Dynamics and GA-Based Stable Control for a Class of Underactuated Mechanical Systems

Diantong Liu, Weiping Guo, and Jianqiang Yi

Abstract: The control of underactuated mechanical system is very complex for the loss of its control inputs. The model of underactuated mechanical systems in a potential field is built with Lagrangian method and its structural properties are analyzed in detail. A genetic algorithm (GA)-based stable control approach is proposed for the class of underactuated mechanical systems. The Lyapunov stability theory and system properties are utilized to guarantee the system stability to its equilibrium. The real-valued GA is used to adjust the controller parameters to improve the system performance. This approach is applied to the underactuated double-pendulum-type overhead crane and the simulation results illustrate the complex system dynamics and the validity of the proposed control algorithm.

Keywords: GA, overhead crane, stable control, system dynamics, underactuated mechanical systems.

1. INTRODUCTION

Underactuated mechanical systems (UMSs) are a class of mechanical systems that have fewer control inputs than generalized coordinate variables. UMS has the advantage over fully-actuated systems in energy saving, cost reducing, manufacturing and installing. The restriction of control inputs of UMS brings a challenging control problem. Moreover, a fully-actuated system may become a UMS because of actuator failure and so the control algorithm for UMSs can be used as a kind of fault-tolerate control algorithm.

Since the early 1990s, the dynamics and control of the UMSs have attracted much attention [1-6]. UMS is interesting because of its structural properties. For fully actuated mechanical systems, a broad range of powerful techniques are used to improve their performance (optimal, robust, adaptive and learning control techniques). These techniques are possible because fully actuated systems possess a number of strong properties that facilitate control design, such as feedback linearizability and matching conditions. For the UMSs, one or more of the above properties are lost and it is hard to find a unique useful theory to solve the control problem. Therefore, the previous related studies mainly focused on the UMS dynamics and control properties. The partial feedback linerization was put forward [1]. Three classes of control problems were analyzed [2]. Nonholonomic constraint and controllability were researched [3]. System dynamics, controllability and stabilizability results were derived [4]. However, from the literature [1-6], it can be seen there are few results that are applicable to design a controller for an entire class of UMSs.

During the last decade, evolutionary computational techniques, such as GA, have been successfully applied to deal with some complex engineering problems, which are difficult to be solved by traditional methods [7]. Since the GA simultaneously evaluates many points in the parameter space, it is more likely to converge toward a global solution. Therefore, the GA has been widely introduced to deal with nonlinear control challenges [8,9] and the GA-based global optimization technique has been integrated into other control methodology, such as neural network or fuzzy control [10]. In view of these previous research results, the destination of favorable control or optimization performance can be achieved owing to its powerful global searching capability. The ordinary form of GA is a binary encoding during operating procedures. However, for a real application, a real-valued encoding is usually preferable and easy

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to be directly implemented in the computer programming. Moreover, binary coding of real number may suffer for the loss of precision and the longer chromosome strings.

This paper has three objectives. The first is to build the model and to analyze some properties of a class of UMSs in a potential field. The second is to propose a GA-based stable control algorithm for the class of UMSs in a potential field based on the Lyapounov theory and the system's properties. The third is to verify the validity of the system dynamics and the proposed control algorithm with an underactuated double-pendulum-type overhead crane (DPTOC) system.

The remainder of this paper is organized as follows. In Section 2, the dynamics and properties of the UMSs in a potential field are analyzed such as its positive definite symmetric inertia matrix and its passivity. In Section 3, a stable control algorithm is proposed and the stability is analyzed with Lyapunov theory. In Section 4, the real-valued GA is designed to improve the system performance. In Section 5, the DPTOC system is used to simulate. The complex dynamics and the proposed algorithm are validated. Finally, in Section 6, conclusions are drawn.

2. DYNAMICS AND PROPERTIES OF UMS

According to the Lagrange mechanics, i.e., Euler-Lagrange equation, the dynamics of the class of UMSs in a potential field can be built as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \overline{\tau}, \qquad (1)$$

where $q = (q_1, \cdots, q_n)^{\mathrm{T}}$ is the generalized coordinate vector, $\overline{\tau} = (\tau_1, \cdots, \tau_m, 0, \cdots, 0)^{\mathrm{T}} = (\tau, 0)^{\mathrm{T}}$ is the control vector. When $m<n$, a system is said to be underactuated. $M(q) \in R^{n \times n}$ is the inertia matrix and $m_{ij}(q)$ is its matrix element. $C(q,\dot{q})$ is Centrifugal terms ($i = j$) and Coriolis terms ($i \neq j$), its matrix element is

$$c_{ij}(q,\dot{q}) = \sum_{k=1}^{n} \Gamma_{kj}^{i}(q)\dot{q}_k. \qquad (2)$$

$\Gamma_{ij}^{k}(q)$ are called Christoffel symbols and defined as

$$\Gamma_{ij}^{k}(q) = \frac{1}{2}\left(\frac{\partial m_{kj}(q)}{\partial q_i} + \frac{\partial m_{ki}(q)}{\partial q_j} - \frac{\partial m_{ij}(q)}{\partial q_k}\right). \qquad (3)$$

$G(q)$ contains the gravity terms

$$G(q) = \partial P(q)/\partial q. \qquad (4)$$

The UMSs have the following structural properties:

Property 1: $M(q)$ is a positive definite symmetric matrix [11].

