

NUMERICAL SOLUTIONS OF AN IMPACT OF NATURAL CONVECTION ON MHD FLOW PAST A VERTICAL PLATE WITH SUCTION OR INJECTION

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ABSTRACT. Because of the importance of suction or injection in the fields of aerodynamics, space science and many other industrial applications, our present study is motivated. The effect of natural convection on MHD flow past a vertical plate with suction or injection is studied. We have tried to solve the dimensionless governing equations by using finite difference scheme. To ensure the validity of our numerical solutions, we have compared our numerical solutions for temperature and velocity for the case of suction and injection for unit Prandtl number with the available exact solutions in the literature. The corresponding codes were written in Mathematica 5.0 for calculating numerical solutions for temperature and velocity and the comparison between the exact and numerical solutions. For the purpose of discussing the results some numerical calculations are carried out for non-dimensional temperature T , velocity u , skin friction τ and the Nusselt number N_u , by making use of it, the rate of heat transfer is studied.

1. INTRODUCTION

The phenomenon free convection has many important technological applications e.g in cooling a nuclear reactor, providing heat sinks in turbine blades etc. On the other hand, the structures of stars and planets are known to be greatly influenced by thermal convection in their interior. Soundalgekar[1] initiated the study of free convection effects on the oscillatory flow past an infinite, vertical porous plate with constant suction. Free convection effects on the Stokes problem for an infinite vertical plate was again investigated by Soundalgekar[2]. Free convection and mass transfer effects on the oscillatory flow of a dissipative fluid past an infinite vertical porous plate was studied by Georgantopoulos et al [3]. Natural convection effects on MHD flow past an impulsively started permeable vertical plate was studied by Revankar[4]. Soundalgekar et al [5] have investigated mass transfer effects on the flow past an impulsively started infinite vertical plate with variable temperature or constant heat flux. MHD thermal –diffusion effects on free convective and mass transfer flow over an infinite vertical moving plate was studied by kafousias [7]. Ching-yung-cheng[9] analyzed the effect of a magnetic field on heat and mass transfer by natural convection from vertical surface in porous medium by an integral

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approach.

An effect of natural convection on MHD flow past a vertical plate was investigated by many researchers as mentioned above. But in presence of suction or injection, an impact of natural convection on MHD flow was not initiated by any one. It is proposed to study the same in this paper. Also the governing equations were solved by finite difference method. To ensure the validity of our numerical solutions, we have compared our numerical solutions for temperature and velocity for the case of suction ($r > 0$) for unit Prandtl number with the available exact solutions in the literature.

2. MATHEMATICAL FORMULATION

An unsteady two-dimensional free convective flow of an electrically conducting viscous and incompressible fluid past an infinite, porous and vertical plate with suction/injection is considered. A magnetic field B_0 is applied perpendicular to the plate. A system of rectangular coordinate axes $ox_1y_1z_1$ is taken such that $y_1=0$ on the plate and z_1 is along its leading edge. All the fluid properties are considered.

