next, the efficacies of statistic in (4.1) are calculated for the LFR, Makeham and Weibull families alternatives as follows :

Since

$$AE\left(\hat{\delta}_{F_n}^{(2)}\right)=\sqrt{3}\left\{-\int_0^\infty F_\theta'(x)e^{-x}dx\right\},$$

then for

LFR family,

$$AE\left(\hat{\delta}_{F_n}^{(2)}\right)=\frac{\sqrt{3}}{2}\left\{\int_0^\infty \frac{t^2}{2}e^{-2t}dt\right\}=\frac{\sqrt{3}}{8}.$$

Weibull family,

$$AE\left(\hat{\delta}_{F_n}^{(2)}\right)=\sqrt{3}\left\{\int_0^\infty e^{-2t}\ln t dt\right\}=0.05019.$$

Similarly

Makeham family

$$AE\left(\hat{\delta}_{F_n}^{(2)}\right)=\sqrt{3}\left\{-\int_0^\infty \left(te^{-2t}+e^{-3t}-e^{-2t}\right)dt\right\}=\sqrt{3}/12.$$

A full powers obtained for test from Tables 4.1, 4.2 and 4.3.

In the following, we derive an expression for testing exponentiality against NBAFR properties with randomly right censored data as follows:

The proposed test here is depend on the following departure taking into consideration incorporting H_0 into the measure of departure it can lead to the following simpler test statistic by using Kaplan Meier estimator of $\overline{F}(x)$ and Tanner failure rate estimatore in the case of randomly censored data.

$$\hat{\delta}_{F_n}^{c(2)}=\int_0^\infty\left(e^{-tf(0)}-\overline{F}_n(t)\right)dF_{0n}(t).$$

For computation use, $\hat{\delta}_{F_n}^{c(2)}$ can be written as follows:

$$\hat{\delta}_{F_n}^{c(2)}=\sum_{i=1}^{n}\left[e^{-Z_{(i)}\hat{r}(0)}-\prod_{k=1}^{i-1}\left(C_{(k)}\right)^{\delta_{(k)}}\right]e^{-Z_{(i)}}\left(Z_{(i)}-Z_{(i-1)}\right). \tag{4.3}$$

Table 4.1. Power calculations for samples from LFR

Sample size	θ					
	0.25	0.5	0.75	1.0	1.5	2.0
10	.985	.998	.999	1.000	1.000	1.000
20	.994	.1000	1.000	1.000	1.000	1.000
30	.997	1.000	1.000	1.000	1.000	1.000

Table 4.2. Power calculations for samples from pareto

Sample size	θ					
	0.25	0.5	0.75	1.0	1.5	2.0
10	0.985	1.000	1.000	1.000	1.000	1.000
20	0.999	1.000	1.000	1.000	1.000	1.000
30	1.000	1.000	1.000	1.000	1.000	1.000

Table 4.3. Power calculations for samples from weibull

Sample size	θ					
	0.25	0.5	0.75	1.0	1.5	2.0
10	0.722	0.780	0.804	0.805	0.781	0.685
20	0.731	0.766	0.790	0.770	0.633	0.403
30	0.752	0.787	0.791	0.771	0.561	0.246

Table A.4 in appendix gives the percentiles of $\hat{\delta}_{F_n}^{c(2)}$ for sample sizes 5(1)50,80,81,86.

5. APPLICATIONS

The following data consists of 86 survival times (in months) with 23 right censored lung cancer patients from Pena (2002):

The whole life times (non-censored data) :

0.99	1.28	1.77	1.97	2.17	2.63	2.66	2.76	2.79	2.86
2.99	3.06	3.15	3.45	3.71	3.75	3.81	4.11	4.27	4.34
4.4	4.63	4.73	4.93	4.93	5.03	5.16	5.17	5.49	5.68
5.72	5.85	5.98	8.15	8.26	8.48	8.61	9.46	9.53	10.05
10.15	10.94	10.94	11.24	11.63	12.26	12.65	12.78	13.18	13.47
13.96	14.88	15.05	15.31	16.13	16.46	17.45	17.61	18.2	18.37
19.06	20.7	22.54	23.36						

The ordered censored observations are:

11.04	13.53	14.23	14.65	14.91	15.47	16.49	17.05	17.28
17.88	17.97	18.83	19.55	19.58	19.75	19.78	19.95	20.04
20.24	20.73	21.55	21.98					

If we deal with a complete 64 survival times and computing the values of statistics (2.1) and (4.1), we get the following conclusions:

1. The value of statistic $\hat{\delta}^{(1)}_{F_n}$ gives the value 0.02595778 which is less than the critical value of the Table A.1 in appendix. Then these data is exponential.
2. The value of $\hat{\delta}^{(2)}_{F_n}$ = 0.01338637 which leads to reject H_1.

