

# Delay-Dependent Guaranteed Cost Control for Uncertain Neutral Systems with Distributed Delays

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**Abstract:** This paper considers the problem of delay-dependent guaranteed cost controller design for uncertain neutral systems with distributed delays. The system under consideration is subject to norm-bounded time-varying parametric uncertainty appearing in all the matrices of the state-space model. By constructing appropriate Lyapunov functionals and using matrix inequality techniques, a state feedback controller is designed such that the resulting closed-loop system is not only robustly stable but also guarantees an adequate level of performance for all admissible uncertainties. Furthermore, a convex optimization problem is introduced to minimize a specified cost bound. By matrix transformation techniques, the corresponding optimal guaranteed controller can be obtained by solving a linear matrix inequality. Finally, a simulation example is presented to demonstrate the effectiveness of the proposed approach.

**Keywords:** Distributed delay, linear matrix inequality, neutral systems, robust guaranteed control, robust stabilization.

## 1. INTRODUCTION

The guaranteed cost control problem of uncertain systems was first put forward in [1] and then has been extensively investigated; the purpose is to design a controller to robustly stabilize an uncertain system while guaranteeing an adequate level of performance. On the other hand, as is well known, time delay is frequently one of the main causes of instability and poor performance of a control system [7,13,14]. Therefore, analysis and synthesis of time-delay systems have attracted a great deal of attention [9,17,19]. It is noted that the guaranteed cost control approach has been extended to various types of uncertain time-delay systems; see, e.g., [10,15,18,20-22,25], and the references therein. When not all the states are available for feedback, dynamic output feedback controllers were designed in [16] to solve

the guaranteed cost control problem.

In practical applications there are numerous control systems depending not only on state delays but also on derivatives of delayed states. Such systems are referred to as neutral delay systems [2,27,28,30]. The guaranteed cost control problem related to neutral delay systems has been studied in [23,24] and [26], respectively. It is worth noting that although the design methods used in these references are delay-independent, the achieved guaranteed costs depend on the size of the time delay. This fact suggests that delay-dependent design method give lower cost value than the delay-independent ones [3,11,14]. When the number of summands in a system equation is increased and the differences between neighboring argument values are decreased, systems with distributed delays will arise. Distributed delays can also be found in the modeling of feeding systems and combustion chambers in a liquid monopropellant rocket motor with pressure feeding [4,6]. Therefore, systems with distributed delays have received much attention in the past years. Results on stability analysis and controller design for such systems can be found in [7]. However, the problem of delay-dependent guaranteed cost control for uncertain neutral systems with distributed delays has not been investigated so far, which is still open and remains challenging. This motivates the present study.

In this paper, by utilizing the free weighing matrix method [12], we consider the problem of delay-dependent guaranteed cost control for uncertain neutral delay systems with time-varying norm-bounded parametric uncertainties and distributed

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delays. The performance index is assumed to be integral quadratic cost functions. The purpose of the problem we address is the design of a state feedback controller such that the closed-loop system is stable and an adequate level of performance is guaranteed for all admissible uncertainties. A sufficient condition for the solvability of this problem is obtained in terms of an LMI. When this LMI is feasible, an explicit expression of the desired guaranteed cost controller is given. It is worth pointing out that the LMI approach developed in the paper does not involve any tuning of parameters and thus can be computed effectively by using interior point algorithm [31].

**Notation:** Throughout this paper, for real symmetric matrices  $X$  and  $Y$ ,  $X \geq Y$  (respectively,  $X > Y$ ) means that the matrix  $X - Y$  is positive semi-definite (respectively, positive definite).  $I$  is an identity matrix with appropriate dimension. The superscript “ $T$ ” represents the transpose of a matrix. The notation “ $*$ ” is used as an ellipsis for terms that are induced by symmetry. Matrices, if the dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

## 2. PROBLEM FORMULATION

Consider a class of uncertain neutral systems with distributed delays described by:

$$\begin{aligned} \Sigma : \dot{x}(t) &= [A + \Delta A(t)]x(t) + [A_h + \Delta A_h(t)] \\ &\quad \times x(t - \tau_1(t)) + [A_{d_1} + \Delta A_{d_1}(t)] \\ &\quad \times \dot{x}(t - \tau_2(t)) + [A_{d_2} + \Delta A_{d_2}(t)] \\ &\quad \times \int_{t-h_3}^t x(s)ds + [B + \Delta B(t)]u(t), \\ x(t) &= \varphi(t), \quad t \in [-h, 0], \end{aligned}$$

where  $x(t) \in R^n$  is the state;  $u(t) \in R^m$  is the control input;  $\varphi(t)$  is the continuously differentiable initial function on  $[-h, 0]$  with  $h = \max\{h_1, h_2, h_3\}$ ;  $A$ ,  $A_h$ ,  $A_{d_1}$ ,  $A_{d_2}$  and  $B$  are known real-valued matrices representing time-varying parameter uncertainties, and are assumed to be of the form:

$$\begin{aligned} &[\Delta A(t), \Delta A_h(t), \Delta A_{d_1}(t), \Delta A_{d_2}(t), \Delta B(t)] \\ &= MF(t)[N_1, N_2, N_3, N_4, N_5], \end{aligned} \quad (1)$$

where  $M, N_1, N_2, N_3, N_4$  are known real constant matrices of appropriate dimensions and  $F(\cdot): R \rightarrow R^{l_1 \times l_2}$  is an unknown time-varying matrix function satisfying:

$$F(t)^T F(t) \leq I. \quad (2)$$

The uncertain matrices  $\Delta A(t)$ ,  $\Delta A_h(t)$ ,  $\Delta A_{d_1}(t)$ ,

$\Delta A_{d_2}(t)$  and  $\Delta B(t)$  are said to be admissible if both (1) and (2) hold.  $\tau_1(t)$  and  $\tau_2(t)$  are the time-varying delays of the system and satisfy:

$$0 \leq \tau_i(t) \leq h_i, \quad i = 1, 2, \quad (3)$$

$$\dot{\tau}_1(t) \leq d_1, \dot{\tau}_2(t) \leq d_2 < 1. \quad (4)$$

Now, consider the following linear state-feedback controller:

$$u(t) = Kx(t), \quad (5)$$

where  $K \in R^{m \times n}$  is the controller gain to be determined. Then the resulting closed-loop system from  $\Sigma$  and (5) as:

$$\begin{aligned} \Sigma^c : \dot{x}(t) &= A(t)x(t) + A_h(t)x(t - \tau_1(t)) \\ &\quad + A_{d_1}(t)\dot{x}(t - \tau_2(t)) + A_{d_2}(t)\alpha(t), \\ x(t) &= \varphi(t), t \in [-h, 0], \end{aligned}$$

where

$$\begin{aligned} A(t) &= A + BK + \Delta A(t) + \Delta B(t)K, \\ A_h(t) &= A_h + \Delta A_h(t), A_{d_1}(t) = A_{d_1} + \Delta A_{d_1}(t), \\ A_{d_2}(t) &= A_{d_2} + \Delta A_{d_2}(t), \alpha(t) = \int_{t-h_3}^t x(s)ds. \end{aligned}$$

