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Multi-Objective Shape Optimization of an Axial Fan Blade

Abdus Samad, Ki-Sang Lee, Kwang-Yong Kim †

Department of Mechanical Engineering, Inha University, 253 Yonghyun-Dong, Nam-Gu, Incheon, 402-751, Rep. of Korea

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Abstract

Numerical optimization for design of a blade stacking line of a low speed axial flow fan with a fast and elitist Non-Dominated Sorting of Genetic Algorithm (NSGA-II) of multi-objective optimization using three-dimensional Navier-Stokes analysis is presented in this work. Reynolds-averaged Navier-Stokes (RANS) equations with k-ε turbulence model are discretized with finite volume approximations and solved on unstructured grids. Regression analysis is performed to get second order polynomial response which is used to generate Pareto optimal front with help of NSGA-II and local search strategy with weighted sum approach to refine the result obtained by NSGA-II to get better Pareto optimal front. Four geometric variables related to spanwise distributions of sweep and lean of blade stacking line are chosen as design variables to find higher performed fan blade. The performance is measured in terms of the objectives; total efficiency, total pressure and torque. Hence the motive of the optimization is to enhance total efficiency and total pressure and to reduce torque.

Key words: Axial fan; Stacking line; RANS analysis; Optimization; NSGA-II.

Nomenclature

f : Objective function
p : Pressure
Q : Flow rate
r : Non-dimensional radius=$(R-R_h)/(R_t-R_h)$
R : Radius
γ : Sweep
ρ : Density
η : Total efficiency
δ : Lean
τ : Torque
ω : Angular velocity
$\underline{\xi}$: Sweep or lean
$\underline{f}$, $\underline{g}$, $\overline{h}$: Objective functions
x : Vector space
$\overline{w}$: Weights associated with the objectives
M : Number of objectives
U,V : Velocity

Subscripts

a : Axial
h : Hub
m : Middle of blade
t : Tip
T : Total
in, out : Inlet and exit, respectively

† Corresponding author. Tel.: +82-32-872-3096, Fax.: +82-32-868-1716
E-mail address: kykim@inha.ac.kr

1. Introduction

Design optimization techniques based on three-dimensional Reynolds-averaged Navier-Stokes equations become practical for the design of turbomachinery blades with the aid of development in computing power in the last decade. Application of optimization methods has reduced the computational cost sharply to predicting the better design of turbomachines to enhance the performance in terms of reducing weight, torque, flow loss, and enhancing efficiency, pressure, surge margin, etc. by changing shape of stacking line, camber profile, etc.

Deformation of stacking line using sweep, lean, and skew has become a matter of interest in the design of turbomachinery blades by many researchers [1-5]. These blade shape parameters, which form a three-dimensional stacking line, are generally introduced to reduce shock losses, corner separation in the blade hub, and tip clearance losses in axial turbomachines. Design of blade stacking line using numerical optimization techniques has been reported by Jang and Kim[6] and Jang et al.[7]. These papers optimized blades using efficiency as single objective optimization by polynomial response surface method (RSM)[8]. Seo et al.[9] reported axial fan blade shape optimization considering efficiency as objective and stacking line parameters as design variables.

Most of practical engineering problems involve multiple disciplines, and need simultaneous optimization of multiple objectives related to each discipline. These design problems usually known as multi-objective problems which require simultaneous consideration of all objective functions to optimize the system. There are numbers of solution methods and algorithms available for solving multi-objective optimization problems[10]. In multi-objective optimizations; efficiency, total pressure, static pressure, pressure loss, weight, stress, etc. are used as objectives, and variables related to camber profile and/or stacking line of blade are employed as design variables[11-13]. A multi-objective optimization problem consists of many optimal solutions called Pareto-optimal solutions; therefore a designer's aim is to find as many optimal solutions within the design range as possible. This helps designer to find a global Pareto-optimal front. Each design set corresponding to optimal solution represents a compromise among design objectives. A fast and elitist Non-dominated Sorting Genetic Algorithm (NSGA-II) given by Deb et al.[14] generates Pareto optimal solution using evolutionary algorithm. Goel et al.[15-16] presented weighted sum strategy combining with NSGA-II algorithm using polynomial based response surface algorithm.

