Minimization of Modeling Error of the Linear Motion System with Voice Coil Actuator

Jin-Dong Hwang, Yong-Kil Kwak, Hong-Jung Jung, Sun-Ho Kim, and Jung-Hwan Ahn

Abstract: This paper presents a method for reducing modelling error in the linear motion system with voicecoil actuator (VCA). A model of linear motion system composed of a mechanism and control was prepared to verify the proposed method. In modeling of the system, the damping coefficient obtained experimentally is applied to the model in order to consider the effect of the viscous friction for the moving part in VCA. The response velocity of VCA for duty ratio of PWM signal was analyzed in the time domain. Consequently, the relation between velocity and duty ratio was obtained. The result from the experiment showed an error of 9% when compared with that of simulation. In order to reduce the modeling error, impedance variation according to input frequency was analyzed, and equivalent impedance with multi-frequency was applied to the control part. As a result, the modeling error decreased to 5%.

Keywords: Friction, modeling error, PWM response, voice coil actuator (VCA).

1. INTRODUCTION

Over 70% of industrial machine systems employ the linear motion system as the operating source. In general, the complex mechanism which changes rotation motion into linear motion has been used in such systems. But this mechanism has high-speed limits due to high inertia force of components, and it is difficult to be applied at ultra precision position control because of the error caused by assembling components. The linear motion system for non-circular cutting is a field requiring ultra precision position control with high speed. The parts requiring such noncircular machining are automotive camshaft, piston head and so on [1-5]. The actuator formerly used to non-circular machining on the lathe was hydraulic actuator. While the hydraulic actuator provides the advantage of high system stiffness and robustness against disturbance due to high stiffness, its application in noncircular machining is limited by the large size of the hydraulic system, low response, and associated high costs of installation and maintenance [6-8].

In case of using piezoelectric actuator with high response, to improve the drawback of having very short stroke, the hybrid mechanism has been applied at the noncircular machining on the lathe. However, this also presents some defects such as the difficulty to control nonlinearity of PZT and low output force [2]. The representative linear motion actuator which has high thrust, long stroke and fast response is linear motor and VCA, however the disadvantage of linear motor is that the device configuration is complex while that of VCA is simple. However, VCA has low stiffness, and as a result, the cutting forces generated during machining cause large deflections and tracking error. Researches to eliminate tracking error of VCA due to low stiffness have been achieved, though. For instance, Reddy used force feedback scheme to enhance tracking performance, and analyzed the effect on tracking performance when system parameters such as Order of contour symmetry, Depth of cut, Feedrate, feedback type and maximum acceleration are changed at noncircular machining [2]. In order to reduce the tracking error caused by cutting load at noncircular machining, Babinski applied acceleration feedback method and developed "model-free" tracking controller [3]. Hwang minimized the tracking error generated during noncircular machining on the lathe by using TSIC (Three-Stage Intelligent Controller). TSIC consists of MZPTC (Modifed Zero Phase Tracking Controller) based on the learned model, fuzzy sliding-mode control and a forecast compensate controller to compensate the cutting error caused by the variant cutting depth [4]. In the control of VCA, however, there have been no reports on

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researches to reduce the modelling error by analyzing the change of velocity according to duty ratio of PWM signal and reflecting the variation of impedance, which is changed according to input frequency. If the characteristic of VCA for the variation of two parameters is analyzed and reflected, the control part to reduce modelling error of the linear motion system using VCA, and the precision and tracking performance of the VCA will be enhanced.

In this paper, a linear motion system composed of mechanism and control was made. The mechanism part consists of VCA, linear motion guide, and position detect device. Meanwhile, the control part consists of a driver to control the mechanism part and computer to communicate with the driver. To consider the effect of viscous friction (dynamic friction) for the moving part, the damping coefficient derived from the experiments was applied at modeling. The relation between duty ratio and velocity was obtained by analyzing the response velocity of moving part for duty ratio variation. And the impedance variation of VCA according to input frequency was analyzed. Consequently, an equivalent inductance and resistance of coil in VCA was determined. Therefore, this paper presents the relation between duty ratio of PWM signal and response velocity of VCA and an equivalent impedance in order to reduce modeling error. The proposed method is verified through comparing experimental results with simulation.

