A Searching Algorithm for Minimum Bandpass Sampling Frequency in Simultaneous Down-Conversion of Multiple RF Signals

Junghwa Bae and Jinwoo Park

Abstract: **Bandpass sampling (BPS) techniques for the direct down-conversion of RF bandpass signals have become an essential technique for software defined radio (SDR), due to their advantage of minimizing the radio frequency (RF) front-end hardware dependency. This paper proposes an algorithm for finding the minimum BPS frequency for simultaneously down-converting multiple RF signals through full permutation over all the valid sampling ranges found for the multiple RF signals. We also present a scheme for reducing the computational complexity resulting from the large scale of the purmutation calculation involved in searching for the minimum BPS frequency. In addition, we investigate the BPS frequency allowing for the guard-band between adajacent down-converted signals, which help lessen the severe requirements in practical implementations. The performance of the proposed method is compared with those of other pre-reported methods to prove its effectiveness.**

Index Terms: **Bandpass sampling (BPS), software-defined radio (SDR), sub-sampling.**

I. INTRODUCTION

Reconfigurable and flexible systems, epitomized by the software defined radio (SDR) system, have come into the spotlight as a potential technology for the next generation of multimedia wireless communications [1]–[7]. The SDR system is a radio receiver that relies on high-speed digital signal processing techniques to perform most of the communication processes using software, such as digital mixing, modulation demodulation, noise-suppression, channel-coding, etc. This allows the hardware dependency of the communication systems to be substantially reduced, as well as permitting higher system flexibility in accommodating various multimedia wireless services on a single receiver platform. The usefulness of this approach is manifested in the system's adaptability, due to its easy reconfigurability and reprogrammability via software that is updated directly over the open-air interface whenever service providers wish to do so. The role of digital signal processing has therefore become very important in replacing the functionalities that were previously implemented with analog components such as mixers, filters and so on. The current trend of the research and development in the field of SDR is minimizing the necessity for components in the radio frequency (RF) front-end part connected to the antenna, and realizing most of the remaining communication processing tasks in the digital processing part.

One of the important factors in the design of digital radio systems is the choice of a suitable sampling frequency. In particular, bandpass sampling (BPS), which is also called sub-sampling, is a core sampling technique in SDR terminals, because the BPS technique directly down-converts analog bandpass signals to baseband or low-intermediate frequency (IF) digital signals without depending on analog components such as mixers and image-rejection filters [8]–[10]. It should also be noted that a lower BPS frequency is directly related to a higher efficiency of digital processing and power consumption of the SDR terminal. Therefore, BPS determines the extent to which the RF front-end can be minimized, as well as the overall system performance.

In BPS, the information bandwidth of the signal is of a major concern rather than the RF carrier frequencies. Finding the minimum BPS frequency is a constrained task, because it has to take account of the negative frequency part of the signal spectrum to avoid unwanted signal spectral folding, namely the overlapping of the signal bandwidths. The avoidance of such overlapping makes it difficult to choose the proper sampling frequency, and this task becomes more complex when multiple RF signals are simultaneously down-converted [11], [12]. A properly-chosen BPS frequency should be able to translate all distinct RF signals received in different wireless standards into digital IF signals without any mutual overlapping of the signal bandwidths in the resultant sampled spectral domain, i.e., the sampled bandwidth. Methods of finding the valid sampling ranges for down-converting two distinct RF signals have been widely investigated in [11]–[13].

The expansion of such methodologies to the down-conversion of multiple RF signals has been also carried out in [14]. In [14], the authors extended the method described in [13] to the general case of N RF signals. However, this has been found to be much more complicated and time-consuming, because the number of possible signal sets allowing for signal placement in the sampled bandwidth produced by the permutation of N bandpass signals can be as large as $N! \times 2^N$ [16]. In order to reduce the scale of the signal permutations, methods of down-converting N RF signals employing certain constraints in the signal permutation have been reported [14]–[16]. However, it should be pointed out that such methods do not provide the true minimum

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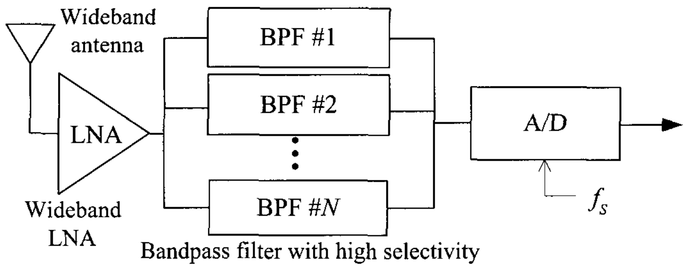

Fig. 1. Basic configuration of the receiver structure for down-conversion of N RF signals.

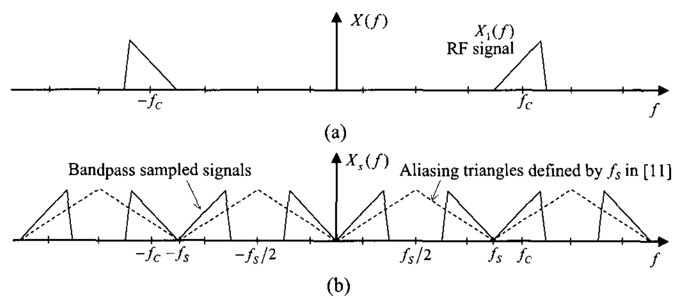

Fig. 2. Signal spectrum sampled by BPS: (a) RF spectrum and (b) spectrum of bandpass-sampled signal.

sampling frequency and wider valid sampling ranges for down-converting N RF signals at the cost of reduced complexity in finding a rational bandpass sampling rate. In this paper, we describe generalized formulas producing the valid BPS ranges and the minimum BPS frequency based on the full permutation of signal placements in the sampled bandwidth, in particular including guard-bands of user-specified size between adjacent down-converted signals. The insertion of guard-bands can lessen the strict requirements in practical down-conversion implementations, especially in the case of a filter design for avoiding adjacent channel interferences in the IF stage and jitter or frequency instability effects [17]. We also investigate how the searching complexity can be reduced in order to achieve the minimum sampling rate faster and more efficiently, unlike in the case of previous papers. In order to demonstrate the validity of the proposed methods, we present practical examples of a mobile SDR receiver application which is assumed to support three different wireless communication standards currently in service. The structure of this paper is as follows. In Section II, we describe the basic equations and the principle of BPS, including hardware implementation issues. In Section III, the proposed method of finding valid sampling frequency ranges is described in detail. A method of reducing the computational complexity is proposed in Section IV. In Section V, we describe our simulation results, complexity comparisons, and examples of a practical wireless SDR receiver application. Finally, Section VI concludes this paper.

