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論 文
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Non-fragile Guaranteed Cost Controller Design for Uncertain Time-delay Systems via Delayed Feedback

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Abstract -In this paper, we propose a non-fragile guaranteed cost controller design method for uncertain linear systems with constant delyas in state. The norm bounded and time-varying uncertainties are subjected to system and controller design matrices. A quadratic cost function is considered as the performance measure for the system. Based on the Lyapunov method, an LMI(Linear Matrix Inequality) optimization problem is established to design the controller which uses information of delayed state and minimizes the upper bound of the quadratic cost function for all admissible system uncertainties and controller gain variations. Numerical examples show the effectiveness of the proposed method.

Key Words : Non-fragile, LMI, Lyapunov method, Time-delay, Uncertain Systems.

1. Introduction

Time delay occurs in many dynamic system such as nuclear reactors, chemical engineering systems, ship stabilization, biological systems, neural network, epidemic systems, population dynamics models and so on [1-25]. In practice, the systems almost have some uncertainties because it is almost impossible to obtain an exact mathematical model due to the difficulty of measuring various parameters, environmental noises, poor plant knowledge and system complexities. Therefore, during the past two decades, considerable attention has been paid towards robust controller design methods for uncertain linear systems with time-delays. These methods can be classified into two categories: delay independent approaches [1-4] and delay-dependent approaches [5-7]. In general, delay-dependent methods provide less conservative results when the size of the delays is small.

There are various efforts to consider system performance in designing a controller for uncertain time delay systems. One kinds of them is guaranteed cost control introduced first by Cheng and Peng [8]. This method not only guarantees closed-loop stability but also maintains an adequate level of performance represented by the quadratic cost function. The advantage of this method provides an upper bound of given cost function and the system degradation is guaranteed to be less than this bound. While many researchers have proposed the delay-independent guaranteed cost controller design methods for uncertain linear systems with delays [9-12], only a few delay-dependent results for the guaranteed cost controller design method can be found [13-15]. In the work [13-15], the design procedures for memoryless state feedback guaranteed cost control laws have been proposed. Although this memoryless state feedback controller is simple and easy to implement, its performance can be more conservative than that of a delayed feedback controller which use the information of the size of the time-delay. Fortunately, in many real systems, information on the size of the time-delay is often available. Thus, if we design a delayed feedback controller, we can provide a better performance. This property has been shown in the literature [16-18].

On the other hand, when we implement a controller in real world, it is also desirable to consider the controller gain variations because it is impossible to design the given controller exactly. It is because the controller may be subject to the inaccuracy such as the error of resistance, A/D and D/A conversion, finite word length, and round-off errors in numerical computation. Therefore, this controller fragility issue have been attracted by some researchers [19-20]. Keel [19] explains the controller fragility in the continuous-time domain by weighted H_∞ and μ synthesis technique, even though these controllers are robust only with respect to system uncertainty. Yee [20] studies a discrete-time non-fragile guaranteed cost controller design that tolerates some forms of controller

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gain uncertainties that are closely related to the actuator fault. Haddad [21] developed multivariable robust control design frameworks such as H_∞ control and μ synthesis for the robust stability and performance control problem.

Recently, in order to reduce the conservatism in stability analysis and controller synthesis, a new integral inequality by adopting the free weighting matrix is introduced by [22]. This may improve the performance of delayed-feedback guaranteed cost control in the closed-loop system. Motivated by the work [22}], we present a new integral inequality for our stability analysis.

In this paper, we propose a delayed feedback non-fragile guaranteed cost controller design method for dynamic systems with time delay and uncertainties. We design a controller with feedback provisions for the current state and the past history of the state. Based on the Lyapunov function method, an optimization problem is formulated to design the controller, which stabilizes the uncertain linear systems with time-delay and minimizes the upper bound value of cost function. This stabilization problem is formulated in terms of LMIs which can be solved efficiently using various convex optimization algorithms. Finally, we include numerical examples to show that our result is less conservative than that of the existing method. Throughout this paper, ★ represents the elements below the main diagonal of a symmetric matrix. The notation $X>0$ ($X\ge 0$) means that X is a real symmetric positive definite matrix (positive semi-definite). I denotes the identy matrix whose dimensions can be determined from the context. R^n is the n-dimensional Euclidean space, $R^{m\times n}$ denotes the set of $m\times n$ real matrix. $diag\{\cdots\}$ denotes the block diagonal matrix. $C_{n,h}([-h,\ 0), R^n)$ denotes the Banach space of continuous vector functions which maps the interval $[-h,0)$ into R^n.