Associated with the delay system ( $\Sigma^c$ ) we define the cost function as:

$$J = \int_0^\infty [x(t)^T R_1 x(t) + u(t)^T R_2 u(t)] dt, \quad (6)$$

where  $R_1$  and  $R_2$  are given matrices with  $R_1 > 0$ ,  $R_2 > 0$ . Throughout this paper, we shall use the following definition:

**Definition 1:** The uncertain neutral delay system ( $\Sigma$ ) is said to be robustly stable if the equilibrium solution of system ( $\Sigma$ ) with  $u(t) = 0$  is globally asymptotically stable for all admissible uncertainties  $\Delta A(t)$ ,  $\Delta A_h(t)$ ,  $\Delta A_{d_1}(t)$ ,  $\Delta A_{d_2}(t)$  and  $\Delta B(t)$ .

The guaranteed cost control problem to be addressed in this paper can be formulated as follows: Given three scalars  $h_i > 0, i = 1, 2, 3$ , design a state-feedback controller in (5) such that for any time-varying delays  $\tau_i(t)$  satisfying (3) and (4),  $i = 1, 2$ , the closed-loop system ( $\Sigma$ ) is robustly stable and the cost-function in (6) has an upper bound for all admissible uncertainties. In this case, (5) is said to be a guaranteed cost state-feedback controller.

Before concluding this section, we introduce the following lemma, which will be used to derive our main results in the next section.

**Lemma 1** [29]: Let  $A, D, S, W$  and  $F$  be real matrices with appropriate dimensions such that

$W > 0$  and  $F^T F \leq I$ . Then we have the following results:

(1) For any scalar  $\varepsilon > 0$  and vectors  $x$  and  $y$  of appropriate dimensions,

$$2x^T DFSy \leq \varepsilon^{-1} x^T DD^T x + \varepsilon y^T S^T S y.$$

(2) For any scalar  $\varepsilon > 0$  such that  $W - \varepsilon DD^T > 0$ ,

$$\begin{aligned} & (A + DFS)^T W^{-1} (A + DFS) \\ & \leq A^T (W - \varepsilon DD^T)^{-1} A + \varepsilon^{-1} S^T S. \end{aligned}$$

### 3. MAIN RESULTS

In this section, we will give a sufficient condition for the robust guaranteed cost control problem formulated in the previous section.

**Theorem 1:** Consider the uncertain neutral time-delay system  $\Sigma^c$  and the cost function (6). Then, for given matrices  $R_1 > 0, R_2 > 0$  and three scalars  $h_i > 0, i = 1, 2, 3$ , the guaranteed cost control problem is solvable if there exist matrices  $X > 0, X_1 > 0, X_{21} > 0, X_{22} > 0, X_3 > 0, P_4 > 0, P_5 > 0, S_1, S_2, W_1, W_2$  and scalars  $\varepsilon_1 > 0, \varepsilon_2 > 0, \varepsilon_3 > 0, \varepsilon_4 > 0$ , such that the following LMI holds:

$$\begin{bmatrix} H_{11} & H_{12} \\ * & H_{22} \end{bmatrix} < 0, \quad (7)$$

where

$$\begin{aligned} \Omega_1 &= XA^T + Y^T B^T, \Omega_2 = XN_1^T + Y^T N_5^T, \\ \Omega_{11} &= AX + BY + \Omega_1 + X_1 + X_{21} + h_3^2 X_3 \\ &\quad - S_1 - S_1^T - W_1 - W_1^T, \\ \Omega_{12} &= A_h X + W_1 - W_2^T, \Omega_{13} = S_1 - S_2^T, \\ \Omega_{22} &= W_2 + W_2^T - (1 - d_1) X_1, \\ \Omega_{33} &= S_2 + S_2^T - (1 - d_2) X_{21}, \\ \Omega_{44} &= -(1 - d_2) X_{22}, \Omega_{55} = -X_3, \\ \Omega_{66} &= h_1 P_4 - 2h_1 X, \Omega_{77} = h_2 P_5 - 2h_2 X, \\ \Omega_{88} &= \varepsilon_2 MM^T - h_1^{-1} P_4, \Omega_{99} = \varepsilon_3 MM^T - h_2^{-1} P_5, \end{aligned}$$

$$H_{11} = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ * & \Omega_{22} & 0 \\ * & * & \Omega_{33} \\ * & * & * \\ * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$$

$$H_{12} = \begin{bmatrix} A_{d_1} X_{22} & A_{d_2} X & h_1 W_1 & h_2 S_1 \\ 0 & 0 & h_1 W_2 & 0 \\ 0 & 0 & 0 & h_2 S_2 \\ \Omega_{44} & 0 & 0 & 0 \\ * & \Omega_{55} & 0 & 0 \\ * & * & \Omega_{66} & 0 \\ * & * & * & \Omega_{77} \end{bmatrix},$$

$$H_{12} = \begin{bmatrix} \Omega_1 & \Omega_1 & \Omega_1 & \Omega_2 \\ XA_h^T & XA_h^T & XA_h^T & XN_2^T \\ 0 & 0 & 0 & 0 \\ X_{22} A_{d_1}^T & X_{22} A_{d_1}^T & X_{22} A_{d_1}^T & X_{22} N_3^T \\ XA_{d_2}^T & XA_{d_2}^T & XA_{d_2}^T & XN_4^T \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \Omega_2 & \Omega_2 & \Omega_2 & M & X & Y^T \\ XN_2^T & XN_2^T & XN_2^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ X_{22} N_3^T & X_{22} N_3^T & X_{22} N_3^T & 0 & 0 & 0 \\ XN_4^T & XN_4^T & XN_4^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$H_{22} = \text{diag}\{\Omega_{88}, \Omega_{99}, \varepsilon_4 MM^T - X_{22}, -\varepsilon_2 I, -\varepsilon_3 I, -\varepsilon_4 I, -\varepsilon_1 I, -\varepsilon_1 I, -R_1^{-1}, -R_2^{-1}\}.$$