2. EXPERIMENTAL APPARATUS AND SYSTEM MODELING

Fig. 1 shows the block diagram of the control system used in this research. The controller is PD controller which is used generally in high speed control. The duty ratio is limited from 5% to 95%. As to the flow of control, first, the error between reference input and output enters PD controller and is then converted into velocity. The duty ratio of PWM signal is decided by the relative formula. Second, the amplified PWM signal is applied to electrical system of VCA. Third, the current to be outputted from electrical system is converted into force by Lorentz law and fourth, the force is worked at the mechanical system. Finally, the linear motion is generated.

Table 1. Experimental system specification.

Components	Specification	Value
Controller (TMS320C2407)	Carrier frequency (kHz)	10
	Control frequency (kHz)	1
Amplifier	RMS 50V, 20A(0~20kHz)	
VCA	L(H)	0.0019
	L_{eq}(H)	0.0131
	$R(\Omega)$	1.9
	$R_{eq}(\Omega)$	1.487
	m(kg)	1.4
	c(Ns/m)	68.3
	k_f(N/A), k_b(V$_s$/m)	24.02

2.1. Experimental apparatus

The system has the control part and mechanism part for linear motion. The control part consists of a computer, a controller and an amplifier. Fig. 2 shows the schematic diagram of the system. The computer of the control part creates velocity profile of moving parts. The controller has functions to generate PWM signal and to convert the two-phase square wave signal of encoder into displacement and it is implemented via TMS320C2407 DSP chip of TI company. The amplifier responds linearly to input frequency of up to 20kHz. PWM signal to be generated at the controller is outputted as 0~3.5V. The output signal is amplified up to ±22.5V and applied to VCA. In this research, the sampling period is 1msec. The carrier frequency of PWM signal is 10kHz. The VCA has 24N/A trust force at the center of full stroke and cylindrical shape. Linear encoder with resolution of 0.5um is used for measuring position. The output of linear encoder is inputted to the counter of the controller. The specifications of hardware and values of parameter used in the experiment are shown at Table 1.

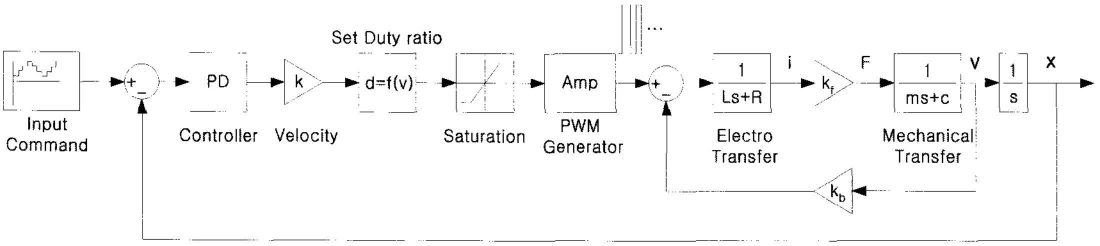


Fig. 1. Block diagram for control of VCA.

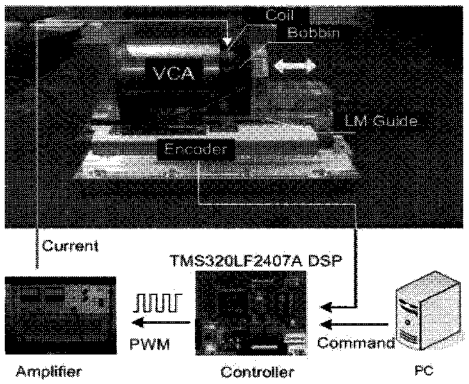


Fig. 2. Experimental setup.

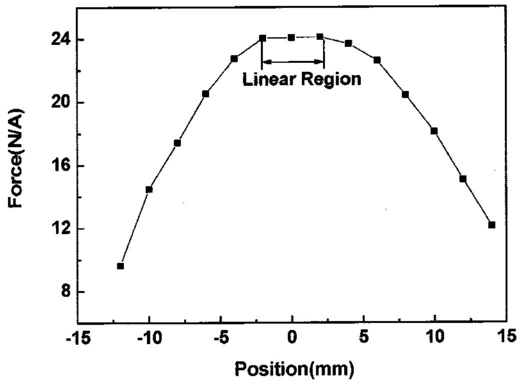


Fig. 3. Force variation according to moving position.