Property 2: $N(q,\dot{q}) = \dot{M}(q) - 2C(q,\dot{q})$ is a skew symmetric matrix.

Proof: Each element of the derivative of the inertia matrix is given by:

$$\dot{m}_{ij}(q) = \sum_{k=1}^{n} \frac{\partial m_{ij}(q)}{\partial q_k}\dot{q}_k.$$

Each element of $N(q,\dot{q})$ can be calculated from (2) and (3):

$$\begin{aligned} n_{ij}(q,\dot{q}) &= \dot{m}_{ij}(q) - 2c_{ij}(q) \\ &= \sum_{k=1}^{n}\left(\frac{\partial m_{kj}(q)}{\partial q_i} - \frac{\partial m_{ik}(q)}{\partial q_j}\right)\dot{q}_k. \end{aligned}$$

Recalling Property 1, it is straightforward to deduce that:

$$\begin{aligned} n_{ji}(q,\dot{q}) &= \sum_{k=1}^{n}\left(\frac{\partial m_{ki}(q)}{\partial q_j} - \frac{\partial m_{jk}(q)}{\partial q_i}\right)\dot{q}_k \\ &= -\sum_{k=1}^{n}\left(\frac{\partial m_{kj}(q)}{\partial q_i} - \frac{\partial m_{ik}(q)}{\partial q_j}\right)\dot{q}_k \\ &= -n_{ij}(q,\dot{q}), \end{aligned} \qquad (5)$$

i.e., $\dot{M}(q) - 2C(q,\dot{q})$ is a skew symmetric matrix. □

Property 3: The UMSs are passive systems

Proof: The total energy function of the UMSs can be written as

$$E(q,\dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + P(q), \qquad (6)$$

where $\frac{1}{2}\dot{q}^T M(q)\dot{q}$ is the system kinetic energy, and $P(q)$ is the system potential energy. A reference point of the potential energy can be chosen to make $P(q) \geq 0$, and so $E(q,\dot{q}) \geq 0$. The differential of $E(q,\dot{q})$ can be computed using (1), (4) and Properties 1-2:

$$\begin{aligned} \dot{E}(q,\dot{q}) &= \dot{q}^T M(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \dot{q}^T \frac{\partial P(q)}{\partial q} \\ &= \dot{q}^T\overline{\tau} + \frac{1}{2}\dot{q}^T(\dot{M}(q) - 2C(q,\dot{q}))\dot{q} \\ &= \dot{\Theta}^T\tau, \end{aligned} \qquad (7)$$

where $\Theta = (q_1, q_2, \ldots, q_m)^T = Zq$, $Z = [I_m\ 0]$, $\tau = (\tau_1, \tau_2, \ldots, \tau_m)^T = Z\overline{\tau}$. Therefore,

$$\begin{aligned} \int_0^t \dot{\Theta}^T\tau dt &= E(q(t),\dot{q}(t)) - E(q(0),\dot{q}(0)) \\ &\geq -E(q(0),\dot{q}(0)), \end{aligned}$$

i.e.,

$$\langle \tau \mid \dot{\Theta} \rangle_t \geq -E(q(0), \dot{q}(0)). \tag{8}$$

Thus, UMSs are passive systems with respect to input τ and output $\dot{\Theta}$ [12]. □

3. STABLE CONTROL FOR UMS

It is supposed that the control objective is to control the system to q_d, one of the equilibrium states, where Θ_d is the control objective of the actuated part. The following Lyapunov function is defined:

$$V(q,\dot{q}) = \frac{1}{j} k_E (E(q,\dot{q}) - P(q_d))^j + \frac{1}{2} \dot{\Theta}^T K_D \dot{\Theta} + \frac{1}{2} (\Theta - \Theta_d)^T K_P (\Theta - \Theta_d), \tag{9}$$

where k_E is a positive constant, $P(q_d)$ is the potential energy at desired position, j is a constant. In order to make $V(q,\dot{q}) \geq 0$, j should be 1 when the desired position is the minimal potential energy point among all the accessible states, and j should be 2 when the desired position is not the minimal potential energy point. Both K_D and K_P are positive definite diagonal matrix.

