Adaptive Fault-Tolerant Dynamic Output Feedback Control for a Class of Linear Time-Delay Systems

Dan Ye and Guang-Hong Yang*

Abstract: This paper considers the problem of adaptive fault-tolerant guaranteed cost controller design via dynamic output feedback for a class of linear time-delay systems against actuator faults. A new variable gain controller is established, whose gains are tuned by the designed adaptive laws. More relaxed sufficient conditions are derived in terms of linear matrix inequalities (LMIs), compared with the corresponding fault-tolerant controller with fixed gains. A real application example about river pollution process is presented to show the effectiveness of the proposed method.

Keywords: Adaptive control, fault-tolerant control, linear matrix inequalities, time-delay.

1. INTRODUCTION

Faults may drastically change the system behavior ranging from performance degradation to instability. Fault tolerant control (FTC) is needed in order to reach the system objectives, or if this turns to be impossible, to assign new (achievable) objectives to avoid to catastrophic behaviors. The study of fault-tolerant control has received much attention over the past few decades [1-8]. In most of these papers, either LMI approach or adaptive method is used to design fault-tolerant controllers. Recently, in [9] and [10] new adaptive fault-tolerant H_∞ controllers are proposed, taking the individual advantages of LMI method and adaptive approach via state feedback and dynamic output feedback controllers, respectively.

Since delay phenomena are frequently encountered in mechanics, physics, applied mathematics, biology, economics and engineering systems [11,12], and time-delay is a source of instability and poor performance, considerable attention has been devoted to the study of different issues related to time-delay systems [13-20]. Many techniques are proposed to reduce the conservatism of delay-dependent criteria. Recently in [15-17], different types of Lyapunov-Krasovskii functionals and some free-weighting matrices are introduced to bring flexibility on solving LMIs. In the presence of time-delay, the design of fault-tolerant controllers becomes more complex and different. Some results about reliable state feedback control for time-delay systems can be found in the literature [21-23] based on either LMI method or adaptive approach. Recently in [24], a reliable delay-dependent H_∞ memory controller via variable gain state feedback is proposed to reduce the inherent conservativeness of fixed gain controllers. However, in many practical situations, the state information is not available. Thus there is a strong need to construct a dynamic controller satisfy practical situations and obtain a better performance and dynamical behavior of the state response [25,26]. Since the dimension of closed-loop system via dynamic output feedback is bigger and the structure of the chosen Lyapunov-Krasovkii functional for time-delay system becomes more complex, the FTC design problem via dynamic output feedback for time-delay systems becomes more challenging. As far as we know, the topic of dynamic output feedback fault-tolerant control for linear time-delay systems has received little attention.

In this paper, we deal with the dynamic output feedback fault-tolerant guaranteed cost controller design problem for a class of linear time-delay systems against actuator faults. The main contribution

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of this paper lies in that indirect adaptive method and LMI approach are combined successfully to construct a new fault-tolerant controller via dynamic output feedback for time-delay systems, whose gains are updating automatically based on the online estimated values of actuator faults. More relaxed conditions than those for the corresponding fault-tolerant dynamic output feedback controllers with fixed gains are derived to guarantee the asymptotically stability of the closed-loop system and an adequate level of performance. Finally, a real application example about river pollution process is provided to show the applicability and effectiveness of the proposed method developed in this paper.

Notation: Throughout this paper, for symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite(respectively, positive definite); I is the identity matrix with appropriate dimension. The superscript " T " represents the transpose. In a symmetric block matrix, the notation * is used to denote the submatrices lying above the diagonal.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following system with time-delay:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_1 x(t-h) + Bu(t), \\ x(t) &= \phi(t); \qquad t \in [-h,\ 0], \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $y(t) \in R^p$ is the measured output, respectively. h is a positive constant delay. $\{\phi(t), t \in [-h, 0]\}$ is a real-valued initial function. A, A_1, and B are known constant matrices of appropriate dimensions.

Since $C \in R^{p \times n}$ and rank(C)=$p_1 \leq p$ then there exists a matrix $T_c \in R^{p_1 \times p}$ such that rank$(T_c C)$=p_1. Furthermore, there exists a matrix C_{cn} such that rank$\begin{bmatrix} T_c C \\ C_{cn} \end{bmatrix}$=n. Denote $T_{cn} = \begin{bmatrix} T_c C \\ C_{cn} \end{bmatrix}^{-1}$.

The following actuator fault model from [6] is adopted in this paper to formulate the reliable control problem:

$$u_{ij}^F(t) = (1 - \rho_i^j) u_i(t), \quad 0 \leq \underline{\rho}_i^j \leq \rho_i^j \leq \overline{\rho}_i^j \leq 1, \quad i = 1 \cdots m, j = 1 \cdots L. \tag{2}$$

Let $u_{ij}^F(t)$ represent the signal from the ith actuator that has failed in the jth fault mode. Here, the index j denotes the jth fault mode and L is the total fault modes. For every fault mode, $\underline{\rho}_i^j$ and $\overline{\rho}_i^j$ represent the lower and upper bounds of unknown constant ρ_i^j, respectively. Note that, when $\underline{\rho}_i^j = \overline{\rho}_i^j = 0$, there is no fault for the ith actuator ui in the jth fault mode. When $\underline{\rho}_i^j = \overline{\rho}_i^j = 1$, the ith actuator u_i is outage in the jth fault mode. When $0 < \underline{\rho}_i^j \leq \overline{\rho}_i^j < 1$, in the jth fault mode the type of actuator faults is loss of effectiveness.

Denote

$$u_j^F(t) = [\ u_{1j}^F(t),\ u_{2j}^F(t), \cdots\ u_{mj}^F(t)]^T = (1 - \rho^j) u(t),$$

where $\rho^j = diag[\rho_1^j, \rho_2^j, \cdots \rho_m^j]$, $j = 1 \cdots L$. Considering the lower and upper bounds the following set can be defined

$$N_{\rho^j} = \{\rho^j \mid \rho^j = \text{diag}\{\rho_1^j, \rho_2^j, \cdots \rho_m^j\}, \quad \rho_i^j = \underline{\rho}_i^j \text{ or } \rho_i^j = \overline{\rho}_i^j\}.$$

Thus, the set N_{ρ^j} contains a maximum of 2^m elements.

For convenience in the following sections, for all possible fault modes L, we use a uniform actuator fault model

$$u^F(t) = (1 - \rho) u(t), \rho \in \{\rho^1, \rho^2, \cdots \rho^L\} \tag{3}$$

and ρ can be described by $\rho \in \text{diag}\{\rho_1, \rho_2, \cdots \rho_m\}$.

