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Surface and Internal Waves Scattering by Partial Barriers in a Two-Layer Fluid
이층유체에서 부분 장벽에 의한 표면파와 내부파의 분산

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Abstract : Water waves are generated mainly by winds in open seas and large lakes. They carry a significant amount of energy from winds into near-shore region. Thereby they significantly contribute to the regional hydrodynamics and transport process, producing strong physical, geological and environmental impact on coastal environment and on human activities in the coastal area. Furthermore an accurate prediction of the hydrodynamic effects due to wave interaction with offshore structures is a necessary requirement in the design, protection and operation of such structures. In the present paper surface and internal waves scattering by thin surface-piercing and bottom-standing vertical barriers in a two-layer fluid is analyzed in two-dimensions within the context of linearized theory of water waves. The reflection coefficients for surface and internal waves are computed and analyzed in various cases. It is found that wave reflection is strongly dependent on the interface location and the fluid density ratio apart from the barrier geometry.

Key words: Partial Barriers, Internal Waves, Surface Waves, Reflection, Orthogonal Relation

요 지 : 파랑은 주로 바람에 의해서 발생하여 많은 에너지를 해안으로 전달하며 각종 수리현상을 야기하고 물질의 이송 등에 의하여 연안환경 뿐만 아니라 인간의 활동에도 큰 영향을 미친다. 또한, 해안 구조물과 파랑의 상호작용에 의한 효과를 정확히 예측하는 것은 구조물의 설계 및 거동특성 파악에 매우 중요하다. 본 논문에서는 이층 유체에서 수표면과 저층에 설치되어 있는 얇은 연직벽에 의한 표면파와 내부파의 분산을 선형파 이론을 이용하여 이차원으로 해석하였다. 반사계수를 계산하여 여러 경우에 대하여 효과를 분석한 결과 반사계수는 구조물의 형상과는 별도로 경계층의 위치와 유체간의 밀도차에 크게 영향을 받는 것으로 밝혀졌다.

핵심용어 : 부분 장벽, 내부파, 표면파, 반사, 연직관계

1. INTRODUCTION

Floating and submerged structures are generally used to reduce the transmitted wave height and protect various types of coastal structures from high wave attack in the downstream of wave motion. In recent years, there is a significant interest in the use of partial breakwaters to attenuate the wave energy. Most of these breakwaters are extended from the bottom up to the water surface, while partial breakwaters only occupy a segment of the whole water depth. Partial barriers as breakwaters are more economical and sometimes more appropriate for engineering applications. These kinds of breakwaters also provide a less expensive means to protect beaches exposed to waves of small or moderate amplitudes, and to reduce the wave amplitude at resonance. Floating and submerged partial breakwaters are also popularly known as surface-piercing and bottom-standing breakwaters respectively. A bottom-standing partial breakwater not only resists the wave propagation but also allows the navigation of vessels over it. They are also used for fish farming. With the environmental concerns, the bottom-standing breakwater resists the