When j=1, the differential of Lyapunov function $V(q,\dot{q})$ can be calculated:

$$\begin{aligned} \dot{V}(q,\dot{q}) &= k_E \dot{E}(q,\dot{q}) + \dot{\Theta}^T K_D \ddot{\Theta} + (\dot{\Theta} - \dot{\Theta}_d)^T K_P (\Theta - \Theta_d) \\ &= \dot{\Theta}^T (k_E \tau + K_D \ddot{\Theta} + K_P (\Theta - \Theta_d)). \end{aligned} \tag{10}$$

In order to ensure the system stability, we take:

$$\dot{V}(q,\dot{q}) = -\dot{\Theta}^T K \dot{\Theta}, \tag{11}$$

where K is a positive definite diagonal matrix. Then following equation holds:

$$k_E \tau + K_D \ddot{\Theta} + K_P (\Theta - \Theta_d) = -K \dot{\Theta}. \tag{12}$$

From $\tau = Z\overline{\tau}$ and $Z = [I_m \;\; 0]$, $\overline{\tau} = Z^T \tau$ holds. Considering $\Theta = Zq$ is a column vector and from (1), the following equation can be obtained

$$\begin{aligned} \ddot{\Theta} &= Z\ddot{q} = ZM^{-1}(q)(\overline{\tau} - C(q,\dot{q})\dot{q} - G(q)) \\ &= ZM^{-1}(q)(Z^T \tau - C(q,\dot{q})\dot{q} - G(q)). \end{aligned}$$

Substituting the above equation into (12), the control law can be obtained:

$$\begin{aligned} \tau = &-(k_E I_m + K_D Z M^{-1}(q) Z^T)^{-1} (K_P (\Theta - \Theta_d) \\ &- K_D Z M^{-1}(q)(C(q,\dot{q})\dot{q} + G(q)) + K\dot{\Theta}). \end{aligned} \tag{13}$$

Theorem 1: For UMSs described by (1), when the equilibrium point q_d is the minimal potential energy point in the system accessible space, controller (13) can make the closed loop system converge to the equilibrium point q_d or stable trajectory $\frac{1}{2}\dot{q}^T M(q)\dot{q} + P(q) = C$, where, C is a positive constant.

Proof: From (9) and (11), $V(q,\dot{q}) \geq 0$ holds. There are two cases:

When $q = q_d$ and $\dot{q} = 0$ hold at the same time, $V(q,\dot{q}) = 0$ holds. From (13), $\tau = 0$ holds. Therefore, the system is stabilized to the desired position.

When $q \neq q_d$ or $\dot{q} \neq 0$, $V(q,\dot{q})$ is a positive definite function. From (11), $\dot{V}(q,\dot{q}) \leq 0$ holds. Therefore, $\dot{\Theta} \in L_2 \cap L_\infty$, $E(q,\dot{q})$ and Θ are bounded. From (6), $q, \dot{q}$ and $M(q)$ are bounded. $M(q)$ is a positive definite symmetric matrix indicating that $M^{-1}(q)$ is bounded. From (1), $\ddot{q}$ is bounded, and then $\ddot{V}(q,\dot{q}) = -2\ddot{\Theta}^T K \dot{\Theta}$ is bounded, which indicates $\dot{V}(q,\dot{q})$ is a uniformly continuous function. According to Barbalat's lemma, $\lim_{t\to\infty} \dot{V}(q,\dot{q}) = 0$, i.e., $\lim_{t\to\infty} \dot{\Theta} = 0$. It can be seen that $\lim_{t\to\infty} V(q,\dot{q})$ and $\lim_{t\to\infty} (\Theta - \Theta_d)$ are constant. From (7) and (13), $\lim_{t\to\infty} E(q,\dot{q})$ and $\lim_{t\to\infty} \tau$ are constant. If $\lim_{t\to\infty} \tau \neq 0$ holds, then Θ will change with $t \to \infty$, which is inconsistent with $\lim_{t\to\infty} (\Theta - \Theta_d)$ is constant. Thus $\lim_{t\to\infty} \tau = 0$ must be true. From (12), $\lim_{t\to\infty} \Theta = \Theta_d$ holds. The stability analysis can be divided into two cases [13]:

When $\lim_{t\to\infty} E(q,\dot{q}) = P(q_d)$, (6) can be written as $\frac{1}{2}\dot{q}^T M(q)\dot{q} + (P(q) - P(q_d)) = 0$ with $t \to \infty$, and both the two terms in the left of the above equation are greater than or equal to zero. Therefore, the system kinetic energy is equal to zero, and potential energy is equal to the potential energy of desired position, i.e., $q = q_d$ and $\dot{q} = 0$. That is to say the system is stabilized to the desired position.

When $\lim_{t\to\infty} E(q,\dot{q}) = C \neq P(q_d)$, the system converges to a stable trajectory $\frac{1}{2}\dot{q}^T M(q)\dot{q} + P(q) = C$, where $\Theta = \Theta_d$, C is a greater than $P(q_d)$ and can be calculated with the above equation, (1) and $\overline{\tau} = 0$. □

When j=2, the following control law can be obtained

$$\tau = -\left(k_E(E(q,\dot{q}) - P(q_d)) + K_D Z M^{-1}(q) Z^T\right)^{-1} \cdot \left(K_P(\theta - \theta_d) - K_D Z M^{-1}(q)(C(q,\dot{q})\dot{q} + G(q)) + K\dot{\Theta}\right).$$

The system stability can be proved in the same way as Theorem 1.