The present work performed a multi-objective optimization for design of a NACA65 axial fan blade. A hybrid multi-objective optimization approach is applied using NSGA-II in combination with a local search strategy. Three competing objectives functions i.e., total pressure, torque on blade, and total efficiency are selected for optimization and four design variables from lean and sweep have been taken to check the fan blade performance. Three-dimensional Reynolds averaged Navier-Stokes equations (RANS) are solved for internal flow analysis to evaluate the efficiency and total pressure ratio and torque on blade.

2. Numerical analysis

The commercial software CFX 5.7[17] is used for the flow analysis in an axial flow fan. Three-dimensional steady incompressible Reynolds-averaged Navier-Stokes equations are solved. Governing equations are discretized using finite volume approximations and standard k-ε turbulence model is used as a turbulence closure. The implementation of wall boundary conditions in turbulent flow is completed by the use of empirical wall function.

The single blade which is used for simulation for reference fan is NACA65 blade section and the shape is shown in Fig.1. Its major specifications are listed in Table 1. Unstructured grid system is employed for the grids, and the optimum grid selected after grid dependence test has 4.3×10^5 grid points as reported by Seo et al.[9]. One of the nine blades is selected for numerical analysis that uses periodic boundary conditions. An example of grid system is shown in Fig. 2. Uniform velocity profiles are assumed at the inlet, and constant pressures are applied at the exit boundary. The working fluid is 20°C air. To obtain a completely converged solution for the present analysis, the CPU time was approximately 12 hours with a Pentium-IV, 3.0 GHz processor.

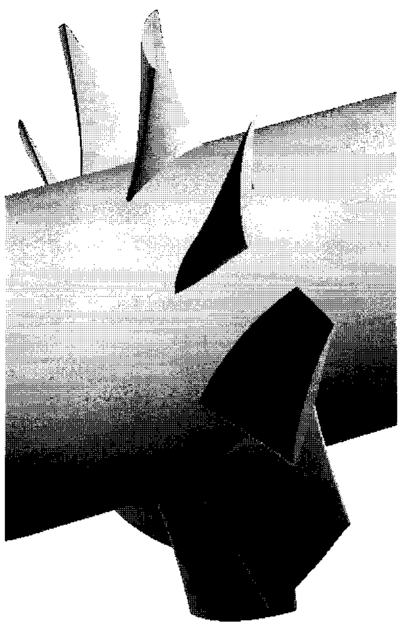


Fig. 1. Shape of the reference fan blade.

Table 1. Specifications of reference fan blade

Flow Coefficient	0.41
Total Pressure Coefficient	0.3
Rotor Rotation Frequency	1000 rpm
Tip Radius	287.5 mm
Hub-Tip Ratio	0.52
Inlet Angle at Rotor Tip	68.8 degree
Outlet Angle at Rotor Tip	63.8 degree

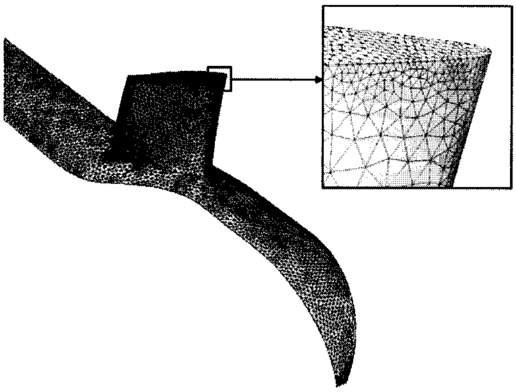


Fig. 2. Example of grid system.

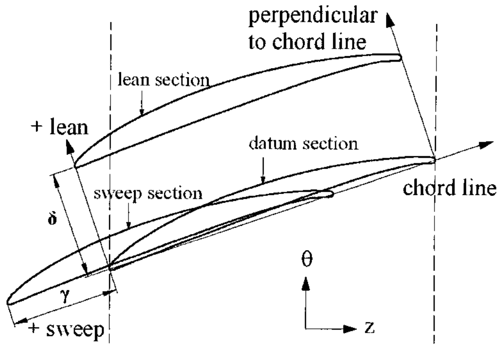


Fig. 3. Definition of sweep and lean.

3. Objective functions and design variables

The present problem is setup for performance enhancement of fan blade by increasing efficiency and total pressure rise and reducing torque on blade. Therefore, the following performance parameters are selected as objective functions for multiobjective optimization by NSGA-II.

