A Realization of Reduced-Order Detection Filters

Yongmin Kim and Jaehong Park

Abstract: In this paper, we deal with the problem of reducing the order of the detection filter for the linear time-invariant system. Even if the detection filter is generally designed in the form of full order linear observer, we show that it is possible to reduce its order when the response of fault signals is limited to a subspace of the estimation state space. We propose a method to extract the subspace using the observer canonical form considering the dynamics related to the remaining subspace acts as a disturbance. We designed a reduced order detection filter to reject the disturbance as well as to guarantee fault detection and isolation. A simulation result for a 5th order system is presented as an illustrative example of the proposed design method.

Keywords: Fault detection filter, invariant zero, model reduction, observer canonical form.

1. INTRODUCTION

The detection filter is one of the efficient fault detection and isolation (FDI) systems that is described in the form of the Luenberg observer with an additional condition that the independent subspace of the error state space, which is called the detection space, is associated with each fault. Several research results on the detection filter with their own definitions of the detection space have been presented by the mid-1990s [1-4], and the robustness issue has become an important research topic for the application of detection filters in the environment where system is affected by disturbances or noises [5-9].

Here we pay attention to the fact that most of the detection filters have been proposed based on full order observers. However, if the range of faults is not equal to the state space, i.e. the number of system modes directly driven by the faults is less than the system order, it is possible to designed a reduced-order detection filter. As far as we know, the research results on this topic have not been actively presented.

In this paper, we propose a method to obtain reduced-order detection filters by extracting the aforementioned range using the observer canonical form. We have already presented the result on the relation between the detection space and the observability indices in [11]. Applying this result, we extract the minimum detection space including the range of the faults and design a detection filter with respect to the subspace.

Since the dynamics corresponding to the subspace not observed by this filter acts as a disturbance, the detection filter must reject this disturbance for the sake of its improved performance. In addition, the observed subsystem should be mutually detectable [1-3] for arbitrary assignment of error dynamics. The two existence conditions of the reduced-order detection filter are as follows: first, disturbance rejection for the unobserved subsystem and second, mutual detectability for observed subsystem.

As an example, we present the design procedure to obtain a 3rd order detection filter for a 5th order system together with its simulation results to show that the fault isolation is available using the reduced-order detection filter.

Since the proposed method is clear and simple from an analytical point of view, it can reduce computing resources required in the implementation of detection filter, in particular, for large-scaled systems.

2. PRELIMINARIES

3.1. Detection filter

In this section, we present preliminary theory of the detection filter before we discuss the main topic. The detection filter is normally described based on the following linear time-invariant system:

$$\dot{x}(t) = Ax(t) + Bu(t) + F\mu(t),\\ y(t) = Cx(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^p$ and $y(t) \in \mathbb{R}^q$ are the state, input and output of the system, respectively. A, B and C have appropriate dimensions. $F\mu(t)$ is the model of faults, where $\mu(t) \in \mathbb{R}^r$ is the function

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vector which describes time evolution of the faults and the column vectors of $F \in \mathbb{R}^{n \times r}$, $f_i (i = 1, \cdots, r)$, represent the directions through which the faults enter the system. We define these vectors as the fault event vectors. For the simplicity of discussion, we assume that the pair (A, C) is observable and not degenerate. We also assume that F is output separable, i.e., $\mathrm{rank}(CF) = r$ [3].

The detection filter is implemented as the following full-order observer:

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + D(y(t) - \hat{y}(t)), \\ \hat{y}(t) &= C\hat{x}(t), \end{aligned} \tag{2}$$

where $\hat{x}(t) \in \mathbb{R}^n$ and $\hat{y}(t) \in \mathbb{R}^q$ are the estimated state and output, respectively. D is the observer gain.