4. PARAMETERS ADJUSTING BY REAL-VALUED GA

The main drawback of the proposed stable approach is that the parameters of the controller have to be predefined and system performance is sensitive to those parameters. That is to say, the values of k_E, K_D, K_P and K in (13) have to be determined firstly and carefully. However, it is very difficult to determine the parameters because these parameters haven't obvious physics meaning and there are complex couplings among them. Moreover, the relation between these parameters and system performance cannot be directly gained. Therefore, it is difficult to set them by the experiences or trial and error, especially in the long-distance transporting. Here a real-valued GA is used to optimize the parameters in the stable controller (13).

4.1. Genetic algorithm

The GA is based on an analogy to the genetic code in our own DNA Structure, where its coded chromosome is composed of many genes [7]. The GA approach involves a population of individuals represented by strings of characters pr digits. Each string is, however, coded with a search point in the hyper search-space. From the evolutionary theory, only the most suited individuals in the population are likely to survive and generate off-spring that passes their genetic material to the next generation. Therefore, the most promising individuals are manipulated by GA in its search for improve performances or solutions. In GA, three basic operators, namely, reproduction, crossover and mutation, form the core of the genetic computation. In the application of GA to an optimizing problem, there are two considerations to be made. First, the choice of fitness function to be used for the measure of performance in the optimization process, and second, the choice of coding to be used to code the design parameters into the chromosome.

4.2. Design of real-valued GA

4.2.1 Encoding

In order to avoid the onerous computation of encoding and decoding in binary GA, the real-valued GA is adopted. A chromosome represents a set of parameters:

$$\Omega = [\, k_E \quad K_D \times T \quad K_P \times T \quad K \times T \,],$$

where $T = [1 \ \ 1 \ \cdots 1]^{\mathrm{T}} \in R^m$.

4.2.2 Fitness function

The fitness function is the key to use the GA. A simple fitness function that reflects small steady-state errors, short rise-time, low oscillations and overshoots with a good stability is given

$$F = 1/J, \quad J = (1 + penalty)\int_0^{\infty} (\mu_1 |e| + \mu_2 |\tau|) dt, \tag{14}$$

where $e = q - q_d$ is tracking errors of coordinate variables, $\mu_1 \in R^n$ and $\mu_2 \in R^m$ are positive coefficients matrix, which respectively represent the importance degree of generalized coordinate variables and control inputs in the fitness function. The penalty is a penalty coefficient when some overshoots arise.

4.2.3 Real-valued GA operations

Select and Reproduction Operation: The operator selects good chromosomes on the basis of their fitness values and produces a temporary population, namely, the mating pool. Here the most common method, Roulette Wheel Selection [7] is used.

Crossover Operation: The P chromosomes in the mating pool are randomly divided into P/2 pairs, which serve as parents and will be crossed. Suppose Ω_1 and Ω_2 are parents of a given pair and c is a number randomly generated in [0, 1]. If $c \le p_c$, then the following crossover operation for Ω_1 and Ω_2 are performed

$$\Omega_1 \leftarrow \Omega_1 + r(\Omega_1 - \Omega_2),$$
$$\Omega_2 \leftarrow \Omega_2 + r(\Omega_2 - \Omega_1),$$

where p_c is the probability of crossover operation, r is a random vector deciding the crossover grade of these two parents, whose elements is in [0, 1]. If $c > p_c$ no crossover operation is performed.

Mutation Operation: The mutation operator follows the crossover and provides a possible mutation on some chromosomes. Supposing the probability of mutation operation is p_m, only $p_m \times P$ chromosomes in current population will be randomly selected to mutate. The formula of mutation operation for selected Ω is give by

$$\Omega \leftarrow \Omega + s\Psi,$$

where s is a positive constant and Ψ is a random vector, whose elements are in [-1, 1]. If the output goes beyond the scope, s will be replaced by the random number between 0 and s.

4.2.4 Steps of real-valued GA

The real-valued GA that we implemented can be described algorithmically as follows:

Step 1: Generate an initial population with P chromosomes randomly.

Step 2: While (generation number <= maximum value) do.

Step 3: Evaluate each chromosome using equation (14) according to the system simulation results.

Step 4: Select and reproduction operation.

Step 5: Crossover operation.

Step 6: Mutation operation.

From the system analysis and controller design, it can be seen the proposed control algorithm is designed for an entire class of complex underactuated system in a potential field but not for an underactuated system, such as the whirling pendulum [9], the cranes [14-17] and a two-freedom underactuated system [6]. The real-valued GA is implemented to overcome some drawbacks of the stable controller, i.e., the difficulty in parameters setting as [6]. However, the evolution process is time consuming in real-time application and the online GA-based control scheme is not considered because the stability cannot be guaranteed directly.