The traditional dynamic output feedback controller with fixed gains is

$$\begin{aligned} \dot{\xi}_f(t) &= A_{Kf} \xi_f(t) + B_{Kf} y(t), \\ u^F(t) &= (I - \rho) C_{Kf} \xi_f(t), \end{aligned} \tag{4}$$

where $\xi_f(t) \in R^n$ is the controller state, A_{Kf} B_{Kf} and C_{Kf} are the controller gains to be designed.

Combing controller (4) with system (1), we have

$$\dot{\overline{x}}_f(t) = \overline{A}_f \overline{x}_f(t) + \overline{A}_{1f} \overline{x}_f(t - h), \tag{5}$$

where $\overline{x}_f(t) = [x^T(t),\ \xi_f^T(t)]^T$,

$$\overline{A}_f = \begin{bmatrix} A & B(I - \rho) C_{Kf} \\ B_{Kf} C & A_{Kf} \end{bmatrix}, \quad \overline{A}_{1f} = \begin{bmatrix} A_1 & 0 \\ 0 & 0 \end{bmatrix}.$$

In order to stabilize system (1) and improve performance of systems, in this paper we introduce the following dynamic output feedback controller with time-varying gains to reduce the conservativeness inherent in the corresponding controller with fixed gains (4)

$$\dot{\xi}(t) = A_K(\hat{\rho})\xi(t) + B_K(\hat{\rho})y(t),$$
$$u^F(t) = (I - \rho)C_{K0}\xi(t), \quad (6)$$

where $\xi(t) \in R^n$ is the controller state, $\hat{\rho}(t)$ is the estimated value of ρ obtained by the adaptive laws, which are determined later.

$$A_K(\hat{\rho}) = A_{K0} + A_{Ka}(\hat{\rho}) + A_{Kb}(\hat{\rho}),$$
$$B_K(\hat{\rho}) = B_{K0} + B_{Ka}(\hat{\rho}) + B_{Kb}(\hat{\rho})$$

with

$$A_{Ka}(\hat{\rho}) = \sum_{i=1}^{m} \hat{\rho}_i A_{Kai},$$
$$A_{Kb}(\hat{\rho}) = \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\rho}_i\hat{\rho}_j A_{Kbij} + \sum_{i=1}^{m} \hat{\rho}_i A_{Kbi}$$
$$B_{Ka}(\hat{\rho}) = \sum_{i=1}^{m} \hat{\rho}_i B_{Kai}, \quad B_{Kb}(\hat{\rho}) = \sum_{i=1}^{m} \hat{\rho}_i B_{Kbi},$$

where $A_{K0}, A_{Kai}, A_{Kbi}, A_{Kbij}, B_{K0}, B_{Kai}, B_{Kbi}$, and C_{K0} are gain matrices with appropriate dimensions to be determined later.

Applying this controller (6) to (1), results in the following closed-loop system

$$\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{A}_1\bar{x}(t-h), \quad (7)$$

where

$$\bar{x}(t) = [x^T(t),\ \xi^T(t)]^T,$$
$$\bar{A}_f = \begin{bmatrix} A & B(I-\rho)C_{K0} \\ B_K(\hat{\rho})C & A_K(\hat{\rho}) \end{bmatrix},$$
$$\bar{A}_1 = \begin{bmatrix} A_1 & 0 \\ 0 & 0 \end{bmatrix}.$$

Given positive definite symmetric matrices Q and S, the following cost function is considered in this paper.

$$J = \int_0^{\infty} (x^T(t)Qx(t) + (u^F)^T Su^F(t)dt) \quad (8)$$

Definitions 1: Consider linear time-delay system (1). If there exist a controller of form (6) and a positive scalar μ such that for both normal and faulty cases, the closed-loop system (7) is asymptotically stable and the closed-loop value of cost function (8) satisfies $J \le \mu$ then μ is said to be a guaranteed cost and the controller (6) is said to be an adaptive reliable guaranteed cost dynamic output feedback controller.

The objective of this paper is to develop a procedure to design an adaptive reliable guaranteed cost dynamic output feedback controller such that, in normal and faulty cases, the resultant closed-loop system is asymptotically stable and give an upper bound for the cost function (8).

Before proceeding further, we introduce the following lemmas, which are essential for the development of our results.

Denote $\Delta_v = \{\delta = (\delta_1 \cdots \delta_N) : \delta_i \in \{\underline{\delta}_i, \bar{\delta}_i\}\}$, where δ_i $(i = 1\cdots N)$ is an unknown constant $\underline{\delta}_i$ and are $\bar{\delta}_i$ the known lower and upper bounds of δ_i respectively.

Lemma 1 [9]: If there exists a symmetric matrix Θ with

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{12}^T & \Theta_{22} \end{bmatrix}$$

and $\Theta_{11},\ \Theta_{22} \in R^{Nn\times Nn}$ such that the following inequalities hold:

$$\Theta_{22ii} \le 0, \quad i = 1\cdots N$$

for $\delta \in \Delta_v$

$$\Theta_{11} + \Theta_{12}\Delta(\delta) + (\Theta_{12}\Delta(\delta))^T + \Delta(\delta)\Theta_{22}\Delta(\delta) \ge 0,$$
$$\begin{bmatrix} Q_0 & E \\ E^T & F \end{bmatrix} + U^TU + G^T\Theta G < 0, \quad (9)$$

then for all $\delta_i \in [\underline{\delta}_i, \bar{\delta}_i]$

$$W(\delta) = Q_0 + \sum_{i=1}^{N}\delta_i E_i + (\sum_{i=1}^{N}\delta_i E_i)^T + \sum_{i=1}^{N}\sum_{j=1}^{N}\delta_i\delta_j F_{ij}$$
$$+ (U_0 + \sum_{i=1}^{N}\delta_i U_i)^T(U_0 + \sum_{i=1}^{N}\delta_i U_i) < 0, \quad (10)$$

where $Q_0 = Q_0^T$ and

$$F_{ij} = F_{ij}^T, \Delta(\delta) = diag\{\delta_1 I \cdots \delta_N I\},$$
$$E = [E_1\ E_2 \cdots E_N], U = [U_1\ U_2 \cdots U_n],$$
$$F = \begin{bmatrix} F_{11} & F_{12} & \cdots & F_{1N} \\ F_{21} & F_{22} & \cdots & F_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ F_{N1} & F_{N2} & \cdots & F_{NN} \end{bmatrix}, \quad G = \begin{bmatrix} \begin{bmatrix} I \\ \vdots \\ I \end{bmatrix} & 0 \\ 0 & I \end{bmatrix}.$$