In this case, a desired guaranteed cost state-feedback controller can be chosen as:

$$u(t) = YX^{-1}x(t), \quad (8)$$

and the corresponding cost function in (6) satisfies:

$$\begin{aligned} J &\leq \varphi(0)^T X^{-1} \varphi(0) + \int_{h_1}^0 \varphi(s)^T X^{-1} X_1 X^{-1} \varphi(s) ds \\ &\quad + \int_{-h_2}^0 \varphi(s)^T X^{-1} X_{21} X^{-1} \varphi(s) ds \\ &\quad + \int_{-h_2}^0 \dot{\varphi}(s)^T X_{22}^{-1} \dot{\varphi}(s) ds \\ &\quad + \int_{-h_3}^0 \left[ \int_s^0 \varphi(\theta)^T d\theta \right]^T X^{-1} \\ &\quad \quad \times X_3 X^{-1} \left[ \int_s^0 \varphi(\theta)^T d\theta \right] ds \\ &\quad + \int_0^{h_3} ds \int_s^0 (\theta + s) \varphi(\theta)^T X^{-1} X_3 X^{-1} \varphi(\theta) d\theta \\ &\quad + \int_{-h_1}^0 \int_\theta^0 \dot{\varphi}(s)^T P_4 \dot{\varphi}(s) ds d\theta \\ &\quad + \int_{-h_2}^0 \int_\theta^0 \dot{\varphi}(s)^T P_5 \dot{\varphi}(s) ds d\theta. \end{aligned} \quad (9)$$

**Proof:** Under the condition of the theorem, we first show the robust stability of the closed-loop system ( $\Sigma^c$ ). To this end, we denote

$$\begin{aligned} P &= X^{-1}, \quad K = YX^{-1}, \quad P_1 = PX_1P, \quad P_{21} = PX_{21}P, \\ P_3 &= PX_3P, \quad \bar{P}_4 = P_4^{-1}, \quad \tilde{W}_1 = PW_1P, \quad \tilde{W}_2 = PW_2P, \\ \bar{S}_1 &= PS_1P, \quad \bar{S}_2 = PS_2P, \quad P_{22} = X_{22}^{-1}, \quad \bar{P}_5 = P_5^{-1}, \end{aligned}$$

then, pre- and post-multiplying (7) by  $\text{diag}\{P, P, P, P_{22}, P, P, P, I, I, I, I, I, I, I, I\}$ , we obtain

$$\begin{bmatrix} \tilde{H}_{11} & \tilde{H}_{12} \\ * & \tilde{H}_{22} \end{bmatrix} < 0, \quad (10)$$

where

$$\begin{aligned} \tilde{\Omega}_1 &= A^T + K^T B^T, \quad \tilde{\Omega}_2 = N_1^T + K^T N_5^T, \\ \tilde{\Omega}_{11} &= PA + PBK + P\Omega_1P + P_1 + P_{21} + h_3^2 P_3 \\ &\quad - \tilde{S}_1 - \tilde{S}_1^T - \tilde{W}_1 - \tilde{W}_1^T, \\ \tilde{\Omega}_{12} &= PA_h + \tilde{W}_1 - \tilde{W}_2^T, \quad \tilde{\Omega}_{13} = \tilde{S}_1 - \tilde{S}_2^T, \\ \tilde{\Omega}_{22} &= \tilde{W}_2 + \tilde{W}_2^T - (1 - d_1)P_1, \\ \Omega_{33} &= \tilde{S}_2 + \tilde{S}_2^T - (1 - d_2)P_{21}, \\ \tilde{\Omega}_{44} &= -(1 - d_2)P_{22}, \quad \tilde{\Omega}_{55} = -P_3, \\ \tilde{\Omega}_{66} &= h_1 PP_4P - 2h_1P, \quad \tilde{\Omega}_{77} = h_2 PP_5P - 2h_2P, \\ \tilde{\Omega}_{88} &= \varepsilon_2 MM^T - h_1^{-1} \bar{P}_4^{-1}, \\ \tilde{\Omega}_{99} &= \varepsilon_3 MM^T - h_2^{-1} \bar{P}_5^{-1}, \\ \tilde{H}_{11} &= \begin{bmatrix} \tilde{\Omega}_{11} & \tilde{\Omega}_{12} & \tilde{\Omega}_{13} & PA_{d_1} & PA_{d_2} & h_1 \tilde{W}_1 & h_2 \tilde{S}_1 \\ * & \tilde{\Omega}_{22} & 0 & 0 & 0 & h_1 \tilde{W}_2 & 0 \\ * & * & \tilde{\Omega}_{33} & 0 & 0 & 0 & h_2 \tilde{S}_2 \\ * & * & * & \tilde{\Omega}_{44} & 0 & 0 & 0 \\ * & * & * & * & \tilde{\Omega}_{55} & 0 & 0 \\ * & * & * & * & * & \tilde{\Omega}_{66} & 0 \\ * & * & * & * & * & * & \tilde{\Omega}_{77} \end{bmatrix}, \end{aligned}$$

$$\tilde{H}_{12} =$$

$$\begin{bmatrix} \tilde{\Omega}_1 & \tilde{\Omega}_1 & \tilde{\Omega}_1 & \tilde{\Omega}_2 & \tilde{\Omega}_2 & \tilde{\Omega}_2 & \tilde{\Omega}_2 \\ A_h^T & A_h^T & A_h^T & N_2^T & N_2^T & N_2^T & N_2^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{d_1}^T & A_{d_1}^T & A_{d_1}^T & N_3^T & N_3^T & N_3^T & N_3^T \\ A_{d_2}^T & A_{d_2}^T & A_{d_2}^T & N_4^T & N_4^T & N_4^T & N_4^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} PM & I & K^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\begin{aligned} \tilde{H}_{22} &= \text{diag}\{\tilde{\Omega}_{88}, \tilde{\Omega}_{99}, \varepsilon_4 MM^T - P_{22}^{-1}, -\varepsilon_2 I, \\ &\quad -\varepsilon_3 I, -\varepsilon_4 I, -\varepsilon_1 I, -\varepsilon_1 I, -R_1^{-1}, -R_2^{-1}\}. \end{aligned}$$

It is easy to verify that

$$(P_4^{-1} - P)P_4(P_4^{-1} - P) \geq 0, \quad (11)$$

$$(P_5^{-1} - P)P_5(P_5^{-1} - P) \geq 0, \quad (12)$$

which imply

$$h_1 PP_4P - 2h_1P \geq -h_1P_4^{-1}, \quad (13)$$

$$h_2 PP_5P - 2h_2P \geq -h_2P_5^{-1}. \quad (14)$$

This together with (10) provides

$$\begin{bmatrix} \hat{H}_{11} & \tilde{H}_{12} \\ * & \tilde{H}_{22} \end{bmatrix} < 0, \quad (15)$$

where

$$\hat{\Omega}_{66} = -h_1 \bar{P}_4, \quad \hat{\Omega}_{77} = -h_2 \bar{P}_5, \quad (16)$$

$$\hat{H}_{11} =$$

$$\begin{bmatrix} \tilde{\Omega}_{11} & \tilde{\Omega}_{12} & \tilde{\Omega}_{13} & PA_{d_1} & PA_{d_2} & h_1 \tilde{W}_1 & h_2 \tilde{S}_1 \\ * & \tilde{\Omega}_{22} & 0 & 0 & 0 & h_1 \tilde{W}_2 & 0 \\ * & * & \tilde{\Omega}_{33} & 0 & 0 & 0 & h_2 \tilde{S}_2 \\ * & * & * & \tilde{\Omega}_{44} & 0 & 0 & 0 \\ * & * & * & * & \tilde{\Omega}_{55} & 0 & 0 \\ * & * & * & * & * & \hat{\Omega}_{66} & 0 \\ * & * & * & * & * & * & \hat{\Omega}_{77} \end{bmatrix}. \quad (17)$$

