# Theoretical Analysis of Wave Energy Converter

Jin-Seok Oh<sup>†</sup> · Toshimitsu Komatsu\* · Yun-Hyung Kim\*\* (Manuscript: Received November 7, 2007; Revised: January 10, 2008)

**Abstract**: Floating devices, such as a cavity resonance device take advantage of both the water motion and the wave induced motions of the floating body itself. The wave energy converter is known commercially as the WAGB(Wave Activated Generator Buoy) and is used in some commercially available buoys to power navigation aids such as lights and horns. This wave energy converter consists of a circular floatation body which contains a vertical center pipe that has free communication with the sea. A theoretical analysis of this power generated by a pneumatic type wave energy converter is performed and the results obtained from the analysis are used for a real wave energy converter for buoy.

This paper presents the analysis results and the design method for the WEC(Wave Energy Converter), and the associate results are application to the commercially available WEC for buoy. Maximum performance of WEC occurs at resonance with driving waves. The analysis of WEC is performed with LabVIEW program, and the design method of WEC for buoy is suggested in this paper.

Key words: Wave height, Wave Frequency, Wave Energy Converter, Buoy, Resonance

### 1. Introduction

For utilizing ocean wave energy has been a challenge to ocean researchers for many years. Ocean waves are mathematically complex: consequently, their complete description requires several parameters. The optimization of WES cannot be dissociated from the resource evaluation at the locations of interest. However, for the purpose of a broad resource assessment, wave height is the

most important parameter. WEC for buoy consists of a circular floatation body which contains a vertical center pipe. The center pipe there is an area contraction through which the air above the internal free surface of the water passes. The air has a relative motion to the buoy caused by both the moving water surface and the heaving of the buoy.

The air is excited by both the heaving motions of the buoy and the motions of the internal free-surface which is in free

<sup>†</sup> Corresponding Author(Division of Mechatronics Engineering, Korea Maritime University), E-mail: ojs@hhu.ac.kr. Tel: +82-(0)51-410-4866

<sup>\*</sup> Department of Civil Engineering, Kyushu University

<sup>\*\*</sup> Division of Mechatronics Engineering, Korea Maritime University

communication with the sea.

In the design of WEC, the most significant item is that an optimum length of the internal water column exists, i.e. a length for which maximum power is converted near the heaving resonance. This paper is described the influence of internal water column length on the converted kinetic energy of the wave. Further, the wave height effect on the energy conversion is also studied. The purpose of this paper is that which performs a theoretical analysis of the WEC with LabView program.

The analysis is utilized a possible way of improving the performance of WEC system.

## 2. Theoretical analysis

For analyzing the air motion above the internal free surface has to study the compressibility of the air, the viscous actions on the flow, and the unsteadiness of the flow. To simplify the analysis it is assumed that the air is incompressible and that the consequences of the viscosity can be summed in one loss term.

The oscillation modes of WEC have a function of several parameters, such as the mass, frequency, height and heaving motion<sup>[1]-[3]</sup>.

The principle of the buoy system can be understood by referring to the sketch in Fig. 1. WEC can be analyzed with water column motion, momentum within the turbine passage, heaving motion of the buoy and power relationship in Fig. 1. The area-averaged internal free-surface displacement is

$$\eta_{1a} = \frac{1}{A_1} \int_{0}^{R_1} \int_{-\pi}^{\pi} \eta_1 r d\theta dr \tag{1}$$