Lemma 2 [1]: For given positive scalar h and any $\bar{A}_1 \in R^{n\times n}$, the operator $D(\bar{x}_t) : C_0 \to R^n$ defined by

$$D(\bar{x}_t) = \bar{x}(t) + \int_{t-h}^{t} \bar{A}_1 \bar{x}(s)ds \tag{11}$$

is stable if there exist a positive definite matrix Γ and a scalar $0 < \alpha_1 < 1$ such that

$$\begin{bmatrix} -\alpha_1 \Gamma & h\bar{A}_1^T \Gamma \\ * & -\Gamma \end{bmatrix} < 0. \tag{12}$$

Lemma 3 [1]: For any constant matrix $M \in R^{n\times n}$, $M = M^T$, scalar vecto $\gamma > 0$, function $v:[0,\gamma] \to R^n$ such that the integrations concerned are well defined, then

$$\left(\int_0^{\gamma} v(s)ds\right)^T M\left(\int_0^{\gamma} v(s)ds\right) \le \gamma\left(\int_0^{\gamma} v^T(s)Mv(s)ds\right). \tag{13}$$

Lemma 4: Consider the closed-loop system described by (5). Then the following statements are equivalent:

(i) there exist a symmetric matrix $P_a > 0$, $R = \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} > 0$ and a controller described by (4) such that for all $\rho \in \{\rho^1, \rho^2, \cdots \rho^L\}$, $\rho^j \in N_{\rho^j}$

$$\begin{bmatrix} \Omega_0 + \Omega_0^T + h\bar{A}_{1f}^T R \bar{A}_{1f} + \Phi_0 & h(\bar{A}_f + \bar{A}_{1f})^T P_a \\ * & -hR \end{bmatrix} < 0, \tag{14}$$

where $\Omega_0 = P_a(\bar{A}_f + \bar{A}_{1f})$ and

$$\Phi_0 = \begin{bmatrix} Q & 0 \\ 0 & C_{Kf}^T (I-\rho)S(I-\rho)C_{Kf} \end{bmatrix}.$$

(ii) there exist symmetric matrices N_1 and Y_1 with $0 < N_1 < Y_1$, $R = \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} > 0$ and a controller described by (4) with $A_{Kf} = A_{Ke0}$, $B_{Kf} = B_{Ke0}$, $C_{Kf} = C_{Ke0}$ such that for all $\rho \in \{\rho^1, \rho^2, \cdots \rho^L\}$, $\rho^j \in N_{\rho^j}$

$$V_{a0} = \begin{bmatrix} \Lambda_0 & \Lambda_1 & \Lambda_3 & \Lambda_5 \\ * & \Lambda_2 & \Lambda_4 & \Lambda_6 \\ * & * & -hR_{11} & -hR_{12} \\ * & * & * & -hR_{22} \end{bmatrix} < 0, \tag{15}$$

where

$$\Lambda_1 = Y_1 B(I-\rho)(I-\rho) - N_1 A_{Ke0} + (-N_1(A+A_1) + N_1 B_{Ke0} C)^T,$$

$$\Lambda_2 = -N_1 B(I-\rho)C_{Ke0} + N_1 A_{Ke0} + (-N_1 B(I-\rho)C_{Ke0} + N_1 A_{Ke0})^T + C_{Ke0}^T (I-\rho)S(I-\rho)C_{Ke0},$$

$$\Lambda_3 = -h(A+A_1)^T Y_1 - hC^T B_{Ke0} N_1,$$

$$\Lambda_4 = hC_{Ke0}^T (I-\rho)B^T Y_1 - hA_{Ke0}^T N_1,$$

$$\Lambda_5 = -h(A+A_1)^T N_1 + hC^T B_{Ke0} N_1,$$

$$\Lambda_6 = -hC_{Ke0}^T (I-\rho)B^T N_1 + A_{Ke0}^T N_1.$$

Proof: (14)$\Leftrightarrow$(15). (14) holds for is equivalent to that there exists

$$P_a = \begin{bmatrix} P_{11} & P_{12}^T \\ P_{12} & P_{22} \end{bmatrix} \tag{16}$$

with $P_{11} \in R^{n\times n}$ and P_{12} nonsingular such that

$$\begin{bmatrix} \Omega_0 + \Omega_0^T + h\bar{A}_{1f}^T R \bar{A}_{1f} + \Phi_0 & h(\bar{A}_f + \bar{A}_{1f})^T P_a \\ * & -hR \end{bmatrix} < 0. \tag{17}$$

Let $A_{Ke0} = (P_{12}^{-1})^T P_{22} A_{Kf} P_{22}^{-1} P_{12}^T$, $B_{Ke0} = -(P_{12}^{-1})^T P_{22} B_{Kf}$, $C_{Ke0} = -C_{Kf} P_{22}^{-1} P_{12}^T$, $Y_1 = P_{11}$ and $N_1 = P_{12} P_{22}^{-1} P_{12}^T$. Then

$$\begin{aligned} P &= \begin{bmatrix} I & 0 \\ 0 & -P_{12}P_{22}^{-1} \end{bmatrix} P_a \begin{bmatrix} I & 0 \\ 0 & -P_{12}P_{22}^{-1} \end{bmatrix}^T \\ &= \begin{bmatrix} Y_1 & -N_1 \\ -N_1 & N_1 \end{bmatrix}, \end{aligned} \tag{18}$$

$P > 0$ is equivalent to $0 < N_1 < Y_1$, and (17) is equivalent to

$$V_{a1} = \begin{bmatrix} \Omega_1 + \Omega_1^T + h\bar{A}_{1e}^T R \bar{A}_{1e} + \Phi_1 & h(\bar{A}_e + \bar{A}_{1e})^T P \\ * & -hR \end{bmatrix} < 0, \tag{19}$$

where $\Omega_1 = P(\bar{A}_e + \bar{A}_{1e})$,

$$\Phi_1 = \begin{bmatrix} Q & 0 \\ 0 & C_{Ke0}^T (I-\rho)S(I-\rho)C_{Ke0} \end{bmatrix},$$

$$\bar{A}_e = \begin{bmatrix} A & B(I-\rho)C_{Ke0} \\ B_{Ke0}C & A_{Ke0} \end{bmatrix}, \quad \bar{A}_{1e} = \begin{bmatrix} A_1 & 0 \\ 0 & 0 \end{bmatrix}.$$

By (15), it follows $V_{a1} = V_{a0}$. The proof is completed.□

3. MAIN RESULTS

In this section, the solvability of the adaptive fault-tolerant control problem via dynamic output feedback in the framework of LMI and adaptive laws for the linear time-delay model (1) is studied.