II. BANDPASS SAMPLING

A. Implementation and Hardware Structure for BPS Receiver

A traditional receiver structure, i.e., super-heterodyne receiver, consists of multiple RF stages such as double or more IF stages using analog mixers, including the functions of amplification, filtering, and image signal rejection. However, according to the SDR concept of moving the ADC as close to the antenna as possible, schematic structures of the BPS receiver for the simultaneous down-conversion of N multiple RF signals can be presented as shown in Fig. 1. This structure represents a minimal configuration set of RF front-end components for a true software radio, in which the RF signals received by a wideband or multiband antenna are amplified by a wideband low-noise amplifier (LNA). The RF signals intended to be received by a user are selected in a bandwidth-limited fashion by the tunable bandpass filter (BPF) bank, and the signal level can be appropriately adjusted by an automatic gain control (AGC) circuit for compatibility with the subsequent ADC. These BPFs are required to be highly selective, because of noise spectrum aliasing [8] as well as adjacent channel interference between the down-converted signals. The reason that the effect of the noise floor aliasing is important is that all frequency energies from DC to the input analog bandwidth of the ADC will alias or fold into the resultant bandwidth [8], [11]. Therefore, the filters must sufficiently reject all frequency band noise outside the RF signal bands of interest. Also, we use ADCs having very low jitter to avoid aliasing effects caused by jitter or frequency instabilities. Therefore, the characteristics of each analog hardware component are required to be close to ideal. The RF devices satisfying the performance criteria of SDR applications are still under development, but are expected to come onto the market in the near future owing to the rapid advancements in RF chip technologies using the micro-electro mechanical systems (MEMS) technique, monolithic microwave ICs (MMICs), superconductor microelectronics and so on [5]. In spite of the immature status of the supporting technologies at present, in this paper, we mainly focus on theoretical studies of the down-conversion of multiple RF bandpass signals and its applicability to SDR systems, assuming that they will become available for practical employment in the near future. It is thus assumed in the following analysis that the RF signals applied to ADC are ideally band-limited and that no RF distortion is present, as assumed in other literatures [8]–[11] and [16].

B. Valid Sampling Range by Using Pre-reported Methods

We assume that N multiple bandpass signals, $x_k(t)$, $k = 1, 2, \cdots, N$ are given for down-conversion. Let f_S, f_{C_k}, f_{U_k}, f_{L_k}, f_{IF_k}, and BW_k denote the sampling frequency, carrier frequency, upper cutoff frequency, lower cutoff frequency, IF, and information bandwidth of $x_k(t)$, respectively. We also assume that $f_{U_k} = f_{C_k} + (BW_k/2)$, $f_{L_k} = f_{C_k} - (BW_k/2)$, $f_{C_i} < f_{C_{i+1}}$, for $i = 1, 2, \cdots, N-1$, and the spectrum of the RF bandpass signal is assumed, as mentioned earlier, to be band-limited as follows

$$|X_k(f)| = 0, \text{ for } |f| \geq f_{U_k} \text{ and } |f| \leq f_{L_k}, \quad k = 1, 2, \cdots, N \quad (1)$$

where $X_k(f)$ denotes the spectrum of $x_k(t)$. Fig. 2 shows an example of a bandpass-sampled signal spectrum. The spectrum

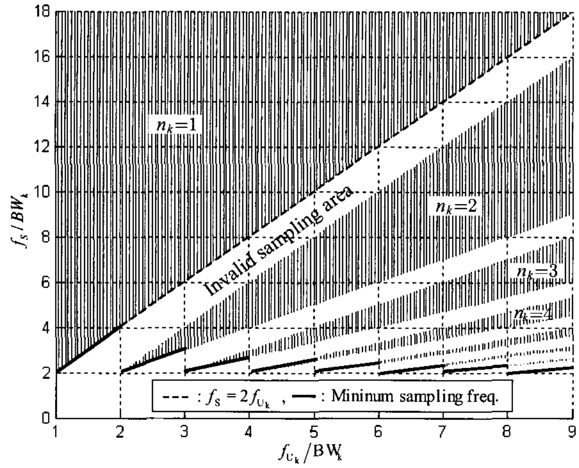

Fig. 3. Valid sampling ranges and minimum sampling rates in BPS.

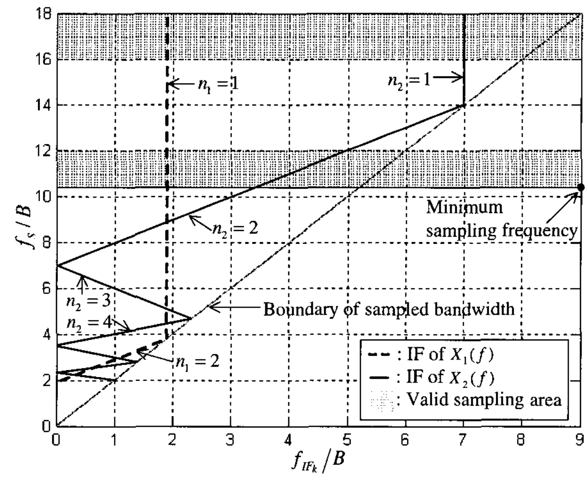

Fig. 4. Valid sampling areas and IFs by BPS when two signals are sampled.

of the RF signal in Fig. 2(a) is periodically repeated in terms of f_S. The bandpass-sampled spectrum is also obtained easily by using the aliasing triangles mentioned in [11], as shown in Fig. 2(b). In this case, the selection of inappropriate sampling rates gives rise to aliasing with the negative frequency part of the RF signal. Thus, a constraint to avoid such aliasing is certainly required, as follows. The equation for the acceptable sampling ranges of a single signal $X_k(f)$ can be expressed by [8]

$$\frac{2f_{U_k}}{n_k} \leq f_S \leq \frac{2f_{L_k}}{n_k - 1} \tag{2}$$

where n_k is an integer number given by

$$1 \leq n_k \leq \left\lfloor \frac{f_{U_k}}{BW_k} \right\rfloor . \tag{3}$$

Here, $\lfloor \cdot \rfloor$ denotes a floor function. In Fig. 3, we depict the valid and invalid sampling areas obtained by (2) and (3), and also present the minimum sampling rate as a function of the signal location. The two axes are normalized by the signal bandwidth, BW_k. In Fig. 3, the area with $n_k = 1$ represents the available sampling area obtained by sampling rates that are larger than twice the highest frequency $2f_{U_k}$. The areas with even integer number of n_k, represent the cases in which the resultant spectrum is inverted in the lowest frequency band [8]. In the unshaded areas in Fig. 3, the signal overlapping with the negative frequency part occurs.