5. SIMULATION STUDIES

When the mass of crane hook can't be ignored, the overhead crane performs as a double pendulum system [14], and the system is a typical UMS with one control input and three generalized coordinate variables.

5.1. Dynamics and properties of DPTOC

Fig. 1 shows the model of DPTOC system: m is trolley mass, m_1 is hook mass, m_2 is load mass, x is trolley position, θ_1 is hook swing angle, θ_2 is load swing angle, l_1 is cable length for hook, l_2 is cable length for load, F is trolley drive force. The friction and cable mass are ignored.

The system dynamics can be described by (1), where

$$\bar{\tau} = [F, 0, 0],\quad q = [x, \theta_1, \theta_2]^T,$$

$$G(q) = [0 \quad (m_1+m_2)gl_1\sin\theta_1 \quad m_2gl_2\sin\theta_2]^T,$$

$$M(q) = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix},$$

$$M_{11} = m + m_1 + m_2, \qquad M_{12} = (m_1+m_2)l_1\cos\theta_1,$$

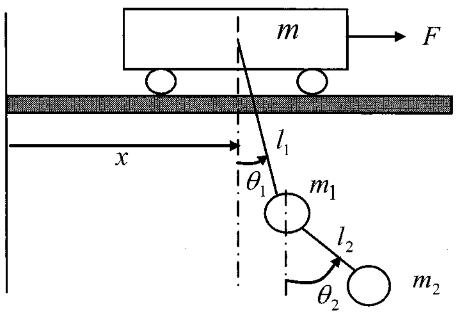

Fig. 1. DPTOC system scheme.

$$M_{13} = m_2 l_2\cos\theta_2, \qquad M_{21} = (m_1+m_2)l_1\cos\theta_1,$$

$$M_{22} = (m_1+m_2)l_1^2, \qquad M_{23} = m_2 l_1 l_2\cos(\theta_1-\theta_2),$$

$$M_{31} = m_2 l_2\cos\theta_2, \qquad M_{32} = m_2 l_1 l_2\cos(\theta_1-\theta_2),$$

$$M_{33} = m_2 l_2^2,$$

$$C(q,\dot{q}) = \begin{bmatrix} 0 & -(m_1+m_2)l_1\dot{\theta}_1\sin\theta_1 & -m_2 l_2\dot{\theta}_2\sin\theta_2 \\ 0 & 0 & m_2 l_1 l_2\dot{\theta}_1\sin(\theta_1-\theta_2) \\ 0 & -m_2 l_1 l_2\dot{\theta}_1\sin(\theta_1-\theta_2) & 0 \end{bmatrix}.$$

In addition to Properties 1-3, the DPTOC system has two different nature frequencies that is calculated through the linearization of (1) around $\theta_1 = 0$ and $\theta_2 = 0$:

$$\bar{M}(q)\ddot{q} + Kq = 0, \tag{15}$$

where $\bar{M}(q)$ is the linearization matrix of $M(q)$,

$$K = \begin{bmatrix} 0 & 0 & 0 \\ 0 & (m_1+m_2)gl_1 & 0 \\ 0 & 0 & m_2gl_2 \end{bmatrix}.$$

The nature frequencies can be obtained with nonzero eigenvalue of matrix $-\bar{M}^{-1}(q)K$ [18]:

$$\omega_{1,2} = \sqrt{\frac{g}{2}\left(\alpha \pm \sqrt{\beta}\right)}, \tag{16}$$

where

$$\alpha = \frac{m_1+m_2}{m_1}\left(\frac{1}{l_1}+\frac{1}{l_2}\right),$$

$$\beta = \left(\frac{m_1+m_2}{m_1}\right)^2\left(\frac{1}{l_1}+\frac{1}{l_2}\right)^2 - 4\left(\frac{m_1+m_2}{m_1}\right)\frac{1}{l_1 l_2}.$$

5.2. Dynamics simulation for DPTOC

In DPTOC system, the parameters that always change in different transport tasks are the hook mass m_1, the payload mass m_2, the cable length l_1 and the cable length l_2. The effects of these parameters'