Denote

$$\bar{A} = [A + BK, A_h, 0, A_{d_1}, A_{d_2}, 0, 0], \quad (18)$$

$$\bar{N} = [N_1 + N_5K, N_2, 0, N_3, N_4, 0, 0], \quad (19)$$

then, by the Schur complement formula and (3), we obtain

$$\begin{aligned} &\bar{H}_{11} + \text{diag}\{R_1 + K^T R_2 K, 0, 0, 0, 0, 0, 0\} \\ &\quad + (\varepsilon_1^{-1} + \varepsilon_2^{-1} + \varepsilon_3^{-1} + \varepsilon_4^{-1}) \bar{N}^T \bar{N} \\ &\quad - \bar{A}^T [(\varepsilon_3 MM^T - h_2^{-1} \bar{P}_5^{-1})^{-1} \end{aligned} \quad (20)$$

$$\begin{aligned}
 & + (\varepsilon_2 MM^T - h_1^{-1} \tilde{P}_4^{-1})^{-1} \\
 & + (\varepsilon_4 MM^T - h_1^{-1} P_{22}^{-1})^{-1} \bar{A} < 0,
 \end{aligned}$$

where

$$\bar{\Omega}_{66} = -\tau_1(t) \tilde{P}_4, \bar{\Omega}_{77} = -\tau_2(t) \tilde{P}_5, \quad (21)$$

$$\bar{H}_{11} =$$

$$\begin{bmatrix}
 \tilde{\Omega}_{11} & \tilde{\Omega}_{12} & \tilde{\Omega}_{13} & PA_{d_1} & PA_{d_2} & \tau_1(t) \tilde{W}_1 & \tau_2(t) \tilde{S}_1 \\
 * & \tilde{\Omega}_{22} & 0 & 0 & 0 & \tau_1(t) \tilde{W}_2 & 0 \\
 * & * & \tilde{\Omega}_{33} & 0 & 0 & 0 & \tau_2(t) \tilde{S}_2 \\
 * & * & * & \tilde{\Omega}_{44} & 0 & 0 & 0 \\
 * & * & * & * & \tilde{\Omega}_{55} & 0 & 0 \\
 * & * & * & * & * & \bar{\Omega}_{66} & 0 \\
 * & * & * & * & * & * & \bar{\Omega}_{77}
 \end{bmatrix}. \quad (22)$$

Now, for the closed-loop system ( $\Sigma^c$ ), we define the following Lyapunov functional candidate:

$$\begin{aligned}
 V(t) = & V_0(t) + V_1(t) + V_{21}(t) + V_{22}(t) \\
 & + V_3(t) + V_4(t) + V_5(t) + V_6(t), \quad (23)
 \end{aligned}$$

where

$$V_0(t) = x(t)^T P x(t),$$

$$V_1(t) = \int_{t-\tau_1(t)}^t x(s)^T P_1 x(s) ds,$$

$$V_{21}(t) = \int_{t-\tau_2(t)}^t x(s)^T P_{21} x(s) ds,$$

$$V_{22}(t) = \int_{t-\tau_2(t)}^t \dot{x}(s)^T P_{22} \dot{x}(s) ds,$$

$$V_3(t) = \int_{t-h_3}^t \left[ \int_s^t x(\theta)^T d\theta \right] P_3 \left[ \int_s^t x(\theta)^T d\theta \right] ds,$$

$$V_4(t) = \int_0^{h_3} ds \int_{t-s}^t (\theta + s - t) x(\theta)^T P_3 x(\theta) d\theta,$$

$$V_5(t) = \int_{-h_1}^0 \int_{t+\theta}^t \dot{x}(s)^T \tilde{P}_4 \dot{x}(s) ds d\theta,$$

$$V_6(t) = \int_{-h_2}^0 \int_{t+\theta}^t \dot{x}(s)^T \tilde{P}_5 \dot{x}(s) ds d\theta,$$

then, the time derivative of  $V(t)$  along the trajectory of system ( $\Sigma^c$ ) is given by

$$\begin{aligned}
 \dot{V}(t) = & \dot{V}_0(t) + \dot{V}_1(t) + \dot{V}_{21}(t) + \dot{V}_{22}(t) \\
 & + \dot{V}_3(t) + \dot{V}_4(t) + \dot{V}_5(t) + \dot{V}_6(t), \quad (24)
 \end{aligned}$$

where

$$\begin{aligned}
 \dot{V}_0(t) = & 2x(t)^T P \dot{x}(t) \\
 = & 2x(t)^T P [A(t)x(t) + A_h(t)x(t - \tau_1(t)) \\
 & + A_{d_1}(t)\dot{x}(t - \tau_2(t)) + A_{d_2}(t)\alpha(t)] \\
 & + 2x(t)^T \tilde{W}_1 \int_{t-\tau_1(t)}^t \dot{x}(s) ds
 \end{aligned}$$

$$\begin{aligned}
 & - 2x(t)^T \tilde{W}_1 [x(t) - x(t - \tau_1(t))] \\
 & + 2x(t - \tau_1(t))^T \tilde{W}_2 \int_{t-\tau_1(t)}^t \dot{x}(s) ds \\
 & - 2x(t - \tau_1(t))^T \tilde{W}_2 [x(t) - x(t - \tau_1(t))] \\
 & + 2x(t)^T \tilde{S}_1 \int_{t-\tau_2(t)}^t \dot{x}(\theta) d\theta \\
 & - 2x(t)^T \tilde{S}_1 [x(t) - x(t - \tau_2(t))] \\
 & + 2x(t - \tau_2(t))^T \tilde{S}_2 \int_{t-\tau_2(t)}^t \dot{x}(\theta) d\theta \\
 & - 2x(t - \tau_2(t))^T \tilde{S}_2 [x(t) - x(t - \tau_2(t))],
 \end{aligned}$$