Where  $A_1$  is cross-sectional area of the water column area,  $\theta$  is the angle measured from the wave direction in cross-sectional area of the center pipe. The internal free-surface displacement from its equilibrium position,  $\eta_1$ , is

$$\eta_1 = \frac{\overline{H_i}}{2} cos(\omega t) \tag{2}$$

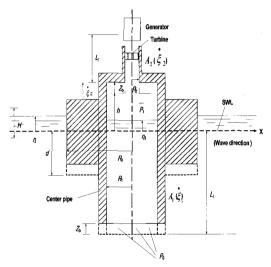


Fig. 1 Schematic diagram of the WEC

Where  $\overline{H_i}$  is the spatially averaged internal wave height.  $\omega$  is the circular wave frequency. The air velocity of the internal free-surface is then

$$\dot{\zeta}_1 = -\frac{\omega \overline{H_i}}{2} \sin(\omega t) \tag{3}$$

Since the air is incompressible, the axial velocity in the turbine passage is

$$\dot{\zeta}_{2} = -\frac{A_{1}}{A_{2}} \frac{\omega \overline{H_{i}}}{2} \sin(\omega t) \tag{4}$$

Thus, the relative unsteady air velocity in the turbine passage is

$$\dot{\zeta}_2 = \dot{\zeta}_1 (D_1/D_2)^2 \tag{5}$$

Where  $D_1$  and  $D_2$  are the inner diameter and turbine passage diameter of the axis symmetry float, respectively.

Fig. 2-Fig. 5 shows the simulation results from equations (2)-(5)

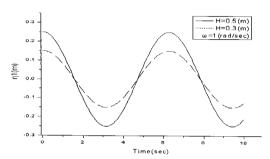


Fig. 2 Variation of internal free-surface displacement(2)

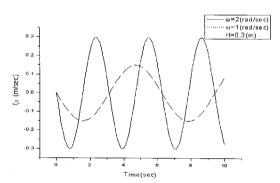


Fig. 3 Transient variation of air velocity on the internal free-surface (3)

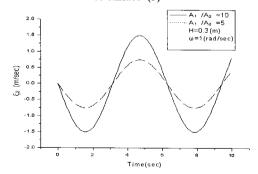


Fig. 4 Transient variation of axial velocity in the turbine passage (4)

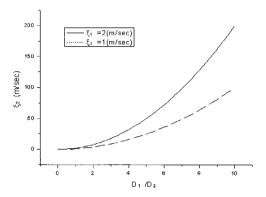


Fig. 5 Variation of air velocity in the turbine passage versus diameter ratio (5)

Equation (1) is based on the assumption that the internal water column motions are independent of the buoy motions. The chamber pressure  $p_1$  can be modified with Bernoulli's equation as equation (6).

$$p_1 = -\rho_w(L_1 + \xi)\ddot{\xi} \tag{6}$$

Where  $\rho_w$  is the mass density of saltwater,  $\xi$  is the relative air displacement within the center column and  $L_1$  is the still water length of the inter water column. The heaving resonance value of the maximum value of the relative air velocity in the turbine passage is seen to increase slightly with increasing values of  $L_1$ .

The power  $P_{out}$  available to the turbine depends on the pressure gradient and volume rate of airflow  $Q_{out}$  across the turbine:

$$P_{out} = (p_2 - p_0) Q_{out} \tag{7}$$

For simplicity, the exhaust pressure  $p_0$  is assumed to be ambient. The upstream pressure  $p_2$  is related to the pressure within the air chamber.

## 3. Application for WEC

The cavity resonance WEC is used in some commercially available buoys to power navigation aids such as lights and horns. If designed properly, this application can take advantage not only of the cavity resonance, but also of the heaving motion of the buoy<sup>[4], [5]</sup>.

In Fig. 1  $\xi_2$  can be expected to have two relative maxima, one at the cavity resonance frequency of equation (8) and the other at the heaving resonance frequency of equation(9). The natural frequency of the internal water column motion is

$$\omega_c = (g/L_1)^{1/2} \tag{8}$$

Also, the heaving resonant frequency is

$$\omega_2 = \sqrt{c/(m+m_w)} \tag{9}$$

Where m and  $m_w$  are the buoy mass and added mass.  $m_w$  is the mass of the excited by the heaving motion. The heaving motions of the buoy are described with the damping coefficient, restoring coefficient, buoy mass, added mass, dynamic pressure and the mass density of the air. The restoring coefficient, c, is

$$c = \rho_w g (D_0^2 - D_1^2) \pi / 4 \tag{10}$$

Where  $\rho_w$  is the mass density of the water, and g is the gravitational constant,  $D_0$  is the outer diameter of the axis symmetry float, and  $A_{wp}$  is the water plane area.