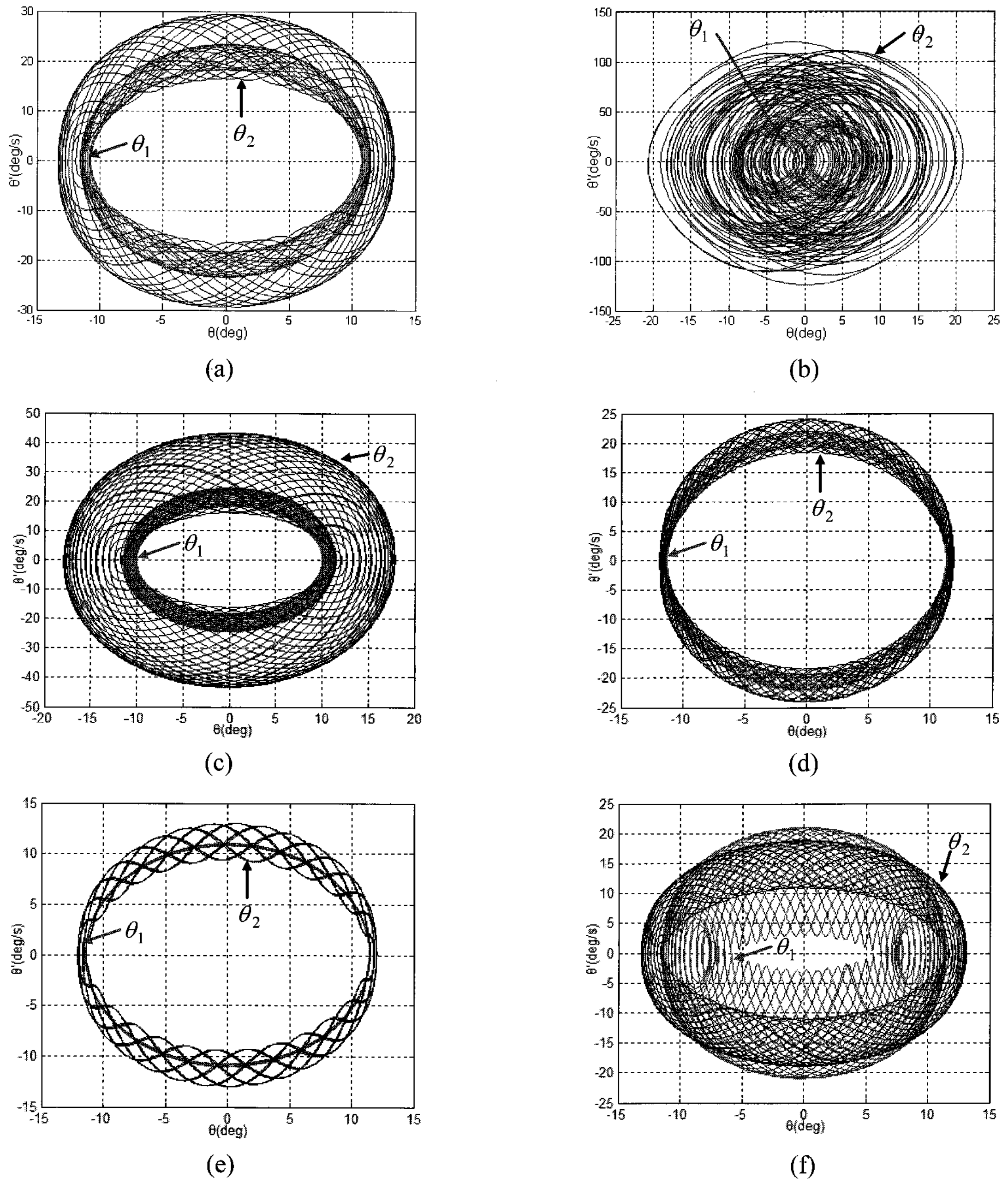

Fig. 2. System dynamics of the DPTOC for different initial condition or system parameters.

change and different initial condition are considered. In the simulations, the basic parameters, m=5Kg, m_1=2Kg, m_2=5Kg, l_1=2m, l_2=1m, are supposed to simulated an actual system. These system parameters can be proportionally set according to an actual DPTOC. Because the swing angles are ordinary less than a constant and the angle velocity is zero as the maximal swing angle arrives, and here 0.2radian (11.465degree) is supposed as the maximal swing angle, the basic initial state, $\theta_1 = 11.465^\circ$, $\dot{\theta}_1 = 0^\circ/\mathrm{s}$, $\theta_2 = 11.465^\circ$, $\dot{\theta}_2 = 0^\circ/\mathrm{s}$, $x = 0\mathrm{m}$, $\dot{x} = 0\mathrm{m/s}$, is adopted. The phase plane under the basic parameters and basic initial state are shown in Fig. 2(a). Only one parameter or one initial state is changed in the following simulations. When parameter θ_2 of initial state is changed to -11.465°, the phase plane is shown in Fig. 2(b). When system parameter m_1 is changed to 10Kg, the phase plane is shown in Fig. 2(c). When system parameter m_2 is changed to 25Kg, the phase plane is shown in Fig. 2(d). When system parameter l_1 is changed to 10m, the phase plane is shown in Fig. 2(e). When system parameter l_2 is changed to 5m, the phase plane is shown in Fig. 2(f). From the simulations of system dynamics, it can be seen: the system dynamics is more complex than the single-pendulum-type overhead crane. The system dynamics is greatly affected by system parameters' change. The heavier the payload becomes, the more the system dynamics is similar to single-pendulum-type overhead crane. Moreover, the system dynamics is greatly affected by the system initial conditions because of system's nonlinearity.