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sediment transport and provides a strong protection against coastal erosion. On the other hand, a surface-piercing breakwater does not require a strong bottom foundation and most suitable for protecting coastal and offshore structures in deep water region. The problems of wave past floating/submerged breakwaters have been studied theoretically by many researchers within the framework of linear wave theory and in general most of the former studies are concerned with determining the reflection and transmission properties for a given incident wave. An explicit solution for the scattering of waves by a pair of surface piercing vertical barriers in deep water has been given by Levine and Rodemich (1958). For bottom-standing rectangular bodies approximate solutions for long waves have been developed by Ogilvie (1960), for long obstacles by Newman (1965), and for low draft structures by Mei (1969). Newman (1965) obtained an approximate solution for surface-waves elevation in the limit of a long submerged obstacle. Levine (1965) studied the interaction of oblique waves with a completely submerged circular cylinder near the free surface based on the Green's function. Transmission and reflection coefficients were calculated. When the obstacle is in the form of a thick barrier with rectangular cross section present in water of uniform finite depth, the corresponding water wave scattering problems for normal incidence of a wave train have been investigated by Mei and Black (1969). They used the variational formulation to solve the problem. For a single floating cylinder of rectangular cross-section, Black et al. (1971) also used the variational method to solve the radiation problem and then used the Haskind relation (Newman, 1976) to deduce the forces due to incident waves. Free surface elevations were obtained for a single cylinder and for two cylinders in series. Garrison (1969) investigated the interaction of an infinite shallow draft cylinder oscillating at the free surface with a train of oblique waves using the boundary integral method. Wave scattering by a circular dock has been considered by Garrett (1971). A number of authors have considered the two-dimensional problems of the radiation and scattering of waves by two parallel circular cylinders in deep water. Wang and Wahab (1971) have extended the multipole method of Ursell (1949) to analyze the heaving of two rigidly connected half-immersed cylinders. Bolton and Ursell (1973) used the multipole expansion method to the interaction of an infinitely long circular cylinder with oblique waves. The added mass, damping coefficients and vertical wave force were calculated. Bai (1975) presented a finite element method to study the diffraction of oblique waves by an infinite cylinder in water of infinite depth. Reflection and transmission coefficients and the diffraction forces and moments were computed for oblique waves incident upon a rectangular cylinder. Leach et al. (1985) investigated the wave diffraction by a floating rigid breakwater and showed better efficiency of such breakwaters compared to fixed rigid breakwaters. Sollitt et al. (1986) examined a system composed of two buoyant flaps clamped at the sea bottom and coupled with weighted mooring lines. Abul-Azm and Williams (1997) used the eigenfunction expansion method to examine oblique wave diffraction by a detached breakwater system consisting of an infinite row of regular spaced thin, impermeable structures located in water of uniform depth. Recently Söylemez and Gören (2003) studied the diffraction of oblique water waves by thick rectangular barrier mounted on seabed. They also studied the diffraction of oblique water waves by thick rectangular barrier floating at the free surface experimentally and investigated theoretically.