$$\begin{aligned}
 \dot{V}_1(t) \leq & x(t)^T P_1 x(t) \\
 & - (1 - d_1)x(t - \tau_1(t))^T P_1 x(t - \tau_1(t)),
 \end{aligned}$$

$$\begin{aligned}
 \dot{V}_{21}(t) \leq & x(t)^T P_{21} x(t) \\
 & - (1 - d_2)x(t - \tau_2(t))^T P_{21} x(t - \tau_2(t)),
 \end{aligned}$$

$$\begin{aligned}
 \dot{V}_{22}(t) \leq & x(t)^T P_{22} x(t) \\
 & - (1 - d_2)\dot{x}(t - \tau_2(t))^T P_{22} \dot{x}(t - \tau_2(t)),
 \end{aligned}$$

$$\begin{aligned}
 \dot{V}_4(t) = & \frac{h_3^2}{2} x(t)^T P_3 x(t) \\
 & - \int_{t-h_3}^t x(\theta)^T P_3 x(\theta) (\theta - t + h_3) d\theta,
 \end{aligned}$$

$$\dot{V}_5(t) \leq \dot{x}(t)^T h_1 \tilde{P}_4 \dot{x}(t) - \int_{t-\tau_1(t)}^t \dot{x}(s)^T \tilde{P}_4 \dot{x}(s) ds,$$

$$\dot{V}_6(t) \leq \dot{x}(t)^T h_2 \tilde{P}_5 \dot{x}(t) - \int_{t-\tau_2(t)}^t \dot{x}(\theta)^T \tilde{P}_5 \dot{x}(\theta) d\theta.$$

By Lemma 2, there exist  $\varepsilon_1 > 0$ ,  $\varepsilon_2 > 0$ ,  $\varepsilon_3 > 0$ , and  $\varepsilon_4 > 0$ , such that

$$\begin{aligned}
 \dot{V}_3(t) \leq & \frac{h_3^2}{2} x(t)^T P_3 x(t) \\
 & + \int_{t-h_3}^t x(\theta)^T P_3 x(\theta) (\theta - t + h_3) d\theta, \\
 2x(t)^T & PA(t)x(t) + 2x(t)^T PA_h(t)x(t - \tau_1(t)) \\
 & + 2x(t)^T PA_{d_1}(t)\dot{x}(t - \tau_2(t)) + 2x(t)^T PA_{d_2}(t)\alpha(t) \\
 \leq & 2x(t)^T P(A + BK)x(t) \\
 & + 2x(t)^T PA_h x(t - \tau_1(t)) + 2x(t)^T PA_{d_1} \dot{x}(t - \tau_2(t)) \\
 & + 2x(t)^T PA_{d_2} \alpha(t) + \varepsilon_1 x(t)^T PMM^T P x(t) \\
 & + \varepsilon_1^{-1} \eta(t, s, \theta)^T \bar{N}^T \bar{N} \eta(t, s, \theta),
 \end{aligned}$$

$$\begin{aligned}
 & \dot{x}(t)^T h_1 \tilde{P}_4 \dot{x}(t) \\
 \leq & \eta(t, s, \theta)^T [\bar{A}^T (h_1^{-1} \tilde{P}_4^{-1} \\
 & - \varepsilon_2 MM^T)^{-1} \bar{A} + \varepsilon_2^{-1} \bar{N}^T \bar{N}] \eta(t, s, \theta),
 \end{aligned}$$

$$\begin{aligned}
 & \dot{x}(t)^T h_2 \tilde{P}_5 \dot{x}(t) \\
 \leq & \eta(t, s, \theta)^T [\bar{A}^T (h_2^{-1} \tilde{P}_5^{-1}
 \end{aligned}$$

$$\begin{aligned}
& -\varepsilon_3 MM^T)^{-1} \bar{A} + \varepsilon_3^{-1} \bar{N}^T \bar{N}] \eta(t, s, \theta), \\
& \dot{x}(t)^T P_{22} \dot{x}(t) \\
& \leq \eta(t, s, \theta)^T [\bar{A}^T (P_{22}^{-1} \\
& \quad - \varepsilon_4 MM^T)^{-1} \bar{A} + \varepsilon_4^{-1} \bar{N}^T \bar{N}] \eta(t, s, \theta),
\end{aligned}$$

where

$$\begin{aligned}
\eta(t, s, \theta) = & [x(t)^T, x(t - \tau_1(t))^T, x(t - \tau_2(t))^T, \\
& \dot{x}(t - \tau_2(t))^T, \alpha(t)^T]^T.
\end{aligned}$$

Therefore, from (20) we obtain

$$\begin{aligned}
\dot{V}(t) \leq & \frac{1}{\tau_1(t)\tau_2(t)} \int_{t-\tau_1(t)}^t \int_{t-\tau_1(t)}^t \eta(t, s, \theta)^T \\
& \times \Xi \eta(t, s, \theta) ds d\theta < 0,
\end{aligned} \quad (25)$$

where

$$\begin{aligned}
\Xi = & \bar{H}_{11} + (\varepsilon_1^{-1} + \varepsilon_2^{-1} + \varepsilon_3^{-1} + \varepsilon_4^{-1}) \bar{N}^T \bar{N} \\
& - \bar{A}^T [(\varepsilon_3 MM^T - h_2^{-1} \bar{P}_5^{-1})^{-1} \\
& + (\varepsilon_2 MM^T - h_1^{-1} \bar{P}_4^{-1})^{-1} + (\varepsilon_4 MM^T - P_{22}^{-1})^{-1}] \bar{A}.
\end{aligned}$$

Hence, by [5,8], it follows from (25) that the closed-loop system ( $\Sigma^c$ ) is asymptotically stable for all admissible uncertainties.

In the next, we note that (20) implies

$$\dot{V}(t) \leq -x(t)^T (R_1 + K^T R_2 K) x(t). \quad (26)$$

Integrating both sides of (26) from 0 to any  $T > 0$  gives:

$$\begin{aligned}
& \int_0^T x(t)^T (R_1 + K^T R_2 K) x(t) dt \\
& \leq \varphi(0)^T P \varphi(0) + \int_{-h_1}^0 \varphi(s)^T P_1 \varphi(s) ds \\
& \quad + \int_{-h_2}^0 \varphi(s)^T P_{21} \varphi(s) ds + \int_{-h_2}^0 \dot{\varphi}(s)^T P_{22} \dot{\varphi}(s) ds \\
& \quad + \int_{-h_3}^0 [\int_s^0 \varphi(\theta)^T d\theta]^T P_3 [\int_s^0 \varphi(\theta)^T d\theta] ds \\
& \quad + \int_0^{h_3} ds \int_{-s}^0 (\theta + s) \varphi(\theta)^T P_3 \varphi(\theta) d\theta \\
& \quad + \int_{-h_1}^0 \int_{\theta}^0 \dot{\varphi}(s)^T P_4 \dot{\varphi}(s) ds d\theta \\
& \quad + \int_{-h_2}^0 \int_{\theta}^0 \dot{\varphi}(s)^T P_5 \dot{\varphi}(s) ds d\theta.
\end{aligned} \quad (27)$$

Therefore (9) is satisfied. This completes the proof.  $\square$

It is worth noting that Theorem 1 gives a set of guaranteed cost controllers characterized in terms of a set of solutions to LMI (7). Each guaranteed cost controller ensures the quadratic stability of the resulting closed-loop system and an upper bound on

the closed-loop cost function is given by (6). In view of this, it is desirable to find an optimal guaranteed cost controller which minimizes the upper bound (9). This problem is dealt with in the following theorem.