- Total efficiency, η
- Total pressure rise, $p_T = p_{T,out} - p_{T,in}$
- Blade torque, τ

where, blade torque is related to input power=$\tau \cdot \omega$ with constant angular velocity (ω) of blade. Two optimization problems have been formulated considering two objectives at a time to construct Pareto optimal solutions. Thus, effects of two objectives on each other can be found in the Pareto optimal curves.

Four design variables are selected for stacking line modification to enhance the fan performance. The stacking line is defined by sweep (γ) and lean (δ) and two variables each from sweep and lean are considered. The definitions of sweep and lean are presented in Fig. 3. The following second order polynomial curve is fitted for radial distribution of sweep or lean, and the blade shapes are determined by assigning values of sweep or lean at mid span (50% span) and tip (100% span):

$$\begin{aligned} &\xi = a\ r^2 + b\,r + c \\ &\xi = 0 \quad at \quad r = 0. \\ &\xi = \xi_m \quad at \quad r = 0.5 \\ &\xi = \xi_t \quad at \quad r = 1.0 \end{aligned} \tag{1}$$

where ξ is sweep or lean, and r indicates non-dimensional radius. Therefore, four variables of γ_t, γ_m, δ_t and δ_m are selected as design variables, which indicate the blade is modified for its mid span and tip and no change in its hub section.

For design optimization, it is important to find the feasible design space which is formed by variable ranges in which the calculations are to be performed. Some preliminary calculations are done to find the design space and D-optimal design[8] is used to find the sampling points for computations and three-level fractional factorial design is used.

4. Multi-objective optimization with hybrid MOEA

Multi-objective evolutionary algorithm (MOEA)[10] optimization procedure which is followed in this paper is described in flowchart shown in Fig. 4. Initially the variables are selected, and the design space is decided for improvement of system performance. The design points are selected using design of experiments (DOE), and the objective functions are calculated at these design points by flow solver. In this work, the DOE is conducted through the three-level fractional factorial design. Evaluations of the objective functions at these design points are carried out by three-dimensional RANS analysis. These RANS analysis data is used to generate polynomial response

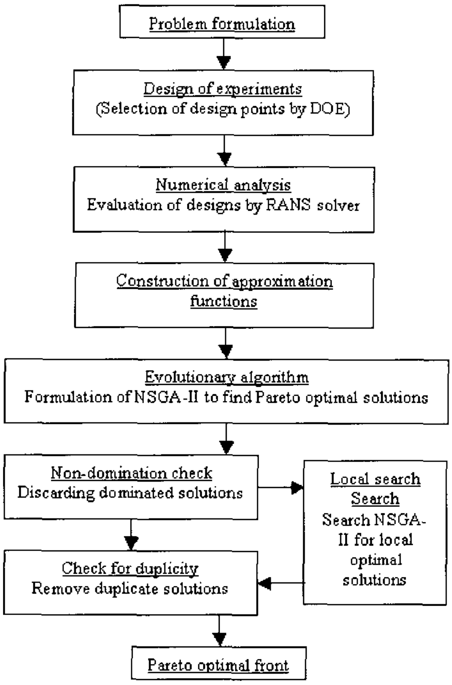


Fig. 4. Optimization procedure for hybrid MOEA.

surface function which produces initial population for evolutionary algorithm based hybrid multi-objective optimization.

Multi-objective approach gives a set of optimal solutions instead of single optimal solution. None of the solutions in this set of optimal solutions can be considered to be better than any other solution with respect to all objectives considered in the problem. These optimal solutions are called Pareto-optimal solutions and their functional space representation is termed as Pareto optimal front(10). There are numbers of methods available(10) for solving multi-objective optimization problems but the classical way of tackling multi-objective problems is to convert multi-objective problem into single objective problem. The methodology of constructing a global Pareto-optimal front is explored to get inside of the trade-off analysis between different conflicting objectives.