2. Problem Statements

Consider the following uncertain linear system with delay in state:

$$\dot{x}(t) = (A+\Delta A)x(t) + (A_1+\Delta A_1)x(t-h) + (B+\Delta B)u(t)$$
$$x(s) = \phi(s),\ \ s\in[-h,0], \tag{1}$$

where $x(t)\in R^n$ is the state, $u(t)\in R^m$ is the control input, A, A_1 and B are known real matrices of appropriate dimensions, ΔA, ΔA_1 and ΔB are norm-bounded time-varying uncertainties, h is a known constant delay, and $\phi(s)\in C_{n,h}$ is a given vector valued initial function. The parameter uncertainties ΔA, ΔA_1 and ΔB have the following form:

$$\Delta A = D_1F_1(t)E_1,\ \Delta A_1 = D_2F_2(t)E_2,\ \Delta B = D_3F_3(t)E_3, \tag{2}$$

where $D_i, E_i (i=1,2,3)$ are known real constant matrices of appropriate dimensions, and $F_i(t)\in R^{k_i\times l_i}$ are known matrices, which satisfy

$$F_i^T(t)F_i(t)\le I, (i=1,2,3). \tag{3}$$

In this paper, it is assumed that the pair $(A+A_1, B)$ is controllable, and the measurement of the state $x(t)$ and the size of time-delay h are always available. In order to evaluate the system performance, we define the following integral quadratic cost function

$$J = \int_0^\infty [x^T(t)W_1x(t) + u^{T(t)}W_2u(t)]dt, \tag{4}$$

where $W_1>0$ and $W_2>0$ are given state and control weighting matrices.

The objective of this paper is to design a controller which minimizes the cost function (4). Thus we consider the controller of the form

$$u(t) = Kz(t) \tag{5}$$

where $K\in R^{m\times n}$ is the nominal controller gain to be designed and

$$z(t) = x(t) + \int_{t-h}^t A_1x(s)ds, \tag{6}$$

which is the neutral model transformation [23].

However, in real controller implemenatation, the controller gain matrix has some amount of error in the neighborhood of K. Theferere, the actual controller implemented is

$$u(t) = (K+\Delta K)z(t), \tag{7}$$

where ΔK represents the multiplicative gain perturbations of the form

$$\Delta K = D_4F_4(t)E_4K, \tag{8}$$

with D_4 and E_4 being known real matrices of appropriate dimensions, and F_4 being unknown matrices satisfying $F_4^TF_4\le I$.

Differentiating $z(t)$ with respect to t leads to

$$\begin{aligned}\dot{z}(t) &= \dot{x}(t) + A_1x(t) - A_1x(t-h)\\ &= (A_0+\Delta A)x(t) + \Delta A_1x(t-h) + (B+\Delta B)u(t)\end{aligned} \tag{9}$$

where $A_0 = A + A_1$.

By applying controller (7) to Eq. (9), we have

$$\begin{aligned}\dot{z}(t) &= (A_0+\Delta A)x(t) + \Delta A_1x(t-h)\\ &\quad + (B+\Delta B)(K+\Delta K)z(t)\\ &= (A_0+\Delta A)x(t) + (B+\Delta B)(K+\Delta K)z(t) + \Delta A_1x(t-h)\\ &\quad - (A_0+\Delta A_1)\int_{t-h}^t A_1x(s)ds.\end{aligned} \tag{10}$$

The following definition, facts and lemmas will be used to derive the main results.

Definition 1. For system (1) and cost function (4), if there exist a control law $u^*(t)$ and a positive scalar J^*, such that for all admissible uncertainties, the closed-loop system is asymptotically stable, and the closed-loop value of the cost function satisfies $J \le J^*$, then $u^*(t)$ is said to be a guaranteed cost controller for system (1), and J^* is said to be a guaranteed cost.

Fact 1. (Schur Complement) Given constant symmetric matrices $\Sigma_1, \Sigma_2, \Sigma_3$ where $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$, then $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$ if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0, \text{ or } \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & -\Sigma_1 \end{bmatrix} < 0$$

Fact 2. For given matrices D, E and F with $F^T F \le I$ and a scalar $\varepsilon > 0$, the following inequality

$$DFE + E^T F^T D^T \le \varepsilon D D^T + \varepsilon^{-1} E^T E$$

is always satisfied.

Lemma 1. [24] For a given positive scalar $\bar{h} > 0$ and α, where $0 < \alpha < 1$, if there exists a positive definite matrix M, such that the LMI

$$\begin{bmatrix} -\alpha M & \bar{h} A_1^{TM} \\ \bar{h} M A_1 & -M \end{bmatrix} < 0$$

holds, then $z(t)$ is a stable operator for any $h \in [0, \bar{h}]$.