If we deal with all data:

3. The value of statistic (3.1) we get the value of departure is 0.007760483 that leads to reject exponentiality hypothesis i.e censored data are NBUFR data.
4. From statistic (4.3) the value of departure is 0.01739595that leads to reject H_0.

ACKNOWLEDGEMENT

The authors acknowledge with gratitude the detailed comments of the referee, which resulted in this much improved version of the work.

REFERENCES

Abouammoh, A. M. and Ahmed, A. N. (1988). The new better than used failure rate class of life distribution. *Adv. Prob*., **20**, 237-240.

Ahmed, I.A. (1994). A class of statistics useful in testing increasing failure rate average and new better than used life distribution. *J. Statist.. Plant. Inf*., **41**, 141-149.

Ahmed. I. A., Ibrahim. A. A. and Mugdadi. A. R. (2001). A goodnees if fit approach to major life testing problems. *International Journal of Reliability & Applications*, **2**, 81-97.

Deshpande, J. V., Kochar, S. C. and Singh, H. (1986). Aspects of positive aging. *J. Appl. Prob.*, **28**, 773-779

Hardle,W. (1991). *Smoothing Techniques with Implementation in S*. Spring-Verlag New York.

Hendi, M. I., Alnachawati, H. and AL-Graian, M. N. (2000). Testing NBUFR and NBAFR classes of life distributions using kernel methods. *Arab J. Math. Sc*., **6**, 37-57.

Kaplan,E. L. and Meier, P. (1958). Nonparametric estimation from incomplete observations. *J. Amer. Assoc*., **53**, 457-481.

Klefsjo, B. (1981). HNBUE survival under some shoch models. *Scand. J. Statist*., **8**, 34-47.

Klefsjo, B. (1982). The HNBUE and HNWUE classes of life distributions. *Naval. Res. Logistics Quartely*, **24**, 331-344.

Lee, A. J. (1990). *U-Statistics* , Marcel Dekker New York, NY.

Loh, W. Y. (1984). A new generalization of the class of NBU distribution. *IEEE Trans. Reli*., **33**, 419-422.

Mahmoud M, A. W. and Abdul Alim, N. A. (2003 a). On testing exponentiality against NBURFR class of life distributions. *International Journal of Reliability and Application*s, **4**, 57-69.

Mahmoud M, A. W. and Abdul Alim, N. A. (2003 b). Moment inequalities of NBURFR and NBARFR classes with hypotheses testing applications. *International Journal of Reliability and Applications*, **4**, 127-142.

Mahmoud M, A. W. and Abdul Alim, N. A. (2002). On testing exponentiality aainst NBARFR life distributions. *STATISTICA*, LXII , **4**, 619-631.

Mahmoud, M. A. W. and Abdul Alim, N. A. (2006). A goodness to fit approach to NBURFR and NBARFR classes. *Economic Quality Control*, **21**, 59-75.

Pena, A. E. (2002). Goodness of fit tests with censored data. *http://statman.stat.sc.edu/~ pena/ta/kspresented/talk atcornel.pdf.*

Tanner, M. A. (1983). A note on the variable kernel estimator of the hazard function from randomly censored data. *Ann. Statist*., **11**, 994-998.

APPENDIX

Table A.1. Critical values of statistic (1)