**Theorem 2:** Consider the uncertain neutral delay system ( $\Sigma$ ) and the cost function (6). Suppose the following optimization problem

$$\min_{\xi, C, D, E, Q, G, L, Z, \Gamma_i, i=1,2,\dots,7} (\xi + tr \sum_{i=1}^7 \Gamma_i), \quad (28)$$

s.t.

(1) LMI in Theorem 1,

$$\begin{aligned}
(2) & \begin{bmatrix} -\xi & \varphi(0)^T \\ * & -X \end{bmatrix} < 0, & (3) & \begin{bmatrix} -\Gamma_1 & C^T \\ * & -XX_1^{-1}X \end{bmatrix} < 0, \\
(4) & \begin{bmatrix} -\Gamma_2 & D^T \\ * & -XX_{21}^{-1}X \end{bmatrix} < 0, & (5) & \begin{bmatrix} -\Gamma_3 & E^T \\ * & -XX_{22}^{-1}X \end{bmatrix} < 0, \\
(6) & \begin{bmatrix} -\Gamma_4 & Q^T \\ * & -XX_3^{-1}X \end{bmatrix} < 0, & (7) & \begin{bmatrix} -\Gamma_5 & G^T \\ * & -XX_3^{-1}X \end{bmatrix} < 0, \\
(8) & \begin{bmatrix} -\Gamma_6 & L^T \\ * & -P_4 \end{bmatrix} < 0, & (9) & \begin{bmatrix} -\Gamma_7 & Z^T \\ * & -P_5 \end{bmatrix} < 0,
\end{aligned}$$

has a solution for  $\xi, C, D, E, Q, G, L, Z, \Gamma_i, i=1,2,\dots,7$ , where

$$\begin{aligned}
& \int_{-h_1}^0 \varphi(s)^T \varphi(s) ds = CC^T, \\
& \int_{-h_2}^0 \varphi(s)^T \varphi(s) ds = DD^T, \\
& \int_{-h_2}^0 \dot{\varphi}(s)^T \dot{\varphi}(s) ds = EE^T, \\
& \int_{-h_3}^0 [\int_s^0 \varphi(\theta)^T d\theta]^T [\int_s^0 \varphi(\theta)^T d\theta] ds = QQ^T, \\
& \int_0^{h_3} ds \int_{-s}^0 (\theta + s) \varphi(\theta)^T \varphi(\theta) d\theta = GG^T, \\
& \int_{-h_1}^0 \int_{\theta}^0 \dot{\varphi}(s)^T P_4 \dot{\varphi}(s) ds d\theta = LL^T \\
& \int_{-h_2}^0 \int_{\theta}^0 \dot{\varphi}(s)^T P_5 \dot{\varphi}(s) ds d\theta = ZZ^T.
\end{aligned}$$

Then, the corresponding guaranteed cost controller in the form of (8) is an optimal guaranteed cost controller in the sense that under this controller the upper bound of the closed-loop cost function (9) is minimized.

**Proof:** It can be done easily by Shur complement formula, and is thus omitted.

**Remark 1:** Note that not all the matrix inequalities in Theorem 2 are LMIs. Some of them are quadratic-matrix inequality (QMI). In order to use the convex

optimization technique, the QMI must be converted to an LMI via some variable changes or transformations. For this purpose, we apply the congruence transformation to Theorem 2 to obtain the following result.

**Theorem 3:** Consider the uncertain neutral delay system ( $\Sigma$ ) and the cost function (6). Suppose the following optimization problem

$$\min_{\xi, \tilde{C}, \tilde{D}, \tilde{E}, \tilde{Q}, \tilde{G}, L, Z, \Gamma_i, i=1,2,\dots,7} (\xi + tr \sum_{i=1}^7 \Gamma_i), \quad (29)$$

s.t.

(1) LMI in Theorem 1,

$$(2) \begin{bmatrix} -\xi & \varphi(0)^T \\ * & -X \end{bmatrix} < 0, \quad (3) \begin{bmatrix} -\Gamma_1 & \tilde{C}^T \\ * & -X_1 \end{bmatrix} < 0,$$

$$(4) \begin{bmatrix} -\Gamma_2 & \tilde{D}^T \\ * & -X_{21} \end{bmatrix} < 0, \quad (5) \begin{bmatrix} -\Gamma_3 & \tilde{E}^T \\ * & -X_{22} \end{bmatrix} < 0,$$

$$(6) \begin{bmatrix} -\Gamma_4 & \tilde{Q}^T \\ * & -X_3 \end{bmatrix} < 0, \quad (7) \begin{bmatrix} -\Gamma_5 & \tilde{G}^T \\ * & -X_3 \end{bmatrix} < 0,$$

$$(8) \begin{bmatrix} -\Gamma_6 & L^T \\ * & -P_4 \end{bmatrix} < 0, \quad (9) \begin{bmatrix} -\Gamma_7 & Z^T \\ * & -P_5 \end{bmatrix} < 0,$$

has a solution for  $\xi, \tilde{C}, \tilde{D}, \tilde{E}, \tilde{Q}, \tilde{G}, L, Z, \Gamma_i, i=1,2,\dots,7$ , where

$$\tilde{C} = X_1 X^{-1} C, \tilde{D} = X_{21} X^{-1} D, \tilde{E} = X_{22} X^{-1} E, \\ \tilde{Q} = X_3 X^{-1} Q, \tilde{G} = X_3 X^{-1} G.$$

Then, the corresponding guaranteed cost controller in the form of (8) is an optimal guaranteed cost controller in the sense that under this controller the upper bound of the closed-loop cost function (9) is minimized.