Lemma 2. For any matrix $Q > 0$, F and scalar $h \ge 0$, the following inequality holds:

$$- \int_{t-h}^{t} x^T(s) Q x(s) ds \le$$
$$\begin{bmatrix} z(t) \\ \int_{t-h}^{t} x(s) ds \end{bmatrix}^T \begin{bmatrix} 0 & 0 \\ \star & F + F^T \end{bmatrix} \begin{bmatrix} z(t) \\ \int_{t-h}^{t} x(s) ds \end{bmatrix}$$
$$+ h \begin{bmatrix} z(t) \\ \int_{t-h}^{t} x(s) ds \end{bmatrix}^T \begin{bmatrix} 0 \\ F \end{bmatrix} Q^{-1} [0 \; F^T] \begin{bmatrix} z(t) \\ \int_{t-h}^{t} x(s) ds \end{bmatrix}^T.$$

Proof. Utilizing Fact 2, we have

$$- \int_{t-h}^{t} x^T(\gamma) Q x(\gamma) d\gamma \le$$
$$2 \int_{t-h}^{t} x^T(\gamma) [0 \; F^T] \begin{bmatrix} z(t) \\ \int_{t-h}^{t} x(s) ds \end{bmatrix} d\gamma$$
$$+ \int_{t-h}^{t} \begin{bmatrix} z(t) \\ \int_{t-h}^{t} x(s) ds \end{bmatrix}^T \begin{bmatrix} 0 \\ F \end{bmatrix} Q^{-1} [0 \; F^T] \begin{bmatrix} z(t) \\ \int_{t-h}^{t} x(s) ds \end{bmatrix}$$
$$= 2 \begin{bmatrix} z(t) \\ \int_{t-h}^{t} x(s) ds \end{bmatrix}^T \begin{bmatrix} 0 \\ I \end{bmatrix} [0 \; F^T] \begin{bmatrix} z(t) \\ \int_{t-h}^{t} x(s) ds \end{bmatrix}$$
$$+ h \begin{bmatrix} z(t) \\ \int_{t-h}^{t} x(s) ds \end{bmatrix}^T \begin{bmatrix} 0 \\ F \end{bmatrix} Q^{-1} [0 \; F^T] \begin{bmatrix} z(t) \\ \int_{t-h}^{t} x(s) ds \end{bmatrix}.$$

3. Design of Nonfragile Guaranteed Cost Controller

In this section, by using the Lyapunov stability theory, we propose the method of designing a delay-dependent non-fragile guaranteed cost controller for system (1).

For simplicity, we define

$$\begin{aligned} \Sigma &= A_0 X + X A_0^T + BY + Y^T B^T + (\varepsilon_1 + \varepsilon_4) D_1 D_1^T + \varepsilon_2 D_2 D_2^T \\ &\quad + \varepsilon_3 D_3 D_3^T + epsilin_5 B D_4 D_4^T B^T, \\ \Xi_1 &= [X E_1^T \; Y^T E_3^T \; Y^T \; Y^T E_4^T \; Y^T E_4^T], \\ \Xi_2 &= diag\{-\varepsilon_1 I, -\varepsilon_3 I + \varepsilon_6 E_3 D_4 D_4^T E_3^T, -W_2^{-1} + \varepsilon_7 D_4 D_4^T, \\ &\quad -\varepsilon_5 I, -\varepsilon_6 I, -\varepsilon_7 I, \} \\ NN^T &= \int_{-h}^{0} \phi(s) \phi^T(s) ds, \; N_d N_d^T = \int_{-h}^{0} \int_{s}^{0} \phi(u) \phi^T(u) du ds. \end{aligned} \tag{11}$$

Now, we give our main result.

Theorem 1. Consider system (1) and cost function (4). For given constant h, if the following optimization problem

$$\min \quad \alpha + Trace(M_1) + Trace(M_2) \quad \text{subject to}$$

$$\begin{bmatrix} \Sigma & -A_0 A_1 R & 0 & 0 & hX & X & X E_2^T & \Xi_1 \\ \star & L + L^T & R A_1^T & L & -hRA_1^T & -RA_1^T & -RA_1^T E_2^T & 0 \\ \star & \star & -\varepsilon_4 I & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & -R & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -R & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -W_1^{-1} & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -\varepsilon_2 I & 0 \\ \star & \star & \star & \star & \star & \star & \star & \Xi_2 \end{bmatrix} < 0 \tag{12}$$

$$\begin{bmatrix} -M & h A_1^T M \\ \star & -M \end{bmatrix} < 0, \tag{13}$$

$$\begin{bmatrix} -\alpha & z^T(0) \\ \star & -X \end{bmatrix} < 0, \tag{14}$$

$$\begin{bmatrix} -M_1 & h N_d^T \\ \star & -hR \end{bmatrix} < 0, \tag{15}$$

$$\begin{bmatrix} -M_2 & N^T E_2^T \\ \star & -\varepsilon_2 I \end{bmatrix} < 0, \tag{16}$$

has the solution $X > 0$, $R > 0$, $M > 0$, $M_1 > 0$, $M_2 > 0$, and matrix Y and L with appropriate dimensions and positive scalar values α, $\varepsilon_i (i = 1, \ldots, 7)$, then system (1) is stabilized by nonfragile guaranteed cost controller (7) with gain $K = YX^{-1}$, and then the guaranteed cost of the cost function (4)is $J^* = \alpha + Trace(M_1) + Trace(M_2)$.