n	.01	.05	.10	.90	.95	.98	.99
5	.1293	.1927	.2262	.5000	.5355	.5757	.6017
6	.1483	.2003	.2335	.4826	.5206	.5605	.5802
7	.1524	.2074	.2349	.4654	.5036	.5422	.5613
8	.1696	.2179	.2450	.4567	.4925	.5258	.5508
9	.1759	.2225	.2491	.4474	.4754	.5114	.5355
10	.1822	.2258	.2522	.4437	.4739	.5077	.5317
11	.1899	.2312	.2566	.4345	.4625	.4916	.5166
12	.1955	.2367	.2577	.4298	.4556	.4883	.5069
13	.2007	.2366	.2594	.4232	.4483	.4778	.4967
14	.2025	.2414	.2621	.4214	.4453	.4738	.4909
15	.2090	.2446	.2647	.4171	.4384	.4657	.4807
16	.2112	.2461	.2667	.4143	.4361	.4636	.4855
17	.2189	.2496	.2666	.4093	.4308	.4569	.4750
18	.2168	.2478	.2682	.4074	.4277	.4498	.4699
19	.2243	.2547	.2717	.4022	.4238	.4469	.4652
20	.2238	.2572	.2722	.4001	.4214	.4469	.4634
21	.2265	.2557	.2731	.4000	.4218	.4444	.4591
22	.2276	.2570	.2726	.3980	.4189	.4429	.4567
23	.2291	.2572	.2725	.3965	.4155	.4365	.4490
24	.2317	.2592	.2759	.3918	.4108	.4326	.4467
25	.2338	.2616	.2760	.3927	.4111	.4308	.4475
26	.2375	.2629	.2783	.3902	.4082	.4284	.4406
27	.2375	.2627	.2784	.3864	.4049	.4267	.4397
28	.2404	.2645	.2786	.3866	.4040	.4228	.4368
29	.2372	.2620	.2764	.3844	.4012	.4205	.4338
30	.2398	.2646	.2782	.3829	.3993	.4181	.4308
31	.2431	.2672	.2812	.3820	.3975	.4138	.4280
32	.2441	.2666	.2795	.3787	.3952	.4140	.4270
33	.2455	.2663	.2796	.3772	.3926	.4121	.4270
34	.2470	.2700	.2817	.3771	.3937	.4101	.4203
35	.2461	.2702	.2834	.3757	.3907	.4101	.4220
36	.2483	.2704	.2814	.3776	.3923	.4092	.4210
37	.2466	.2683	.2814	.3746	.3896	.4068	.4200
38	.2507	.2700	.2831	.3740	.3884	.4041	.4185
39	.2490	.2706	.2832	.3714	.3841	.4009	.4143
40	.2514	.2716	.2822	.3704	.3842	.4009	.4106
41	.2489	.2723	.2844	.3713	.3836	.3975	.4086
42	.2513	.2728	.2839	.3694	.3839	.3983	.4086
43	.2540	.2741	.2847	.3692	.3831	.3978	.4091
44	.2531	.2741	.2849	.3681	.3799	.3952	.4068
45	.2527	.2730	.2845	.3664	.3790	.3937	.4045
46	.2535	.2742	.2848	.3652	.3775	.3908	.4033
47	.2554	.2745	.2850	.3667	.3796	.3941	.4060
48	.2538	.2746	.2850	.3650	.3771	.3908	.4010
49	.2591	.2764	.2856	.3653	.3771	.3904	.3985
50	.2566	.2758	.2850	.3628	.3752	.3923	.4015

Table A.2. Critical values of statistic (3)

n	.01	.05	.10	.90	.95	.98	.99
5	-5.1708	-1.2228	-.7428	.0823	.1550	.2247	.3382
6	-2.6302	-.8037	-.4245	.2062	.2863	.3709	.4182
7	-2.2517	-.9135	-.5256	.1812	.2353	.3252	.3939
8	-1.2125	-.4836	-.2597	.2929	.3572	.4320	.5158
9	-1.7961	-.4932	-.2892	.2367	.3000	.3726	.4029
10	-1.4331	-.5886	-.3233	.2271	.3003	.3740	.4152
11	-1.6638	-.5811	-.3422	.2331	.2838	.3591	.4032
12	-1.4024	-.4502	-.2685	.2002	.2487	.3259	.3583
13	-1.7678	-.6122	-.3294	.1877	.2366	.3019	.3431
14	-1.2281	-.6337	-.3869	.1592	.2116	.2691	.2980
15	-1.4224	-.6558	-.4236	.1507	.2055	.2633	.3076
16	-1.4755	-.6360	-.3944	.1275	.1659	.2151	.2483
17	-2.5910	-.8841	-.5205	.1215	.1584	.2153	.2452
18	-1.8175	-.7222	-.4490	.0982	.1385	.1906	.2329
19	-1.4888	-.6904	-.4399	.0900	.1416	.1892	.2233
20	-1.4001	-.6688	-.4138	.0752	.1075	.1447	.1715
21	-2.1683	-.7806	-.4661	.0598	.1050	.1502	.1821
22	-1.6312	-.7239	-.4570	.0606	.0974	.1435	.1723
23	-1.5535	-.7084	-.4642	.0491	.0889	.1301	.1671
24	-1.6285	-.7125	-.4570	.0421	.0838	.1167	.1482
25	-1.9977	-.8042	-.5002	.0422	.0717	.1156	.1377
26	-1.8270	-.7560	-.4744	.0349	.0636	.1007	.1260
27	-1.7385	-.7839	-.5105	.0348	.0704	.0973	.1153
28	-1.5856	-.7794	-.4903	.0226	.0578	.0934	.1109
29	-1.9698	-.7706	-.4928	.0191	.0446	.0867	.1117
30	-2.2536	-.8464	-.5551	.0127	.0393	.0733	.0952
31	-1.5805	-.7035	-.4600	.0072	.0347	.0732	.0996
32	-2.4034	-.8559	-.5025	.0064	.0294	.0458	.0693
33	-2.6496	-.7500	-.5395	.0000	.0225	.0600	.0815
34	-1.9422	-.7975	-.5721	.0000	.0205	.0401	.0593
35	-2.9604	-.9205	-.6068	.0000	.0219	.0473	.0668
36	-2.5989	-.9839	-.6270	.0000	.0195	.0430	.0672
37	-1.9711	-.8871	-.5710	.0000	.0109	.0465	.0666
38	-2.4134	-.9364	-.6340	.0000	.0121	.0408	.0494
39	-2.1602	-.9833	-.6353	.0000	.0025	.0268	.0390
40	-2.0472	-.9333	-.6093	-.0002	.0000	.0288	.0400
41	-2.1584	-1.0605	-.6687	.0000	.0032	.0246	.0404
42	-1.6635	-.8941	-.5743	.0000	.0000	.0299	.0516
43	-2.6654	-.9710	-.6413	-.0121	.0000	.0241	.0399
44	-2.3692	-.8707	-.6012	-.0107	.0000	.0125	.0235
45	-2.1147	-.8938	-.6276	-.0120	.0000	.0184	.0372
46	-1.9614	-.9458	-.6322	.0000	.0000	.0212	.0342
47	-2.5053	-.9667	-.6533	-.0177	.0000	.0046	.0177
48	-2.0045	-.8824	-.5949	-.0056	.0000	.0067	.0184
49	-2.1464	-.9696	-.6473	-.0204	.0000	.0062	.0244
50	-2.2291	-.9624	-.5956	-.0181	.0000	0061	.0222