2.2. General VCA model

The model of a linear motion system using VCA can be expressed as (1).

$$m\ddot{x} + c\dot{x} + kx = F = k_f i, (k_f = NBl)$$
$$L\frac{di}{dt} + Ri = V - k_b v \tag{1}$$

Here, the parameters $m, c, k, L, R, k_f, k_b, N, B, l$ are mass of moving parts, damping coefficient, spring constant, inductance of coil, coil resistance, motor constant, counter-electromotive force constant, coil winding number, magnetic flux density and length of coil, respectively. Force generated by Lorentz law does not vary linearly with the position of operating region about a constant current input. Because magnetic flux density is not uniform according to the position of operating region, such phenomenon makes the precision control difficult. Fig. 3 shows experimental results about thrust force generated according to position when a constant current is flowed to VCA. If the operating region is limited to ±2mm from the center to control, the thrust force can be assumed to be linear in the input current (k_f= 24.02)). In this research, modelling works at such region. Therefore, (1) can be Laplace transformed, and transfer function from the input voltage(V) from the output displacement(X) becomes (2).

$$\frac{X(s)}{V(s)} = \frac{k_f}{Lms^3 + (Lc + Rm)s^2 + (Lk + Rc + k_f k_b)s + kR} \tag{2}$$

The spring constant can be set to zero in that the LM guide with ball-bearing is used as a moving mechanism. Theoretically, to find the damping coefficient is very difficult because it is altered by the assembly status, a lubrication condition, and temperature. This research tries to decrease the modeling error caused by disregarding dynamic friction through applying damping coefficient to modeling after finding it in the experiment. Fig. 4 shows the velocity measured after applying a constant force (F_v =14.68N) to VCA in order to measure damping coefficient. When a constant force is worked, the change of velocity appears in (3), and velocity convergences a constant value. The damping coefficient was observed to be 68.3Ns/m from such relational formula.

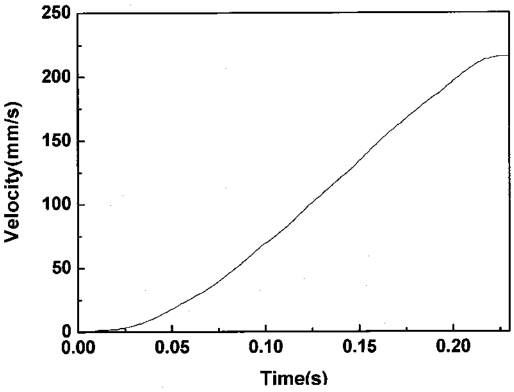


Fig. 4. Velocity according to constant force.

$$v(t) = F_v\left(\frac{1}{c} - \frac{1}{c}e^{-\frac{c}{m}t}\right), \quad v(t) = \frac{F_v}{c}(t \to \infty) \tag{3}$$

2.3. Relation of velocity and PWM duty ratio

Fig. 5 shows block diagram on the relation between the velocity to track the desired path and the duty ratio of PWM signal to drive VCA. The relation can be derived by analyzing the response of the moving part for PWM signal in the time domain. Fig. 6 shows PWM signal during a control cycle. The duty ratio for the cycle is decided by a, b. The response for the signal at the s-plane is represented as (4). It is difficult to derive the relation between velocity and duty ratio from (4) in the time domain because the variable a is represented by exponential function. But if the PWM signal is assumed to be a consecutive step input which has different initial conditions, it can be analyzed in

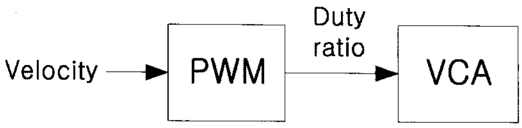


Fig. 5. Block diagram for velocity and duty ratio.