In the recent time, it is observed that there is an increasing interest in understanding internal waves because often in an ocean, internal waves are observed and are a time cause of heavy damages experienced by many onshore and offshore structures. For ocean engineers, interest in internal-waves is due to their role in submarine detection and the generation of anomalous drag on ships in fjords and estuaries. Such anomalous drag occurs when fresh water from rivers and runoff forms a thin layer of light fluid which lies above the cold saline water in a narrow fjord. The passage of a ship can then generate internal waves which radiate energy away from the neighborhood of the ship. This lost energy is an additional wave drag for the ship. Earlier the source of this energy loss was not easy to identify and was regarded as mysterious. Regions having this drag came to be known as "dead water" regions. The more general scientific interest comes from the general role of internal-waves in energy transport in lakes and oceans. Moreover, the major challenge in case of internal-waves is that they are not visible to the naked eye hence it is difficult to detect them and take precautionary measures. The simplest model of an internal-wave is when the density of the liquid is taken to be piecewise constant. This means, the liquid can be described as two-layers of constant density over each layer. In many natural bodies of water, stratification of either temperature or salinity may take place which can lead to large density differences with depth. A sharp change in the fluid density at a certain water depth owing to variation in salinity and/or temperature may be observed in a lake, an estuary or Norwegian fjords. Another example of sharp density change is a thin layer of muddy water at the bottom of harbors or channels with relatively shallow water depth. These density changes may significantly alter the hydrodynamic characteristics of waves past coastal structures. The propagation of waves in a two-layer fluid with both a free surface and an interface (in the absence of any obstacles) was first investigated by Stokes (1847) and the classical problem of this type of two-layer fluid separated by a common interface with the upper fluid having a free surface is given in Lamb (1932) Art. 231 and Wehausen and Laitone (1960). Most of the past wave structure interaction studies are carried out with the assumption that the fluid is of constant density over the entire fluid domain. Until recently, very little work has been done on wave/structure interaction studies in a two-layer fluid. Sturova (1994) approximated the free surface as a rigid lid and studied the radiation of waves by an oscillating cylinder, moving uniformly in a direction perpendicular to its axis. Linton and McIver (1995) developed a general theory for two-dimensional wave scattering by horizontal cylinders in an infinitely deep two-layer fluid. They derived the reciprocity relations that exist between the various hydrodynamic characteristics of the cylinders. It is well-known that a circular cylinder submerged in an infinitely-deep uniform fluid reflects no wave energy, and it was shown in Linton and McIver (1995) that this is also true for a cylinder in the lower layer of a two-layer fluid. Zilman and Miloh (1995, 1996) analyzed the effect of a shallow layer of fluid mud on the hydrodynamics of floating bodies. Sturova (1999) considered the radiation and scattering problem for a cylinder both in a two-layer as well as in a three-layer fluid bounded above and below by rigid horizontal walls. For the three-layer case the middle layer was linearly stratified representing a smooth pycnocline. Using the method of multi modes Sturova was able to calculate the hydrodynamic characteristics of the cylinder. Gavrilov et al. (1999) investigated the effects of a smooth pycnocline on wave scattering for a horizontal circular cylinder where the fluid is bounded above and below by rigid walls. Their work included a comparison between theoretical and experimental results, with reasonable qualitative agreement but notable quantitative disagreement. In the absence of obstacles, the appropriate dispersion relation for such a two-layer fluid has two solutions for a given frequency (Lamb, 1932, Art. 231). One of these solutions corresponds to waves where the majority of the disturbance is close to the free surface and the other to waves on the interface between the two fluid layers. Work on three-dimensional scattering can be found in Yeung and Nguyen (1999) and Cadby and Linton (2000). The motivation for their work came from a plan to build an underwater pipe bridge across one of the Norwegian fjords. In the Norwegian fjords typically, bodies of waters consists of a layer of fresh water of about 10 m thick lies on the top of a very deep body of salt water. Suresh Kumar et al. (2004) and Suresh Kumar and Sahoo (2004) initiated the studies on wave past flexible porous breakwaters in a two-layer fluid. Manam and Sahoo (2005) used a generalized orthogonal relation to investigate wave past porous structures in a two-layer fluid. Later, the wave trapping in SM and IM by porous and flexible barriers near the end of a semi-infinitely long channel is also analyzed by Suresh Kumar and Sahoo (2005) in a two-layer fluid of finite depth. Suresh Kumar et al. (2006) studied the wave scattering by submerged and floating rigid structures in a two-layer fluid of finite depth. Recently, Suresh Kumar and Sahoo (2006), and Suresh Kumar et al. (2007a) carried out a detailed analysis to study the performance of a flexible porous plate and membrane breakwaters respectively in a two-layer fluid, where each fluid is assumed to be of finite depth and the breakwater is extended over the entire water depth. Suresh Kumar et al. (2007b) investigated the surface and internal waves scattering by cylindrical dikes. The problems are analyzed in two dimensions with the assumption of small amplitude wave theory and breakwater response to study the effect of both surface and internal waves. The associated mixed boundary value problems are reduced to a linear system of equations by utilizing a more general orthogonal relation along with least squares approximation method. They observed that wave reflection and transmission in a two-layer fluid by a flexible porous breakwater is strongly dependent on the interface location and the fluid density ratio apart from the structural properties. Studies on a class of floating and submerged breakwaters in a two-layer fluid can be found in Suresh Kumar (2007).

In the present study, the wave scattering by thin partial barriers is investigated in a two-layer fluid of finite depth. The study includes both the cases of surface-piercing and bottom-standing barriers. The reflection characteristics of the system subjected to normal incident waves (one in the surface mode (SM) at the free surface and the other in the internal mode (IM) at the interface) are investigated. The boundary value problem is solved by utilizing an orthogonal relation suitable for the two-layer fluid domain. The reflection coefficients for the surface and internal modes are computed for various physical parameters like the fluid density ratio, ratio of water depths of the two fluids and mean wetted draft to analyze the two-layer fluid wave scattering phenomena.