We define the estimation error as $e(t) \triangleq x(t) - \hat{x}(t)$ and the output of the detection filter as $\varepsilon(t) \triangleq y(t) - \hat{y}(t)$. Then, they satisfy the following equation.

$$\begin{aligned} \dot{e}(t) &= (A - DC)e(t) + F\mu(t) \\ \varepsilon(t) &= Ce(t) \end{aligned} \tag{3}$$

If all the eigenvalues of $(A - DC)$ are in the LHP and $\mu(t) = 0$, $e(t) \to 0$ and $\varepsilon(t) \to 0$ as $t \to \infty$ for any initial condition of $e(0)$. Meanwhile, if $\mu(t) \neq 0, e(t)$ and $\varepsilon(t)$ have non-zero values, which enables us to detect the occurrence of faults. In the detection filter, the gain D is designed to make $\varepsilon(t)$ maintain fixed directions in the output space with respect to each element of $\mu(t)$. Here we utilize the smallest controllable space of f_i with respect to $(A - DC)$, which we define as the detection space of f_i.

When single fault is considered, the necessary and sufficient condition for satisfying the aforementioned condition is the observability of the pair (A, C) [1]. However, an additional condition is required to accommodate multiple faults. For the case where it is possible to assign detection spaces for all f_i and to assign all the eigenvalues of $(A - DC)$, we say that the faults are mutually detectable. This condition can be described using the invariant zero as follows [2]:

$$\sigma(A, F, C) - \biguplus_{i=1}^{r} \sigma(A, f_i, C) = \varnothing, \tag{4}$$

where $\sigma(\cdot)$ is the set of invariant zeros of the triple and $\uplus$ is the union with any common elements repeated. The gain D for this case is defined as the detection gain.

3.2. Observer canonical form

The observer canonical form, the dual problem of the controller canonical form, can be used to define equivalent class of linear time-invariant systems by showing the dynamics that can be associated with each output [10]. In regards to the detection filter, we analyzed the characteristics of the detection space in relation with the invariant zero with this canonical form [11]. Here we present basic the result that will be referred to in the following discussion. For more details, refer to [10,11].

If we define the observability indices of the respective row vectors of C, $c_i (i = 1, \cdots, q)$, as δ_i, the n row vectors $\{c_i A^j,\ i = 1, \cdots, q,\ j = 1, \cdots, \delta_i\}$ are linearly independent if the pair (A, C) is observable. The equivalent transform matrix to obtain its observer canonical form can be derived from these vectors as follows:

$$Q = \tilde{Q}\left[c_1^T, \cdots, (c_1 A^{\delta_1 - 1})^T, \cdots, c_q^T, \cdots, (c_q A^{\delta_q - 1})^T\right]^T, \tag{5}$$

where $\tilde{Q}$ nth order square matrix whose row vectors represent appropriate linear combination $c_i A^j$. $\tilde{Q}$ may be calculated as the dual problem of obtaining the controller canonical form in [10]. We use $(\bar{A}, \bar{C})$ as the transformation result.

$$\bar{A} = QAQ^{-1} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} & \cdots & \bar{A}_{1q} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{A}_{q1} & \bar{A}_{q2} & \cdots & \bar{A}_{qq} \end{bmatrix}, \tag{6}$$

$$\bar{C} = CQ^{-1} = \begin{bmatrix} \bar{c}_{11} & \bar{c}_{12} & \cdots & \bar{c}_{1q} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{c}_{q1} & \bar{c}_{q2} & \cdots & \bar{c}_{qq} \end{bmatrix}, \tag{7}$$

where the respective submatrices have the following structure:

$$\bar{A}_{ii} = \begin{bmatrix} \times & 1 & 0 & \cdots & 0 \\ \times & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \times & 0 & 0 & \cdots & 1 \\ \times & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad \bar{A}_{ij} = \begin{bmatrix} \times & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \times & 0 & \cdots & 0 \end{bmatrix}, \tag{8}$$

$$\bar{c}_{ii} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad \bar{c}_{ij} = \begin{bmatrix} \times & 0 & \cdots & 0 \end{bmatrix}, \tag{9}$$

where the elements represented by $\times$ are appropriate real numbers. For $\bar{c}_{ij}$, they are equal to zero if $i < j$.