**Remark 2:** The model discussed in our article contains distributed delay and variable delays both in state item and neutral item. If we set  $A_{d_2} = 0$ ,  $A_{d_2} = A_d$ ,  $h_1 = h$ ,  $h_2 = d$  in system ( $\Sigma$ ), it becomes an uncertain neutral time-delay system which has been proposed in reference [24]:

$$\Sigma': \dot{x}(t) = [A + \Delta A(t)]x(t) + [A_h + \Delta A_h(t)] \\ \times x(t-h) + [A_d + \Delta A_d(t)] \\ \times \dot{x}(t-d) + [B + \Delta B(t)]u(t), \quad (30) \\ x(t) = \varphi(t), t \in [-h, 0].$$

For this system, by using the same method as that in Theorem 1 and cost function (6), we can easily give a delay-dependent sufficient condition for the solvability of the robust guaranteed cost control problem, which is different from that in reference [24].

## 4. AN ILLUSTRATION EXAMPLE

In this section we present an example to illustrate the theory in the previous sections. Consider system ( $\Sigma$ ) with:

$$A = \begin{bmatrix} -0.9 & 0.8 & 0.9 \\ 0.5 & 0.9 & -0.3 \\ 0.7 & 0.3 & 0.2 \end{bmatrix}, \quad M = \begin{bmatrix} 0 \\ 0.2 \\ 0.1 \end{bmatrix}, \\ A_h = \begin{bmatrix} -1 & 1 & 0.2 \\ 0.2 & 0 & 0.3 \\ 0.8 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 1.2 & 0.5 \\ 0 & -1 \end{bmatrix}, \\ A_{d_1} = \begin{bmatrix} -0.2 & 0.1 & 0.2 \\ 0.6 & 0 & 0.3 \\ 0.2 & 0.3 & -0.5 \end{bmatrix}, \quad A_{d_2} = \begin{bmatrix} -0.2 & 0.5 & 0.2 \\ 0.6 & 0 & 0.3 \\ 0.3 & 0.3 & -0.1 \end{bmatrix}, \\ R_1 = \begin{bmatrix} 0.5 & 0.2 & 0.1 \\ 0.2 & 0.3 & 0 \\ 0.1 & 0 & 0.4 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.2 \end{bmatrix}, \\ N_1 = [0.1, 0, 0.1], \quad N_2 = [0, 0.1, 0.1], \\ N_3 = [0.1, 0, 0.1], \quad N_4 = [0, 0.2, 0.2], \quad N_5 = [0, 0.1], \\ h_1 = 0.5, \quad h_2 = 0.6, \quad h_3 = 0.6, \quad d_1 = 1.2, \quad d_2 = 0.6,$$

the initial condition is assumed to be  $\varphi(t) = [1, 0, \exp(-t)]^T$  for all  $t \in [-0.6, 0]$ . Then, by solving the LMIs in Theorem 3, the optimal guaranteed cost controller gain is

$$K = \begin{bmatrix} -10.2547 & -13.2759 & -4.4138 \\ 1.8173 & -0.9256 & 3.2401 \end{bmatrix}.$$

Furthermore, the corresponding closed-loop optimal cost function is  $J = 18.3923$ . The simulation results of the state responses of both the open-loop and closed-loop systems are shown in Figs. 1 and 2,

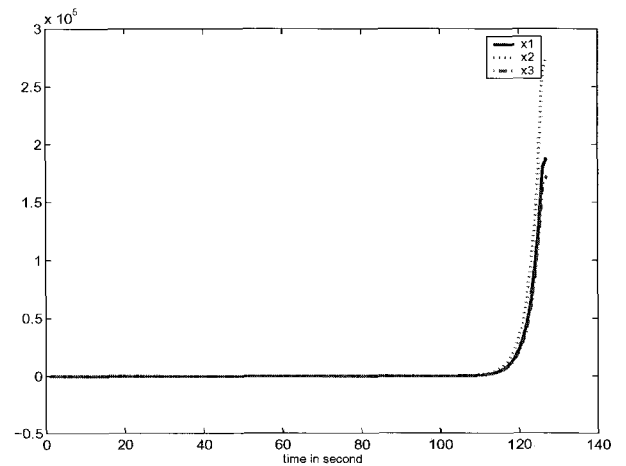


Fig. 1. State response of the open-loop system.

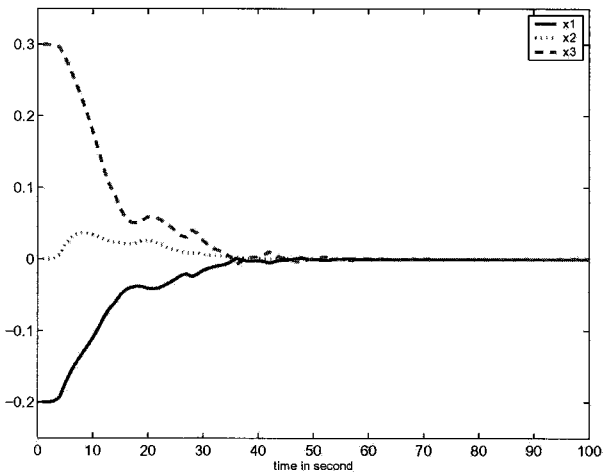


Fig. 2. State response of the closed-loop system.

respectively. From these simulation results, it can be seen that the designed guaranteed controller satisfies the specified requirements.

**Remark 3:** If using Theorem 3 to the system  $(\Sigma')$  proposed in reference [24], we can obtain

$$K = \begin{bmatrix} -3.9755 & -6.3323 & -1.8054 \\ 0.9193 & -0.6922 & 2.3969 \end{bmatrix},$$

and the corresponding closed-loop optimal cost function is  $J = 19.4339$ . But we using the condition in theorem 3 of the article in [24] to our Example, the corresponding LMI is not feasible. This demonstrates that our method is somewhat better than that in [24].

## 5. CONCLUSIONS

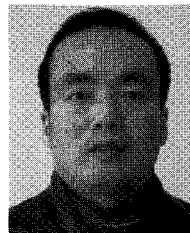
In this paper, we have studied the problem of guaranteed cost control via memoryless state feedback controllers for uncertain neutral systems with norm-bounded time-varying parametric uncertainties and distributed delays. A sufficient condition for the existence of guaranteed cost controllers has been presented. An optimal guaranteed cost controller can be constructed by solving a certain LMI. It has been shown that the proposed guaranteed cost controller guarantees not only the quadratic stability of the closed-loop system, but also an adequate level of a quadratic cost function. An illustrative example has demonstrated the applicability of the proposed approach.

