상수관망의 파이프 파괴확률 산정을 위한 신뢰성 해석

Reliability Analysis for Probability of Pipe Breakage in Water Distribution System

권혁재*·이철응
Hyuk Jae Kwon* · Cheol Eung Lee
강원대학교 토목공학과

(2008년 9월 26일 접수; 2008년 11월 3일 수정; 2008년 11월 5일 채택)

Abstract

Water pipes are supposed to deliver the predetermined demand safely to a certain point in water distribution system. However, pipe burst or crack can be happened due to so many reasons such as the water hammer, natural pipe ageing, external impact force, soil condition, and various environments of pipe installation. In the present study, the reliability model which can calculate the probability of pipe breakage was developed regarding unsteady effect such as water hammer. For the reliability model, reliability function was formulated by Barlow formula. AFDA method was applied to calculate the probability of pipe breakage. It was found that the statistical distribution for internal pressure among the random variables of reliability function has a good agreement with the Gumbel distribution after unsteady analysis was performed. Using the present model, the probability of pipe breakage was quantitatively calculated according to random variables such as the pipe diameter, thickness, allowable stress, and internal pressure. Furthermore, it was found that unsteady effect significantly increases the probability of pipe breakage. If this reliability model is used for the design of water distribution system, safe and economical design can be accomplished. And it also can be effectively used for the management and maintenance of water distribution system.

Key words : Distribution system, Pipe breakage, Reliability analysis, Unsteady flow

주 제 어 : 상수관망, 파이프 파괴, 신뢰성 해석, 부정류

1. INTRODUCTION

Water distribution system is one of the important essential links in modern society. However, pipe breakage or leakage can be happened due to so many reasons such as external impact force, pipe ageing and erosion or corrosion of pipe wall. Furthermore, various environments of the ground and given conditions of the pipe installation can cause the pipe breakage. Therefore, causes of pipe breakage should

*Corresponding author Tel : +82-33-250-6241, Fax : +82-33-244-6239, E-mail : hyukjae68@hotmail.com(Kwon, H.J.)

be defined and used for the design of water distribution system. These causes of pipe breakage should be considered as the uncertain factors. So, the accurate reliability model should be developed to estimate the probability of pipe breakage.

Some of researchers presented statistical models which can predict the probability of pipe breakage during the service period as the evaluations of structural status for water distribution system. However, these models failed to accurately predict the probability of pipe breakage because of the absence of practically observed data. To overcome this limitation, Mailhot et al. (2000) presented a statistical model for urban water distribution system which has the short period of record for pipe breakage. And they also suggested the application of model for the real city.

Kirmeyer et al. (1994) reported that 16 times of pipe breakage were happened in 100km of pipe annually in United States of America. The report of National Research Council Canada (1995) also showed that 9.5 times of pipe breakage in 100km of pipe were happened from the year 1992 to 1993.

Sudden stoppage of pump station due to electrical shortage or valve opening/closure can make the excessively high pressure or low pressure. The excessive high pressure can make huge economical damage because of the loss of drinking water due to pipe breakage or leakage. It also makes the citizen must put up with the unpleasant pipe repair or replacement time and increased price of drinking water. However, it is hard to predict the breakage or leakage due to natural pipe ageing, unsteady flow, or external impact force. Therefore, the accurate method to estimate the probability of pipe breakage has to be developed using the statistical approach (Modarre, 1999; Frankel, 1988). It is now very difficult to study these researches effectively and efficiently since these researches are confronted by so many engineering obstacles. For example, it is hard to determine the status of erosion or corrosion of pipe installed at underground and difficult to analyze the unsteady effect in pipe. Therefore, the development of reliability model is indispensable to accurately calculate the probability of pipe breakage. In the present study, the reliability model which can calculate the probability of pipe breakage was developed regarding unsteady effect. The present reliability model can contribute to the planning and design of water distribution system. And it also can contribute to management and maintenance of water distribution system by determining which pipe is prior to be replaced or repaired.