2. MATHEMATICAL FORMULATION

In the present study wave past thin rigid partial barrier is considered. Schematic diagrams of bottom standing and surface piercing barrier in a two-layer fluid are shown in the Figs. 1 and 2 respectively. The fluid is assumed to be inviscid and incompressible and the wave motion is considered in the linearized theory of water waves neglecting the effect of surface tension. In the two-layer fluid, the upper fluid has a free surface (undisturbed free surface located at $y = 0$) and the two fluids are separated by a common interface (undisturbed interface located at $y = h$), each fluid is of

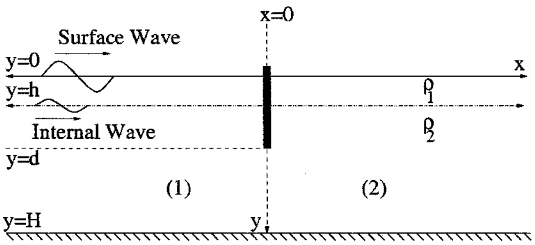

Fig. 1. Definition sketch for surface-piercing barrier.

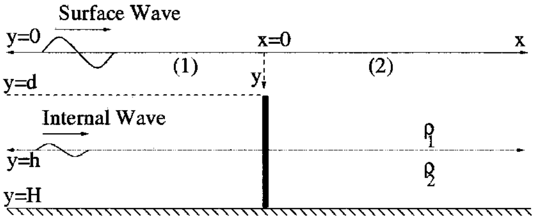

Fig. 2. Definition sketch for bottom-standing barrier.

infinite horizontal extent occupying the region $-\infty<x<\infty$; $0<y<h$ in case of the upper fluid of density ρ_1, and $-\infty<x<\infty$; $h<y<H$ in case of the lower fluid of density ρ_2. The flow is assumed to be irrotational and simple harmonic in time with angular frequency ω and hence the velocity potential $\Phi(x,y,t)$ exists such that $\Phi(x,y,t)$ = Re $[\phi(x, y)\exp(-i\omega t)]$. The spatial velocity potential ϕ satisfy the Laplace equation.

$$\nabla^2\phi_j = 0 \text{ in the fluid region } j \tag{1}$$

with subscript 1 refers to the fluid region 1 ($0<x<\infty$, $0<x<H$) and 2 refers to the fluid region 2 ($0<x<\infty$, $0<x<H$) as shown in Figs. 1 and 2. The linearized free surface boundary condition is

$$\frac{\partial\phi_j}{\partial y}+K\phi_j = 0 (j=1,2) \text{ on } y=0 \tag{2}$$

where $K = \omega^2/g$, and g is the gravitational constant. The boundary condition at the interface requires that

$$\phi_{jy} \text{ is continuous across } y=h, \text{ and} \tag{3}$$

$$(\phi_{jy}+K\phi_j)_{y=h_+} = s(\phi_{jy}+K\phi_j)_{y=h_-}, (j=1,2) \text{ for } y=h \quad -\infty<x<\infty \tag{4}$$

where s is the ratio of the densities of the upper fluid and the lower fluid, i.e $s = \rho_1/\rho_2$ and has range $0<s<1$. The condition on the rigid bottom is given by

$$\phi_{jy} = 0 (j=1,2) \text{ on } y=H \tag{5}$$

The radiation conditions are given by

$$\phi_1 \to (I_I e^{ip_I x}+R_I e^{-ip_I x}) f_0(p_I, y)+(I_{II} e^{ip_{II} x}+R_{II} e^{-ip_{II} x}) f_0(p_{II}, y) \text{ as } x\to-\infty \tag{6}$$

and

$$\phi_2 \to T_I f_0(p_I, y)e^{ip_I x}+T_{II} f_0(p_{II}, y)e^{ip_{II} x} \text{ as } x\to-\infty \tag{7}$$

where I_I, I_{II} represent the incident wave amplitudes in SM (fast mode) and IM (slow mode) respectively. R_I, T_I and R_{II}, T_{II} are the unknown reflected and transmitted wave amplitudes in SM and IM respectively.