Proof. This proof is composed of two parts. The first part is the method for obtaining controller gain matrix and the second one is to derive the condition, which minimizes the upper bound value of cost function (4).

First, consider the Lyapunov functional candidate as

$$\begin{aligned} V(z, t) &= z^T(t) P z(t) + \int_{t-h}^{t} \int_{s}^{t} x^T(u) Q x(u) du ds \\ &\quad + \int_{t-h}^{t} x^T(s) T x(s) ds, \end{aligned} \tag{17}$$

where the matrices P, Q, and T are positive definite matrices.

Taking the derivative of $V(z, t)$ and defining

$$\begin{aligned} V_1(z, t) &= 2 z^T(t) P (A_0 + \Delta A + (B + \Delta B)(K + \Delta K)) z(t) \\ &\quad - 2 z^T(t) P (A_0 + \Delta A) \int_{t-h}^{t} A_1 x(s) ds \\ &\quad + 2 z^T(t) P \Delta A_1 x(t-h) + h x^T(t) Q x(t) \\ &\quad - \int_{t-h}^{t} x^T(s) Q x(s) ds + x^T(t) T x(t) \end{aligned}$$

$$\begin{aligned}&-x^T(t-h)Tx(t-h)+x^T(t)W_1x(t)\\&+z^T(t)(K+\Delta K)^TW_2(K+\Delta K)z(t)\end{aligned} \tag{18}$$

leads to

$$\begin{aligned}V(z,t) =\; & V_1(z,t)-x^T(t)W_1x(t)\\ & -z^T(t)(K+\Delta K)W_{2(K+\Delta}K)z(t).\end{aligned} \tag{19}$$

By using Fact 2, we obtain

$$\begin{aligned}&2z^T(t)PD_1F_1(t)E_1z(t)\le\\&\varepsilon_1 z^T(t)PD_1D_1^TPz(t)+\varepsilon_1^{-1}z^T(t)E_1^TE_1z(t),\end{aligned} \tag{20}$$

$$\begin{aligned}&2z^T(t)PD_2F_2(t)E_2x(t-h)\le\\&\varepsilon_2 z^T(t)PD_2D_2^TPz(t)+\varepsilon_2^{-1}x^T(t-h)E_2^TE_2x(t-h),\end{aligned} \tag{21}$$

$$\begin{aligned}&2z^T(t)PD_3F_3(t)E_3(K+\Delta K)z(t)\le\\&\varepsilon_3 z^T(t)PD_3D_3^TPz(t)\\&+\varepsilon_3^{-1}z^T(t)(K+\Delta K)^TE_3^TE_3(K+\Delta K)z(t),\end{aligned} \tag{22}$$

$$\begin{aligned}&-2z^T(t)PD_1F_1(t)E_1\int_{t-h}^{t}A_1x(s)ds\le\\&\varepsilon_4 z^T(t)PD_1D_1^TPz(t)\\&+\varepsilon_4^{-1}\left(\int_{t-h}^{t}x(s)ds\right)^TA_1^TE_1^TE_1A_1\left(\int_{t-h}^{t}x(s)ds\right).\end{aligned} \tag{23}$$

Let us define $T_1=hQ+W_1+T$, and then we have

$$\begin{aligned}x^T(t)T_1x(t) =\; & z^T(t)T_1z(t)-2z^T(t)T_1\int_{t-h}^{t}A_1x(s)ds\\ & +\left(\int_{t-h}^{t}x(s)ds\right)^{TA_1^T}T_1A_1\left(\int_{t-h}^{t}x(s)ds\right).\end{aligned} \tag{24}$$

Substituting Eqs. (20)-(24) into Eq. (18) gives that

$$\begin{aligned}&V_1(z,t)=\\&z^T(t)[PA_0+A_0^TP+PB(K+\Delta K)+(K+\Delta K)^TB^TP]z(t)\\&+\varepsilon_4 z^T(t)PD_1D^{T_1}z(t)+\\&+\varepsilon_4^{-1}\left(\int_{t-h}^{t}x(s)ds\right)^TA_1^TE_1^TE_1A_1\left(\int_{t-h}^{t}x(s)ds\right)\\&-2z^T(t)PA_0\int_{t-h}^{t}A_1x(s)ds\\&+x^T(t-h)(-T+\varepsilon_2^{-1}E_2^TE_2)x(t-h)\\&+\varepsilon_2 z^T(t)PD_2D_2^TPz(t)+z^T(t)T_1z(t)\\&-2z^T(t)T_1\int_{t-h}^{t}A_1x(s)ds\\&+\left(\int_{t-h}^{t}x(s)ds\right)^TA_1^TT_1A_1\left(\int_{t-h}^{t}x(s)ds\right)\\&+\varepsilon_1 z^T(t)PD_1D_1^TPz(t)+\varepsilon_1^{-1}z^T(t)E_1^TE_1z(t)\\&+\varepsilon_3PD_3D_3^TPz(t)\\&+\varepsilon_3^{-1}z^T(t)(K+\Delta K)^TE_3^TE_3(K+\Delta K)z(t)\\&+z^T(t)(K+\Delta K)^TW_2(K+\Delta K)z(t)\\&+\begin{bmatrix}z(t)\\ \int_{t-h}^{t}x(s)ds\end{bmatrix}^T\begin{bmatrix}0 & 0\\ \star & F+F^T\end{bmatrix}\begin{bmatrix}z(t)\\ \int_{t-h}^{t}x(s)ds\end{bmatrix}\\&+h\begin{bmatrix}z(t)\\ \int_{t-h}^{t}x(s)ds\end{bmatrix}^T\begin{bmatrix}0\\ F\end{bmatrix}Q^{-1}[0\;F^T]\begin{bmatrix}z(t)\\ \int_{t-h}^{t}x(s)ds\end{bmatrix}^{T.}\end{aligned} \tag{25}$$