Table A.3. Critical values of statistic (4)

n	.01	.05	.10	.90	.95	.98	.99
5	.0151	.0864	.1279	.4660	.5129	.5566	.5903
6	.0322	.0946	.1315	.4433	.4882	.5362	.5658
7	.0426	.1006	.1363	.4229	.4663	.5135	.5432
8	.0559	.1112	.1429	.4101	.4543	.4946	.5182
9	.0567	.1170	.1482	.3991	.4360	.4778	.5070
10	.0656	.1212	.1512	.3946	.4309	.4706	.4979
11	.0771	.1256	.1564	.3824	.4138	.4541	.4816
12	.0826	.1314	.1601	.3761	.4094	.4414	.4665
13	.0883	.1323	.1596	.3670	.3995	.4352	.4595
14	.0899	.1367	.1621	.3634	.3971	.4299	.4472
15	.0952	.1411	.1660	.3591	.3877	.4148	.4303
16	.1003	.1423	.1685	.3553	.3843	.4163	.4394
17	.1059	.1444	.1679	.3488	.3767	.4075	.4282
18	.1000	.1433	.1696	.3489	.3732	.3973	.4205
19	.1088	.1518	.1753	.3417	.3663	.3961	.4171
20	.1131	.1547	.1733	.3392	.3648	.3954	.4144
21	.1166	.1537	.1753	.3403	.3661	.3961	.4080
22	.1146	.1529	.1739	.3354	.3628	.3886	.4083
23	.1170	.1556	.1742	.3335	.3566	.3814	.3994
24	.1225	.1574	.1780	.3276	.3495	.3758	.3981
25	.1247	.1597	.1783	.3288	.3513	.3798	.4034
26	.1245	.1600	.1796	.3258	.3483	.3730	.3889
27	.1295	.1604	.1805	.3219	.3444	.3676	.3876
28	.1316	.1636	.1800	.3218	.3419	.3673	.3823
29	.1284	.1603	.1778	.3185	.3386	.3611	.3789
30	.1315	.1618	.1805	.3166	.3377	.3606	.3753
31	.1353	.1669	.1845	.3164	.3359	.3572	.3734
32	.1361	.1655	.1822	.3137	.3309	.3566	.3745
33	.1368	.1649	.1820	.3107	.3304	.3519	.3714
34	.1397	.1679	.1847	.3099	.3320	.3520	.3641
35	.1384	.1683	.1858	.3090	.3275	.3488	.3641
36	.1407	.1691	.1838	.3106	.3283	.3487	.3659
37	.1365	.1674	.1831	.3067	.3271	.3461	.3597
38	.1441	.1701	.1869	.3069	.3255	.3466	.3620
39	.1409	.1695	.1860	.3037	.3212	.3399	.3547
40	.1423	.1710	.1855	.3005	.3174	.3418	.3549
41	.1420	.1716	.1879	.3017	.3190	.3367	.3491
42	.1421	.1721	.1874	.3002	.3193	.3380	.3493
43	.1473	.1751	.1882	.3002	.3180	.3352	.3491
44	.1471	.1735	.1886	.2998	.3164	.3356	.3474
45	.1434	.1720	.1876	.2979	.3136	.3334	.3456
46	.1456	.1755	.1886	.2954	.3123	.3302	.3447
47	.1496	.1747	.1891	.2961	.3137	.3322	.3452
48	.1480	.1751	.1886	.2950	.3109	.3292	.3419
49	.1521	.1774	.1898	.2959	.3116	.3275	.3379
50	.1508	.1754	.1897	.2931	.3074	.3281	.3422