The boundary condition on the rigid barrier surface is given by

$$\frac{\partial\phi_j}{\partial x} = 0 (j=1,2) \text{ on } x=0,$$

($0<y<H-b$, for surface-piercing structure and

$H-b<y<H$, for bottom-standing structure) (8)

The continuity of pressure and normal velocity along the gap L_g ($L_g \in H-b<y<H$ on $x=0$ for surface-piercing barrier and $L_g \in 0<y<H-b$ on $x=0$ for bottom-standing barrier) gives

$$\phi_1 = \phi_1, \text{ and } \frac{\partial\phi_1}{\partial x} = \frac{\partial\phi_2}{\partial x} \text{ on } x=0,\ y\in L_g \quad (9)$$

3. METHOD OF SOLUTION

The spatial velocity potentials ϕ_j for $j=1, 2$ satisfying Eq. (1) along with conditions (2), (3), (4), (5), (6) and (7) are expressed as

$$\phi_1 = I_I f_0(p_I, y)e^{ip_I x} + I_2 f_0(p_{II}, y)e^{ip_{II}x} + \sum_{n=I,II,1}^{\infty} R_n f_n(p_m,y)e^{-ip_n x}, \text{ for } x=0 \quad (10)$$

$$\text{and } \phi_2 = \sum_{n=I,II,1}^{\infty} T_n f_n(p_n,y)e^{ip_n x}, \text{ for } x>0 \quad (11)$$

where f_0 (p_n, y) =

$$\begin{cases} \dfrac{\sinh p_n(h-H)[p_n\cosh m_i y - K\sinh p_n y]}{p_n\sinh p_n h + K\cosh p_n h}, & \text{for } 0<y<h \\ \cosh p_n(y-H), & \text{for } h<y<H \end{cases}$$

$$(n=I, II, 1,2,...) \quad (12)$$

R_n, T_n's ($n=I$, II, 1, 2, ...) are unknown constants to be determined. It may be noted that R_I and R_{II} are related with the reflections coefficients in SM Kr_I and IM Kr_{II} respectively (see Suresh Kumar and Sahoo, 2006 and Suresh Kumar et al., 2007)). The wave numbers p_I, p_{II} are the positive real roots, and p_n's ($n=1, 2, 3,...$) are the purely imaginary roots of the dispersion relation in p as given by

$$(1-s)p^2\tanh p(H-h)\tanh ph - pK[\tanh ph + \tanh p(H-h)] + K^2[s\tanh p(H-h)\tanh ph + 1] = 0 \quad (13)$$

The eigenfunctions $f_n(p_n, y)$ are integrable in $0<y<H$ having a single discontinuity at $y=h$ and are orthogonal (see Suresh Kumar and Sahoo (2006) and Suresh Kumar et al. (2007)) with respect to the inner product as defined below

$$\langle f_n, f_m\rangle = \rho_1\int_0^h f_n(y)f_m(y)dy + \rho_2\int_h^H f_n(y)f_m(y)dy \quad (14)$$

On $x=0$ applying the continuity of ϕ_x along the barrier and the continuity of ϕ and ϕ_x along the gap L_g and invoking the orthogonality relation of the eigenfunctions over $0<y<H$, we obtain the required linear systems of equations (see, Suresh Kumar et al., 2004, Suresh Kumar and Sahoo, 2004, Suresh Kumar and Sahoo, 2005, Manam and Sahoo, 2005, Suresh Kumar and Sahoo, 2006 and Suresh Kumar et al., 2007). These systems of equations are solved to obtain the various physical quantities of interest.

4. NUMERICAL RESULTS AND DISCUSSION

Numerical results are computed and analyzed for surface and internal waves scattering by partial barriers in a two-layer fluid. The effects of various non-dimensional physical parameters on wave reflection in both SM and IM are investigated. For convenience, the wave parameters are given in terms of the non-dimensional wave number $p_I d$, depth ratio h/H, fluid density ratio s and the non-dimensional depth of submergence of barrier H/d.