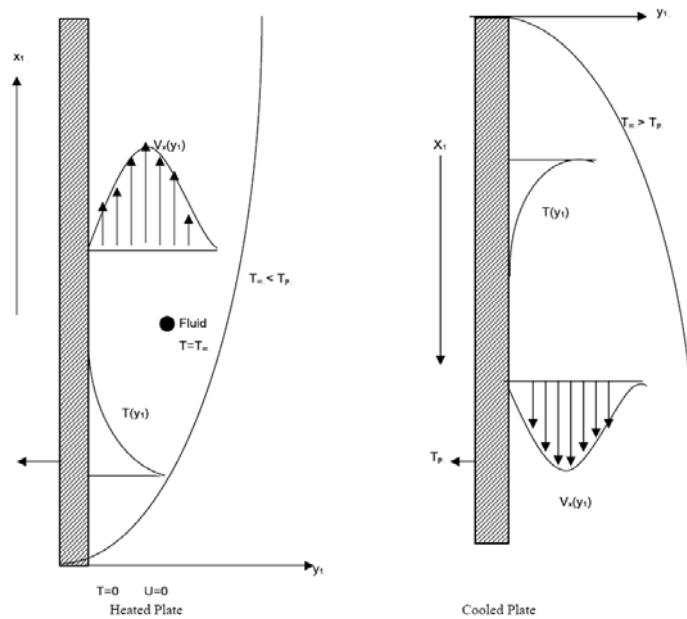


FIGURE 0. Geometry of the problem

An influence of the density variation with temperature is considered only in the body force term. Its influence in other terms of the momentum and energy equation is neglected. This is the well-known Boussinesq approximation. Thus, under these assumptions, the physical variables are functions of y_1 and t_1 only. The problem is governed by the following equations

Continuity equation $\frac{\partial v_1}{\partial y_1} = 0,$ (1)

Momentum equation $\frac{\partial u_1}{\partial t_1} + v_1 \frac{\partial u_1}{\partial y_1} = g\beta(T_1 - T_\infty) + \frac{v\partial^2 u_1}{\partial y_1^2} - \frac{\sigma B_0^2 u_1}{\rho},$ (2)

Energy equation $\frac{\partial T_1}{\partial t_1} + v_1 \frac{\partial T_1}{\partial y_1} = \frac{\alpha\partial^2 T_1}{\partial y_1^2}.$ (3)

The initial and boundary conditions of the problem are

$$\left. \begin{aligned} t_1 \leq 0, u_1(y_1, t_1) = 0, T_1(y_1, t_1) = T_\infty ; \\ t_1 > 0, u_1(0, t_1) = V_0, T_1(0, t_1) = T_P, \text{ at } y_1 = 0 ; \\ t_1 > 0, u_1(\infty, t_1) = 0, T_1(\infty, t_1) = T_\infty, \text{ as } y_1 \rightarrow \infty. \end{aligned} \right\} \quad (4)$$

Since the plate is assumed to be porous and through it suction with uniform velocity occurs, equation (1) integrates to $v_1 = -v_0(v_0 > 0)$ where $v_0 (> 0)$ is the constant suction velocity. From equation (1) we observe that v_1 is independent of space co-ordinates and may be taken as constant. We define the following non-dimensional variables and parameters.

$$\left. \begin{aligned} t &= \frac{t_1 V_0^2}{v}, y = \frac{V_0 y_1}{v}, \\ u &= \frac{u_1}{V_0}, T = \frac{T_1 - T_\infty}{T_P - T_\infty}, \\ r &= \frac{v_1}{V_0}, P_r = \frac{v}{\alpha}, \\ M &= \frac{\sigma B_0^2 v}{\rho V_0^2}, G_r = \frac{v g \beta (T_P - T_\infty)}{V_0^3} \end{aligned} \right\} \quad (5)$$

In view of Eqs. (4) and (5), Equations (2) and (3) reduces to the following

$$\frac{\partial u}{\partial t} - r \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_r T - M u, \quad (6)$$

$$P_r \frac{\partial T}{\partial t} - P_r r \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2}. \quad (7)$$

With

$$\left. \begin{aligned} t \leq 0, u(y, t) = 0, T(y, t) = 0; \\ t > 0, u(0, t) = 1, T(0, t) = 1; \\ t > 0, u(\infty, t) = 0, T(\infty, t) = 0. \end{aligned} \right\} \quad (8)$$

where the parameter ‘r’ represents suction or injection depending on whether it is positive or negative. The Grashof number $G_r > 0$ represents external cooling of the plate and $G_r < 0$ denotes external heating of the plate.

3. METHOD OF SOLUTION

It is not possible to find analytical solution for equations (6) and (7) when $P_r \neq 1$ by known method which is the Laplace transform technique. Hence we sought a solution by finite difference technique of implicit type namely Crank- Nicolson implicit finite difference method which is always convergent and stable. This method has been used to solve equations (6) and (7) subject to the conditions given by Eqn. (8). To obtain the difference equations, the region of the flow is divided into a grid or mesh of lines parallel to y and t axes. Solution of difference equations are obtained at the intersection of these mesh lines called nodes (as in FIG. (1)). The values of the dependent variables T and u at the nodal points along the planes $y = 0$ are given by $T(0, t)$ and $u(0, t)$ and hence are known from the boundary conditions.