Based on (2) and (3), we can obtain the bandpass sampling equations for two RF signals as follows [8], [11]

$$\left(\frac{2f_{U_1}}{n_1} \leq f_S \leq \frac{2f_{L_1}}{n_1 - 1}\right) \cap \left(\frac{2f_{U_2}}{n_2} \leq f_S \leq \frac{2f_{L_2}}{n_2 - 1}\right) \tag{4}$$

$$|f_{IF_1} - f_{IF_2}| \geq \frac{BW_1 + BW_2}{2}. \tag{5}$$

These two conditions, namely a non-aliasing condition with the negative frequency part of each RF signal and a non-overlapping condition among the different RF signals, respectively, are certainly required with each n_k. The IF of the down-converted RF signal in (5) is also represented by

$$f_{IF_k} = \begin{cases} \text{rem}(f_{C_k}, f_S), \text{ if } \left\lfloor \frac{f_{C_k}}{f_S/2} \right\rfloor \text{ is even} \\ f_S - \text{rem}\,(f_{C_k}, f_S)\,, \text{ if } \left\lfloor \frac{f_{C_k}}{f_S/2} \right\rfloor \text{ is odd} \end{cases} \tag{6}$$

where $\text{rem}(f_{C_k}, f_S)$ is the remainder after the division of f_{C_k} by f_S. Therefore, by searching in the common frequency ranges satisfying (4), (5), and (6) simultaneously, we can obtain the resulting valid sampling ranges.

Let us assume that two bandpass signals, such as $X_1(f)$ with $f_{C_1} = 1.9B$ and $BW_1 = 1B$, and $X_2(f)$ with $f_{C_2} = 7B$ and $BW_2 = 2B$, are given. Fig. 4 shows the valid sampling areas in the form of shaded regions, which are calculated using the above equations, and presents the down-converted center frequencies of $X_1(f)$ and $X_2(f)$ as dashed and solid-lines, respectively. The sampled bandwidth is defined as the range of 0 to $f_S/2$. From this example, we can recognize that the minimum sampling frequency is 10.4 B, and the two valid sampling ranges can also be obtained.

It is noted that the task of obtaining the intersection ranges from (4), (5), and (6) is too complicated, and moreover it is difficult to derive one equation with respect to f_S. Especially, the difficulty becomes greater when the number of RF signals is more than three, unless the limitation of one particular signal placement in the sampled bandwidth is assumed as in [15] and [16]. In the next subsection, we thus derive generalized equations for the down-conversion of N RF signals, and describe a novel method of acquiring all of the valid sampling ranges and minimum bandpass sampling frequency through all possible permutations

III. A SCHEME TO OBTAIN VALID SAMPLING RANGES FOR DOWN-CONVERSION OF N RF SIGNALS

To begin with, the notations used for the signal descriptions are as follows. An RF signal $X_k(f)$ consists of two spectral components, i.e., $X_{k+}(f)$ in the positive frequency part and

$X_{k-}(f)$ in the negative frequency part. A signal spectrum containing N RF signals can thus be expressed as $2N$ spectral components, $X_j(f)$ for $j = 1\pm, 2\pm, \cdots, N\pm$, as shown in Fig. 5(b).

The frequency relationship between the signals can also be described as follows; $f_{L_{k-}} = -f_{U_k}$, $f_{C_{k-}} = -f_{C_k}$, $f_{U_{k-}} = -f_{L_k}$, $f_{L_{k+}} = f_{L_k}$, $f_{C_{k+}} = f_{C_k}$, and $f_{U_{k+}} = f_{U_k}$, for $k = 1, 2, \cdots, N$. We also assume that $f_{C_{N-}} < f_{C_{(N-1)-}} < \cdots < f_{C_{1-}} < f_{C_{1+}} < \cdots < f_{C_{(N-1)+}} < f_{C_{N+}}$.

Let us first consider the sampling ranges for any two RF signals, $X_m(f)$ and $X_n(f)$, where $m, n \in \{1\pm, 2\pm, \cdots, N\pm\}$, as shown in Fig. 5(c). Two observations should now be made regarding the determination of the valid sampling ranges. First, an upper limit for the valid sampling ranges can be defined based on the fact that the lower bound $f_{L_{n,r}}$ of the $(r_{m,n})$th left-shifted replica of $X_n(f)$ should be larger than the upper bound f_{U_m} of $X_m(f)$, as shown in Fig. 5(d). Similarly, a lower limit can be defined based on the fact that the upper bound $f_{U_{n,r+1}}$ of the $(r_{m,n}+1)$th left-shifted replica of $X_n(f)$ should be smaller than the lower bound f_{L_m} of $X_m(f)$. These findings can be described by

$$f_{C_n} - \frac{BW_n}{2} - r_{m,n} f_S \geq f_{C_m} + \frac{BW_m}{2} \tag{7}$$

and

$$f_{C_n} + \frac{BW_n}{2} - (r_{m,n}+1) f_S \leq f_{C_m} - \frac{BW_m}{2}. \tag{8}$$

Combining (7) and (8), the resultant sampling ranges $f_{S_{m,n}}$ are as follows

$$\frac{f_{C_{n-m}} + (BW_{m+n}/2)}{r_{m,n}+1} \leq f_{S_{m,n}} \leq \frac{f_{C_{n-m}} - (BW_{m+n}/2)}{r_{m,n}} \tag{9}$$

where $f_{C_{n-m}} = f_{C_n} - f_{C_m}$, $BW_{m+n} = BW_m + BW_n$, and $r_{m,n}$ is an integer number given by

$$0 \leq r_{m,n} \leq \left\lfloor \frac{f_{C_{n-m}} - (BW_{m+n}/2)}{BW_{m+n}} \right\rfloor. \tag{10}$$

In (10), $r_{m,n}$, which means a frequency shift parameter (FSP), means how many times the two signals can be placed in the interval $f_{L_n} - f_{U_m}$ without any mutual overlapping of the signal bands. If $r_{m,n} = 0$, the valid sampling range is simply $f_S \geq f_{U_n} - f_{L_m}$.

Let us consider the valid sampling ranges for a single signal $X_1(f)$. Owing to the fact that $f_{C_{1+}} = f_{C_1}$, $f_{C_{1-}} = -f_{C_1}$, and $BW_{1+} = BW_{1-} = BW_1$, we can obtain the following equation from (9)

$$\frac{2f_{U_1}}{r_{1-,1+}+1} \leq f_{S_{1-,1+}} \leq \frac{2f_{L_1}}{r_{1-,1+}} \tag{11}$$

and $0 \leq r_{1-,1+} \leq \lfloor f_{L_1}/BW_1 \rfloor$ from (10). Here, we find that (11) coincides with the result described in [8].