Define an operator $D(\overline{x}_t): C_{n,h} \to R^n$ as

$$D(\overline{x}_t) = \overline{x}(t) + \int_{t-h}^{t} \overline{A}_1 \overline{x}(s)ds, \tag{20}$$

where $\overline{x}_t = \overline{x}(t+s), s \in [-h, 0]$

Theorem 1: Suppose that there exist Γ, $\alpha_1 > 0$ and $\alpha_2 > 0$ satisfying (12). If there exist a controller of form (6), matrices $0 < N_1 < Y_1$, R_{12}, $R_{11} > 0$, $R_{22} > 0$, $A_{K0}, A_{Kai}, A_{Kbi}, A_{Kbij}$, $B_{K0}, B_{Kai}, B_{Kbi}, C_{K0}$, $i, j = 1 \cdots m$ and a symmetric matrix Θ with

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{12}^T & \Theta_{22} \end{bmatrix}$$

and Θ_{11} $\Theta_{22} \in R^{4mn \times 4mn}$ such that the following inequalities hold:

$$\Theta_{22ii} \le 0, \quad i = 1 \cdots m$$

for $\hat{\rho} \in \Delta_{\hat{\rho}}$

$$\Theta_{11} + \Theta_{12}\Delta(\delta) + (\Theta_{12}\Delta(\delta))^T + \Delta(\delta)\Theta_{22}\Delta(\delta) \ge 0$$

for all $\rho \in \{\rho^1, \rho^2, \cdots \rho^L\}$, $\rho^j \in N_{\rho^j}$

$$\begin{bmatrix} Q_1 & E \\ E^T & F \end{bmatrix} + U^T U + G^T \Theta G < 0, \tag{21}$$

where

$$Q_1 = \begin{bmatrix} \Delta_0 & \Delta_1 & h\Delta_2 & h\Delta_5 \\ * & \Delta_3 & h\Delta_4 & h\Delta_6 \\ * & * & -hR_{11} & -hR_{12} \\ * & * & * & -hR_{22} \end{bmatrix},$$

$$E = [E_1 \; E_2 \cdots E_m],$$

$$F = [F_{ij}], \quad i, j = 1 \cdots m,$$

$$E_i = \begin{bmatrix} -N_1 B_{Kbi} C - N_1 B_{Kai} C & \Delta_7 & \Delta_8 & -\Delta_8 \\ N_1 B_{Kbi} C + N_1 B_{Kai} C M_2 & N_1 A_{Kbi} & \Delta_9 & -\Delta_9 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$F_{ij} = \begin{bmatrix} 0 & -N_1 A_{Kbij} & 0 & 0 \\ -A_{Kbij}^T N_1 & N_1 A_{Kbij} + (N_1 A_{Kbij})^T & -hA_{Kbij}^T N_1 & hA_{Kbij}^T N_1 \\ 0 & -hN_1 A_{Kbij} & 0 & 0 \\ 0 & hN_1 A_{Kbij} & 0 & 0 \end{bmatrix},$$

$$\Delta_0 = Y_1(A + A_1) - N_1 B_{K0} C + (Y_1(A + A_1) - N_1 B_{K0} C)^T + Q + hA_1^T R_{11} A_1,$$

$$\Delta_1 = Y_1 B(I - \rho)C_{K0} - N_1 A_{K0} - N_1 A_{Ka}(\rho) + M_2^T N_1 A_{Ka}(\rho) - M_2^T C^T B_{Ka}(\rho) N_1 + [-N_1(A + A_1) + N_1 B_{K0} C + N_1 B_{Ka}(\rho) C]^T,$$

$$\Delta_2 = (A + A_1)^T Y_1 - C^T B_{K0}^T N_1,$$

$$\Delta_3 = -N_1 B(I - \rho)C_{K0} + (-N_1 B(I - \rho)C_{K0})^T + N_1 A_{K0} + N_1 A_{Ka}(\rho) + [N_1 A_{K0} + N_1 A_{Ka}(\rho)]^T + C_{K0}^T (I - \rho) S (I - \rho) C_{K0},$$

$$\Delta_4 = C_{K0}^T (I - \rho) B^T Y_1 - A_{K0}^T N_1,$$

$$\Delta_5 = -(A + A_1)^T N_1 + C^T B_{K0}^T N_1,$$

$$\Delta_6 = -C_{K0}^T (I - \rho) B^T N_1 + A_{K0}^T N_1,$$

$$\Delta_7 = -N_1 A_{Kbi} - M_2^T N_1 A_{Kai},$$

$$\Delta_8 = -hC^T [B_{Kai} + B_{Kbi}]^T N_1,$$

$$\Delta_9 = -h[A_{Kai} + A_{Kbi}]^T N_1,$$

$$\Delta(\hat{\rho}) = \mathrm{diag}[\hat{\rho}_1 I \cdots \hat{\rho}_m I],$$

$$M_1 = T_{cn} \begin{bmatrix} T_c \\ 0 \end{bmatrix}, \; M_2 = T_{cn} \begin{bmatrix} 0 \\ C_{cn} \end{bmatrix},$$

and also $\hat{\rho}_i(t)$ is determined according to the adaptive law

$$\dot{\hat{\rho}}_i(t) = \mathrm{Proj}_{[\min_j \{\underline{\rho}_i^j\}, \, \max_j \{\overline{\rho}_i^j\}]} \{L_{2i}\}$$
$$= \begin{cases} 0, \text{ if} & \hat{\rho}_i = \min_j \{\underline{\rho}_i^j\} \text{ and } L_{2i} \le 0 \\ & \text{or } \hat{\rho}_i = \max_j \{\overline{\rho}_i^j\} \text{ and } L_{2i} \ge 0; \\ L_{2i} & \text{otherwise,} \end{cases} \tag{22}$$

where

$$L_{2i} = -l_i[\xi^T N_1 A_{Kai} \xi - y^T M_1^T A_{Kai} \xi + \xi^T N_1 B_{Kai} C M_1 y],$$

and $l_i \ge 0$ $(i = 1 \cdots m)$ is the adaptive law gain to be chosen according to practical applications. Proj{·} denotes the projection operator [27], whose role is to project the estimates $\hat{\rho}_i(t)$ to the interval $[\min_j \{\underline{\rho}_i^j\}, \max_j \{\overline{\rho}_i^j\}]$. Then the closed-loop system

(7) is asymptotically stable and the cost function (8) satisfies the following bound:

$$J \le D^T(0)PD(0) + h\int_{-h}^{0}(s+h)\bar{x}^T(s)A_1^T RA_1\bar{x}(s)ds + \sum_{i=1}^{m}\frac{\tilde{\rho}_i^2(0)}{l_i} \quad (23)$$

with $R = \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix}$.