A multi-objective problem may be defined as:

Minimize $\bar{f}(\bar{x})$ (M function to be optimize)

Subject to

$\bar{g}(\bar{x}) \le 0$ (m inequality constraints)

$\bar{h}(\bar{x}) = 0$ (p equality constraints)

where $\bar{f}(\bar{x}) = \{f_1(\bar{x}), f_2(\bar{x}), f_3(\bar{x}), \ldots\ldots\ldots f_M(\bar{x})\}$ is a vector of n real valued objective functions and x is a vector of n design variables. $\bar{x} \in R^n$, $\bar{g}(\bar{x}) \in R^m$, $\bar{h}(\bar{x}) \in R^p$. In general, engineering problems are associated with some conflicting objectives in which improvement of one objective leads to deterioration of other objectives. Each feasible solution of the set $\bar{x}$ of multi-objective problem is either dominated or non-dominated solution, in which all non-dominated solutions are called Pareto-optimal solutions. Vector $\bar{x}_i$ dominates a vector $\bar{x}_j$ if $\bar{x}_i$ is at least as good as $\bar{x}_j$ for all objectives and $\bar{x}_i$ is strictly better than $\bar{x}_j$ for at least one objective.

Objective functions and constraints are defined mathematically and evaluated on the data obtained from numerical calculations at the specified points. Polynomial based RSA model is constructed for the objective functions to reduce the complexity of the multi-objective optimization problem. Real coded NSGA-II is used for MOEA to produce Pareto optimal front.

The different parameters are adjusted one by one to suit the nature of the problem. Population size=100, Generation=250, Crossover=20, and Mutation =200. NSGA-II gives a set of approximate Pareto-optimal solutions and therefore a weighted sum strategy of local search method is used to improve the quality of Pareto optimal solutions. In weighted sum strategy, all the objectives are combined into one objective. As this strategy is easy to implement, and hence is used in this problem. Weights associated with each objective are computed using the formula:

$$\bar{w} = \frac{(f_j^{\max} - f_j(X)/(f_j^{\max} - f_j^{\min})}{\sum_{k=1}^{M}(f_k^{\max} - f_k(X))/(f_k^{\max} - f_k^{mim})} \qquad (2)$$

And, the objective becomes:

$$f = \sum_{k=1}^{M} f_k w_k \qquad (3)$$

where $\bar{W}$ is the weight for j^{th} objective, M is the number of objectives. f_j^{min}, f_j^{max} and $f_j(X)$ are the scaled minimum, maximum and initial values of j^{th} objectives, respectively. This composite objective is locally optimized using Sequential Quadratic Programming

(SQP)[18]. These optimized solutions are merged with NSGA-II obtained solutions and dominated solutions are discarded. The global Pareto-optimal solutions are achieved after removing duplicate solutions from the non-dominated solutions.

5. Result and discussion

This numerical problem is formulated for stacking line shape optimization for the performance enhancement of fan blade. The polynomial equations are used to define the four shape variables. After grid test and preliminary calculation to decide the design space, D-optimal design is used to find the sampling points at which the computations are performed. The RANS equations are solved to evaluate the objective functions values.

The evaluated objective functions are used to generate polynomial response surface curve for each objectives. A second order curve is fitted using the design variables and the evaluated objective functions. At next step, these curves were used to generate population in the design space by NSGA-II. The generated population forms a Pareto optimal curve and the optimal points are refined using the weighted sum strategy of local search method. After preliminary calculations, the design space is decided and presented in table 2.

Two problems are formulated for two objectives:

- Efficiency and total pressure
- Efficiency and torque.

Pareto optimal front is generated for these problems. For verification purpose, a randomly selected point is calculated using RANS-solver for each problem. The curves shown in Figs. 5 and 6 are weighted sum refined curves. Here, obviously the curves showed the improvement of performance.

Fig. 5 shows the Pareto optimal front generated by NSGA-II coupled with weighted sum strategy results. The confliction of objectives proved as the efficiency and total pressure rise behaves inversely with each other.

Table 2. Ranges of design variables.

Variables	Lower Bounds	Upper Bounds
γ_t	-0.02	0.04
γ_m	-0.03	0.03
δ_t	-0.04	0.02
δ_m	-0.01	0.01

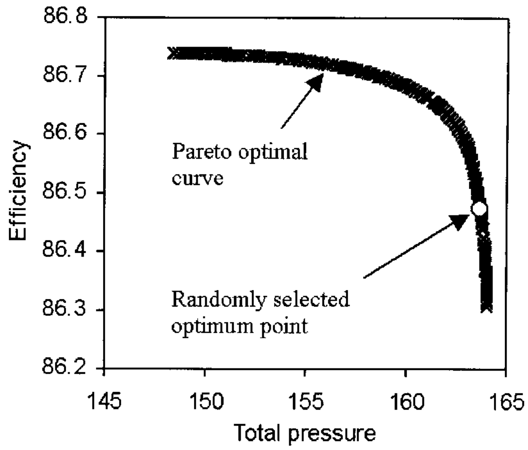


Fig. 5. Pareto optimal solutions for efficiency and total pressure.