2. ANALYSIS OF UNSTEADY FLOW

In the previous study, many numerical schemes were developed and utilized for analysis of transient flow. The method of characteristics model (Wylie, 1984; Chauhdry, 1979; Karney and McInnis, 1992;

Table 1. Properties of pipe network

Pipe No.	Length(m)	Diameter(cm)	Junction No.	Demand(m^3/s)	Elevation(m)
1	300	35	1	0	150
2	50	35	2	0.08	140
3	300	35	3	0.08	130
4	200	40	4	0.06	140
5	300	30	5	0.06	140
6	400	35	6	0.1	130
7	400	35	7	0	130
8	300	30	8	0.11	30
9	300	40	9	0.12	140
10	250	30	10	0.12	140
11	200	30	11	0	135
12	300	30	12	0	130
13	300	25	13	0	300
14	150	25	14	0	300
15	250	30			

상하수도학회 논문
22권 6호, pp. 609-617, 12월, 2008

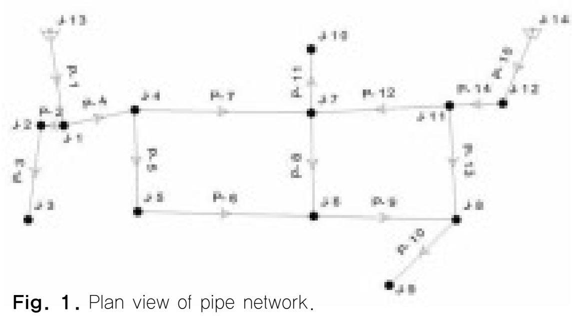

Fig. 1. Plan view of pipe network.

Kwon and Lee, 2008) was chosen for the present study since the method of characteristics model is numerically stable, accurate, and has short computation time. Equation of motion and continuity equation (Kwon, 2005) for the method of characteristics model can be summarized as follows:

$$\frac{\partial Q}{\partial t}+gA\frac{\partial H}{\partial x}+\frac{f}{2DA}Q|Q|=0 \quad (1)$$

$$\frac{c^2}{gA}\frac{\partial Q}{\partial x}+\frac{\partial H}{\partial t}=0 \quad (2)$$

where, Q represents the flow rate, H is the pressure head, A is the cross-sectional area of the pipe, c is the speed of the pressure wave, and f is the Darcy-Weisbach friction coefficient. For the present study, only the friction loss was considered for the head loss coefficient. Therefore, it was assumed that any other head loss effects such as minor losses are negligible for the present computer simulations. 0.03 of Darcy-Weisbach friction coefficient was used for the entire pipes. In the present study, $c\Delta t/\Delta x$=1 was used for the stability condition through out the whole computations. Δx=1.524m and Δt=0.0015s were used for the computations. Therefore, 1,016m/s of the speed of pressure wave was used. **Table 1** shows the pipe length, pipe diameter, junction demand, and junction elevation used for the present study.

Fig. 1 shows the small pipe network chosen for the present study. The small pipe network is consisted of 2 reservoirs, 15 pipes, and 12 junctions as presented in **Table 1** Total 78 simulations of unsteady analysis for 4 cases have been conducted as presented in **Table 2** For the unsteady analysis, water hammer has been intentionally generated by sudden valve closure to simulate the unsteady flow. Valve at junctions which have a certain demand was suddenly closed with three different valve closure time as the Case 1. Therefore, Case 1 contains 24 simulations. And then, valve at J-5 was closed together with valve at the other junction having the three different valve closure times as the Case 2 which contains 18 simulations. Valve at J-6 was closed together with valve at the other junction having three different valve closure times as the Case 3. Similarly, valve at J-10 was closed together with valve at the other junction having three different valve closure times as the Case 4.

Fig. 2 shows the pressure time history at J-7 as the results of unsteady analysis. **Fig. 2(a)** shows the pressure time history at J-7 when the control valve at J-2 was closed in 0.3, 0.6, and 1.2 seconds as the Case 1. When the valve was closed in 0.3s, the maximum pressure wave height was 40m which is 20m higher than when the valve was closed in 1.2s. As a result of the Case 2, **Fig. 2(b)** shows the pressure time history at J-7 when the control valves at J-5 and J-2 were simultaneously closed in 0.3, 0.6, and 1.2s. It was also observed that there's large difference in pressure wave height depending on the speed of valve closure. The maximum pressure wave height was about 70m when the valves were closed in

Table 2. Total cases of simulation

Valve closure (0.3s, 0.6s, 1.2s)			
CASE 1	CASE 2	CASE 3	CASE 4
J-2	J-5, J-2	J-6, J-2	J-10, J-2
J-3	J-5, J-3	J-6, J-3	J-10, J-3
J-4	J-5, J-4	J-6, J-4	J-10, J-4
J-5	J-5, J-8	J-6, J-5	J-10, J-6
J-6	J-5, J-9	J-6, J-8	J-10, J-8
J-8	J-5, J-10	J-6, J-9	J-10, J-9
J-9			
J-10			