Proof: Take Lyapunov-Krasovkii functional as

$$V = V_1 + V_2 + V_3, \quad (24)$$

where

$$V_1 = D^T(\bar{x}_t)PD(\bar{x}_t),\ V_3 = \sum_{i=1}^{m}\frac{\tilde{\rho}_i^2(0)}{l_i},$$

$$V_2 = \int_{t-h}^{t}(s-t+h)\bar{x}^T(s)\bar{A}_1^T R\bar{A}_1\bar{x}(s)ds$$

with $P > 0,\ R > 0.$

From the derivative of V along the closed-loop system (7}), it follows

$$\begin{aligned}\dot{V}_1 &= 2D^T(\bar{x}_t)P(\bar{A}+\bar{A}_1)\bar{x}(t) \\ &= \bar{x}^T(t)[P(\bar{A}+\bar{A}_1)+(\bar{A}+\bar{A}_1)^T P]\bar{x}(t) \\ &\quad + 2(\int_{t-h}^{t}\bar{A}_1\bar{x}(s)ds)^T P(\bar{A}+\bar{A}_1)\bar{x}(t),\end{aligned}$$

$$\begin{aligned}\dot{V}_2 &\le h\bar{x}^T(t)\bar{A}_1^T R\bar{A}_1\bar{x}(t) \\ &\quad -(\int_{t-h}^{t}\bar{A}_1\bar{x}(s)ds)^T h^{-1}R(\int_{t-h}^{t}\bar{A}_1\bar{x}(s)ds),\end{aligned}$$

$$\dot{V}_3 = \sum_{i=1}^{m}\frac{\tilde{\rho}_i(t)\dot{\hat{\rho}}_i(t)}{l_i},$$

where Lemma 3 is used to get $\dot{V}_2$.

Here, the following equalities are obtained by using

$$\tilde{\rho}_i(t) = \hat{\rho}_i(t) - \rho_i,$$
$$A_{Ka}(\tilde{\rho}) = A_{Ka}(\rho) + A_{Ka}(\hat{\rho}),$$
$$B_{Ka}(\tilde{\rho}) = B_{Ka}(\rho) + B_{Ka}(\hat{\rho}).$$

Then $\bar{A}$ can be written as $\bar{A} = \bar{A}_a + \bar{A}_b$, where

$$\bar{A}_a = \begin{bmatrix} A & B(I-\rho)C_{K0} \\ [B_{K0}+B_{Ka}(\rho)+B_{Kb}(\hat{\rho})]C & A_{K0}+A_{Ka}(\rho)+A_{Kb}(\hat{\rho}) \end{bmatrix},$$

$$\bar{A}_b = \begin{bmatrix} 0 & 0 \\ B_{Ka}(\tilde{\rho})C & A_{Ka}(\tilde{\rho}) \end{bmatrix}.$$

Let P is the following form, that is

$$P = \begin{bmatrix} Y_1 & -N_1 \\ -N_1 & N_1 \end{bmatrix} \quad (25)$$

with $0 < N_1 < Y_1$, which implies $P > 0$.
From (1), it follows $T_c Cx = T_c y$. Then

$$x = T_{cn}\begin{bmatrix} T_c Cx \\ C_{cn}x \end{bmatrix} = M_1 y + M_2 y \quad (26)$$

with $M_1 = T_{cn}\begin{bmatrix} T_c \\ 0 \end{bmatrix}$, $M_2 = T_{cn}\begin{bmatrix} 0 \\ C_{cn} \end{bmatrix}$.

Notice that

$$P\bar{A}_a = \begin{bmatrix} Y_1 A - N_1[B_{K0}+B_{Ka}(\rho)+B_{Kb}(\hat{\rho})]C & T_1 \\ -N_1 A + N_1[B_{K0}+B_{Ka}(\rho)+B_{Kb}(\hat{\rho})]C & T_2 \end{bmatrix}$$

with

$$T_1 = Y_1 B(I-\rho)C_{K0} - N_1[A_{K0}+A_{Ka}(\rho)+A_{Kb}(\hat{\rho})],$$
$$T_2 = -N_1 B(I-\rho)C_{K0} + N_1[A_{K0}+A_{Ka}(\rho)+A_{Kb}(\hat{\rho})],$$

and

$$P\bar{A}_b = \begin{bmatrix} -N_1 B_{Ka}(\tilde{\rho})C & -N_1 A_{Ka}(\tilde{\rho}) \\ N_1 B_{Ka}(\tilde{\rho})C & N_1 A_{Ka}(\tilde{\rho}) \end{bmatrix},$$

which follows

$$\begin{aligned}\bar{x}^T(t)P\bar{A}_b\bar{x}(t) &= -x^T N_1 B_{Ka}(\tilde{\rho})Cx - x^T N_1 A_{Ka}(\tilde{\rho})\xi \\ &\quad + \xi^T N_1 B_{Ka}(\tilde{\rho})Cx + \xi^T N_1 A_{Ka}(\tilde{\rho})\xi.\end{aligned} \quad (27)$$

Thus, by (26) it is easy to see

$$\begin{aligned}-x^T N_1 A_{Ka}(\tilde{\rho})\xi &= -y^T M_1^T N_1 A_{Ka}(\tilde{\rho})\xi \\ &\quad - x^T M_2^T N_1 A_{Ka}(\tilde{\rho})\xi,\end{aligned}$$

$$\begin{aligned}\xi^T N_1 B_{Ka}(\tilde{\rho})Cx &= \xi^T N_1 B_{Ka}(\tilde{\rho})CM_1 y \\ &\quad + \xi^T N_1 B_{Ka}(\tilde{\rho})CM_2 x.\end{aligned}$$

So $\bar{x}^T(t)P\bar{A}_b\bar{x}(t) = \bar{x}^T(t)M_a\bar{x}(t) + M_b$, where

$$M_a = \begin{bmatrix} -N_1 B_{Ka}(\tilde{\rho})C & -M_2^T N_1 A_{Ka}(\tilde{\rho})\xi \\ N_1 B_{Ka}(\tilde{\rho})CM_2 & 0 \end{bmatrix},$$

$$\begin{aligned}M_b &= -y^T M_1^T N_1 A_{Ka}(\tilde{\rho})\xi + \xi^T N_1 B_{Ka}(\tilde{\rho})CM_1 y \\ &\quad + \xi^T N_1 A_{Ka}(\tilde{\rho})\xi.\end{aligned}$$

Then from the derivative of V(t) along the closed-loop system (7), it follows

$$\dot{V}_1 = \bar{x}^T(t)[P(\bar{A}_a+\bar{A}_1)+(\bar{A}_a+\bar{A}_1)^T P]\bar{x}(t) \quad (28)$$

$$+\bar{x}^T(t)(M_a + M_a^T)\bar{x}(t) + 2M_b$$
$$+2(\int_{t-h}^{t} \bar{A}_1\bar{x}(s)ds)^T P(\bar{A} + \bar{A}_1)\bar{x}(t).$$

So

$$\dot{V}(t) \le \chi^T W_0 \chi + 2M_b + \sum_{i=1}^{m} \frac{\tilde{\rho}_i(t)\dot{\hat{\rho}}_i(t)}{l_i}, \quad (29)$$

where

$$\chi = \begin{bmatrix} \bar{x}(t) \\ \int_{t-h}^{t} \bar{A}_1\bar{x}(s)ds \end{bmatrix},$$

$$W_0 = \begin{bmatrix} \Phi + \Phi^T + h\bar{A}_1^T R\bar{A}_1 & (\bar{A} + \bar{A}_1)^T P \\ * & -h^{-1}R \end{bmatrix}$$

with $\Phi = P(\bar{A}_a + \bar{A}_1) + M_a$.