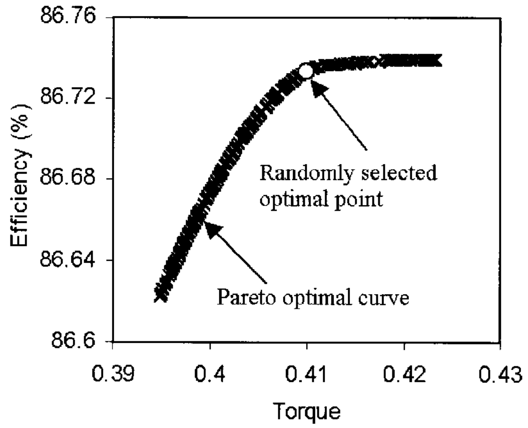


Fig. 6. Pareto optimal solutions efficiency and torque.

Similarly, Fig. 6 shows the Pareto optimal front of objectives, efficiency and torque. This curve, obviously, shows the confliction between torque and efficiency. Here, the torque increases with the increase in efficiency.

The results for randomly selected optimal points on Pareto optimal curves in Figs. 5 and 6 are shown in tables 3 and 4. The optimal points shown in the figures show the points predicted by the surrogate which are somewhat different from those obtained by RANS computation because of the second-order approximation in data fitting in RSA model, random function used in NSGA function, and approximation used in optimal point search algorithm. The RANS computed objective function values are also presented. The comparison between the reference and the optimized shapes shows the improvement in performance. The efficiency is increased by 1.34% and torque is reduced by 9.88% when Pareto optimal curve is generated for efficiency and torque (Fig. 6). Similarly, efficiency is increased by 1.08% and total pressure is increased by 3.07% when optimized for efficiency and total pressure (Fig. 5). Seo et al.[9] performed a single objective optimization employing efficiency as objective function using the same data set as in this work. Their results of optimization show 1.75% improvement in efficiency. Randomly selected optimum point which improves both objectives is used instead of using best design for any single objective, which may not give all objectives improved. Due to the above reason, the present result shows little less efficiency than the previous result.[9]

Fig.7 shows the stream lines for reference and optimized blades. Hub corner separation zone is reduced for torque-efficiency optimized blade as compared to reference one. In Figs. 7(b) and 7(c), the separation zone is washed out. The enhancement of efficiency is due to the reduction of separation zone on blade.

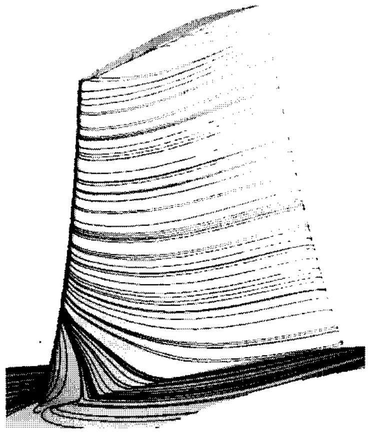
Blade loading implies the work done by the blade, and loading curves at 35%, 50% and 85% span are presented in Fig. 8. It is visible from these figures that the blade loading has been increased for the optimized blades as compared to the reference one.

Table 3. Design variables for reference and optimal shapes.

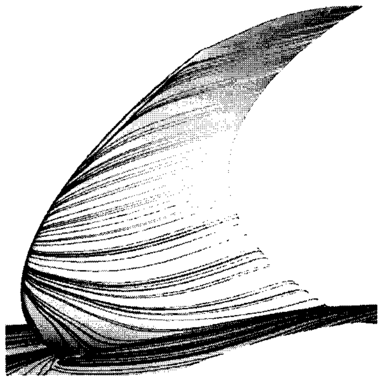
Blade shapes	Sweep		Lean	
	Tip	Mid	Tip	Mid
Reference	0	0	0	0
η-τ optimum	0.04	-0.03	-0.029	0.01
η-P_T optimum	0.034	0.021	0.006	0.01

Table 4. Reference and optimized blade efficiency, torque and total pressure.