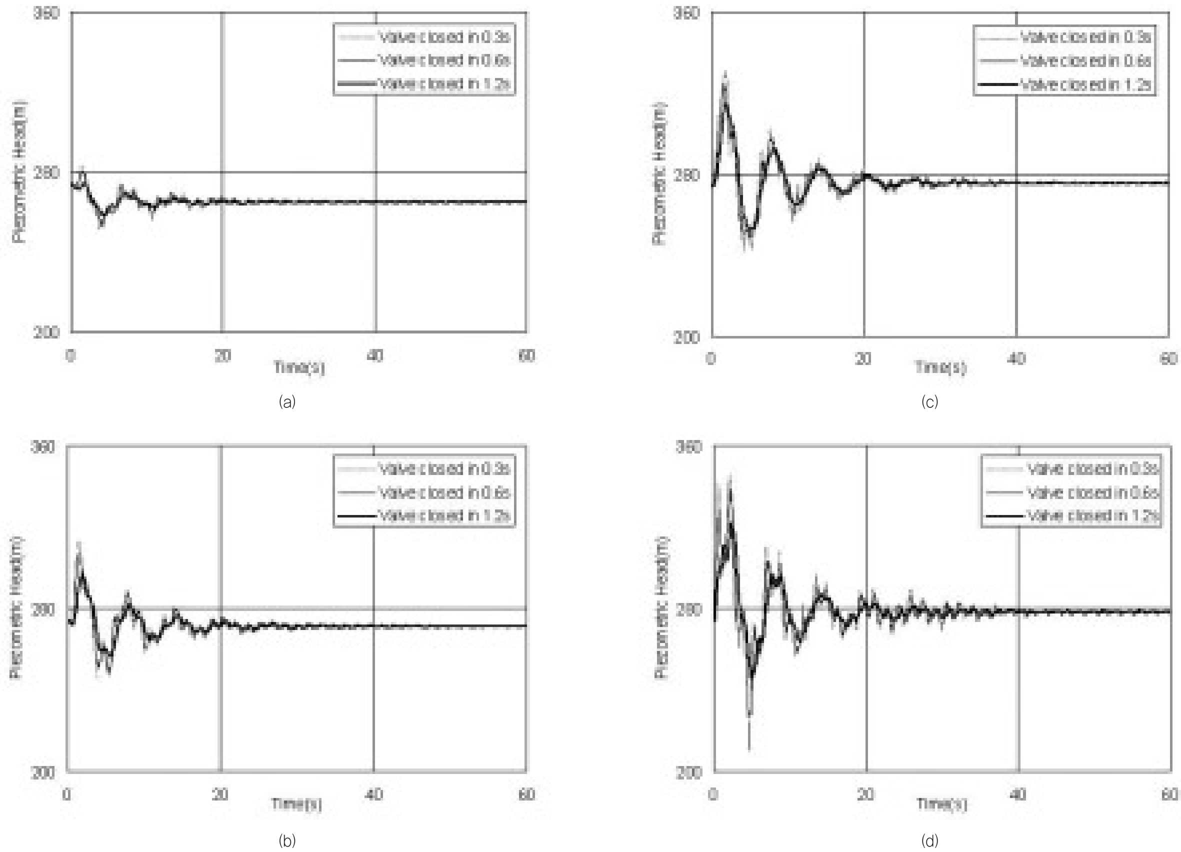

Fig. 2. Pressure time history at J-7 for (a) case 1, (b) case 2, (c) case 3, and (d) case 4.

0.3s and it was originally 45m when the valves were closed in 1.2s. **Fig. 2(c)** shows that the pressure time history at J-7 when the control valves at J-6 and J-2 were closed in 0.3, 0.6, and 1.2s as the Case 3. Similarly, **Fig. 2(d)** shows the pressure time history at J-7 when the control valves at J-10 and J-2 were closed in 0.3, 0.6, and 1.2s as the Case 4. When the two control valves were simultaneously closed, the higher pressure fluctuations were observed. Furthermore, the maximum pressure wave height of the Case 4 was the highest because the demand at J-10 is the biggest. From the results of unsteady analysis, it was observed that the biggest pressure fluctuation was observed during the first 20 seconds of simulation. For the next 20 seconds, it reached to reduction period. Finally, it reached almost the steady state for the last 20 seconds of simulation. The results of unsteady simulations were used to find the statistical distribution of internal pressure for reliability model.