We try now to extend the results developed for two signals to the general case of N RF signals. To accomplish this, it is necessary to find the intersection ranges $f_{S,all}$ among the sampling ranges $f_{S_{m,n}}$ of any two signals $X_m(f)$ and $X_n(f)$ where $m, n \in \{1\pm, 2\pm, \cdots, N\pm\}$ as (9). The resultant equation can be expressed by

$$f_{S,all} = f_{S\_N-} \cap f_{S\_(N-1)-} \cap \cdots \cap f_{S\_1-} \cap f_{S\_1+} \cap \cdots \cap f_{S\_(N-1)+} \tag{12}$$

where

$$f_{S\_N-} = \left[\bigcap_{k=(N-1)-}^{1-} f_{S_{N-,k}} \right] \cap \left[\bigcap_{k=1+}^{N+} f_{S_{N-,k}} \right],$$

$$f_{S\_(N-1)-} = \left[\bigcap_{k=(N-2)-}^{1-} f_{S_{(N-1)-,k}} \right] \cap \left[\bigcap_{k=1+}^{N+} f_{S_{(N-1)-,k}} \right],$$

$$f_{S\_1-} = \bigcap_{k=1+}^{N+} f_{S_{1-,k}}, \quad f_{S\_1+} = \bigcap_{k=2+}^{N+} f_{S_{1+,k}}$$

and

$$f_{S\_(N-1)+} = f_{S_{(N-1)+,N+}}.$$

Here, the total number of $f_{S_{m,n}}$ required for $2N$ RF components is

$$\text{The number of } f_{S_{m,n}} = {}_{2N}C_2 = \frac{(2N)!}{(2N-2)!\ 2!}. \tag{13}$$

Once one finds a valid sampling range $f_{S,all}$, the minimum sampling frequency for N RF signals is the minimum one in $f_{S,all}$.

For example, if we obtain the valid sampling range with respect to two RF signals, owing to the presence of four distinct RF components such as $X_{2-}(f)$, $X_{1-}(f)$, $X_{1+}(f)$, and $X_{2+}(f)$, the intersection ranges among all six equations $f_{S_{m,n}}$ is needed, as follows

$$f_{S,two} = f_{S_{2-,1-}} \cap f_{S_{2-,1+}} \cap f_{S_{2-,2+}} \cap f_{S_{1-,1+}} \cap f_{S_{1-,2+}} \cap f_{S_{1+,2+}}. \tag{14}$$

It is noticed that determining $f_{S,all}$ in (12) is computationally intensive, because it can only be found through an exhaustive searching process over all the valid sampling ranges $f_{S_{m,n}}$ for any two signal sets among $2N$ RF spectrum components. However, the searching complexity can be reduced by considering two parameters, as follows. Firstly, we attempt to reduce the total number of $f_{S,all}$ in (13). If we look at (9) in detail, we can recognize that the equation consists of a function with two parameters, namely the sum of the bandwidths and the difference in the carrier frequencies of the two RF components. Therefore, except in the case $m = -n$ where (where '$-$' means the counterpart of the signal), $f_{S,all}$ and $f_{S_{-n,-m}}$ have exactly the same range. For instance, $f_{S_{2-,1-}} = f_{S_{1+,2+}}$ and $f_{S_{2-,1+}} = f_{S_{1-,2+}}$ in (14). That is, (14) is changed to the following equation

$$f_{S,two} = f_{S_{2-,1-}} \cap f_{S_{2-,1+}} \cap f_{S_{2-,2+}} \cap f_{S_{1-,1+}}. \tag{15}$$

Consequently, the modified version of (12) can be expressed by

$$f_{S,all} = f_{S\_N-} \cap f_{S\_(N-1)-} \cap f_{S\_(N-2)-} \cap \cdots \cap f_{S\_2-} \cap f_{S\_1-} \tag{16}$$

where

$$f_{S\_N-} = \left[\bigcap_{k=(N-1)-}^{1-} f_{S_{N-,k}} \right] \cap \left[\bigcap_{k=1+}^{N+} f_{S_{N-,k}} \right],$$

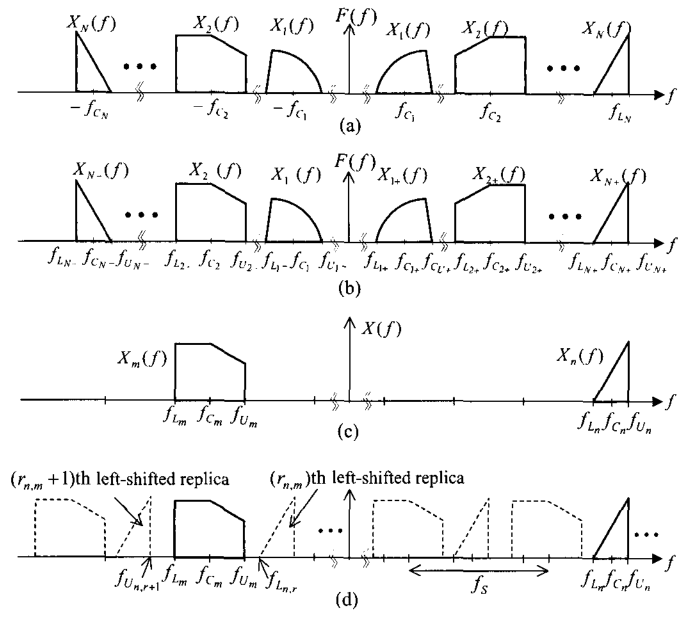

Fig. 5. (a) Spectrum of N RF signals, (b) notations of signal components for equation derivation; $2N$ spectral components, $X_j(f)$ for $j = 1\pm, 2\pm, \cdots, N\pm$, (c) an example of two RF components selected among $2N$ spectral components in (b) for equation generalization, (d) The spectrum of bandpass-sampled signals in (c).