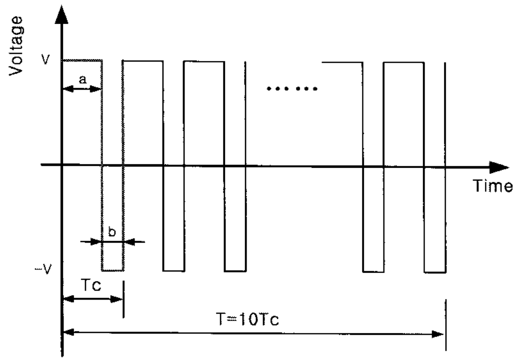


Fig. 6. PWM signal with period Tc.

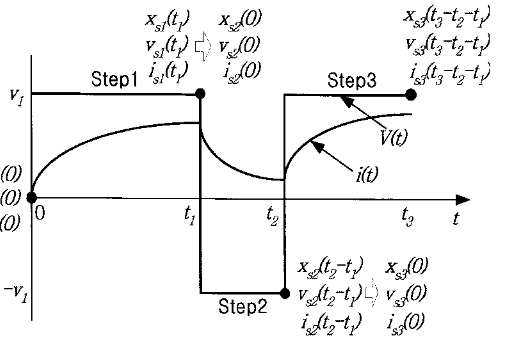


Fig. 7. Simplification of PWM signal.

the time domain. Fig. 7 shows the schematic diagram. If initial condition is considered, the Laplace transform of (1) is given by (5).

In the first step, the initial condition $x_1(0)$, $\dot{x}_1(0)$, $i_1(0)$ is set to zero and the response for step input by time t_1 can be obtained by Inverse Laplace transform of (5). In the second step, $x_1(t_1)$, $\dot{x}_1(t_1)$, $i_1(t_1)$, which are values for first step input at time t_1, we set a new initial condition ($x_2(0), v_2(0), i_2(0)$), so that the response for second step input can be obtained.

$$V(s) = \int_0^{\infty} f(t)e^{-st}dt$$

$$= \sum_{n=0}^{\infty} \int_{nT_c}^{(n+1)T_c} f(t)e^{-st}dt \qquad (4)$$

$$= \frac{1}{1-e^{-T_c s}} \left(\frac{1-2e^{-as}+e^{-T_c s}}{s} \right),$$

$$Y(s) = V(s)G(s) + \frac{1}{1-e^{-T_c s}} \left(\frac{1-2e^{-as}+e^{-T_c s}}{s} \right)$$

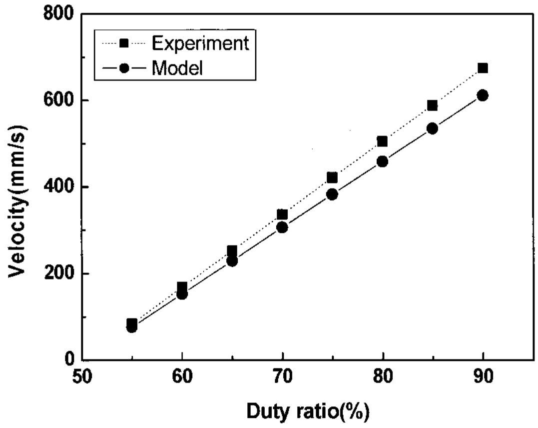
$$\left(\frac{k_f}{Lms^3 + (Rm+Lc)s^2 + (k_f k_b + Rc)s} \right),$$

$$X(s) = \frac{1}{s}x(0) + \frac{(Lms+Rm)\dot{x}(0) + k_f V + Lk_f i(0)}{Lms^3 + (Rm++Lc)s^2 + (k_f k_b + Rc)s}, \qquad (5)$$