3. RELIABILITY MODEL

Reliability analysis can be categorized as Level II and Level III according to the given assumptions and methodology. Level III is the method that can directly calculate the probability of failure using random variables which can affect the stability of structures without any assumptions. Monte-Carlo Simulation is one of Level III methods. Level II is the method that calculates reliability index, using load and resistance function and estimate the probability of failure. In this method, it is assumed that random variables are following the certain statistical distribution function such as normal distribution (Ang and Tang, 1984; Modarre, 1999; Frankel, 1988). Level II can be classified as FORM (First-Order Reliability Method) and SORM (Second-Order Reliability Method) according to existence of non-linearity for the calculations of mean, variance, and statistical properties of load and resistance functions. FORM

상하수도기술 논문
22권 6호, pp. 609-617, 12월. 2008

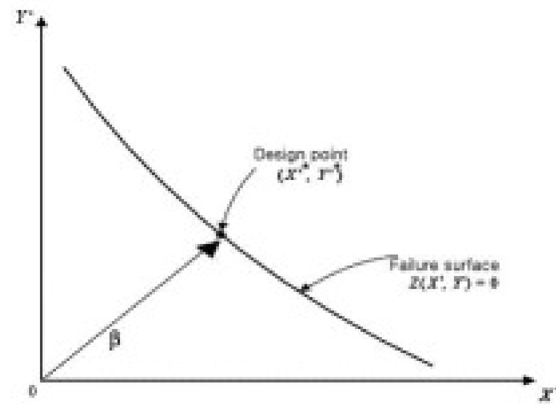

Fig. 3. Failure surface and design point.

can be classified as FMA (First-Order Mean Value Approach) and FDA (First-Order Design Point Approach) according to existence of repeat calculation for the design point on failure surface. However, FDA and FMA can be applied only if load and resistance functions are independent and variables should follow normal distribution. Therefore, AFDA (Approximate Full Distribution Approach) was developed to analyze the random variables which follow non-normal distribution function.

For the present study, reliability function was established by the Barlow formula as shown in Eq. (3).

$$Z = 2\sigma_{\alpha} t - pD \tag{3}$$

σ_{α} is allowable stress of pipe, t is pipe thickness, D is pipe diameter, and p is internal pressure. At the reliability function, Z<0 means failure state, Z<0 means safe state, and Z=0 means limit state. Reliability function can quantitatively estimate the probability of failure for Z≤0 using Eq. (4).

$$P_f = P(Z \le 0) \tag{4}$$

For the present study, AFDA method of FORM (Level II) was applied to calculate the probability of pipe breakage since the non-linearity effect is not significant. As shown in **Fig. 3**, the reliability index β is the minimum distance from the origin to the design point on the failure surface. Therefore, the reliability index β should be determined by repeat calculations. When the reliability index is calculated, normalization procedure is necessary since the invariability of reliability index is required. At the first, the directional cosines should be defined by Eq. (5) to calculate the new design points as shown in Eq. (6). For the first iteration, the mean values of random variables were used for design points to calculate Eq. (5).

$$\alpha^*_{xi} = \frac{\left(\frac{\partial Z}{\partial x'_i}\right)_*}{\sqrt{\Sigma_i \left(\frac{\partial Z}{\partial x'_i}\right)_*}} \tag{5}$$

where, $x'_i = (x_i - \mu_{x_i})/\sigma_{xi}$, $x_i = \sigma_{\alpha}, t, p, D$, μ_{x_i}, and are the means and standard deviations of each random variables. Now, the design points can be found by Eq. (6).

$$\sigma^*_{\alpha} = \mu_{\sigma_\alpha} - \alpha^*_{\sigma_\alpha} \beta \sigma_{\sigma_\alpha} \tag{6a}$$

$$t^* = \mu_1 - \alpha^*_1 \beta \sigma_1 \tag{6b}$$

$$p^* = \mu_p - \alpha^*_p \beta \sigma^N_p \tag{6c}$$

$$D^* = \mu_D - \alpha^*_D \beta \sigma_D \tag{6d}$$

where, μ^N_p and σ^N_p are the mean and standard deviation of equivalent normal distribution of internal pressure. μ^N_p and σ^N_p can be determined by Rosenblatt transform as shown in Eq. (7).

$$\mu^N_p = p^* - \sigma^N_p \Phi^{-1}[F_p(x^*)] \tag{7a}$$

$$\sigma^N_p = \frac{\phi\{\Phi^{-1}[F_p(x^*)]\}}{f_p(x^*)} \tag{7b}$$

where, $F_p(x)$ and $f_p(x)$ are the Gumbel distribution function for the internal pressure as shown in Eq. (8).