4.1 Case of a Surface-Piercing Barrier

The wave reflection by a surface-piercing barrier are analyzed in both SM and IM. For the sake of simplicity, all the results are plotted versus the normalized wave number $p_I d$ in SM by allowing $p_{II}d$ (normalized wave number in IM) to vary based on the two-layer fluid dispersion relation (Eq. 13). It is observed that in general for all values of H/d, with an increase in $p_I d$, the wave reflection in SM increases and attains maximum in the deep water region. On the other hand, the reflection coefficients in IM increase with an increase in $p_I d$ and attain a maximum value in the intermediate water depth and then decrease to zero.

The variation of single reflection coefficients in SM and IM versus $p_I d$ is plotted in Fig. 3(a and b) respectively for different values of H/d. In Fig. 3(a), the wave reflection in SM increases with an increase in the value of H/d. However, lowest wave reflection is observed when $h/H=d/H$ in the case of wave motion in SM. On the other hand, a highest peak is observed for wave reflection in IM in the case of $h/H=d/H$ (Fig. 3(b)). Moreover, the wave reflection in IM increases with a decrease in the value of H/d.

The effect of depth ratio h/H on the barrier reflection coefficients in SM and IM is shown in Fig. 4(a and b) respectively. In Fig. 4(a), it is observed that the reflection

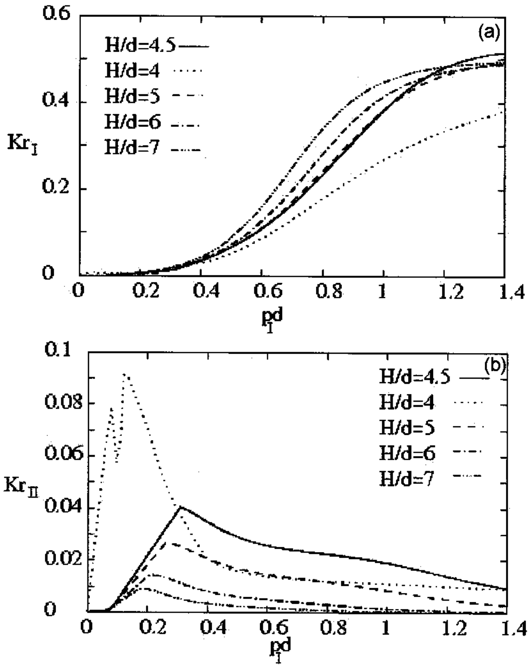

Fig. 3. Reflection coefficients in (a) SM, Kr_I and (b) IM, Kr_{II} versus p_Id for a surface-piercing barrier at different H/d values, $h/H = 0.25$ and $s = 0.75$.

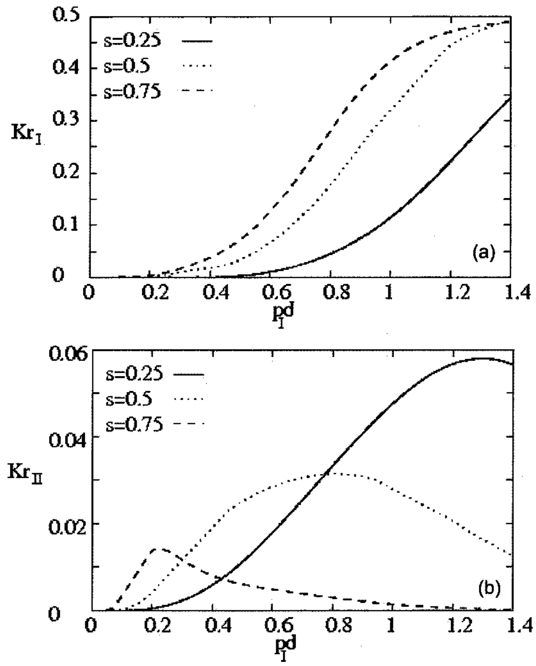

Fig. 5. Reflection coefficients in (a) SM, Kr_I and (b) IM, Kr_{II} versus p_Id for a surface-piercing barrier at different s values, $H/d = 6.0$ and $h/H = 0.25$.