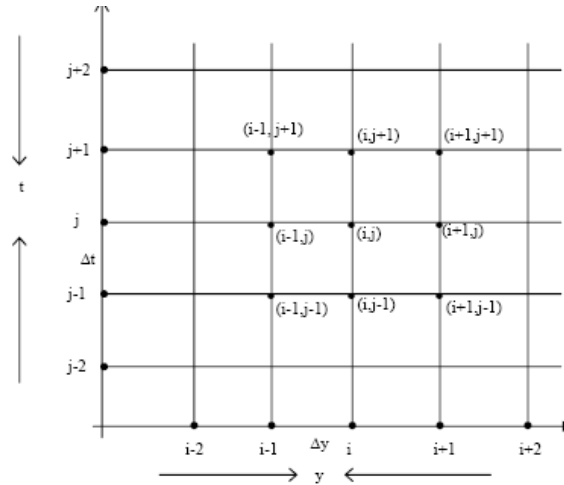


FIGURE 1. Finite difference grid

In the above FIG.1, Δy and Δt are constant mesh sizes along y and t directions respectively. We need a scheme to find single values at next time level in terms of known values at an earlier time level. A forward difference approximation for the first order partial derivatives of T and u w.r.t. t and y and a central difference approximation for the second order partial

derivative of u and T w.r.t. y are used.

$$\left. \begin{aligned} \left(\frac{\partial T}{\partial y}\right)_{i,j} &= \frac{T_{i+1,j} - T_{i-1,j} + T_{i+1,j+1} - T_{i-1,j+1}}{4(\Delta y)}, \\ \left(\frac{\partial u}{\partial y}\right)_{i,j} &= \frac{u_{i+1,j} - u_{i-1,j} + u_{i+1,j+1} - u_{i-1,j+1}}{4(\Delta y)}, \\ \left(\frac{\partial T}{\partial t}\right)_{i,j} &= \frac{T_{i,j+1} - T_{i,j}}{\Delta t}, \\ \left(\frac{\partial u}{\partial t}\right)_{i,j} &= \frac{u_{i,j+1} - u_{i,j}}{\Delta t}, \\ \left(\frac{\partial^2 T}{\partial y^2}\right)_{i,j} &= \frac{T_{i+1,j} + T_{i-1,j} - 2T_{i,j} + T_{i+1,j+1} + T_{i-1,j+1} - 2T_{i,j+1}}{2(\Delta y)^2}, \\ \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} &= \frac{u_{i+1,j} + u_{i-1,j} - 2u_{i,j} + u_{i+1,j+1} + u_{i-1,j+1} - 2u_{i,j+1}}{2(\Delta y)^2}, \end{aligned} \right\} \quad (9)$$

The finite difference approximation of equations (6) and (7) are obtained on substituting equation (9) into equations (6) and (7) .

$$\left(\frac{u_{i,j+1} - u_{i,j}}{\Delta t}\right) - r \left(\frac{u_{i+1,j} - u_{i-1,j} + u_{i+1,j+1} - u_{i-1,j+1}}{4(\Delta y)}\right) = \left(\frac{u_{i+1,j} + u_{i-1,j} - 2u_{i,j} + u_{i+1,j+1} + u_{i-1,j+1} - 2u_{i,j+1}}{2(\Delta y)^2}\right) + G_r T_{i,j} - M u_{i,j} \quad (10)$$

$$P_r \left(\frac{T_{i,j+1} - T_{i,j}}{\Delta t}\right) - P_r r \left(\frac{T_{i+1,j} - T_{i-1,j} + T_{i+1,j+1} - T_{i-1,j+1}}{4(\Delta y)}\right) = \left(\frac{T_{i+1,j} + T_{i-1,j} - 2T_{i,j} + T_{i+1,j+1} + T_{i-1,j+1} - 2T_{i,j+1}}{2(\Delta y)^2}\right) \quad (11)$$