5.3. Stable control simulation

In the process of control simulations and optimization, the initial position $x = 0$m of the basic initial state is changed to $x = -30$m, the other initial state and system parameters of each simulation are same as the above system dynamics simulations. The corresponding controller parameters are: $I_m = [1]$, $T = [1]$, $Z = [1\ 0\ 0;\ 0\ 1\ 0]$, and in order to simplify the controller, only the position term in (9) is considered, namely make $K_D = 0$. The controller (13) becomes $\tau = -(K_P\Theta + K\dot{\Theta})/k_E$. Since the system only have one actuated coordinate variable x, make $k_p = K_p/k_E$ and $k = K/k_E$, the controller can be rewritten as

$$\tau = -k_p x - k\dot{x}. \quad (17)$$

Thus the chromosome consists of two real numbers $[k_p\ k]$.

In the fitness function (14) of GA, the coefficients represent the importance degrees of generalized coordinate variables and control inputs. In an actual transport, the anti-sway control is more important than the position control and the control of θ_2 is more difficult. In order to set these coefficients, the following states are considered: the transport is a long-distance transport; since the unit of both θ_1 and θ_2 is radian in computer programs, they should be transformed to degree through their coefficients to calculate the fitness function. Here, the coefficient μ_1 is set to [0.1 50 60]. In an actual transport, a large position overshoot maybe makes the payload collide with other equipments. Moreover, it is more dangerous because the swing angle and the position overshoot have the same direction around the desired position. Therefore, when the system is in the state where the overshoot of x arises, the coefficient *penalty* is added to decrease the fitness function of the corresponding chromosome. The longer the system stays in the overshoot state, the faster the chromosome vanishes in the evolution process. Here, *penalty* = 0.5 is adopted, and a small energy saving coefficient $\mu_2 = 0.01$ is set.

The convergence of fitness function is normally linked to the search space, the size of the population and the probabilities of crossover and mutation operations. Therefore, before introducing genetic operations, the search space is first set according to some rated values of the mechanical system, such as rated transport velocity and rated driving force. Here, $k_p \in (0,\ 20)$ and $k \in (0,\ 60)$ are adopted. Moreover, the population number and the probabilities is respectively set to $P = 30$, $p_c = 0.9$ and $p_m = 0.1$.

Using real-valued GA, the evolution of the best solution is illustrated in Fig. 3 for controlling DPTOC. After the evolution process, the best parameters of stable controller respectively are: $k_p = 5.7033$, $k = 16.3616$. From the convergence of fitness function and controller parameters, the system performance is improved through the real-valued GA.

After the controller (17) with the best parameters is added to the DPTOC system, where the desired position is chosen as the reference point of the potential energy, the time responses of the DPTOC are shown in Fig. 4. It is clear the proposed control law can transport the payload to the desired position while damping swing angle, and has some robustness to the parameters changes. The control performance is improved with a larger load mass and is degraded with a larger cable length l_1, where the system may be stabilized to a stable trajectory. Because the penalty term in (14) is included, there aren't the overshoots of trolley position around the desired position from the simulation results as Fig. 4(a), 4(b), 4(c), 4(e), and 4(f). Only when the payload mass m_2 becomes large enough, the overshoot arises as Fig. 4(d).

From the simulations, the dynamics of DPTOC is very complex and a stable controller is implemented with the proposed method. Comparing with other control methods [14-16], the controller has the following advantages: In addition to the ensured

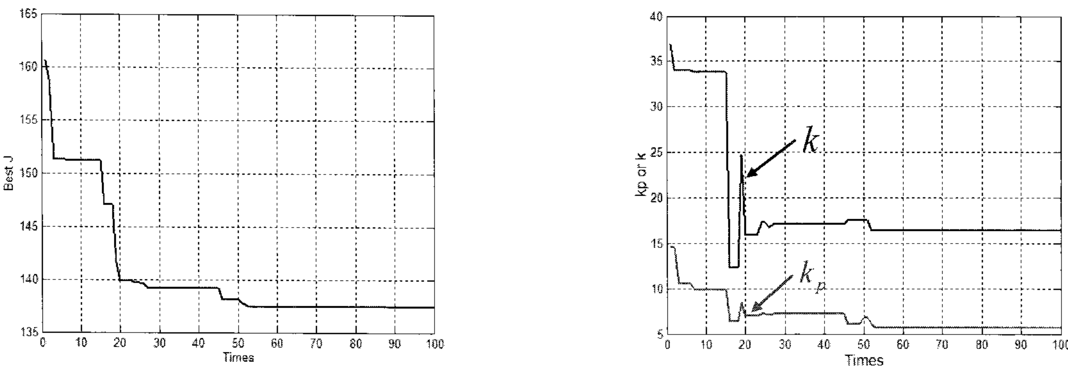

Fig. 3. Fitness function and gene for the best solution as a function of the number of generations.