The observer canonical form shows some facts of detection filters such as the relation between observability indices and detection order [11]. In particular, in relation with the main topic, it is possible to determine the system dynamics directly driven by a fault event vector.

3. REDUCED-ORDER DETECTION FILTER

3.1. System reduction

When we transform a system into the observer canonical form, it is possible to extract the subspace which is directly driven by a fault 1 problem matrix defined in the above section and divide it according to the observability indices as follows:

$$\bar{F} = Q F = \begin{bmatrix} \bar{F}_1^T & \bar{F}_2^T & \cdots & \bar{F}_q^T \end{bmatrix}^T, \quad (10)$$

where the row dimensions of $\bar{F}_1, \cdots, \bar{F}_q$ are equal to $\delta_1, \cdots, \delta_q$, respectively.

For the simplicity of discussion, assume that $\bar{F}_i \neq 0 \quad (i = 1, \cdots, q_\delta)$ and $\bar{F}_i = 0 \quad (i = q_\delta + 1, \cdots, q)$. Then, we divide the system into two parts of dimensions $n_1 = \sum_{i=1}^{q_\delta} \delta_i$ and $n_2 = n - n_1$.

$$\bar{A} = QAQ^{-1} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}, \ \bar{C} = CQ^{-1} = \begin{bmatrix} \tilde{C}_1 & \tilde{C}_2 \end{bmatrix},$$
$$\bar{B} = QB = \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \end{bmatrix}, \ \bar{F} = QF = \begin{bmatrix} \tilde{F}_1 \\ 0 \end{bmatrix}, \quad (11)$$

where $\tilde{A}_{11} \in \mathbb{R}^{n_1 \times n_1}$ and the rest of submatrices have appropriate dimensions. Note that we always transform (6), (7) and (10) into the above using an appropriate unitary matrix.

Define a new state vector as $\bar{x}(t) \triangleq Qx(t) = [\bar{x}_1(t)^T \ \bar{x}_2(t)^T]^T$, where $\bar{x}_1(t) \in \mathbb{R}^{n_1}$. Then, the new system is given as follows:

$$\dot{\bar{x}}_1(t) = \tilde{A}_{11}\bar{x}_1(t) + \tilde{A}_{12}\bar{x}_2(t) + \tilde{B}_1 u(t) + \tilde{F}_1 \mu(t), \quad (12a)$$
$$\dot{\bar{x}}_2(t) = \tilde{A}_{21}\bar{x}_1(t) + \tilde{A}_{22}\bar{x}_2(t) + \tilde{B}_2 u(t), \quad (12b)$$
$$y(t) = \tilde{C}_1 \bar{x}_1(t) + \tilde{C}_2 \bar{x}_2(t). \quad (12c)$$

Since $\dot{\bar{x}}_1(t) \in \text{range}(\tilde{F}_1)$, $\mu(t)$ directly drives the trajectory of $\bar{x}_1(t)$, while $\mu(t)$ is filtered by the subsystem of $\bar{x}_2(t)$ in (12b).

Now we propose a detection filter that observes only $\bar{x}_1(t)$.

$$\dot{\hat{\bar{x}}}_1(t) = \tilde{A}_{11}\hat{\bar{x}}_1(t) + \tilde{B}_1 u(t) + \tilde{D}_1(y(t) - \tilde{C}_1 \hat{\bar{x}}_1(t)), \quad (13)$$

where $\hat{\bar{x}}_1(t)$ is the estimated state of $\bar{x}_1(t)$. Even if the order of the detection filter is reduced from n to n_1, there is no restriction on implementation since we use the system input $u(t)$ and the output $y(t)$. Moreover, since $CF = \tilde{C}_1 \tilde{F}_1$, the output separability of the original system is preserved.