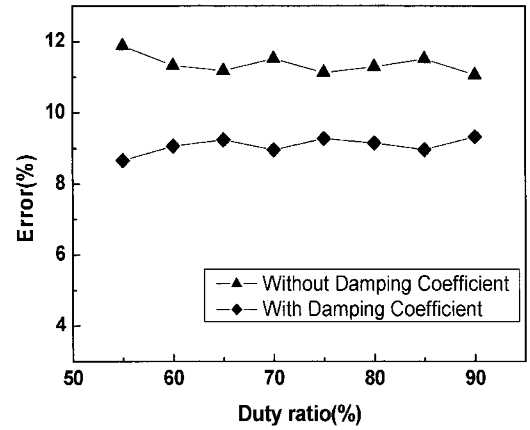
$$I(s) = \frac{-k_b \dot{x}(0) + (ms+c)V + (Lms+Lc)i(0)}{Lms^2 + (Rm++Lc)s + k_f k_b + Rc},$$

$$d = 50 + 0.0654v, \qquad (6)$$
$$(d: \text{duty ratio}(\%),\ v: \text{velocity}(\text{mm/s})).$$

The relation between velocity and duty ratio can be derived by the method mentioned above. Equation (6) shows the relation. Fig. 8(a) shows the change of velocity which is measured and simulated according to duty ratio. When the dynamic friction is considered, the modeling error is found to decrease from 11.3% to about 9%. The inductance and resistance used in this model is 1.9mH, 1.9Ω in each. In case that duty ratio is below 50%, the change of velocity according to



(a) Velocity vs. duty ratio.



(b) Modelling error.

Fig. 8. Velocity according to duty ratio and modeling error by considering damping coefficient.

duty ratio is almost same with the case that duty ratio is above 50% except that the sign of velocity is negative.

3. MODELLING ERROR MINIMIZATION BY CONSIDERING IMPEDANCE VARIATION

Inductance and resistance of the coil in VCA are altered by input frequency. But in general, these are treated as constant values. It is important to consider the change of impedance according to input frequency in order to reduce the model error. Fig. 9 shows the circuit to measure the inductance and resistance of VCA

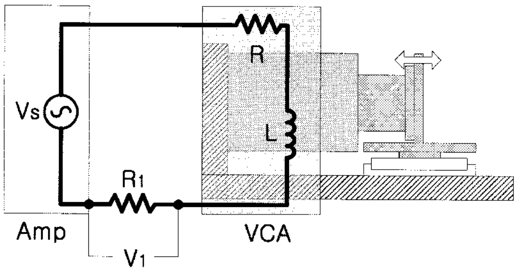
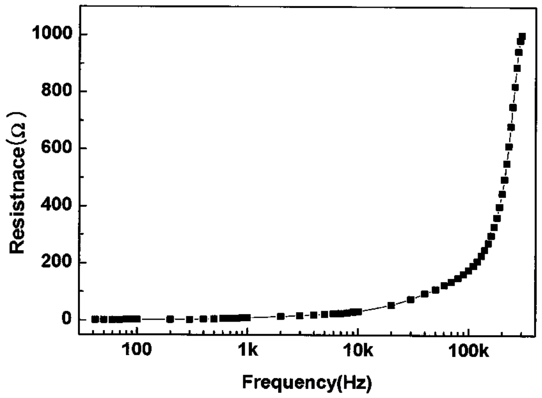
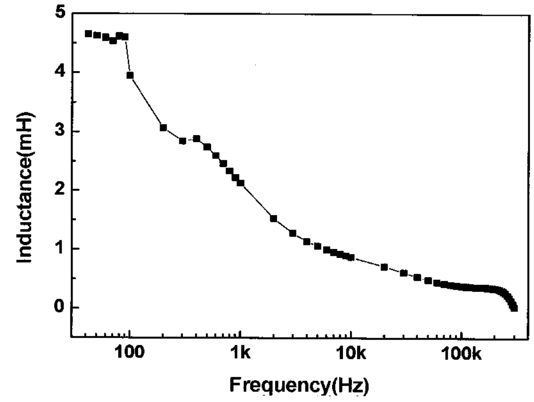


Fig. 9. An equivalent circuit for measuring the impedance of coil.



(a) Inductance.



(b) Resistance.

Fig. 10. Impedance variation according to input frequency.

$$\theta = \tan^{-\left(\frac{wL}{R+R_1}\right)},$$
$$\left|\frac{V_s(jw)}{V_1(jw)}\right| = \frac{\sqrt{(R+R_1)^2+(wL)^2}}{R_1}. \tag{7}$$

The values of L and R for input frequency can be obtained from (7). Equation (7) represents the change of amplitude and phase about input signal. Here, the parameters θ, w, R_1, R, L are phase angular, angular velocity, inner resistance of VCA, measurement resistance, inductance of VCA, respectively.