On multiplying both sides of equations (10) and (11) by (Δt) and after simplifying, we obtain

$$u_{i,j+1} - \frac{\Delta t}{(\Delta y)^2} u_{i,j+1} - \frac{r\Delta t}{4\Delta y} (u_{i+1,j+1} - u_{i-1,j+1}) - \frac{\Delta t}{2(\Delta y)^2} u_{i+1,j+1} - \frac{\Delta t}{2(\Delta y)^2} u_{i-1,j+1} = \frac{\Delta t}{2(\Delta y)^2} (u_{i+1,j} + u_{i-1,j} - 2u_{i,j}) + u_{i,j} + \frac{r\Delta t}{4\Delta y} (u_{i+1,j} - u_{i-1,j}) - M\Delta t u_{i,j} + G_r \Delta t T_{i,j}$$

$$T_{i,j+1} + \frac{1}{P_r} \frac{\Delta t}{(\Delta y)^2} u_{i,j+1} - \frac{r\Delta t}{4\Delta y} (T_{i+1,j+1} - T_{i-1,j+1}) - \frac{\Delta t}{2P_r(\Delta y)^2} (T_{i+1,j+1} + T_{i-1,j+1}) =$$

$$\frac{1}{2P_r} (T_{i+1,j} + T_{i-1,j} - 2T_{i,j}) + T_{i,j} + \frac{r\Delta t}{4\Delta y} (T_{i+1,j} - T_{i-1,j})$$

Now for Crank- Nicolson implicit method, let $\frac{\Delta t}{(\Delta y)^2} = r' = 1$ (method is always stable and convergent), under this condition the above equations reduces as

$$\left(\frac{1}{2} - \frac{r\Delta t}{4\Delta y}\right) u_{i+1,j+1} - \left(\frac{1}{2} - \frac{r\Delta t}{4\Delta y}\right) u_{i-1,j+1} = \left(\frac{1}{2} + \frac{r\Delta t}{4\Delta y}\right) u_{i+1,j} + \left(\frac{1}{2} - \frac{r\Delta t}{4\Delta y}\right) u_{i-1,j} +$$

$$-M\Delta t u_{i,j} + G_r \Delta t T_{i,j} \quad (12)$$

$$\left(1 + \frac{1}{P_r}\right) T_{i,j+1} + \left(r \frac{\Delta t}{4\Delta y} - \frac{1}{2}\right) T_{i-1,j+1} - \left(r \frac{\Delta t}{4\Delta y} + \frac{1}{2}\right) T_{i+1,j+1} = \left(r \frac{\Delta t}{4\Delta y} + \frac{1}{2}\right) T_{i+1,j} + \left(-r \frac{\Delta t}{4\Delta y} + \frac{1}{2}\right) T_{i-1,j}$$

$$(13)$$

4. NUMERICAL SOLUTIONS AND THEIR ACCURACY

To get the numerical solutions of the temperature T and velocity u, we have taken the aid of the computer by developing a code (program) in Mathematica5.0. The logic of the program is divided into 3 modules as follows:

Module 4.1: main, initially it creates two tables to hold the numerical solutions of temperature and velocity whose coefficients are allotted in the Module 2. After this, it calculates the numerical values at the next time step level. In order to do this, it uses another sub module named, TriDiagonal, which solves the tri-diagonal matrix by using Gauss-Elimination method. Further it moves to the Module 3, for comparison of numerical solutions with analytical solutions.

Module 4.2: CoeffMat, we know that all the terms and their coefficients on RHS of eqn.(13) are known values from initial and boundary conditions. At every time step, for different values of 'i', the finite difference approximation of equation (13) gives a linear system of equations. Then, for j = 0 and i = 1, 2,.. n-1, equation (13) gives a linear system of (n-1) equations for the (n-1) unknown values of 'T' in the first time row in terms of known initial and boundary values. This module maintains coefficients of this linear system of equations.

Module 4.3: Comparison, It compares the numerical solution with the analytical solution at every time step level.