Since y and ξ are available on line, we choose the adaptive laws as (22). Then it follows

$$M_b + \sum_{i=1}^{m} \frac{\tilde{\rho}_i(t)\dot{\hat{\rho}}_i(t)}{l_i} \le 0. \quad (30)$$

Thus

$$\dot{V}(t) \le \chi^T W_0 \chi. \quad (31)$$

Furthermore,

$$J \le \int_0^\infty \left(\bar{x}^T(t)\Psi\bar{x}^T(t) + \dot{V}\right)dt + V(0)$$
$$\le \int_0^\infty \chi^T W_1 \chi dt + V(0), \quad (32)$$

where

$$\Psi = \begin{bmatrix} Q & 0 \\ 0 & C_{K0}^T(I-\rho)S(I-\rho)C_{K0} \end{bmatrix},$$

$$W_1 = \begin{bmatrix} \Phi + \Phi^T + h\bar{A}_1^T R A_1 + \Psi & (\bar{A} + \bar{A}_1)^T P \\ * & -h^{-1}R \end{bmatrix}.$$

By pre-and-post multiplying inequalities $W_1 < 0$ by $\mathrm{diag}\{I, h\}$ then $W_1 < 0$ is equivalent to

$$W_2 = \begin{bmatrix} \Phi + \Phi^T + h\bar{A}_1^T R A_1 + \Psi & h(\bar{A} + \bar{A}_1)^T P \\ * & -hR \end{bmatrix} < 0. \quad (33)$$

Furthermore (33) can be described by

$$W_2(\hat{\rho}) = Q_1 + \sum_{i=1}^{m} \hat{\rho}_i E_i + (\sum_{i=1}^{m} \hat{\rho}_i E_i)^T + \sum_{i=1}^{m}\sum_{j=1}^{m} \hat{\rho}_i \hat{\rho}_j F_{ij} < 0,$$

where Q_1, E_i, F_{ij} are defined in (21). By Lemma 1, we can get $W_2(\hat{\rho}) < 0$ if (21) holds, which implies $W_1 < 0$ and $W_0 < 0$. Then the closed-loop system (7) is asymptotically stable in both normal and faulty cases. Moreover

$$J \le D^T(0)PD(0) + h\int_{-h}^{0} (s+h)\bar{x}^T \bar{A}_1^T R\bar{A}_1\bar{x}(s)ds$$
$$+\sum_{i=1}^{m} \frac{\tilde{\rho}_i^2(0)}{l_i}. \quad \square$$

Remark 2: Theorem 1 presents sufficient conditions for adaptive fault-tolerant guaranteed cost controller design via dynamic output feedback. Generally, (21) is not LMIs. But when C_{K0} is given, $N_1A_{K0}, N_1A_{Kai}, N_1A_{Kbi}, N_1A_{Kbij}, N_1B_{K0}, N_1B_{Kai}$ and N_1B_{Kbi} are defined as new variables, (21) becomes LMIs and linearly depends on uncertain parameters ρ and $\hat{\rho}$.

Remark 3: By (2) and (22), it follows that $\tilde{\rho}_i(0) \le \max_j\{\bar{\rho}_i^j\} - \min_j\{\underline{\rho}_i^j\}$. We can choose l_i relatively large so that $\sum_{i=1}^{m} \frac{\tilde{\rho}_i^2(0)}{l_i}$ is sufficiently small.

Theorem 2: Consider the closed-loop system (7) with cost function (8). If the following optimization problem

$$\min\{\alpha + tr(\Gamma_1)\} \text{ subject to}$$

(i) LMI (12), (21)

(ii) $\begin{bmatrix} -\alpha & D^T(0)P \\ * & -P \end{bmatrix} < 0$

(iii) $\begin{bmatrix} -\Gamma_1 & hV_0^T\bar{A}_1^T R \\ * & -hR \end{bmatrix} < 0 \quad (34)$

has a solution set, the controller (6) ensures the minimization of the guaranteed cost (8) for the closed-loop system (7) against actuator faults, where $\int_{-h}^{0}(s+h)\bar{x}(s)\bar{x}^T(s)ds = V_0V_0^T$.

Proof: By Theorem 1, (i) in (34) is clear. Also, it follows form the Schur complement that (ii) and (iii) in (34) are equivalent to $D^T(0)PD(0) < \alpha$ and $hV_0^T\bar{A}_1^T R\bar{A}_1V_0 \le \Gamma_1$, respectively. On the other hand

$$\int_{-h}^{0}(s+h)\bar{x}^T(s)\bar{A}_1^T R\bar{A}_1\bar{x}(s)ds$$
$$= \int_{-h}^{0} \mathrm{tr}((s+h)\bar{x}^T(s)\bar{A}_1^T R\bar{A}_1\bar{x}(s))ds$$
$$= \mathrm{tr}(V_0^T\bar{A}_1^T R\bar{A}_1V_0) < tr(\Gamma_1).$$

Hence, it follows from (32) that

$$J^* < \alpha + \mathrm{tr}(\Gamma_1) + \sum_{i=1}^{m} \frac{\tilde{\rho}_i^2(0)}{l_i}.$$

Thus, the minimization of $\alpha + \mathrm{tr}(\Gamma_1)$ implies the minimization of the guaranteed cost for the system (7).

Remark 4: It should be noted that a matrix C_{cn} satisfying rank $\begin{bmatrix} T_c C \\ C_{cn} \end{bmatrix} = n$ is not unique in general, which can be used to regulate C_{cn} for obtaining better performance in adaptive fault-tolerant guaranteed control design.