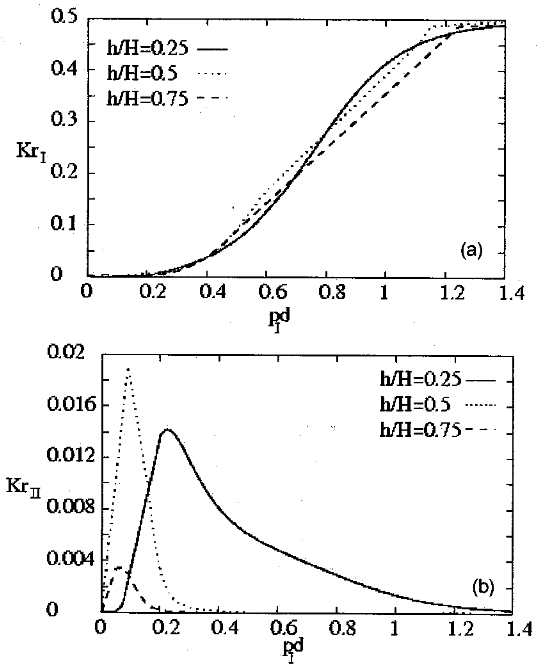

Fig. 4. Reflection coefficients in (a) SM, Kr_I and (b) IM, Kr_{II} versus p_Id for a surface-piercing barrier at different h/H values, $H/d = 6.0$ and $s = 0.75$.

coefficients in SM are less sensitive to the depth ratio h/H. On the other hand, the reflection coefficient in IM is found to be increasing with a decrease in h/H ratio (Fig. 4(b)). However, highest reflection peak is observed when the interface at the center of free surface and seabed.

The reflection coefficients versus p_Id are plotted in SM and IM for various values of s in Fig. 5(a and b) respectively. In general it is observed that wave reflection in SM increases with an increase in the value of s and a reverse trend is observed in case of IM wave reflection.

4.2 Case of a Bottom-Standing Barrier

In the present subsection, the reflection coefficients in SM and IM for bottom-standing barrier are analyzed for various physical parameters of interest.

The variation of reflection coefficients in SM and IM versus $p_I(H-d)$ (it may be noted that d is the barrier length in case of bottom-standing barrier) are plotted in Fig. 6(a and b) respectively for different values of H/d. In general, it is observed that the wave reflection in both SM and IM are found to be increasing with a decrease in H/d. However, when the barrier tip approaches towards the interface the

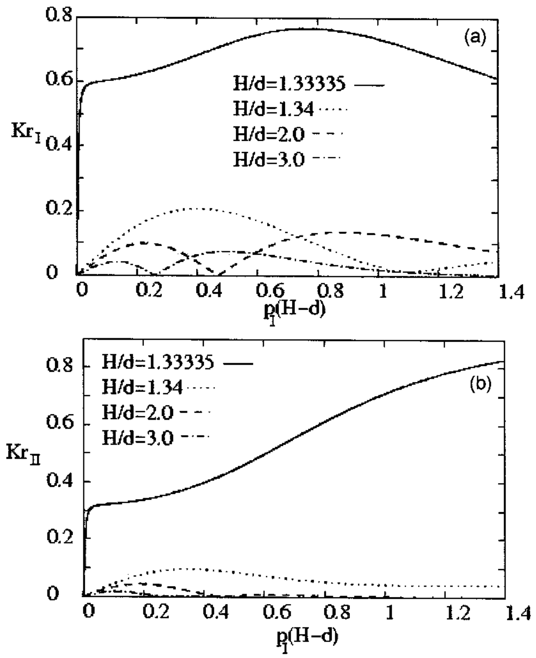

Fig. 6. Reflection coefficients in (a) SM, Kr_I and (b) IM, Kr_{II} versus $p_I d$ for a bottom-standing barrier at different H/d values, $s = 0.75$ and $h/H = 0.25$.