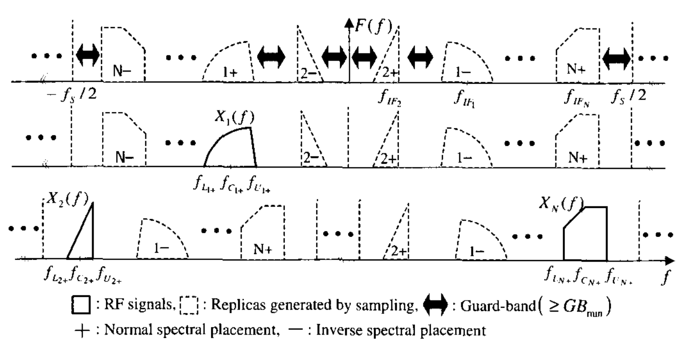

Fig. 6. An example of a signal spectrum after bandpass sampling.

$$fs\_(N-1)- = \left[\bigcap_{k=(N-2)-}^{1-} fs_{(N-1)-,k}\right] \cap \left[\bigcap_{k=1+}^{(N-1)+} fs_{(N-1)-,k}\right],$$

$$fs\_(N-2)- = \left[\bigcap_{k=(N-3)-}^{1-} fs_{(N-2)-,k}\right] \cap \left[\bigcap_{k=1+}^{(N-2)+} fs_{(N-2)-,k}\right],$$

$$fs\_2- = fs_{2-,1-} \cap \left[\bigcap_{k=1+}^{2+} fs_{2-,k}\right],$$

and

$$fs\_1- = fs_{1-,1+}.$$

Also, the number of $fs_{m,n}$ is diminished as in the following equation

$$\text{Number of } fs_{m,n} = \left\{\frac{({}_{2N}C_2) - N}{2}\right\} + N = N^2. \quad (17)$$

In above equation, the first term denotes the total number of $fs_{m,n}$ for $m \neq -n$ with removing overlapping ranges, and the second term means for $m \neq -n$. Finally, the total number simply becomes $N \times N$. Secondly, by reducing the range the FSP $r_{m,n}$ in (10), the searching task can be made more efficient. First we note that $r_{m,n}$ is directly related to the searching range corresponding to the spectral bands of only two RF components $X_m(f)$ and $X_n(f)$. Therefore, it can be reduced considerably if we preclude the FSP ranges related to only the two RF signals themselves in (10). We can thus redefine the FSP $r_{m,n}$ as $r_{b\_m,n}$ as follows

$$0 \leq r_{b\_m,n} \leq \left\lfloor \frac{f_{C_{n-m}} - (BW_{m+n}/2)}{f_{bound}} \right\rfloor \quad (18)$$

where $f_{bound} = 2(BW_1 + BW_2 + \cdots + BW_N)$, which also means the possible minimum sampling frequency. Generally, the minimum sampling frequency is always equal to or slightly greater than f_{bound}. We note that the determination of this minimum sampling frequency is dependent on the locations and bandwidths of the RF signals. Since f_{bound} is greater than BW_{m+n}, the range of $r_{b\_m,n}$ becomes smaller than that of $r_{m,n}$, resulting in a reduction of the search range. It is implied by (18) that values $fs_{m,n}$ of less than the lowest possible sampling frequency bound f_{bound} would be disregarded in calculating (16). With a smaller $r_{b\_m,n}$, the number of $fs_{m,n}$ in (9) also becomes smaller and, thus, less work is required in finding common sampling ranges.

IV. MINIMUM BANDPASS SAMPLING RATE FOR SUPPORTING A USER-SPECIFIED MINIMUM GUARD-BAND

In practical applications, it is crucial for designers to take account of the existence of a guard-band or marginal spacing between closely-located signals in the down-conversion process, in order to reduce the adverse effects caused by timing jitter and the performance variation of the components due to aging and degradation, as well to lessen the severe requirements in the component and system design. Also, it should be noted that each IF signal has one guard-band on each side and the size of the guard-band appears somewhat arbitrary because it can be determined only after placing the RF signal spectrums in the sampled bandwidth with a properly chosen sampling frequency, as shown in Fig. 6, which shows an example of a signal spectrum after bandpass sampling. To reduce such ambiguity of the guard-band, we refer to the user-defined minimum guard-band between adjacent IF signals in the sampled bandwidth as $GB_{\min}$ (Hz).

Now let us find one minimum sampling frequency to support $GB_{\min}$ between adjacent down-converted signals. To accomplish this, we use a scheme in which some bandwidth or space for the guard-band is added on both sides of each signal. That is to say, this is the same as finding the available sampling rate after redefining the signal bandwidth that becomes larger due to the insertion of the guard-band. For the derivation of the equation, two conditions should be taken into consideration, as follows; First, in the case where m and n are different RF signals, namely $m \neq -n$ (e.g., the replicas of $X_{2+}(f)$ and $X_{1-}(f)$ in Fig. 6), $GB_{\min}/2$ should be added on both sides of each signal in order to support $GB_{\min}$. By substituting BW_{m+n} with

$BW_{m+n+2GB} = BW_m + BW_n + 2 \cdot GB_{\min}$, we can obtain new modified versions of (9) and (10) as follows

$$\frac{f_{C_{n-m}} + BW_{m+n+2GB}/2}{r_{GB\_m,n} + 1} \le f_{S_{m,n}} \le \frac{f_{C_{n-m}} - BW_{m+n+2GB}/2}{r_{GB\_m,n}} \quad \text{for } m \ne -n \quad (19)$$

$$\left\lfloor \frac{f_{C_{n-m}} - BW_{m+n+2GB}/2}{2f_{GB\_bound}} \right\rfloor \le r_{GB\_m,n} \le \left\lfloor \frac{f_{C_{n-m}} - BW_{m+n+2GB}/2}{f_{GB\_bound}} \right\rfloor \text{ for } m \ne -n \quad (20)$$

where $f_{GB\_bound} = 2(BW_1 + BW_2 + \cdots + BW_N + N \cdot GB_{\min})$, which also means the possible minimum sampling frequency bound for supporting $GB_{\min}$. The maximum value of $r_{b\_m,n}$ in (18) is more reduced, as shown in (20), owing to the larger bandwidth sum caused by the insertion of the guard-band, which *results in the exhaustive searching task being relaxed*. Furthermore, the lower bound of the range in (20) can also be considerably limited by finding one minimum sampling frequency supporting the user-specified minimum guard-band. The lower bound of $r_{GB\_m,n}$ can be confined by a proper value, $2f_{GB\_bound}$, of the denominator on the left-side, as shown in (20). This value is large enough to obtain the minimum sampling rate, because the resulting minimum sampling rate is equal to or close to f_{GB_bound}. These facts will be confirmed in the discussion of the simulation results below. In addition, the range of $r_{GB\_m,n}$ can be compared with the guard-band-added form obtained from (10) as

$$0 \le r_{F\_m,n} \le \left\lfloor \frac{f_{C_{n-m}} - (BW_{m+n+2GB}/2)}{BW_{m+n+2GB}} \right\rfloor. \quad (21)$$

Hence, $r_{GB\_m,n}$ in (20) is a subset of the full range $r_{F\_m,n}$. These complexity comparisons used for obtaining the sampling rate will also be examined in Sec. V.