With the definition of the estimation error as $\bar{e}_1(t) \triangleq \bar{x}_1(t) - \hat{\bar{x}}_1(t)$, we obtain the following error equation:

$$\dot{\bar{e}}_1(t) = (\tilde{A}_{11} - \tilde{D}_1\tilde{C}_1)\bar{e}_1(t) + (\tilde{A}_{12} - \tilde{D}_1\tilde{C}_2)\bar{x}_2(t) + \tilde{F}_1 \mu(t). \quad (14)$$

In comparison with (3), the additional term of $\bar{x}_2(t)$ exists, which can be considered as a disturbance of this filter. To obtain fault detection and isolation with the reduced-order detection filter in (13), it should satisfy the following two conditions:

1) Disturbance rejection:

$$\tilde{A}_{12} - \tilde{D}_1\tilde{C}_2 = 0.$$

2) Mutual detectability

$$\sigma(\tilde{A}_{11}, \tilde{F}_1, \tilde{C}_1) - \uplus_{i=1}^{r} \sigma(\tilde{A}_{11}, \tilde{f}_{1i}, \tilde{C}_1) = \varnothing,$$

where $\tilde{f}_{1i}$ is the ith column vector of $\tilde{F}_1$.

3.2. Detection filter design

In this section, we discuss the existence condition of the reduced-order detection filter presented in Section 3.1 and present its design procedure. We divide $\bar{C}$ and $\tilde{D}_1$ according to n_1 and n_2 as follows:

$$\bar{C} = \begin{bmatrix} \tilde{C}_{11} & 0 \\ \tilde{C}_{21} & \tilde{C}_{22} \end{bmatrix}, \ \tilde{D}_1 = \begin{bmatrix} \tilde{D}_{11} & \tilde{D}_{12} \end{bmatrix}. \quad (15)$$

As in (8) and (9), since most of the column vectors of $\bar{C}$ are zero vectors, we define the following index set to refer the non-zero vectors.

$$\Im_{m_1, m_2} = \left\{ i \mid i = 1 + \sum_{j=1}^{k} \delta_j, k = m_1 - 1, \cdots, m_2 - 1 \right\} \quad (16)$$

Then, the column vectors of $\bar{C}$ corresponding to $\Im_{1,q}$ are the left-most vectors in the respective blocks.

With the division in (15), the condition of disturbance rejection is changed into

$$\tilde{A}_{12} - \tilde{D}_1\tilde{C}_2 = \tilde{A}_{12} - \tilde{D}_{12}\tilde{C}_{22} = 0. \quad (17)$$

In order to simplify the calculation, we modify $\tilde{A}_{12}$ and $\tilde{C}_{22}$ as follows:

$$\breve{A}_{12} \triangleq \tilde{A}_{12} Q_T, \quad \breve{C}_{22} \triangleq \tilde{C}_{22} Q_T, \quad (18)$$

where $Q_T \in \mathbb{R}^{n_2 \times (q - q_\delta)}$ is defined as

$$Q_T \triangleq \left\{ e_k^{n_2} \mid k \in \Im_{q_\delta + 1, q} \right\}, \quad (19)$$

where $e_k^{n_2}$ is a unit vector of order n_2 that only kth element is equal to one. Q_T is used to collecting the column vectors in $\tilde{A}_{12}$ that are denoted by $\times$ in (8) and non-zero column vectors in $\tilde{C}_{22}$. Since $\breve{\tilde{C}}_{22}$ is a $(q-q_\delta)$ th order square matrix whose diagonal elements are equal to one, it is invertible.
Therefore, the unknown $\tilde{D}_{12}$ can be calculated as follows:

$$\tilde{D}_{12} = \breve{\tilde{A}}_{12}\breve{\tilde{C}}_{22}^{-1}. \tag{20}$$

From the characteristics of the observer canonical form, it can be shown that the structure of $(\tilde{A}_{11} - \tilde{D}_1\tilde{C}_1)$ is the same as that of $\bar{A}_{11}$ except that the elements corresponding to $\times$ in (8), which means that the observability of $(\tilde{A}_{11}, \tilde{C}_{11})$ is preserved irrespective of $\tilde{D}_{12}\tilde{C}_{21}$.