$$F_p(x) = \exp[-e^{-\kappa(x-\lambda)}] \tag{8a}$$

$$f_p(x) = \kappa \exp[-\kappa(x-\lambda) - e^{-\kappa(x-\lambda)}] \tag{8b}$$

where, scale parameter $\kappa = \pi/(\sqrt{6}\sigma_p)$, location

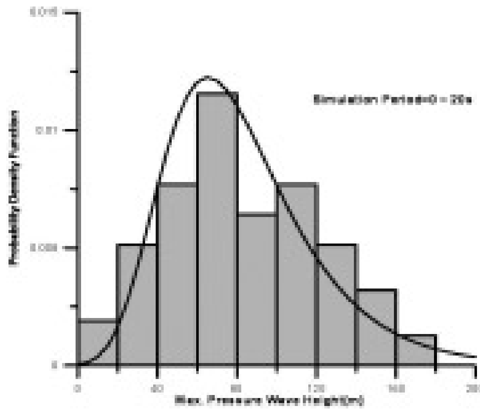

Fig. 4(a). Statistical distribution of the maximum pressure wave heights after water hammer is produced in pipe network (0 ~ 20s).

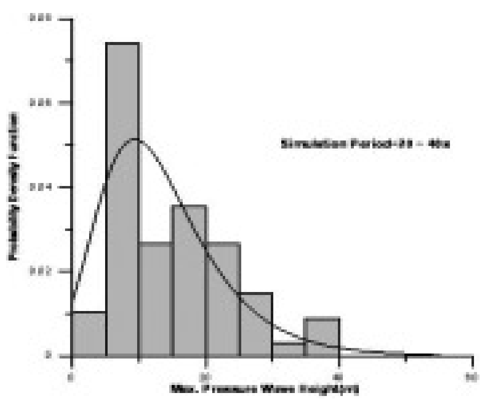

Fig. 4(b). Statistical distribution of the maximum pressure wave heights after water hammer is produced in pipe network (20 ~ 40s).

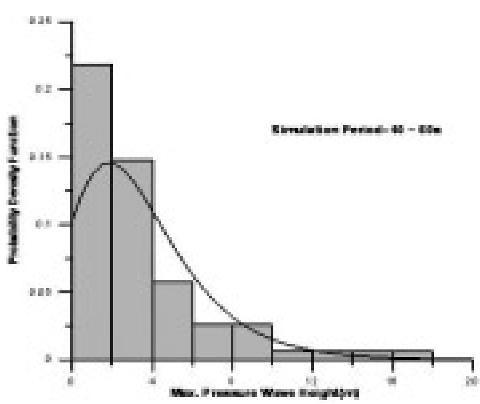

Fig. 4(c). Statistical distribution of the maximum pressure wave heights after water hammer is produced in pipe network (40 ~ 60s).

parameter $\lambda=\mu_p-(0.577/\kappa)$.

If Eq. (6) is inserted into Eq. (3), the limit-state equation can be defined as Eq. (9).

$$2(\mu_{\sigma_\alpha} - \alpha^*_{\sigma_\alpha}\beta\sigma_{\sigma_\alpha})(\mu_t - \alpha^*_t\beta\sigma_t)-(\mu_p - \alpha^*_p\beta\sigma^N_p)(\mu_D - \alpha^*_D\beta\sigma_D) = 0 \quad (9)$$

Now, the pertinent solution for β should be found. And the new design points can be obtained by inserting β into Eqs. (5) and (6). Iteration should be repeated until β is sufficiently converged.

In the present study, 78 cases of unsteady simulations were performed as shown in **Fig. 2** Results of unsteady simulations were used to define the statistical distribution for internal pressure. The statistical distribution for unsteady internal pressure was found by the maximum pressure wave heights in the first 20 seconds of simulation as shown in **Fig. 4(a)** From the results, it was found that the statistical distribution for the maximum pressure wave heights is very well matched with Gumbel distribution. **Fig. 4(b)** shows the statistical distribution for the maximum pressure wave heights for the next 20 seconds of simulation. **Fig. 4(c)** shows the statistical distribution for the maximum pressure wave heights during the last 20 seconds of simulation. From these results, it was found that the probability distribution function for the unsteady pressure has a good agreement with the Gumbel distribution function. It was found that the mean of internal pressure μ_p=18.3kg/cm^2, COV=0.566, and standard deviation of internal pressure σ_p=10.36kg/cm^2.