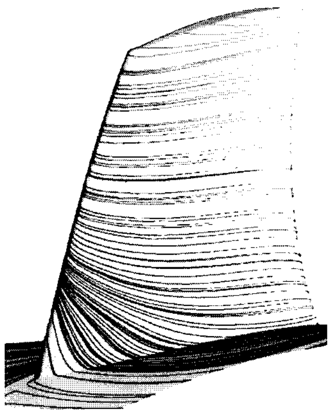
Blade shape	Result	τ (kg m^{2}s^{-2})	η (%)	P_T
Reference	RANS Calculation	0.447	85.10	153.3
η-τ optimum	RANS Calculation	0.403	86.24	-
	Improvement	9.88%↓	1.34%↑	-
η-P_T optimum	RANS Calculation	-	86.02	158.0
	Improvement	-	1.08%↑	3.07%↑



(a) Reference blade

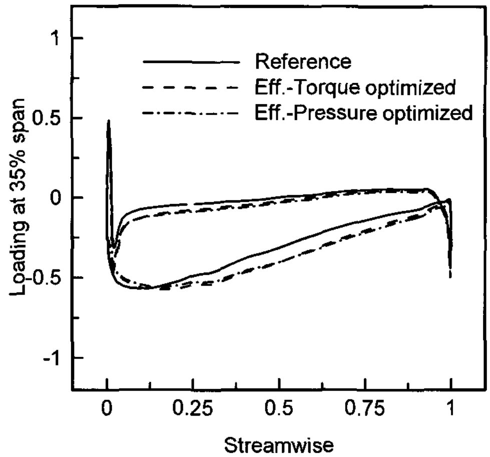


(b) Torque-efficiency optimized blade

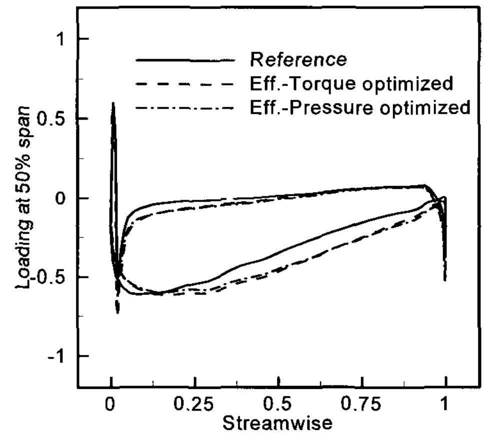


(c) Pressure-efficiency optimized blade

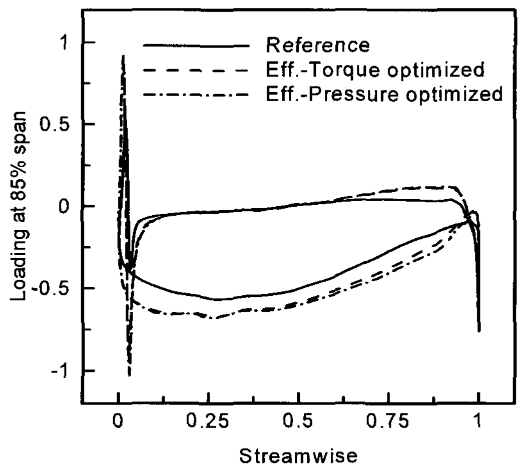
Fig. 7. Streamlines on suction surfaces on reference and optimum blades.



(a) At 35% span



(b) At 50% span



(c) At 85% span

Fig. 8. Blade loading curves.

6. Conclusions

A shape optimization has been performed numerically for three objective functions, namely efficiency, total pressure and torque and four design variables. Two optimization problems each with two objective functions have been formulated for multi-objective optimization. NSGA-II enhanced with local search strategy is used to design Pareto optimal front. By the present simulation and optimization, a set of optimal designs has been generated. The hybrid MOEA strategy applied in this work can be applied in other turbomachinery area when, in general, performance is evaluated for multiple terms (objectives) instead of single objective. The flow analysis shows the reduction of separation zone and increase in blade loading for optimal designed blades as compared to reference one.

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