Rewriting (14) as

$$\dot{\bar{e}}_1(t) = ((\tilde{A}_{11} - \tilde{D}_{12}\tilde{C}_{21}) - \tilde{D}_{11}\tilde{C}_{11})\bar{e}_1(t) + \tilde{F}_1\mu(t), \tag{21}$$

we have $\tilde{D}_{11}$ as the design parameter to obtain reduced-order detection filter since $(\tilde{A}_{11} - \tilde{D}_{12}\tilde{C}_{21})$ is determined with (20).

In this stage, the second condition of the existence of the reduced-order detection filter can be presented, which is summarized by the following theorem.

Theorem 1: $\tilde{F}_1$ in $(\tilde{A}_{11}, \tilde{F}_1, \tilde{C}_{11})$ is mutually detectable if and only if F in (A, F, C) is mutually detectable.

Proof: The proof can be given if we show that the same solution for the Rosenbrock system equation to obtain invariant zeros and zero vectors. As given in [13], since the set of invariant zeros are independent of the equivalent transform, we consider matrix using $(\bar{A}, \bar{F}, \bar{C})$ instead of (A, F, C).

$$\begin{bmatrix} \tilde{A}_{11} - \lambda I_{n_1} & \tilde{A}_{12} & \tilde{F}_1 \\ \tilde{A}_{21} & \tilde{A}_{22} - \lambda I_{n_2} & 0 \\ \tilde{C}_1 & \tilde{C}_2 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ w \end{bmatrix} = 0, \tag{22}$$

where I_k is the kth order identity matrix. A complex number λ satisfying the above matrix equation is an invariant zero of the triple $(\bar{A}, \bar{F}, \bar{C})$.
$[v_1^T \ v_2^T]^T \in \mathbb{R}^n$ is the invariant zero vector (direction) associated with λ and $w \in \mathbb{R}^r$ contains the corresponding linear combination coefficients of $\tilde{F}_1$.

Since $[\tilde{C}_1 \ \tilde{C}_2]$ is a lower triangular matrix, if we exclude the column vectors corresponding to $\Im_{1,q}$, the elements of v_1 and v_2 corresponding to $\Im_{1,q}$ are equal to zero from $\tilde{C}_1 v_1 + \tilde{C}_2 v_2 = 0$.
With this result, $\tilde{A}_{21}v_1 = 0$ and $\tilde{A}_{12}v_2 = 0$ since the column vectors of $\tilde{A}_{21}$ and $\tilde{A}_{12}$ excluding $\Im_{1,q}$ are equal to zero. Extracting the term including v_2 in (22), we obtain

$$\begin{bmatrix} \tilde{A}_{22} - \lambda I_{n_2} \\ \tilde{C}_2 \end{bmatrix} v_2 = 0.$$

We know that $v_2 = 0$ if the pair (A, C) is observable considering the characteristics of the observer canonical form.

$$\begin{bmatrix} \tilde{A}_{11} - \lambda I_{n_1} & \tilde{F}_1 \\ \tilde{C}_1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ w \end{bmatrix} = 0 \tag{23}$$

Note that $\tilde{C}_1 v_1 = \tilde{C}_{11} v_1 = 0$ from the fact that the column ranks of $\tilde{C}_{11}$ and $\tilde{C}_{21}$ are the same; we can replace $\tilde{C}_1$ with $\tilde{C}_{11}$. This means that the solution of the system matrix for (A,F,C) can also be applied to $(\tilde{A}_{11}, \tilde{F}_1, \tilde{C}_{11})$, which completes the proof. □

This theorem means that the mutual detectability of the triple (A, F, C) is inherited to the reduced triple $(\tilde{A}_{11}, \tilde{F}_1, \tilde{C}_{11})$. As for the second existence condition, we can conclude that the original system should be mutually detectable in order to arbitrarily assign the eigenvalues of the reduced-order detection filter proposed in (21).