Table A.4. Critical values of statistic (6)

n	.01	.05	.10	.90	.95	.98	.99
5	.0000	.0000	.0353	.2231	.2517	.2789	.2902
6	.0000	.0310	.0599	.2483	.2719	.3023	.3166
7	.0000	.0631	.0749	.2584	.2879	.3179	.3318
8	.0126	.0686	.0863	.2707	.2957	.3239	.3318
9	.0401	.0724	.0972	.2831	.3048	.3359	.3496
10	.0584	.0929	.1186	.2865	.3101	.3349	.3508
11	.0568	.1088	.1278	.3031	.3237	.3459	.3625
12	.0579	.1087	.1337	.2995	.3239	.3481	.3652
13	.0759	.1200	.1443	.3124	.3372	.3593	.3711
14	.0833	.1249	.1483	.3176	.3400	.3688	.3864
15	.0904	.1330	.1602	.3257	.3444	.3635	.3757
16	.1067	.1367	.1585	.3304	.3505	.3749	.3882
17	.1181	.1488	.1743	.3353	.3529	.3706	.3843
18	.1157	.1562	.1836	.3362	.3527	.3738	.3862
19	.1239	.1633	.1829	.3399	.3601	.3830	.3953
20	.1306	.1710	.1926	.3462	.3656	.3887	.4083
21	.1389	.1746	.1951	.3478	.3634	.3878	.3967
22	.1478	.1813	.2019	.3487	.3645	.3788	.4006
23	.1487	.1866	.2041	.3475	.3697	.3967	.4174
24	.1353	.1827	.2075	.3549	.3734	.3893	.4063
25	.1517	.1957	.2157	.3543	.3695	.3870	.3992
26	.1571	.1956	.2163	.3534	.3718	.3887	.4013
27	.1618	.2009	.2193	.3592	.3780	.3989	.4122
28	.1646	.2060	.2219	.3674	.3849	.4014	.4249
29	.1721	.2089	.2247	.3650	.3822	.3957	.4096
30	.1852	.2120	.2341	.3676	.3846	.4052	.4133
31	.1637	.2112	.2344	.3694	.3869	.4068	.4184
32	.1766	.2208	.2404	.3693	.3871	.4070	.4205
33	.1878	.2215	.2406	.3713	.3918	.4050	.4184
34	.1837	.2232	.2451	.3762	.3903	.4074	.4172
35	.1935	.2281	.2450	.3727	.3888	.4171	.4279
36	.1997	.2290	.2512	.3819	.4026	.4121	.4217
37	.1820	.2271	.2484	.3770	.3893	.4125	.4256
38	.1943	.2381	.2575	.3841	.3947	.4108	.4193
39	.2133	.2455	.2587	.3843	.4023	.4169	.4335
40	.1989	.2413	.2609	.3828	.4009	.4174	.4287
41	.2102	.2397	.2578	.3849	.4001	.4123	.4251
42	.2067	.2493	.2658	.3889	.4032	.4194	.4308
43	.2145	.2470	.2655	.3874	.4021	.4174	.4244
44	.2108	.2490	.2711	.3896	.4041	.4201	.4258
45	.2232	.2530	.2706	.3889	.4032	.4188	.4287
46	.2215	.2530	.2755	.3902	.4041	.4169	.4296
47	.2250	.2593	.2755	.3912	.4064	.4217	.4363
48	.2210	.2603	.2776	.3884	.4042	.4244	.4300
49	.2282	.2596	.2775	.3976	.4130	.4290	.4424
50	.2360	.2672	.2812	.3956	.4081	.4244	.4393
80	.2756	.3061	.3187	.4154	.4272	.4392	.4454
81	.2767	.3056	.3240	.4178	.4270	.4420	.4500
86	.2920	.3139	.3266	.4182	.4294	.4402	.4468