Secondly, if $m = -n$ for a signal, BW_{m+n} can be substituted by $BW_{m+n+4GB} (= BW_m + BW_n + 4 \cdot GB_{\min})$ instead of $BW_{m+n+2GB}$. This is because $GB_{\min}$ must be provided between the down-converted signals and the both ends of the sampled bandwidth, i.e., 0 Hz and $f_S/2$ Hz (e.g., the replicas of $X_{2-}(f)$ and $X_{2+}(f)$, or $X_{N-}(f)$ and $X_{N+}(f)$ in Fig. 6). We can thus obtain the following equations

$$\frac{f_{C_{n-m}} + (BW_{m+n+4GB}/2)}{r_{GB\_m,n} + 1} \le f_{S_{m,n}} \le \frac{f_{C_{n-m}} - (BW_{m+n+4GB}/2)}{r_{GB\_m,n}} \quad \text{for } m = -n \quad (22)$$

$$\left\lfloor \frac{f_{C_{n-m}} - (BW_{m+n+4GB}/2)}{2f_{GB\_bound}} \right\rfloor \le r_{GB\_m,n} \le \left\lfloor \frac{f_{C_{n-m}} - (BW_{m+n+4GB}/2)}{f_{GB\_bound}} \right\rfloor \text{ for } m = -n. (23)$$

Similarly, the range of (23) can be compared with

$$0 \le r_{F\_m,n} \le \left\lfloor \frac{f_{C_{n-m}} - (BW_{m+n+4GB}/2)}{BW_{m+n+4GB}} \right\rfloor. \quad (24)$$

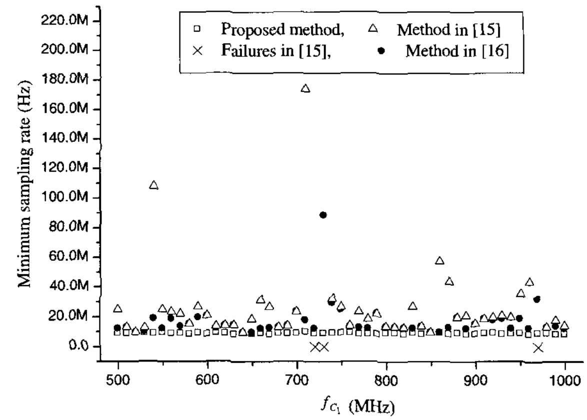

Fig. 7. Minimum sampling rates required in several BPS methods.

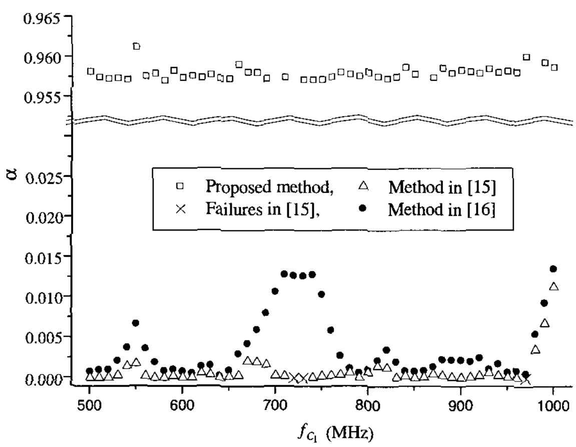

Fig. 8. α required in several BPS methods.

Based on the above descriptions, we can arrange the procedure used for obtaining the optimum minimum sampling frequency for the user-specified minimum guard-bands, as follows

1. *Specify the size of the minimum guard-band.*
2. Calculate the ranges of $r_{GB\_m,n}$ for each $f_{S_{m,n}}$ using (20) and (23).
3. Calculate the ranges of $f_{S_{m,n}}$ corresponding to every $r_{GB\_m,n}$ using (19) and (22).
4. Finally, search for the lowest common sampling frequency among the range of $f_{S_{m,n}}$, starting from f_{GB_bound} and from the largest $r_{GB\_m,n}$, namely in descending order, and iterating until a value is found that works. The last step can be described by

$$f_{S,GB} = \min \left\{ f_{S\_N-} \cap f_{S\_(N-1)-} \cap \cdots \cap f_{S\_1-} \right\}. \quad (25)$$

V. COMPARISONS AND NUMERICAL RESULTS

In this section, we discuss the performance tests of the proposed method involving several simulations. In addition, the derived formulae are verified by applying them to a practical wireless communication example currently in service.

First, in order to compare the results of the proposed method with those obtained under the limitation of one particular permutation from the previous studies in [15] and [16], we consider an example of a system receiving three signals each with different bandwidths. The parameters are assumed to be as follows:

- $X_1(f)$: f_{C_1} varying from 500 MHz to 1000 MHz at intervals of 10 MHz, $BW_1 = 0.7$ MHz,
- $X_2(f)$: $f_{C_2} = 1100$ MHz, $BW_2 = 2$ MHz
- $X_3(f)$: $f_{C_3} = 1455$ MHz, $BW_3 = 1.25$ MHz

Fig. 7 shows the comparisons of the minimum sampling rates obtained by three methods with respect to various f_{C_1}: The first one is the method described in [15], the second is the method described in [16] using $f_{S,GB\ \min}$ with $GB_{\min} = 0$ Hz, and the third is the method proposed in this paper represented by $f_{S,GB}$ with $GB_{\min} = 0$ Hz in (25). It is noticed from Fig. 7 that the proposed method consistently gives rise to lower minimum bandpass sampling rates for all ranges of f_{C_1}, while a large variation in the minimum sampling rate is observed in the other methods. This is because our method does not apply the two constraints imposed by the other methods, namely the constraint of one particular signal placement in the sampled bandwidth as in both [15] and [16], and the constraint that each replica of the RF signal is to be placed at each segment of the sampled bandwidth divided proportional to the size of the signal's bandwidth as shown in [15], respectively. Moreover, the constraints in [15] result in the failure to find the minimum sampling rate for a certain frequency, f_{C_2}, marked by 'X' in Fig. 7. It is also noticed that the larger the RF signal number, the lower the probability that the minimum sampling rates of the proposed method and other methods are equal, because the possible number of signal sets for the signal placements produced by the permutation of N RF signals is $N! \times 2^N$, as mentioned earlier. Fig. 8 shows the sum of the total valid bandpass sampling range that can be provided by the three previously mentioned methods, in terms of a parameter α defined as

$$\alpha = \frac{\text{Sum of total valid bandpass sampling ranges}}{2 f_{U_N}} \quad (26)$$