4. DESIGN PROCEDURE

In this section, we summarize the arguments in Section III and present a design procedure for the reduced-order detection filter.

1) Examine the mutual detectability of (A, F, C). If not, modify the system to be mutually detectable.
2) Calculate Q in (5).
3) Calculate $\bar{F}$ in (10). Check that there exists $\bar{F}_i = 0$. If not, stop.
4) Rearrange (A, F, C) as in (11) with Q and an appropriate equivalent transform.
5) Calculate Q_T in (19) and $\tilde{D}_{12}$ in (20).
6) With the triple $(\tilde{A}_{11} - \tilde{D}_{12}\tilde{C}_{21}, \tilde{F}_1, \tilde{C}_{11})$, calculate $\tilde{D}_{11}$ using the well-known design methods.

In the case that (A, F, C) is not mutually detectable, more than two column vectors of F are

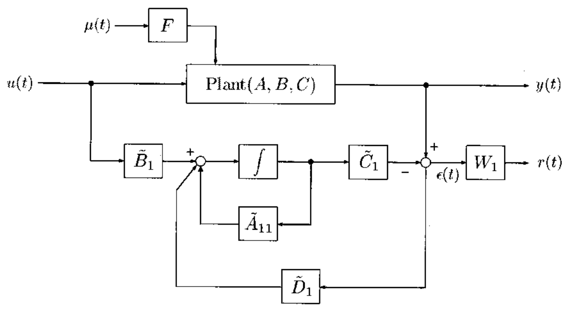

Fig. 1. Structure of the reduced-order detection filter.

associated with one invariant zero. If this invariant zero, which will be a fixed closed-loop eigenvalue that can not be moved by the detection gain, is located in the RHP, the dynamics depicted in (14) become unstable. In this case, we can use the method to expand the system dynamics to increase the geometric multiplicity of the zero. For the details on the invariant zero and mutual detectability, refer to [3].

The structure of the proposed detection filter is given as in Fig. 1. In this figure, W_1 is a transformation making the final residual, generally given as $(\tilde{C}_1\tilde{F}_1)^{\dagger}$ where $\dagger$ is the pseudo- or left inverse.

5. AN ILLUSTRATIVE EXAMPLE

In this section, we present an illustrative example to show the design procedure, we consider the following 5th order system.

$$A=\begin{bmatrix} -0.750 & -0.375 & 1.875 & -1.125 & -0.625 \\ 3.250 & 1.125 & 1.375 & -0.625 & -0.125 \\ 1.750 & -0.125 & -0.375 & 2.625 & 1.125 \\ 0.750 & -0.625 & 1.125 & 0.125 & -0.375 \\ -0.750 & 1.125 & -4.625 & 5.375 & 3.875 \end{bmatrix}, \tag{24}$$

$$C=\begin{bmatrix} 1 & 1 & -1 & 0 & 1 \\ 1 & 1 & -2 & 2 & 2 \\ 0 & -1 & 0 & 3 & 0 \end{bmatrix},\ F=\begin{bmatrix} -0.250 & -0.375 \\ 1.750 & 1.125 \\ 0.250 & -0.125 \\ 0.250 & 0.375 \\ -0.250 & 0.125 \end{bmatrix}. \tag{25}$$

The observability indices of the pair (A,C) are given for the respective row vectors as $\delta_1=2$, $\delta_2=1$ and $\delta_3=2$. The index set in (16) is given as $\Im_{1,3}=\{1,3,4\}$. Since the sum of the indices is equal to 5, this system is observable. The observer canonical form of this system is given as follows:

$$\tilde{A}=\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 3 & 0 \\ 0 & 0 & 2 & -1 & 0 \\ -1 & 0 & -1 & 1 & 1 \\ 1 & 0 & 4 & 0 & 0 \end{bmatrix}, \tag{26}$$

$$\tilde{C}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 \end{bmatrix},\ \tilde{F}=\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \tag{27}$$

where we omit the detailed calculation with the transformation matrix Q.