Table 3 shows the statistical properties and distributions of random variables for reliability function. Gumbel distribution function was used for the statistical distribution of internal pressure in the present study. AFDA model for the probability of pipe breakage was re-evaluated by MCS (Monte Carlo Simulation). The statistical properties and distributions in **Table 3** were used to realize random

Table 3. Statistical properties and distributions for random variables (P-8)

	σ_α(kg/cm^2)	t(cm)	D(cm)	p(kg/cm^2)	
				w/o unsteady effect	w/ unsteady effect
Mean	1000	0.38	30	10	18.3
COV	0.1	0.1	0.1	0.1	0.566
Distribution	Normal	Normal	Normal	Gumbel	Gumbel

상하수도학회 논문
22권 6호, pp. 609-617, 12월. 2008

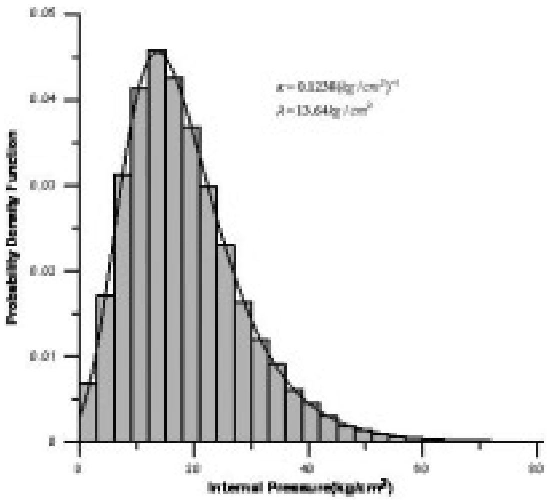

Fig. 5. Comparison of theoretical distribution for pressure with simulated pressure data.

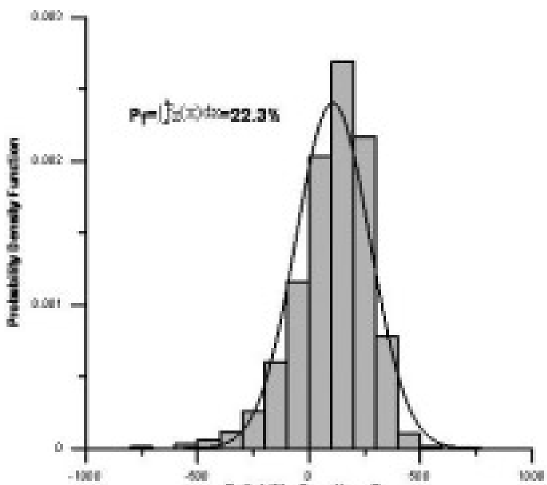

Fig. 6. The probability distribution of reliability function calculated by MCS model.

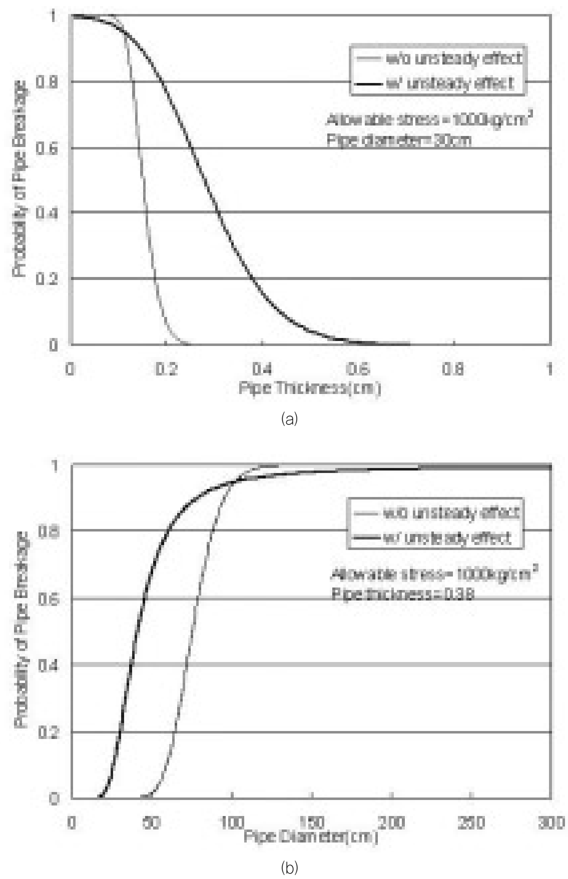

Fig. 7. Probability of pipe breakage for P-8 according (a) pipe thickness (b) pipe diameter.

variables of reliability function. Therefore, 20,000 random numbers for each 4 random variables of reliability function were generated in this study. In the Gumbel distribution function, κ was defined as $0.1238(kg/cm^2)^{-1}$ and λ was defined as $13.64kg/cm^2$.