Fig. 10 shows the change of L, R according to frequency when the sinusoidal signal from 40Hz to 300kHz is applied to VCA. The reason why frequency is limited from 40Hz to 300kHz is that it is difficult to get the change of phase and amplitude experimentally below 40Hz and the induced frequency from the basic frequency (10kHz) does not happen almost over the 300kHz. If the PWM signal is completely a square wave, the frequency applied to VCA by the signal can be found by the analysis of Fourier coefficient. The Fourier coefficient of the square wave signal in which the duty ratio is changed appears in (8).

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jkw_0 t},$$
$$x(t) = \begin{cases} V, & 0 < t < a \\ -V, & a < t < T_c, \end{cases}$$
$$C_k = \frac{1}{T_0}\left\{ \int_0^a x(t)e^{-jkw_0 t}dt + \int_0^{T_0} x(t)e^{-jkw_0 t}dt \right\}$$
$$= \frac{jV}{2\pi k}\left(2e^{-jkw_0 a} - e^{-j2\pi k} - 1\right) \tag{8}$$
$$= \frac{V}{\pi k}\sin\left(\frac{2\pi ka}{T_0}\right) + j\frac{V}{\pi k}\left(\cos\left(\frac{2\pi ka}{T_0}\right) - 1\right),$$
$$k = 0,1,2,\ldots$$
$$|C_k| = \frac{2V}{\pi k}\left|\sin\left(\frac{\pi ka}{T_0}\right)\right|, \; if \;\; 0.05T_0 < a < 0.95T_0,$$
$$\therefore \; 0 \le |C_k| \le \frac{2V}{\pi k}.$$

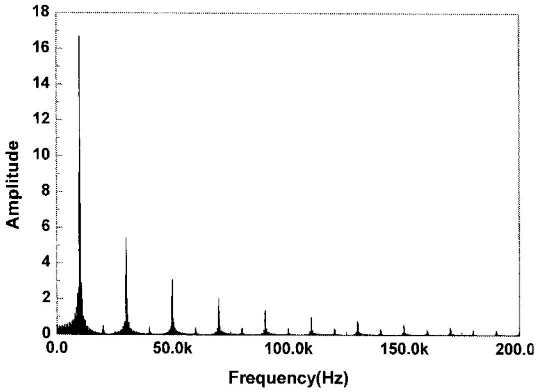
Here, the parameters a, V are time to correspond to duty ratio and amplitude of applied voltage, respectively. In case of completely square wave (the duty ratio is 50%), the frequencies to be odd times larger than carry frequency (f_c) are applied to VCA. However, in control, duty ratio is not only changed by feedback control but the PWM signal is also not a complete square wave. As shown in (8), Input frequency (f_r) and the frequencies to be corresponded to a positive integer times larger than carry frequency are applied to VCA. The fact is verified by analyzing frequency applied to VCA. Fig. 11(a) shows the result of the fast Fourier transform (FFT) regarding the signal applied to VCA when VCA is only controlled without any input. In this case, the induced frequency is almost odd times larger than carry frequency. Fig. 11(b) shows the result of the FFT when VCA is controlled by input frequency (38.5Hz). When multi-frequency is applied to VCA at the same time, the method to determine equivalent L, R is as follows: the equivalent inductance can be obtained by applying the superposition principle if the circuit is linear; therefore the inductances (L_r, L_{T_c}, L_{2T_c}, ..., L_{nT_c}) to correspond to each input frequency (f_r, f_{T_c}, f_{2T_c}, ..., f_{nT_c}) can be regarded as being the serial because it acts on the component of energy storage. The amplitude of resistance (R_r, R_{T_c}, R_{2T_c}, ..., R_{nT_c}) to correspond to input frequency increases as frequency becomes higher. Meanwhile, the current of each frequency is lower as the frequency becomes higher, therefore the current flowed in VCA increases rather than the case in which there is only frequency (f_r). This is identical with the phenomenon in which a total resistance of VCA comes to be small, thus it can be assumed that the resistances to correspond to each frequency is connected to the parallel. This can be expressed in (9).