where Lemma 2 is utilized in obtaining the upper bound of $-\int_{t-h}^{t}x^T(s)Qx(s)ds.$

Here, let us choose the parameter T as

$$T=\varepsilon_2^{-1}E_2^TE_2 \tag{26}$$

and define

$$\begin{aligned}\Sigma_1 =\; & PA_0+A_0^TP+PB(K+\Delta K)+(K+\Delta K)^TB^TP\\ & +(\varepsilon_1+\varepsilon_4)PD_1D_1^TP+\varepsilon_2PD_2D_2^{TP}+\varepsilon_1^{-1}E_1^TE_1\\ & +\varepsilon_3^{-1}(K+\Delta K)^TE_3^TE_3(K+\Delta K)\\ & +(K+\Delta K)^TW_2(K+\Delta K).\end{aligned} \tag{27}$$

Then, we have the following inequality

$$V_1(z,t)=\begin{bmatrix}z(t)\\ \int_{t-h}^{t}x(s)ds\end{bmatrix}^TG\begin{bmatrix}z(t)\\ \int_{t-h}^{t}x(s)ds\end{bmatrix} \tag{28}$$

where

$$G=\begin{bmatrix}\Sigma_1+T_1 & -PA_0A_1-T_1A_1\\ \star & G_{22}\end{bmatrix}+h\begin{bmatrix}0\\ F\end{bmatrix}Q^{-1}[0\;F^T] \tag{29}$$

and $G_{22}=A_1^TT_1A_1+\varepsilon_4^{-1}A_1^TE_1^TE_1A_1+F+F^T.$

If $G<0$, then there exists a positive scalar λ, which satisfies

$$\dot V(z,t)\le-\lambda\|z(t)\|^2. \tag{30}$$

Also, if the inequality (13) holds, then we can prove that a positive scalar δ which is less than one exists such that

$$\begin{bmatrix}-\delta M & hA_1^TM\\ \star & -M\end{bmatrix}<0 \tag{31}$$

according to matrix theory. From Lemma 1, if LMI (13) holds, the operator $z(t)$ is stable. By Theorem 9.8.1 in [25], we can conclude that if $G<0$ and LMI (13) hold, then system (10) is asymptotically stable. From Eq. (29), $G<0$ can be represented as

$$\begin{aligned}&\begin{bmatrix}\Sigma_1 & -PA_0A_1\\ \star & \varepsilon_4^{-1}A_1^TE_1^TE_1A_1+F+F^T\end{bmatrix}+h\begin{bmatrix}0\\ F\end{bmatrix}Q^{-1}[0\;F^T]\\&+\begin{bmatrix}I\\ -A_1^T\end{bmatrix}T_1\begin{bmatrix}I\\ -A_1^T\end{bmatrix}<0.\end{aligned} \tag{32}$$

By Fact 1, the inequality (32) is equivalent to

$$\begin{aligned}&\begin{bmatrix}\Sigma_2 & -PA_0A_1 & 0 & hI & I & E_2^T & K^TE_3^T & K^T\\ \star & \begin{pmatrix}\varepsilon_4^{-1}A_1^TE_1^TE_1A_1\\ +F+F^T\end{pmatrix} & F & -hA_1^T & -A_1^T & -A_1^TE_2^T & 0 & 0\\ \star & \star & -h^{-1}Q & 0 & 0 & 0 & 0 & 0\\ \star & \star & \star & -h^{-1}Q & 0 & 0 & 0 & 0\\ \star & \star & \star & \star & -W_1^{-1} & 0 & 0 & 0\\ \star & \star & \star & \star & \star & -\varepsilon_2I & 0 & 0\\ \star & \star & \star & \star & \star & \star & -\varepsilon_3I & 0\\ \star & \star & \star & \star & \star & \star & \star I & -W_2^{-1}\end{bmatrix}\\&+\begin{bmatrix}PB\Delta K+\Delta K^TB^TP & 0 & 0 & 0 & 0 & 0 & \Delta K^TE_3^T & \Delta K^T\\ \star & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \star & \star & 0 & 0 & 0 & 0 & 0 & 0\\ \star & \star & \star & 0 & 0 & 0 & 0 & 0\\ \star & \star & \star & \star & 0 & 0 & 0 & 0\\ \star & \star & \star & \star & \star & 0 & 0 & 0\\ \star & \star & \star & \star & \star & \star & 0 & 0\\ \star & \star & \star & \star & \star & \star & \star I & 0\end{bmatrix}<0\end{aligned} \tag{33}$$