Fig. 5 shows the comparison of the theoretical Gumbel distribution and distribution of generated random numbers for the internal pressure. As shown in **Fig. 5** it was found that the generated internal pressure data have a good agreement with the theoretical Gumbel distribution. Furthermore, the probability distribution for internal pressure generated by MCS is almost the same with the distribution for maximum pressure wave heights. **Fig. 6** shows the probability density function of reliability function, Z. If the probability density function in **Fig. 6** is integrated from $-\infty$ to 0, the probability of pipe breakage for P-8 can be obtained. When the reliability analysis of MCS was performed, the probability of pipe breakage was 22.3%. When the reliability analysis using AFDA was performed with the same conditions of MCS, the probability of pipe breakage was 22.5%. Therefore, it was confirmed that the results of AFDA have a good agreement with the results of MCS. The AFDA developed in the present study was re-evaluated and confirmed by MCS as shown in **Table 4**.

Table 4. Probability of pipe breakage with unsteady effect

	AFDA	MCS
P_f(%)	22.5	22.3

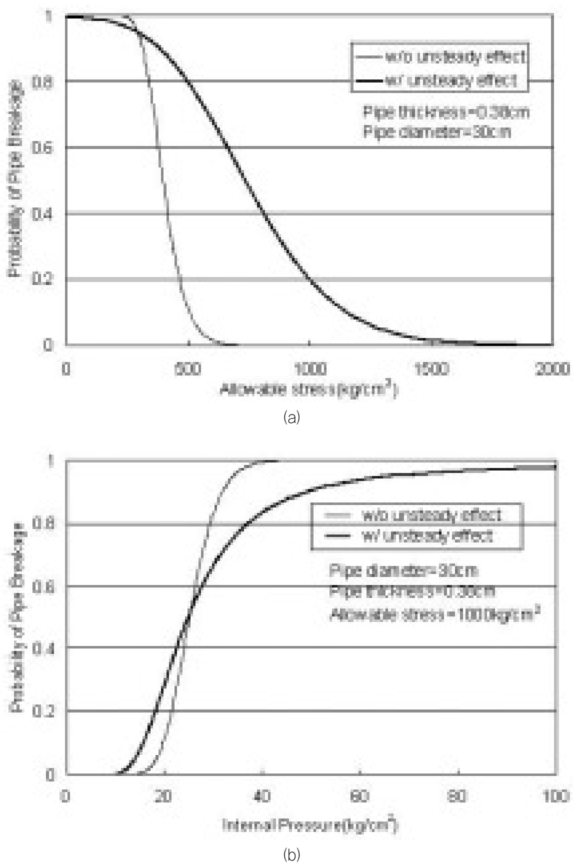

Fig. 8. Probability of pipe breakage for P-8 according (a) allowable stress (b) internal pressure.

4. PROBABILITY OF PIPE BREAKAGE

Fig. 7(a) shows that the probability of pipe breakage for P-8 according to pipe thickness when allowable stress is 1000kg/cm^2 and pipe diameter is 30cm. The thick and thin lines show the probability of pipe breakage with and without unsteady effect, respectively. In this case, internal pressure is 18.3kg/cm^2 added by 8.3kg/cm^2 which is the mean value of the maximum pressure wave heights. And 0.566 of COV for unsteady internal pressure was used for the present study. The probability of pipe breakage is 0% when the pipe thickness is 0.38cm without unsteady effect. However, the probability of pipe breakage reached to 22.5% when the pipe thickness is 0.38cm with unsteady effect. **Fig. 7(b)** shows the probability of pipe breakage for P-8 according to pipe diameter when the allowable stress is 1000kg/cm^2 and pipe thickness is 0.38cm. The probability of pipe breakage is 0% when the pipe diameter is 30cm without unsteady effect but it is 22.5% with unsteady effect. **Fig. 8(a)** shows the probability of pipe breakage for P-8 according to the allowable stress when the pipe diameter is 30cm and pipe thickness is 0.38cm. With unsteady effect, the probability of pipe breakage is 22.5% when the allowable stress is 1000kg/cm^2. But without unsteady effect, the probability of pipe breakage is 0% when the allowable stress is 1000kg/cm^2. **Fig. 8(b)** shows the probability of pipe breakage for P-8 according to internal pressure when the pipe diameter is 30cm, thickness is 0.38cm, and allowable stress is 1000kg/cm^2. With unsteady effect, the probability of pipe breakage is 10% when the internal pressure is 15kg/cm^2.