Remark 5: If we choose the Lyapunov functional candidate $V = V_1 + V_2$, where V_1 are V_2 defined in (24), then it is easy to see conditions (14) can guarantee the closed-loop system (6) is asymptotically stable and the cost function (8) satisfied the following bound:

$$J \le D^T(0)PD(0) + h\int_{-h}^{0} (s+h)\bar{x}^T(s)\bar{A}_1^T R\bar{A}_1\bar{x}(s)ds.$$

From Lemma 4, it follows condition (14) is equivalent to (15). That is, there is no conservativeness brought by the chosen special structure $P = \begin{bmatrix} Y_1 & -N_1 \\ -N_1 & N_1 \end{bmatrix}$ when we deal with the design problem of dynamic output feedback controllers with fixed gains. It should be noted that conditions (15) also are not convex. But when C_{Ke0} is given, $N_1 A_{Ke0}$ and $N_1 B_{Ke0}$ are defined as new variables, they become LMIs. Also the upper bound of J with fixed gains controller can be obtained by solving the following optimization:

$$\min\{\alpha + \mathrm{tr}(\Gamma_1)\}$$
(i) LMI (12) and (15)
$$\text{(ii)} \quad \begin{bmatrix} -\alpha & D^T(0)P \\ * & -P \end{bmatrix} < 0$$
$$\text{(iii)} \quad \begin{bmatrix} -\Gamma_1 & hV_0^T \bar{A}_1^T R \\ * & -hR \end{bmatrix} < 0. \qquad (35)$$

Theorem 3: If the conditions in Lemma 4 hold for the closed-loop system (5) with fixed gain dynamic output feedback controller (4), then the conditions in Theorem 1 hold for the closed-loop system (7) with adaptive dynamic output feedback controller (6).

Proof: Notice that if $V_{al} < 0$ for the actuator failure cases and normal case, then the conditions in Theorem 1 is feasible with $A_{K0} = A_{Ke0}$, $B_{K0} = B_{Ke0}$, $C_{K0} = C_{Ke0}$, $A_{Kai} = A_{Kbi} = A_{Kbij} = B_{Kai} = B_{Kbi} = 0$, $i, j = 1 \cdots m$. The proof is complete.

Remark 6: Theorem 3 shows that the method for the adaptive fault-tolerant guaranteed cost controllers design given in Theorem 1 is less conservative than that given in Lemma 4 for the fault-tolerant guaranteed cost controllers design with fixed gains. The following two-step algorithm gives a method for the fault-tolerant dynamic output controllers design with fixed gains.

Algorithm 1:

Step 1: Given a fixed controller gain C_{Ke0} which may be chosen from a feasible solution for stabilization problem via state feedback using the same Lyanpunov functional $V = V_1 + V_2$

$$\begin{bmatrix} \Upsilon & hX(A+A_1)^T + hY_0^T B^T \\ * & -hR \end{bmatrix} < 0,$$

where

$$\Upsilon = (A+A_1)X + BY_0 + [(A+A_1)X + BY_0]^T + hA_1^T RA_1$$

and conditions (12) holds for $\bar{A}_1 = A_1$. Then the feasible solutions are denoted as X and Y_0. Let $C_{K0} = Y_0 X^{-1}$.

Step 2: Let $N_1 A_{K0} = \bar{A}_{K0}$ and $N_1 B_{K0} = \bar{B}_{K0}$

$\{\alpha + tr(\Gamma_1)\}$ s.t. $0 < N_1 < Y_1$ (35).

Then the controller gains can be obtained by $A_{K0} = N_1^{-1}\bar{A}_{K0}$, $B_{K0} = N_1^{-1}\bar{B}_{K0}$ and $C_{K0} = Y_0 X^{-1}$.

From Theorem 2, we have the following algorithm to optimize the adaptive fault-tolerant guaranteed cost performances in normal and fault cases.

Algorithm 2:

Step 1: The procedure is the same as Step 1 in Algorithm 1.

Step 2: Let $N_1 A_{K0} = \bar{A}_{K0}$, $N_1 A_{Kai} = \bar{A}_{Kai}$, $N_1 A_{Kbi} = \bar{A}_{Kbi}$, $N_1 A_{Kbij} = \bar{A}_{Kbij}$, $N_1 B_{K0} = \bar{B}_{K0}$, $N_1 B_{Kai} = \bar{B}_{Kai}$, $N_1 B_{Kbi} = \bar{B}_{Kbi}$

$\{\alpha + tr(\Gamma_1)\}$ s.t. $0 < N_1 < Y_1$ and (34).

The corresponding controller gains A_{K0}, A_{Kai}, A_{Kbi}, A_{Kbij}, B_{K0}, B_{Kai}, B_{Kbi} and C_{K0} can be obtained.

Remark 7: By Theorem 3, it follows that Algorithm 2 can give less conservative design than Algorithm 1, which will be illustrated by example in Section 4.

4. EXAMPLE

In this section, a real application example about river pollution control [26] is proposed to showthe effectiveness of our approach.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_1 x(t-h) + Bu(t), \\ x(t) &= \phi(t), \quad t \in [-h, 0], \\ y(t) &= Cx(t), \end{aligned} \tag{36}$$

where

$$A = \begin{bmatrix} -k_{10} - \eta_1 - \eta_2 & 0 \\ -k_{30} & -k_{20} - \eta_1 - \eta_2 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} \eta_2 & 0 \\ 0 & \eta_2 \end{bmatrix}, \ B = \begin{bmatrix} \eta_1 & 0 \\ 0 & \eta_1 \end{bmatrix}, \ C = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

Here $u = [u_1(t) \ \ u_2(t)]^T$ is the control variable of river pollution. $k_{i0} (i = 1,2,3)$, η_1 and η_2 are known constants. The physical meaning of these parameters can be found in [1].

In the simulation, we choose $h = 0.7$, $\eta_1 = 2$, $\eta_2 = 1$, $k_{10} = 3$, $k_{20} = 1$, $k_{30} = 1$, $C_{cn} = [1 \ \ 0]$ and $T_c = 1$. The initial state is $\phi(t) = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$. And the matrices in the performance index (8) are $Q = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ and $S = \begin{bmatrix} 1 & 0 \\ 0 & 1.5 \end{bmatrix}$.

Besides normal mode that is, $\rho_1^1 = \rho_2^1 = 0$, the following possible fault modes are considered:

Fault mode 1: The first actuator is outage and the second actuator may be normal or loss of effectiveness, that is, $\rho_1^2 = 1, 0 \le \rho_2^2 \le 0.4$.

Fault mode 2: The second actuator is outage and the first actuator may be normal or loss of effectiveness, that is, $\rho_2^3 = 1, 0 \le \rho_1^3 \le 0.5$.

Using LMI tool box and Algorithms 1-2, it follows that the cost performance index is 4.4836 with adaptive dynamic output feedback controller while that of fixed gain controller is 5.1858. The considered faulty cases in the following simulations are as follows. Faulty case 1 is at 0 second, the first actuator becomes outage. Faulty case 2 is at 0.5 second, the second actuator becomes outage. Then after 1 second, the first actuator becomes loss of effectiveness of 50%.

Figs. 1, 2, and 3 are the state responses with adaptive and fixed gain dynamic output feedback controllers in normal and fault cases, respectively. It is easy to see our adaptive fault-tolerant guaranteed cost controller performs better than the one with fixed gains in both normal and faulty cases just as theory has proved.