From the result, we can include the first three row vectors of $\tilde{F}$ $(\delta_1+\delta_2=3)$ for the reduced-order detection filter.

$$\tilde{A}_{11}=\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix},\ \tilde{C}_1=\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix},\ \tilde{F}_1=\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 0 & 1 \end{bmatrix}. \tag{28}$$

For the extracted triple $(\tilde{A}_{11},\tilde{F}_1,\tilde{C}_1)$, the invariant zero associated with the first column vector of $\tilde{F}_1$ is equal to -2 while there exists no zero associated with the second vector. We define the two fault event vectors as $\tilde{f}_{11}$ and $\tilde{f}_{12}$ and their detection spaces as $D_{\tilde{f}_{11}}$ and $D_{\tilde{f}_{12}}$, respectively. It is easy to check $\tilde{F}_1$ is mutually detectable.

Since $\tilde{C}_{22}=[1\ \ 0]$, $\breve{C}_{22}^{-1}=1$, $\tilde{D}_{12}$ and the resulting closed-loop system are given as follows:

$$\tilde{D}_{12}=\breve{A}_{12}=\begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix},\ (\tilde{A}_{11}-\tilde{D}_{12}\tilde{C}_{21})=\begin{bmatrix} 1 & 1 & 1 \\ 5 & 0 & -3 \\ -1 & 0 & 3 \end{bmatrix},$$

$D_{\tilde{f}_{11}}=\text{span}([\tilde{f}_{11}\ \ \tilde{v}_z])$ and $D_{\tilde{f}_{12}}=\text{span}(\tilde{f}_{12})$, where $\tilde{v}_z$ is the invariant zero vector of $(\tilde{A}_{11},\tilde{F}_1,\tilde{C}_1)$. We assign two closed-loop eigenvalues of -3 and -4 in associated with $D_{\tilde{f}_{11}}$, while the one of -5 with $D_{\tilde{f}_{12}}$. Let us calculate the corresponding detection gain using eigenvalue assignment method as in [9]. One of the right eigenvectors associated with $D_{\tilde{f}_{11}}$ is given as linear combinations of fault event vector and the invariant zero vector while the eigenvector for $D_{\tilde{f}_{12}}$ is $\tilde{f}_{12}$ itself.

$$\bar{v}_1^1=\tilde{f}_{11}+(z-\lambda_1^1)\tilde{v}_z=\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}+((-2)-(-3))\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}=\begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix},$$

where λ_1^1 and $\bar{v}_1^1$ are the eigenvalue and eigenvector associated with $D_{\tilde{f}_{11}}$, respectively. z is the invariant zero associated with $\tilde{v}_z$. Then the detection gain for fault detection and isolation is given as follows:

$$\tilde{D}_{11} = ((\tilde{A}_{11} - \tilde{D}_{12}\tilde{C}_{21})V - V\Lambda)(\tilde{C}_{11}\tilde{F}_1)^{\dagger} = \begin{bmatrix} 8 & 0 \\ 27 & -10 \\ -9 & 8 \end{bmatrix},$$

where V and Λ are the matrices made up of the eigenvectors and eigenvalues, respectively.

$$V = \begin{bmatrix} 1 & 1 \\ 3 & 1 \\ 0 & 1 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix}.$$

Finally, the detection gain for the reduced-order detection filter is given as

$$[\tilde{D}_{11} \ \ \tilde{D}_{12}] = \begin{bmatrix} 8 & 0 & 0 \\ 27 & -10 & 3 \\ -9 & 8 & -1 \end{bmatrix}.$$