From the results of reliability analysis, it was found that the unsteady effect significantly increase the probability of pipe breakage in every occasion. Therefore, when the distribution system is planned and designed, it is necessary to reduce the unsteady effect for minimizing the probability of pipe breakage. One of the alternatives to minimize the unsteady pressure oscillation could be the installation of artificial damper for water hammer in water distribution system. Furthermore, it is necessary to educate the water industry agencies, engineers, and managers to be extremely careful when they are working with the water valves.

5. CONCLUSIONS

The reliability analysis model which can quantitatively calculate the probability of pipe breakage was developed. In the present study, the statistical distribution for the maximum pressure wave heights was defined by the results of 78 unsteady simulations. It was found that the Gumbel distribution is very well matched with the distribution of the maximum pressure wave heights in unsteady flow. Therefore, the Gumbel distribution was used for the internal pressure of reliability function to calculate the probability of pipe breakage. In the present study, AFDA was used for the reliability model and confirmed by MCS.

상하수도학회 논문
22권 6호, pp. 609-617, 12월. 2008

The probability of pipe breakage in water distribution system was calculated. When the pipe thickness is 0.38cm, the probability of pipe breakage was about 22.5% with unsteady effect and 0% without unsteady effect. When the pipe diameter is 30cm, the probability of pipe breakage was 22.5% with unsteady effect and 0% without unsteady effect. When the allowable stress is 1000kg/cm^2, the probability of pipe breakage was 22.5% with unsteady effect and 0% without unsteady effect. Furthermore, it was found that unsteady effect significantly increases the probability of pipe breakage. Therefore, unsteady effect must be considered for the reliability analysis of the water distribution system. Reliability analysis model developed in the present study can be used for the various fields such as the design, planning, management, and maintenance of water distribution system.

Using the present reliability analysis model, it can be possible to find the specific pipe which contains the high probability of pipe breakage in water distribution system. If this model is used for the design of water distribution system, safe design can be accomplished finding the pipes which can be easily burst. This model also can be used for the maintenance and management of water distribution system. Therefore, it can be decided that which pipe has a priority to be replaced or repaired.

KSWW

REFERENCES

1. Ang, A. and Tang, W. H. (1984) ***Probability Concepts in Engineering Planning and Design***. John Wiley and Sons, Inc. New York.
2. Chaudhry, H. M. (1979) ***Applied Hydraulic Transients***. Van Nostrand Reinhold, New York.
3. Frankel, E. G. (1988) ***Systems Reliability and Risk Analysis***. Klume Academic Publishers.
4. Karney, B. W. & McInnis, D. (1992) "Efficient calculation of transient flow in simple pipe networks." ***Journal of Hydraulic Engineering***, **118** (7), 1014-1031.
5. Kirmeyer, G., Richards, W., and Smith, C. D. (1994) ***An Assessment of Water Distribution Systems and Associated Research Needs. Denver***, CO: AWWARF
6. Kwon, H. J. & Lee, J. J. (2008) "Computer and experimental Models of transient flow in a piping system involving backflow preventers." ***Journal of Hydraulic Engineering***, Vol. **134**, No. 4, pp. 426-434.
7. Kwon, H. J. (2005) "***Transient flow in water distribution system***." Ph..D. Thesis, University of Southern California, Los Angeles, CA.
8. Mailhot, A., Pelletier, G., Noel, J-F., and Villeneuve, J-P. (2000) "Modeling the evolution of the structural state of water pipe networks with brief recorded pipe break histories: Methodology and application." ***Water Resources Research***, Vol. **36**, No. 10, pp. 3053-3062.
9. Modarres, M. (1999) ***Reliability Engineering and Risk Analysis***. Marcel Dekker.
10. National Research Council Canada (1995) Water Main Breaks Data on Different Pipe Materials for 1992-1993. Canada.
11. Watters, G. W. (1984) ***Analysis and Control of Unsteady Flow in Pipe lines***. Butterworths, Boston.
12. Wylie, B. E. and Streeter, V. L. (1993) ***Fluid Transients in Systems***. FEB Press, Ann Arbor, Mich.