In the next simulations, some time-varying uncertainties $\Delta A(t) = 0.25A\sin t, \Delta A_1(t) = 0.25A_1 \cos 3t$ and $\Delta B(t) = 0.25B \sin 2t$ are added into the system matrices A, A_1 and B, respectively, which aims to demonstrate the robustness of designed controllers. The corresponding state curves are given in Figs. 4-6. It is easy to see that the designed controllers are robust to these uncertainties.

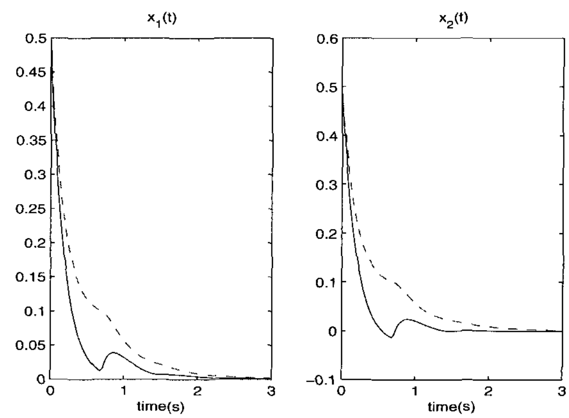

Fig. 1. Response curves in normal case with adaptive controller (solid) and controller with fixed gains (dashed).

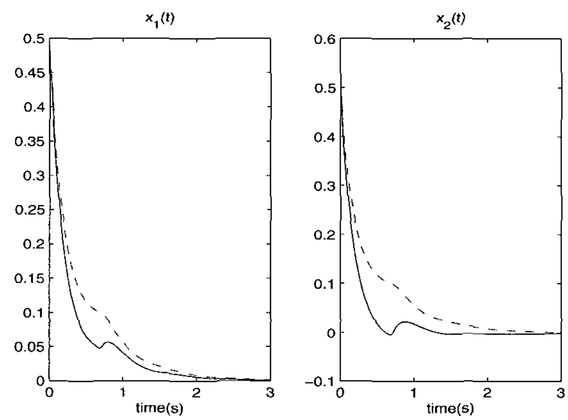

Fig. 2. Response curves in fault case 1 with adaptive controller (solid) and controller with fixed gains (dashed).

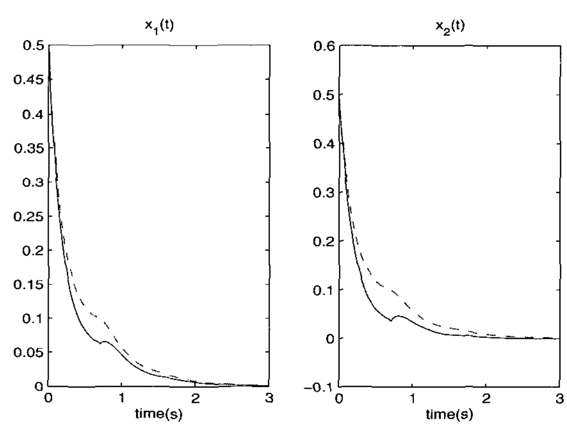

Fig. 3. Response curves in fault case 2 with adaptive controller (solid) and controller with fixed gains (dashed).

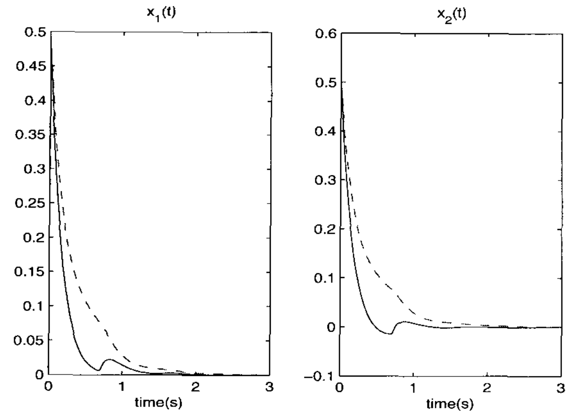

Fig. 4. Robust response curves in normal case with adaptive controller (solid) and controller with fixed gains (dashed).

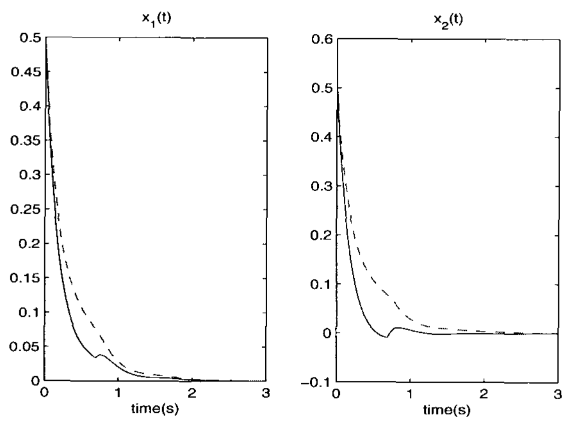

Fig. 5. Robust response curves in fault case 1 with adaptive controller (solid) and controller with fixed gains (dashed).

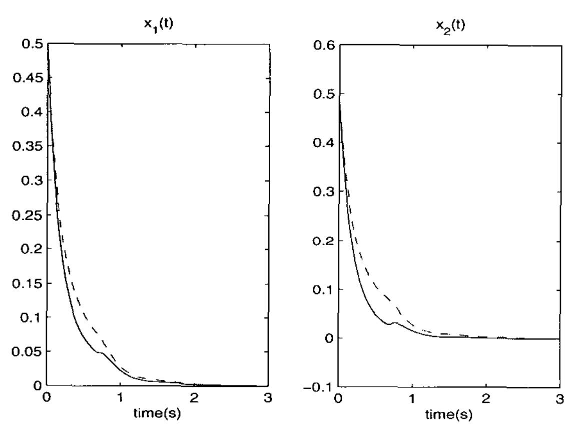

Fig. 6. Robust response curves in fault case 2 with adaptive controller (solid) and controller with fixed gains (dashed).

5. CONCLUSIONS

We have investigated the problem of adaptive fault-tolerant guaranteed cost control via dynamic output feedback against actuator faults for a class of linear time-delay systems. A new fault-tolerant dynamic output feedback controller with variable gains is proposed, based on the online estimation of fault parameters. Sufficient conditions with less conservativeness than the corresponding fault-tolerant controllers with fixed gains are derived in the framework of LMIs, such that in both normal and faulty cases the system can be stabilized and has a sub-optimal performance. A real application example about river pollution process is given, which illustrates the effectiveness of the new controller design method.

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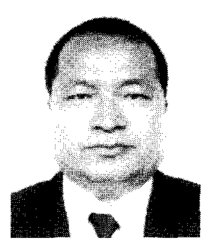

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