$$\begin{aligned} L_{eq} &= L_r + \sum_{i=1}^{n} L_{iT_c}, \\ R_{eq} &= \frac{1}{1/R_{DC} + 1/R_r + \sum_{i=1}^{n} 1/R_{iT_c}}. \end{aligned} \tag{9}$$

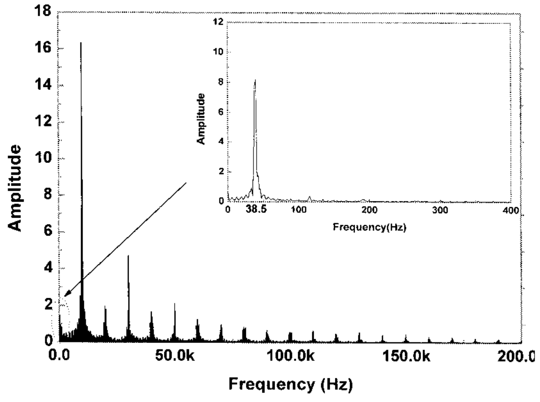
Here, R_{DC} is resistance at DC. In case of inductance, the reason not to consider the value of L at DC is that the influence of inductance does not exist when the variation of current is zero. In real operation, the equivalent inductance calculated from Equation (9) is 13.1mH, the equivalent resistance is 1.487Ω. In case of considering an equivalent impedance, the relation between velocity and duty ratio is expressed in (10).

$$\begin{aligned} &d = 50 + 0.06275v \\ &(d: \text{duty ratio}(\%),\ v: \text{velocity}(\text{mm/s})) \end{aligned} \tag{10}$$

Fig. 12(a) shows the change in velocity which is measured and simulated according to duty ratio. When the equivalent impedance is considered, the modeling

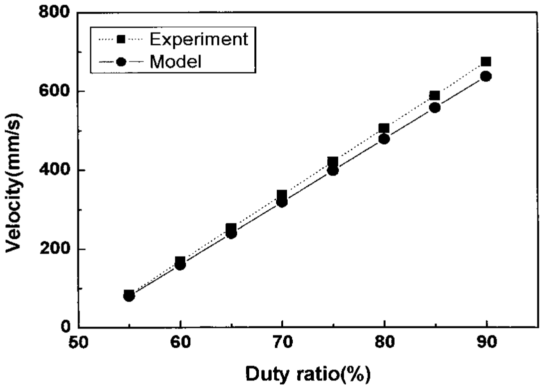


(a) In only control without no input.

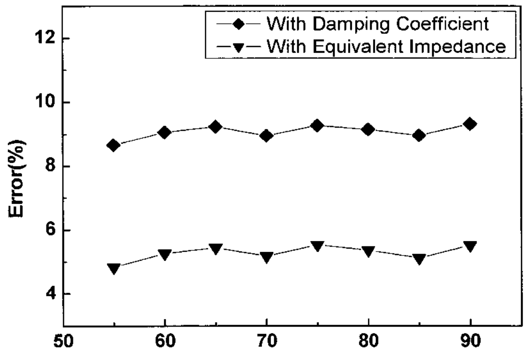


(b) In control for 38.5Hz sinusoidal input.

Fig. 11. Frequencies applied to VCA.



(a) Velocity vs. duty ratio.



(b) Modeling error.

Fig. 12. Velocity according to duty ratio in applying equivalent impedance.

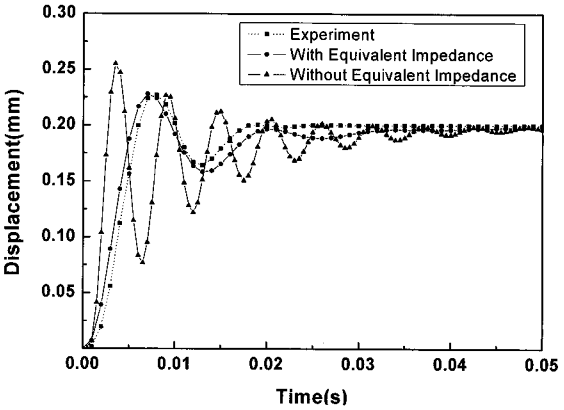


Fig. 13. Step response of the model and the system.