where $2 f_{U_N}$ is twice the highest frequency ($N = 3$ in Fig. 8). The total sum of the valid sampling ranges of the proposed method is remarkably large compared to the other methods, due to there being 48 permutations generated by the 3 RF signals, which means that there is either a higher probability of finding a low minimum sampling rate or more freedom in choosing acceptable sampling rates. Next, in order to demonstrate the usage of the proposed method, we consider another example of a mobile device receiving three practical wireless communication standards; a channel of GSM-900 with $f_{C_1} = 940.1$ MHz and $BW_1 = 200$ kHz, a channel of DAB (Eureka-147 L-Band) with $f_{C_2} = 1473.054$ MHz and $BW_2 = 1.536$ MHz, and a channel of WCDMA with $f_{C_3} = 2121.5$ MHz and $BW_3 = 5$ MHz. Here, let us find the minimum sampling frequency while supporting $GB_{\min} = 5$ MHz. Fig. 9(a) shows the spectrum of the bandpass filtered RF signals. First, in the case of the proposed method, 129 available sampling ranges can be found below $2 f_{U_3}$, i.e., 4248 MHz, and the total size of the valid bandpass sampling ranges is 3655 MHz. In addition, a resultant minimal sampling frequency of 61.065 MHz is obtained by (25), and each corresponding FSP $r_{GB\_m,n}$ with respect to the two RF components is also shown in Table I. However, through the removal of the same FSP ranges in Table I by using (16), the number of $f_{S_{m,n}}$ required can be reduced to 9 ($N^2 = 3^2 = 9$ from (17)), as shown in Table 2. Also, we can observe in this Table II that the ranges of $r_{GB\_m,n}$ form part of the full range, $r_{F\_m,n}$. When the minimum sampling rate is found, the corresponding FSP values are closer to the maximum values in the limited FSP ranges. Consequently, the minimum sampling rate can be quickly found with low-complexity, due to the search being started from the maximum values in each FSP range. Also, the IF of each signal can be obtained using (6) and is found to be $f_{IF_1} = 24.128$ MHz, $f_{IF_2} = 7.499$ MHz, and $f_{IF_3} = 15.767$ MHz, respectively. In Fig. 9 (b), it is also observed that the guard-band between $X_3(f)$ and $X_2(f)$ is 5 MHz, which is the value is specified in the design stage. Next, in the comparison with the results obtained under the constraint of one particular permutation, Figs. 9(c) and 9(d) display the IF spectrum obtained by the methods in [15] and [16], respectively. The minimal sampling rate 104.313 MHz can be calculated by the method in [15]. In [16], we find the sampling frequency of 103.854 MHz by setting the minimum guard-band $GB_{\min} = 5$ MHz. These two results are much larger than our result of 61.065 MHz. Even though the resultant sampling rate in the method described in [16] is smaller, it is sufficient to distinguish between adjacent channels owing to the setting of the desired guard-band in [16]. This is, as explained earlier, due to the additional constraints imposed in [15].

Table 1. Complexity comparison of the parameters for full $f_{S_{m,n}}$ in (12).

Two RF components m, n	Full range of FSP $r_{F\_m,n}$	Limited range of FSP $r_{GB\_m,n}$	FSP $r_{GB\_m,n}$ when the result of (25) is found.
3−,2−	$0 \le r_{F\_3-,2-} \le 38$	$5 \le r_{GB\_3-,2-} \le 11$	10
3−,1−	$0 \le r_{F\_3-,1-} \le 77$	$10 \le r_{GB\_3-,1-} \le 21$	19
3−,1+	$0 \le r_{F\_3-,1+} \le 200$	$28 \le r_{GB\_3-,1+-} \le 57$	50
3−,2+	$0 \le r_{F\_3-,2+} \le 216$	$28 \le r_{GB\_3-,2+} \le 67$	58
3−,3+	$0 \le r_{F\_3-,3+} \le 140$	$39 \le r_{GB\_3-,3+} \le 79$	69
2−,1−	$0 \le r_{F\_2-,1-} \le 44$	$4 \le r_{GB\_2-,1-} \le 9$	8
2−,1+	$0 \le r_{F\_2-,1+} \le 205$	$22 \le r_{GB\_2-,1+} \le 45$	39
2−,2+	$0 \le r_{F\_2-,2+} \le 127$	$27 \le r_{GB\_2-,2+} \le 54$	48
2−,3+	$0 \le r_{F\_2-,3+} \le 216$	$33 \le r_{GB\_2-,3+} \le 67$	58
1−,1+	$0 \le r_{F\_1-,1+} \le 91$	$17 \le r_{GB\_1-,1+} \le 34$	30
1−,2+	$0 \le r_{F\_1-,2+} \le 205$	$22 \le r_{GB\_1-,2+} \le 45$	39
1−,3+	$0 \le r_{F\_1-,3+} \le 200$	$28 \le r_{GB\_1-,3+} \le 57$	50
1+,2+	$0 \le r_{F\_1+,2+} \le 44$	$4 \le r_{GB\_1+,2+} \le 9$	8
1+,3+	$0 \le r_{F\_1+,3+} \le 77$	$10 \le r_{GB\_1+,3+} \le 21$	19
2+,3+	$0 \le r_{F\_2+,3+} \le 38$	$5 \le r_{GB\_2+,3+} \le 11$	10

VI. CONCLUSIONS

It has been well accepted that BPS will be one of the core elements for realizing SDR systems, owing to its ability to allow for direct down-conversion by reducing the heavy dependency on analog hardware such as mixers and filters. In this paper, we derived the equations for effectively finding the valid sampling frequency ranges when bandpass signals are simultaneously to be down-converted, which in turn leads to a minimum bandpass sampling frequency. Recognizing the importance of guard-band provisioning between the down-converted signals in the practical design procedure, the proposed searching algorithm has