where

$$\begin{aligned}\Sigma_2 =\; & PA_0+A_0^T+PBK+K^TB^TP+(\varepsilon_1+\varepsilon_4)PD_1D_1^TP\\ & +\varepsilon_2PD_2D_2^TP+\varepsilon_3PD_3D_3^TP+\varepsilon_1^{-1}E_1^TE_{1.}\end{aligned}$$

By applying Fact 2 to second matrix on left-hand sides of Eq. (33), we have

$$\begin{bmatrix}PBD_4\\ 0\\ \vdots\\ 0\end{bmatrix}F_4(t)[E_4K\;0\;\cdots\;0]+\begin{bmatrix}K^TE_4^R\\ 0\\ \vdots\\ 0\end{bmatrix}F_4^T(t)[D_4^TB^TP\;0\;\cdots\;0]$$

$$\leq \begin{bmatrix} \begin{pmatrix} \varepsilon_5 PBD_4D_4^TB^TP \\ +\varepsilon_5^{-1}K^TE_4^TE_4K \end{pmatrix} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \tag{34}$$

$$\begin{bmatrix} 0 \\ \vdots \\ 0 \\ E_3D_4 \\ 0 \end{bmatrix} F_4(t)[\,E_4K \;\; 0 \;\cdots\; 0\,] + \begin{bmatrix} K^TE_4^R \\ 0 \\ \vdots \\ 0 \end{bmatrix} F_4^T(t)[\,0 \;\cdots\; 0 \;\; D_4^TE_3^T \;\; 0\,]$$

$$\leq \begin{bmatrix} \varepsilon_6^{-1}K^TE_4^TE_4K & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \varepsilon_6E_3D_4D_4^TE_3^T & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \tag{35}$$

$$\begin{bmatrix} 0 \\ \vdots \\ 0 \\ D_4 \end{bmatrix} F_4(t)[\,E_4K \;\; 0 \;\cdots\; 0\,] + \begin{bmatrix} K^TE_4^R \\ 0 \\ \vdots \\ 0 \end{bmatrix} F_4^T(t)[\,0 \;\cdots\; 0 \;\; D_4^T\,]$$

$$\leq \begin{bmatrix} \varepsilon_7^{-1}K^TE_4^TE_4K & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \varepsilon_7D_4D_4^T \end{bmatrix}. \tag{36}$$

Using the inequalities (34), (35) and (36), the new bound of the inequality (33) is

$$\begin{bmatrix} \Sigma_3 & -PA_0A_1 & 0 & hI & I & E_2^T & K^TE_3^T & K^T \\ \star & \Sigma_4 & F & -hA_1^T & -A_1^T & -A_1^TE_2^T & 0 & 0 \\ \star & \star & -h^{-1}Q & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & -h^{-1}Q & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -W_1^{-1} & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -\varepsilon_2I & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \Sigma_5 & 0 \\ \star & \star & \star & \star & \star & \star & \star I & \Sigma_6 \end{bmatrix} < 0 \tag{37}$$

where

$$\begin{aligned} \Sigma_3 &= \Sigma_2+\varepsilon_5PBD_4D_4^TB^TP+\varepsilon_5^{-1}K^TE_4^TE_4K, \\ &\quad ++\varepsilon_6^{-1}K^TE_4^TE_4K+\varepsilon_7^{-1}K^TE_4^TE_4K, \\ \Sigma_4 &= \varepsilon_4^{-1}A_1^TE_1^TE_1A_1+F+F^T \\ \Sigma_5 &= -\varepsilon_3I+\varepsilon_6E_3D_4D_4^TE_3, \\ \Sigma_6 &= -W_2^{-1}+\varepsilon_7D_4D_4^T. \end{aligned} \tag{38}$$

Let

$$X=P^{-1},\;\; R=hQ^{-1},\;\; Y=KX,\;\; RFR=L. \tag{39}$$

Pre-and post-multiplying both sides of Eq. (37) by $diag\{P,R,R,I,I,I,I,I\}$ leads to

$$\begin{bmatrix} \Sigma_7 & -A_0A_1R & 0 & hX & X & XE_2^T & Y^TE_3^T & Y^T \\ \star & \Sigma_8 & L & -hRA_1^T & -RA_1^T & -RA_1^TE_2^T & 0 & 0 \\ \star & \star & -R & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & -R & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -W_1^{-1} & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -\varepsilon_2I & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \Sigma_5 & 0 \\ \star & \star & \star & \star & \star & \star & \star I & \Sigma_6 \end{bmatrix} < 0$$