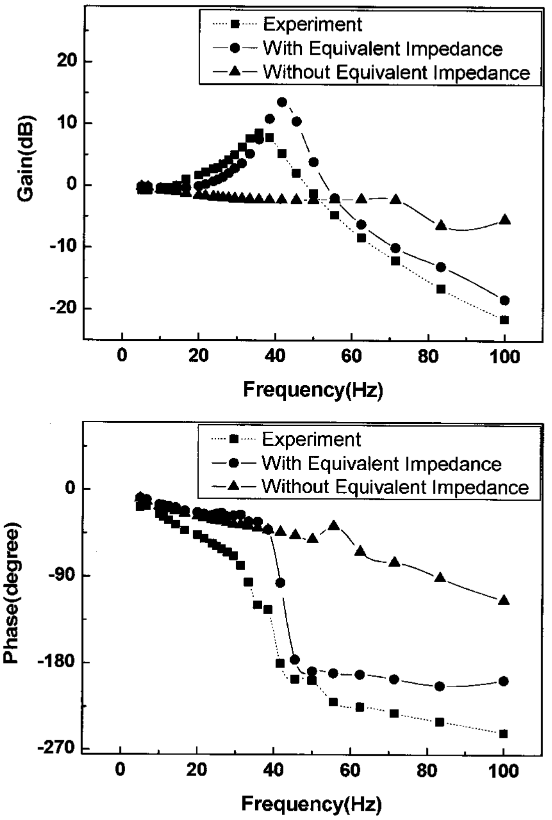


Fig. 14. Frequency response of the model and the system.

error decreases from 9% to 5%.

Fig. 13 shows the step (0.2mm) response of model according to the existence of an equivalent impedance and the response of system. The percent overshoot, delay time and rising time of the system from experiment are 13.5%, 3.8msec, 6.1msec respectively. the percent overshoot according to the existence of an equivalent impedance are 14%, 27.5%, the delay time are 3.2msec, 1.96msec and the rising time are 5.46msec, 2.75msec.

Fig. 14 compares the frequency response of model according to the existence of an equivalent impedance with the frequency response of system at air cutting. The first resonance is generated at 42Hz in the model with an equivalent impedance, and is generated at 36Hz in the experiment. But the first resonance is not generated at the model without an equivalent impedance. From Figs. 13 and 14, The response of model which considers an equivalent impedance is more accurate than the other in comparing with that of real system.

4. CONCLUSIONS

It is essential to analyze the characteristic of the parameter in VCA in order to improve the tracking performance and the precision of the linear motion system using VCA. In this paper, the damping coefficient was obtained from experiment in order to consider the effect of dynamic friction. The relation between response velocity and duty ratio of PWM signal is obtained by analyzing response of VCA to the PWM signal in the time domain. The model error is about 9% when the result of simulation is compared with that of the experiment. To reduce modeling error, the variation of inductance and resistance to inner parameters of VCA according to input frequency has been analyzed and the method to determine an equivalent impedance is presented, and the simulation model with an equivalent impedance is compared with the experimental results. As a result, the modeling error decreased by 5%. The proposed model with equivalent impedance is better than the model, without considering an equivalent impedance at the point of view real system approximation.

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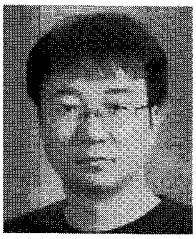

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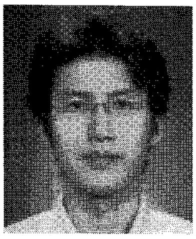

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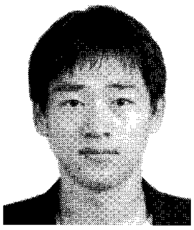

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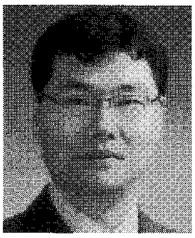

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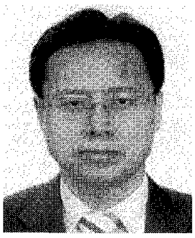

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