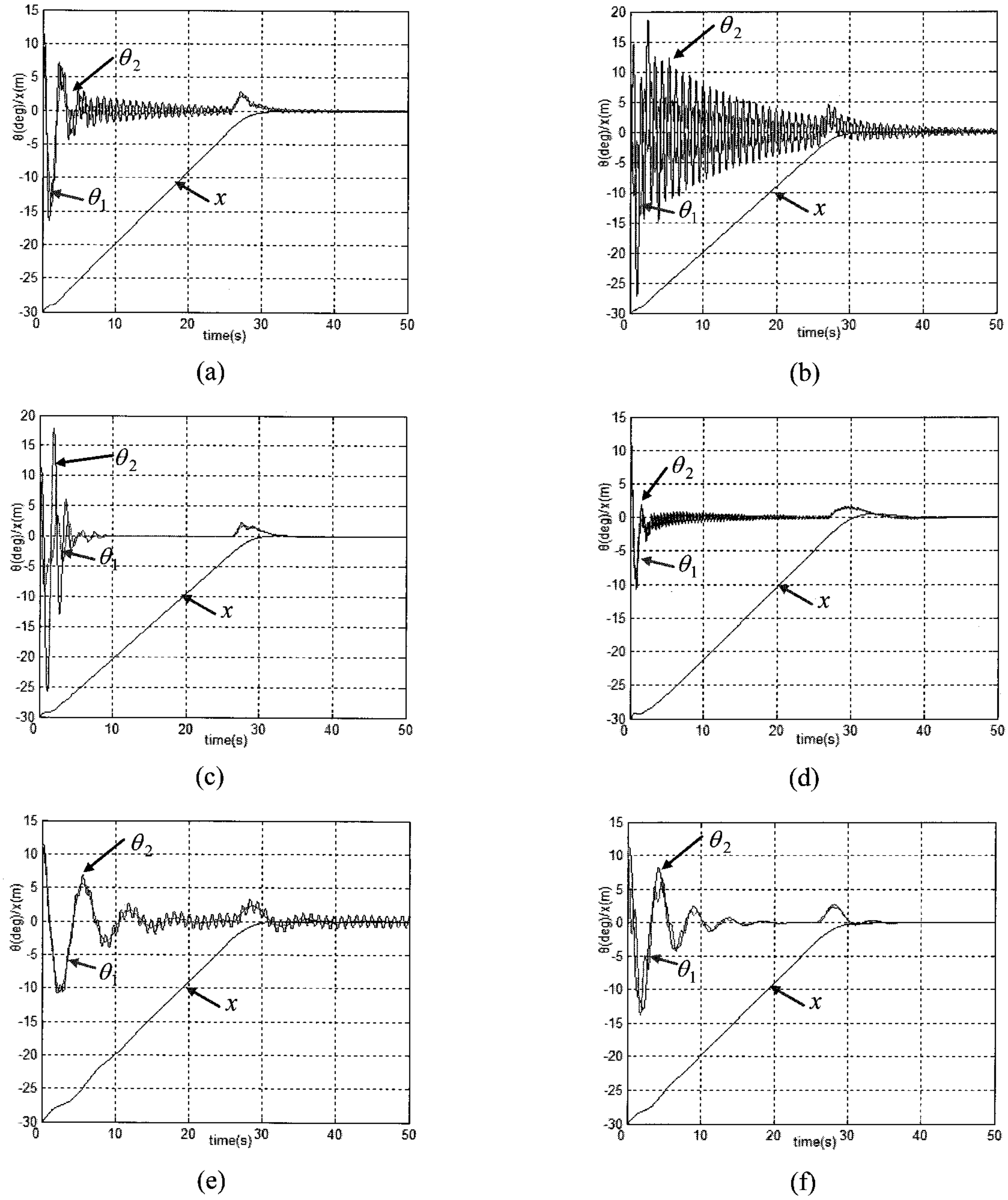

Fig. 4. System dynamics of the DPTOC with the proposed control algorithm for different initial condition and system parameters.

stability, it is very simple and easy to implement because only one actuated coordinate variable, trolley position is needed to be measured. However, the control performance is not so good as sliding mode fuzzy control method [15].

6. CONCLUSIONS

Control of UMSs is currently an active field of research due to their broad applications while the restriction of control inputs of UMSs brings forward a challenging control problem. The dynamic model of the UMS in a potential field is built with Lagrangian method and their several structural properties such as the positive definite symmetric inertia matrix and the passivity are analyzed. A real- valued GA-based stable control method is proposed for a class of UMSs. With the choice of the controller parameters, the method can greatly reduce the number of system states that need measured. The underactuated DPTOC system is used to verify the validity of the proposed control algorithm. Simulation results illustrate the complex dynamics of the DPTOC and the effectiveness of proposed control algorithm.

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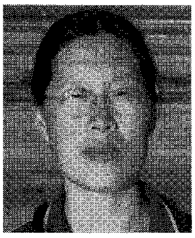

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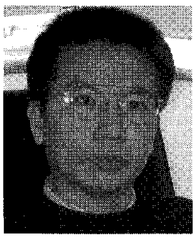

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