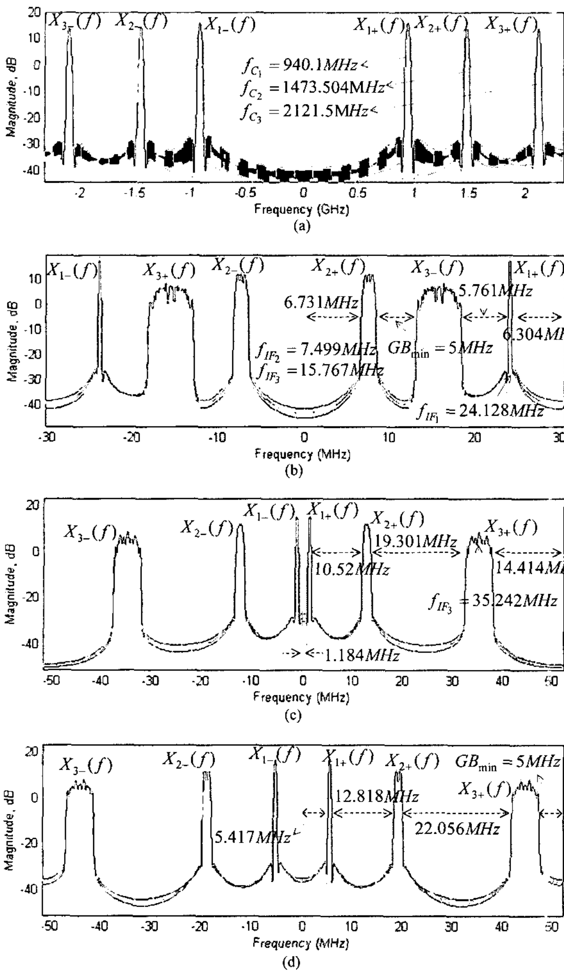

Fig. 9. Down-conversion of three standards using BPS: (a) RF spectrum before sampling, (b) IF spectrum sampled by minimum sampling rate 61.065 MHz supporting $GB_{min} = 5$ MHz using (25), (c) IF spectrum sampled by the minimum sampling rate 104.313 MHz obtained by the method in [15], and (d) IF spectrum sampled by the sampling rate 103.854 MHz obtained by the method in [16] with $GB_{\min} = 5$ MHz.

Table 2. Complexity comparison of the parameters for $f_{S_{m,n}}$ described in (16).

Two RF components m, n	Full range of FSP $r_{F\_m,n}$	Limited range of FSP $r_{GB\_m,n}$	FSP $r_{GB\_m,n}$ when the result of (25) is found.
3−,2 − (2+,3+)	$0 \le r_{F\_3-,2-} \le 38$	$5 \le r_{GB\_3-,2-} \le 11$	10
3−,1 − (1+,3+)	$0 \le r_{F\_3-,1-} \le 77$	$10 \le r_{GB\_3-,1-} \le 21$	19
3−,1 + (1−,3+)	$0 \le r_{F\_3-,1+} \le 200$	$28 \le r_{GB\_3-,1+-} \le 57$	50
3−,2 + (2−,3+)	$0 \le r_{F\_3-,2+} \le 216$	$28 \le r_{GB\_3-,2+} \le 67$	58
3−,3 +	$0 \le r_{F\_3-,3+} \le 140$	$39 \le r_{GB\_3-,3+} \le 79$	69
2−,1 − (1+,2+)	$0 \le r_{F\_2-,1-} \le 44$	$4 \le r_{GB\_2-,1-} \le 9$	8
2−,1 + (1−,2+)	$0 \le r_{F\_2-,1+} \le 205$	$22 \le r_{GB\_2-,1+} \le 45$	39
2−,2 +	$0 \le r_{F\_2-,2+} \le 127$	$27 \le r_{GB\_2-,2+} \le 54$	48
1−,1 +	$0 \le r_{F\_1-,1+} \le 91$	$17 \le r_{GB\_1-,1+} \le 34$	30

been proposed to be modified in order to take account of the user-specified guard-band. The superior performance of the proposed method was verified in simulation experiments of a practical mobile SDR application example by comparing with the other BPS frequency searching algorithms.

REFERENCES

[1] J. Mitola, "The software radio architecture," *IEEE Commun. Mag.*, pp. 26–38, May 1995.
[2] S. P. Reichhart, B. Youmans, and R. Dygert, "The software radio development system," *IEEE Pers. Commun.*, pp. 20–24, Aug. 1999.
[3] E. Buracchini, "The software radio concept," *IEEE Commun. Mag.*, pp. 138–143, Sept. 2000.
[4] T. Yokoi, Y. Iki, J. Horikoshi, K. Miwa, Y. Karasawa, A. Fukuda, T. Shiokawa, Y. Furuya, and K. Araki, "Software receiver technology and its applications," *IEICE Trans. Commun.*, vol. E83-B, no. 6, pp. 1200–1209, June 2000.
[5] W. Tuttlebee, *Software Defined Radio: Enabling Technologies*. John Wiley & Sons, 2002.
[6] T. Hentschel, M. Henker, and G. Fettweis, "The digital front-end of software radio terminals," *IEEE Pers. Commun.*, vol. 6, no. 4, pp. 40–46, Aug. 1999.
[7] H. Zamat and C. R. Nassar, "Introducing Ssoftware defined radio to 4G wireless : Necessity, advantage, and impediment," *J. Commun. Networks (JCN)*, vol. 4, no. 4, pp. 334–349, Dec. 2002.
[8] R. G. Vaughan, N. L. Scott, and D. R. White, "The theory of bandpass sampling," *IEEE Trans. Signal Process.*, vol. 39, no. 9, pp. 1973–1983, Sept. 1991.
[9] D. Akos and J. B. Y. Tusi, "Design and implementation of a direct digitization GPS receiver front end," *IEEE Trans. Microwave Theory Tech.*, vol. 44, no. 12, pp. 2334–2339, Dec. 1996.
[10] J. Liu, X. Zhou, and Y. Peng, "Spectral arrangement and other topics in first-order bandpass sampling theory," *IEEE Trans. Signal Process.*, vol. 49, no. 6, pp. 1260–1263, June 2001.
[11] D. M. Akos, M. Stockmaster, J. B. Y. Tsui, and J. Caschera, "Direct bandpass sampling of multiple distinct RF signals," *IEEE Trans. Commun.*, vol. 47, no. 7, pp. 983–988, July 1999.
[12] N. Wong and T. S. Ng, "An efficient algorithm for down-converting multiple bandpass signals using bandpass sampling," in *Proc. IEEE ICC 2001*, vol. 3, June 2001, pp. 910–914.
[13] C. H. Tseng and S. C. Chou, "Direct down-conversion of multiple RF signals using bandpass sampling," in *Proc. IEEE ICC 2003*, vol. 3, May 2003, pp. 2003–2007.
[14] C. H. Tseng and S. C. Chou, "Direct down-conversion of multiband RF signals using bandpass sampling," *IEEE Trans. Wireless Commun.*, vol. 5, no. 1, pp. 72–76, Jan. 2006.
[15] M. Choe and K. Kim, "Bandpass Ssampling algorithm with normal and inverse placements for multiple RF signals," *IEICE Trans. Commun.*, vol. E88, no. 2, pp. 754–757, Feb. 2005.
[16] J. Bae and J. Park, "An efficient algorithm for bandpass sampling of multiple RF signals," *IEEE Signal Process. Lett.*, vol. 13, no. 4, pp. 193–196, Apr. 2006.
[17] R. Qi, F. P. Coakley, and F. G. Evans, "Practical consideration for bandpass sampling," *IEE Elect. Lett.*, vol. 32, no. 20, pp. 1861–1862, Sept. 1996.

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