By using these resonance conditions, we can now see that the design condition is

$$\omega_c = \omega_z \tag{11}$$

The peak power values are affected by the system damping such that as the damping increases the power decreases. In the buoy system, the damping is due to the creation of waves by both the heaving motion of the buoy and the vertical motion of the water column.

In addition, friction on the wetted body surfaces and friction and turbulence losses in the airflow add to the damping. To optimize the design of the buoy system, use the design condition given in equation (11). The equation (11) can be written as

$$L_1 + L_1' = \frac{4(m + m_w)}{\rho_w \pi (D_0^2 - D_1^2)} \tag{12}$$

Where  $L_1$  is an effective length due to the added mass excited by the water column. The design condition,  $T_c = T_z$ , is obtained from the equation (12).

Fig. 6 is shown the results for the effect of water column with the period T. The generating power as predicted by the theory is seen to increase with increasing values of wave height and decrease with increasing values of oscillating water column period T.

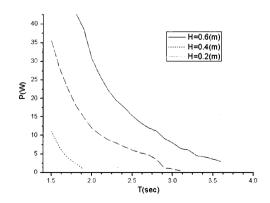


Fig. 6 Generating power with the water column period variation

We choose  $T_c$  to be the design period. We keep the dimensions in equation (12) the some as originally used and simply change the mass 'm' of the system by adding ballast.  $m_{\omega}$  is the added mass, that is, the mass of the water excited by the heaving motion.  $m_{\omega}$  is a function of geometry and will slightly change with the additional draft  $d=m/(\rho_w A_{wp})$ . The added mass,  $m_{\omega}$ , excited by the heaving circular floatation body is

$$m_{\omega} = C_{ce} \rho_{w} \pi (R_{0} - R_{1})^{3} \tag{13}$$

Where  $C_e$  is a numerical coefficient corresponding to the buoy geometry. Fig.7 shows the simulation results from equation (13).

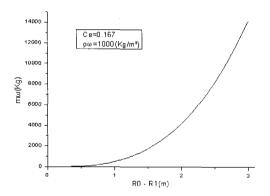


Fig. 7 Variation of added mass versus floating body radius (13)

### 4. Conclusion

The generating power as predicted by the theory is seen to increase with increasing values of wave height and decrease with increasing values of oscillating water column period T. WEC is most effective for waves near the resonant period of the buoy and water column system, as determined from equations (8) and (9). By increasing the length of the internal water column, an additional fluid mass is excited. The mass increasing with water column length will eventually result in an optimum length of the water column for which maximum power will be obtained near the heaving resonance period of the buoy. Also, we confirmed that of generating power with a period and wave height

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# <u>저</u>자 소 개



#### Jin-Seok Oh

He received the B. E. degree in Marine Engineering from Korea Maritime University in 1983. He has been with the Zodiac (England Company) as system engineer about 4 years. He received the M.E. and Ph.D. degrees from Korea Maritime University, Busan, Korea in 1989 and 1996, respectively. He had been with the Agency for Defense Development(ADD) researcher from 1989 to 1992. In 1996. he joined the Division of Mechatronics Engineering at Korea Maritime University.



#### Toshimitsu Komatsu

He was born in Ooita Prefecture, Japan. He received Ph. D from Kyushu university. He had been JSCE council member in 2002. From 2003 to present, he is a chairman in Hydraulic Engineering Committee JSCE. He is currently a professor in Department of civil engineering at Kyushu University.



### Yun-Hyung Kim

He received the B.S in 2003 from korea maritme university . He studies the energy and control for M.S degree in the department of Mechatronics Engineering of Korea Maritime University.