To obtain fault isolation intuitively, we apply the following projection matrix, which corresponds to W_1 in Fig. 1 as follows:

$$\begin{bmatrix} r_{CH1}(t) \\ r_{CH2}(t) \end{bmatrix} \triangleq (\tilde{C}_1\tilde{F}_1)^{\dagger}\varepsilon(t). \tag{29}$$

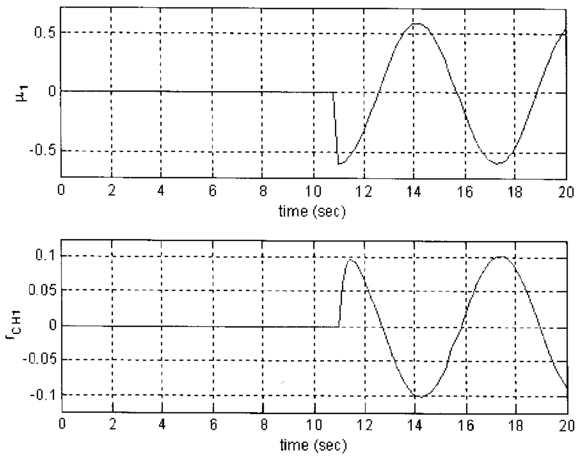

Fig. 2. Fault signal and corresponding residual for f_1.

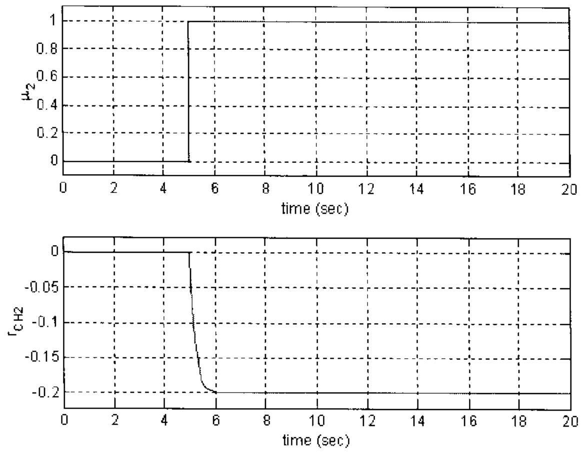

Fig. 3. Fault signal and corresponding residual for f_2.

If we use the above residual, there exists one-to-one correspondence of $f_1 \mapsto r_{CH1}$ and $f_2 \mapsto r_{CH2}$.

Figs. 2 and 3 show the simulation results, where the proposed 3rd order detection filter is applied as in Fig, 1. $\mu_1(t)$ is chosen as a sinusoidal function of amplitude 0.6 starting from 11 seconds; $\mu_2(t)$ is a step function of amplitude 1 starting from 5 seconds. In these figures, the upper graphs correspond to the fault signals and the lower ones to the respective residuals. We see that fault detection and isolation is perfectly obtained with this detection filter except that the phase of the residuals is inverted with respect to the corresponding fault signals.

6. CONCLUDING REMARKS

We have presented a method to reduce the order of detection filter for linear time-invariant systems. We have shown that a part of system directly affected by a fault event vector can be determined by the observer canonical form, and proposed a general form of detection filter to diagnose that system part. We consider the dynamics associated with the unobserved subsystem as a disturbance, the solution of disturbance rejection as well as the condition of mutual detectability has been presented. As an illustrative example, we designed a 3rd order detection filter for a 5th order system following the procedure summarized here, and showed by simulation that its performance is satisfactory. The proposed method is clear and simple from an analytical point of view and so is deemed applicable in the case of large-scaled system diagnosis, for reducing the computing resources required in implementation of detection filter.

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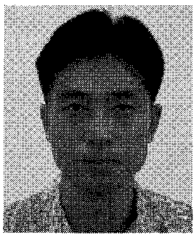

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