where

$$\begin{aligned} \Sigma_7 &= A_0X+XA_0+BY+Y^TB^T+(\varepsilon_1+\varepsilon_4)D_1D_1^T+\varepsilon_2D_2D_2^T \\ &\quad +\varepsilon_3D_2D_2^T+\varepsilon_1^{-1}XE_1^TE_1X+\varepsilon_5BD_4D_4^TB^T \\ &\quad +\varepsilon_5^{-1}Y^TE_4^TE_4Y+\varepsilon_6^{-1}Y^TE_4^TE_4Y+\varepsilon_7^{-1}Y^TE_4^TE_4Y, \\ \Sigma_8 &= \varepsilon_4^{-1}RA_1^TE_1^TE_1A_1R+L+L^T. \end{aligned} \tag{41}$$

By Fact 1, Eq. (40) is equivalent to LMI (12). Therefore, system (1) under controller (7) is asymptotically stable if LMIs (12) and (13) hold.

Next, we derive the sufficient condition, which minimizes the upper bound value of cost function (4). If inequality (12) holds, Eq. (19) can be

$$V(z,t) \leq -(x^T(t)W_1x(t)+u^T(t)W_2u(t)) < 0. \tag{42}$$

Integrating both sides of (42) from 0 to t_f and using initial conditions, we get

$$\begin{aligned} J &\leq V(z,0)-V(z,t_f) \\ &= z^T(0)Pz(0)+\int_{-h}^{0}\int_{s}^{0}\Phi^T(u)Q\Phi(u)duds \\ &\quad +\int_{-h}^{0}\Phi^T(s)T\Phi(s)ds-z^T(t_f)Pz(t_f) \\ &\quad -\int_{t_f-h}^{t_f}\int_{s}^{t_f}x^T(u)Qx(u)duds-\int_{t_f-h}^{t_F}x^T(s)Tx(s)ds \end{aligned} \tag{43}$$

Since we already established the asymptotic stability of closed-loop system (10), when $t_f\to\infty$, the following terms go to zero:

$$z^T(t_f)Pz(t_f)\to 0, \tag{44}$$

$$\int_{t_f-h}^{t_f}\int_{s}^{t_f}x^T(u)Qx(u)duds\to 0, \tag{45}$$

$$\int_{t_f-h}^{t_f}x^T(s)Tx(s)ds\to 0. \tag{46}$$

Therefore, we get the upper bound of cost function (4)

$$\begin{aligned} J &< z^T(0)Pz(0)+\int_{-h}^{0}\int_{s}^{0}\Phi^T(u)Q\Phi(u)duds \\ &\quad +\int_{-h}^{0}\Phi^T(s)T\Phi(s)ds. \end{aligned} \tag{47}$$

In order to obtain the optimum value of guaranteed cost (47), we establish a new upper bound of right terms and minimize it. To do this, we find the upper bound of each term or right hand side in (47).

First of all, we consider the first term of right hand side in Eq. (47). Then we select $\alpha>0$ which satisfies

$$-\alpha+z^T(0)Pz(0)<0. \tag{48}$$

For the second and third terms of right hand side in Eq. (47), it is easy to show that

$$\begin{aligned} \int_{-h}^{0}\int_{s}^{0}\Phi^T(u)Q\Phi(u)duds &= Trace(N_dN_d^TQ) \\ &= Trace(hN_d^TR^{-1}N_d), \end{aligned} \tag{49}$$

$$\begin{aligned} \int_{-h}^{0}\Phi^T(s)Q\Phi(s)ds &= Trace(NN^T(\varepsilon_2^{-1}E_2^TE_2)) \\ &= Trace(N^T(\varepsilon_2^{-1}E_2^TE_2)N). \end{aligned} \tag{50}$$

Then by introducing two design variables $M_1>0$ and $M_2>0$, the minimization of two values given (49) and (50) can be solved by the following inequalities:

$$-M_1+hN_d^TR^{-1}N_d<0, \tag{51}$$

$$-M_2+N^T(\varepsilon_2^{-1}E_2^TE_2)_N<0. \tag{52}$$

By Fact 1, inequalities (48), (51), and (52) are equivalent to LMIs (14), (15), and (16), respectively. Therefore, the guaranteed cost is $J^*=\alpha+Trace(M_1)+Trace(M_2)$. This completes our proof. ■

Remark 1. The matrices D_4 and E_4 determine the non-fragility of the controller gain matrices.

Remark 2. The LMIs (12)-(16) in Theorem 1 can be easily solved by various efficient convex algorithms. In this paper, we utilize Matlab's LMI Control Toolbox [27] which implements interior-point algorithms. These algorithms are significantly faster than classical convex optimization algorithms [26].