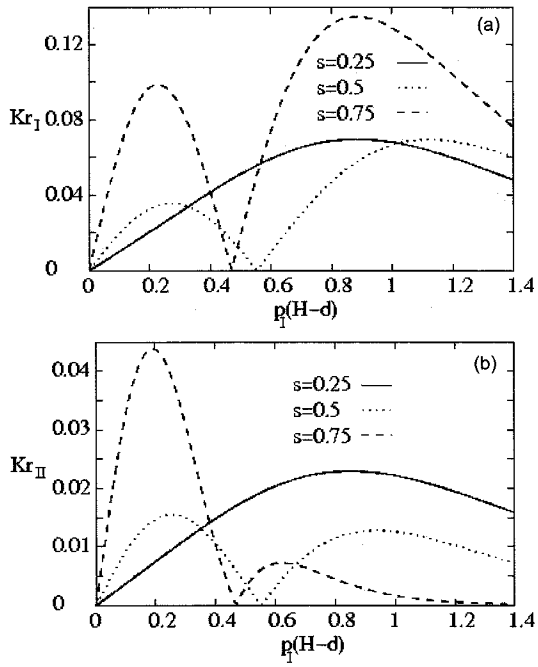

Fig. 8. Reflection coefficients in (a) SM, Kr_I and (b) IM, Kr_{II} versus $p_I d$ for a bottom-standing barrier at different s values, $h/H = 0.25$ and $H/d = 2.0$.

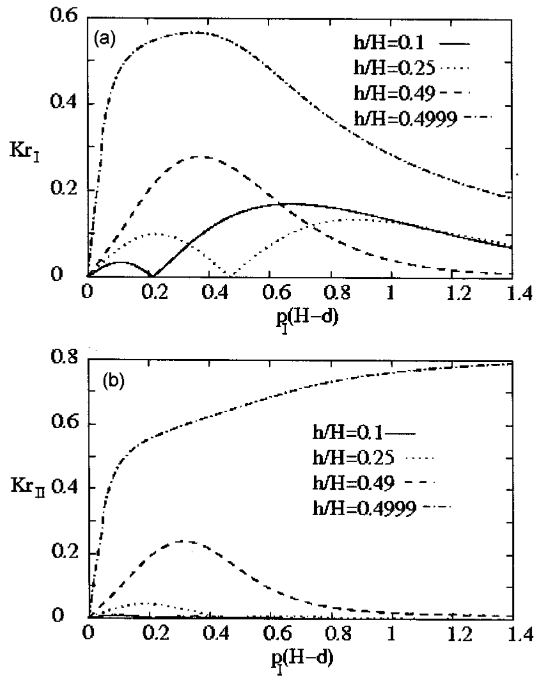

Fig. 7. Reflection coefficients in (a) SM, Kr_I and (b) IM, Kr_{II} versus $p_I d$ for a bottom-standing barrier at different h/H values, $s = 0.75$ and $H/d = 2.0$.

wave reflection in both SM and IM increases very sharply.

The effect of interface location h/H on reflection coefficients in SM and IM are shown in Fig. 7(a and b) respectively. It is observed that the reflection in both SM and IM increases with an increase in h/H ratio and high wave reflection is observed when the tip of the barrier is approaching towards the interface.

Reflection coefficients are plotted versus $p_I(H-d)$ in SM and IM for various values of s in Fig. 8(a and b) respectively. It is observed that the wave reflection in both SM and IM has higher reflection peaks as $s \rightarrow 1$.

5. CONCLUSIONS

The wave scattering by surface-piercing and bottom-standing barriers in a two-layer fluid is investigated. Orthogonal relation suitable for the two-layer fluid is utilized to solve the problems. The wave reflection for both surface-piercing and bottom-standing barriers are found to be strongly dependent on the interface location and the fluid density ratio apart from the barrier geometry. These observations are similar to those observed by Suresh Kumar and Sahoo

(2006) and Suresh Kumar et al. (2007) where they found that wave reflection and transmission in a two-layer fluid by a flexible porous breakwater is strongly dependent on the interface location and the fluid density ratio apart from the structural properties. A similar approach can be utilized to study more general problems in a two-layer fluid having a free surface.

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