## REFERENCES

- [1] S. S. L. Chang and T. K. C. Peng, "Adaptive guaranteed cost control of systems with uncertain parameters," *IEEE Trans. on Automatic Control*, vol. 17, no. 4, pp. 474-483, 1972.
- [2] Q. Han, "On robust stability of neutral systems with time-varying discrete delay and norm-bounded uncertainty," *Automatica*, vol. 40, no. 6, pp. 1087-1092, 2004.
- [3] Y. He, Q. Wang, C. Lin, and M. Wu, "Augmented Lyapunov functional and delay-Dependent stability criteria for neutral systems," *International Journal of Robust and Nonlinear Control*, vol. 15, no. 18, pp. 923-933, 2005.
- [4] S. Xu and T. Chen, "An LMI approach to the  $H_\infty$  filter design for uncertain system with distributed delays," *IEEE Trans. on Circuits and Systems-II*, vol. 51, no. 4, pp. 195-201, 2004.
- [5] J. Hale, *Theory of Functional Differential Equations*, Springer, New York, 1977.
- [6] Y. A. Fiagbedzi and A. E. Pearson, "A multistage reduction technique for feedback stabilizing distributed time-lag systems," *Automatica*, vol. 23, no. 3, pp. 311-326, 1987.
- [7] B. Zhang, S. Zhou, and S. Xu, "Delay-dependent  $H_\infty$  controller design for linear neutral systems with discrete and distributed delays," *International Journal of Systems Science*, vol. 38, no. 8, pp. 611-621, 2007.
- [8] V. B. Kolmanovskii and A. D. Myshkis, *Introduction to The Theory and Applications of Functional Differential Equations*, Kluwer Academic Publishers, Dordrecht, 1999.
- [9] J. Lee, S. Kim, and W. Kwon, "Memoryless  $H_\infty$  controllers for state delayed systems," *IEEE Trans. on Automatic Control*, vol. 39, no. 1, pp. 159-162, 1994.
- [10] L. Xie and Y. Soh, "Guaranteed cost control of uncertain discrete-time systems," *Control Theory and Advanced Technology*, vol. 10, pp. 1235-1251, 1995.
- [11] C. Lin, Q. Wang, and T. Lee, "A less conservative robust stability test for linear uncertain time-delay systems," *IEEE Trans. on Automatic Control*, vol. 51, no. 1, pp. 87-91, 2006.
- [12] Y. He, M. Wu, J. She, and G. Liu, "Parameter-dependent Lyapunov functional for stability of time-delay systems with polytopic type uncertainties," *IEEE Trans. on Automatic Control*, vol. 49, no. 8, pp. 828-832, 2004.
- [13] D. Yue and Q. Han, "Delayed feedback control of uncertain systems with time-varying input delay," *Automatica*, vol. 41, no. 2, pp. 233-240, 2005.
- [14] S. I. Niculescu, *Delay Effects on Stability: A Robust Control Approach*, Springer, Berlin, 2001.
- [15] S. H. Eshfahani, S. O. R. Moheimani, and I. R. Petersen, "LMI approach suboptimal quadratic guaranteed cost control for uncertain time-delay systems," *IEE Proceedings-Control Theory Applications*, vol. 145, no. 6, pp. 491-498, 1998.
- [16] S. H. Eshfahani and I. R. Petersen, "An LMI

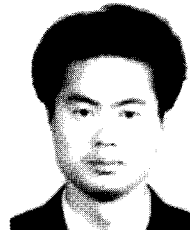


- approach to output feedback guaranteed cost control for uncertain time-delay systems,” *International Journal of Robust and Nonlinear Control*, vol. 10, pp. 157-174, 2000.
- [17] H. Li, S. I. Niculescu, L. Dugard, and J. M. Dion, “Robust guaranteed cost control for uncertain linear time-delay systems,” *Proc. of the 35th Conference on Decision Control*, Kobe, Japan, pp. 3179-3184, December 1996.
- [18] N. Xie and F. Jiang, “Guaranteed-cost control of a linear uncertain system with multiple time-varying delays: An LMI approach,” *IEE Proceedings-Control Theory Applications*, vol. 150, no. 1, pp. 17-22, 2003.
- [19] E. T. Jeung, J. H. Kim, and H. B. Park, “ $H_\infty$  output feedback controller design for linear systems with time-varying delayed state,” *IEEE Trans. on Automatic Control*, vol. 43, no. 7, pp. 971-974, 1998.
- [20] H. Mukaidani, “An LMI approach to guaranteed cost control for uncertain delay systems,” *IEEE Trans. on Circuits and Systems-I*, vol. 50, no. 6, pp. 795-800, 2003.
- [21] L. Yu and J. Chu, “An LMI approach to guaranteed cost control of linear uncertain time-delay systems,” *Automatica*, vol. 35, no. 6, pp. 1155-1159, 1999.
- [22] M. Mahmoud and L. Xie, “Guaranteed cost control of uncertain discrete systems with delays,” *International Journal of Control*, vol. 73, no. 2, pp. 105-114, 2000.
- [23] B. Chen, J. Lam, and S. Xu, “New observer-based guaranteed cost control for neutral delay systems,” *Journal of Systems and Control Engineering*, vol. 218, pp. 227-235, 2004.
- [24] S. Xu, J. Lam, C. Yang, and E. Verriest, “An LMI approach to guaranteed cost control for uncertain linear neutral delay systems,” *International Journal of Robust and Nonlinear Control*, vol. 13, no. 1, pp. 35-53, 2003.
- [25] S. Xu, J. Lam, and Y. Zou, “Delay dependent guaranteed cost control for uncertain systems with state and input delays,” *IEE Proceedings-Control Theory Applications*, vol. 153, no. 3, pp. 307-313, 2006.
- [26] B. Chen, J. Lam, and S. Xu, “Memory state feedback guaranteed cost control for neutral delay systems,” *International Journal of Innovative Computing, Information and Control*, vol. 2, no. 2, pp. 293-303, 2006.
- [27] S. Xu, J. Lam, and C. Yang, “Robust  $H_\infty$  control for uncertain linear neutral delay systems,” *Optimal Control Applications Methods*, vol. 23, no. 3, pp. 113-123, 2002.
- [28] S. Xu, J. Lam, and C. Yang, “Quadratic stability and stabilization of uncertain linear discrete-time systems with state delay,” *System Control Letter*, vol. 43, no. 2, pp. 77-84, 2001.
- [29] Y. Wang, L. Xie, and Carlos E. de Souza, “Robust control of a class of uncertain nonlinear systems,” *System Control Letter*, vol. 19, no. 2, pp. 139-149, 1992.
- [30] J. Park and S. Won, “Stability analysis for neutral delay-differential systems,” *Journal of the Franklin Institute*, vol. 337, no. 1, pp. 1-9, 2000.
- [31] S. Boyd, R. E. El. Ghaoui, R. E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in Systems and Control Theory*, SIAM Studies in Applied Mathematics, vol. 15, Philadelphia, Paris, 1994.

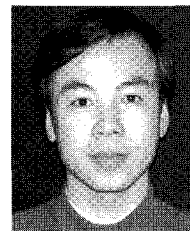


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