4. Numerical Examples

Example 1: Consider the uncertain time-delay system studied in [15]:

$$\dot{x}(t)=(A+\Delta A)x(t)+(A_1+\Delta A_1)x(t-1)+(B+\Delta B)u(t), \tag{53}$$

where system matrices are

$$A=\begin{bmatrix}0 & 1\\ 1 & -2\end{bmatrix},\ A_1=\begin{bmatrix}0 & 0\\ 0.1 & 0.1\end{bmatrix},\ B=\begin{bmatrix}0\\ 1\end{bmatrix}, \tag{54}$$

and parameter uncertainties are

$$D_1=D_2=\begin{bmatrix}0.1 & 0.1\\ 0 & 0\end{bmatrix},\ D_3=\begin{bmatrix}0 & 0\\ 0 & 0.1\end{bmatrix},\quad E_1=E_2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ E_3=\begin{bmatrix}0\\ 1\end{bmatrix}. \tag{55}$$

Let's choose the weighting matrices of cost functino (4) as

$$W_1=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ W_2=1. \tag{56}$$

Assume that the initial functions are $x_1(s)=0.5e^{s/2},\ x_{2(2)}=-e^{s/2}$, for $s\in[-1,0]$. For fair comparison with the work in [15], we take that ΔK is zero matrix, In [15], the gain matrix of guaranteed cost controller and guaranteed cost are $K=[-6.2851\ \ -5.4812]$ and $J^*=3.5073$. However, by applying Theorem 1, the guaranteed cost is $J^*=1.3161$, and the corresponding controller, which guarantees the system (53) is asymptotically stable and minimizes the upper bound value of cost function, is

$$u(t)=[-10.3914\ \ -4.3206]\left(x(t)+\int_{t-1}^{t}A_1x(s)ds\right). \tag{57}$$

This result shows that our method gives a less upper bound of cost function compared with the method proposed by Chen [15] via the delayed feedback.

Example 2: Consider the following uncertain time delay systems:

$$\begin{aligned}\dot{x}(t) &= (A+\Delta A)x(t)+(A_1+\Delta A_1)x(t-0.5)+(B+\Delta B)u(t),\\ \phi(s) &= \begin{bmatrix}e^{s+1}\\ 0\end{bmatrix},\ s\in[-0.5,0],\end{aligned} \tag{58}$$

where system matrices are

$$A=\begin{bmatrix}0 & 0\\ 0 & -1\end{bmatrix},\ A_1=\begin{bmatrix}-1 & -1\\ 0 & -0.9\end{bmatrix},\ B=\begin{bmatrix}0\\ 1\end{bmatrix}, \tag{59}$$

and parameter uncertainties are

$$D_1=D_2=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix},\ D_3=\begin{bmatrix}0\\ 0.2\end{bmatrix},\quad E_1=E_2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ E_3=1. \tag{60}$$

We also consider the weighting matrices of the cost function as

$$W_1=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ W_2=1. \tag{61}$$

Table 1 shows the results of controller gain matrices with respect to the controller uncertainties by applying Theorem 1. From table 1, if we increase the controller gain variations, then the guaranteed cost becomes large, which means the stabilization condition becomes conservative due to the controller gain perturbations ΔK. In table 2, we showed the real cost function value with respect to controller gain uncertainties. From table 2, we can see that even if the controller uncertainties exists, the real cost function values do not have heavily different ones. This shows the proposed method provides non-fragility of the proposed controllers.

Table 1 The guaranteed cost and controller gain matrices with respect to controller uncertainties

Controller Uncertainties	J^*	K
$D_4=0,\ E_4=0$	5.5896	$[0.3079\ \ -27.2925]$
$D_4=0.1,\ E_4=1$	5.6473	$[0.1024\ \ -11.4240]$
$D_4=0.2,\ E_4=1$	5.7753	$[0.0708\ \ -2.9207]$
$D_4=0.3,\ E_4=1$	5.9003	$[0.0474\ \ -1.4267]$
$D_4=0.4,\ E_4=1$	6.0036	$[0.0297\ \ -0.7750]$

Table 2 Real cost function value with respect to controller uncertainties

Controller Uncertainties	Controller Gain	J
$D_4=0,\ E_4=0$	$[0.3079\ \ -27.2925]$	3.5907
$D_4=0.1,\ E_4=1$	$[0.1024\ \ -11.4240]+\Delta K$	3.5911
$D_4=0.2,\ E_4=1$	$[0.0708\ \ -2.9207]+\Delta K$	3.5872
$D_4=0.3,\ E_4=1$	$[0.0474\ \ -1.4267]+\Delta K$	3.5883
$D_4=0.4,\ E_4=1$	$[0.0297\ \ -0.7750]+\Delta K$	3.5907

5. Conclusions

In this paper, a delayed feedback non-fragile guaranteed cost controller design method for uncertain linear systems with time delay has been proposed. An optimization problem, which can be solved effectively by optimizatin algorithms, is expressed in terms of LMIs to design the controller with feedback of the current state ans past history of the state. This controller stabilizes the closed-loop system and minimizes the upper bound value of cost function. Two examples showed the effectiveness of our proposed method.

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