To ensure the validity of our numerical solutions, we have compared our numerical solutions for temperature and velocity for the case of suction ($r > 0$) for unit Prandtl number with the available exact solutions in the literature. Table 1 and Table 2 show comparisons between the numerical values of temperature and velocity for $PB_{rB} = 1$ obtained from the present study and

analytical solution obtained by Revankar [4]. It was clearly seen from these tables that the percentage error decreases as the value of y approaches 1 from 0 for fixed time. Hence the results are in excellent agreement. The corresponding code (program) is written in Mathematica 5.0 for calculating numerical solutions for temperature and velocity and the comparison between the exact and numerical solutions. The comparison tables, Table 1 and Table 2 have been plotted and shown in FIG 2 and FIG 3. As the accuracy of the numerical solutions is very good, the curves corresponding to exact and numerical solutions are laying very close to the other. To ensure the efficiency of our code for velocity, we have given a table of numerical solution for velocity for water ($Pr_B = 6.75$) for both the cases of suction and injection. These values have been plotted under FIG 3 and 4 respectively.

Code for comparison of temperature profiles for $Pr = 1$ for the case of suction

```

CNgrid[n_,m_]:=
Module[{i,j},
u=Table[1,{n},{m}];
For[ i=1,i≤n,i++, uB[[i,1]]B = f[i]; ];
For[ j=1,j≤m,j++, uB[[1,j]]B = gB1B[j]; uB[[n,j]]B = gB2B[j]; ];

TriDiagonal[a0_,d0_,c0_,b0_]:=
Module[{a=a0,b=b0,c=c0,d=d0,k,m,n=Length[b0],x},
For[k=2,k≤n,k++,
dB[[k]]B = dB[[k]]B - (aB[[k-1]]B / dB[[k-1]]B) * cB[[k-1]]B ;
bB[[k]]B = bB[[k]]B - (aB[[k-1]]B / dB[[k-1]]B) * bB[[k-1]]B ;
x=Table[ 0, {n} ]; xB[[n]]B = bB[[n]]B / dB[[n]]B;

For[ k=n-1; 1 ≤ k; k--, XB[[k]]B = ( bB[[k]]B - cB[[k]]B * xB[[k+1]]B) / dB[[k]]B; ];
Return [x]; ];

Comparison[n_,m_]:=
Module[{}],
Print["Complete Table"];
Print[" t y Exact Numerical Error"];
Print[" Solution Solution % "];
Print["====="];
result=Table["-----",{(m*n)+m-20},{5}];
row=1;t=0;
For[i=2,i≤m,i++,
t=t+k; y=-0.05;
For[j=1,j≤n,j++,
y=y+h; eta=(y/(2*√t));
answer=0.5*(eP-(R*y)P * Erfc[eta - 0.5 *R * √t ]+ Erfc [eta + 0.5 *R * √t ]);
resultB[[row,1]]B = t;resultB[[row,2]]B = y; resultB[[row,3]]B = answer;
resultB[[row,4]]B = uB[[j,i]]; BresultB[[row,5]]B = Abs[(answer - uB[[j,i]]B) / 100];
row=row+1;];

```

```

resultB[[row,1]]B = “ -----“; resultB[[row,2]]B = “ -----“; resultB[[row,3]]B = “ ----
--“;
resultB[[row,4]]B = “ -----“; resultB[[row,5]]B = “ -----“; row=row+1;
]; ];

a=1.0; b=0.1; c=1; n=21; m=41; R=1;
F[x_]=0; GB1B[t_]=1.0; GB2B[t_]=0.0;
h = a / (n-1); k = b/(m-1);
f[i_]=F[h(i-1)]; gB1B[j_]=GB1B[k ( j - 1 ) ]; gB2B[j_]=GB2B[k ( j - 1 ) ];

CNgrid[n,m]; r=(cP2P * k) / hP2P ;
Va=Vc=Table[-1, {n-1}]; VaB[[n-1]]B = VcB[[1]]B = 0; Vd = Table[ 2 + (2 / r) , {n} ];
VdB[[1]]B = Vd B[[n]]B = 1;

b=Table[0, {n}];
For[j=2, j ≤ m, j++, bB[[1]]B = gB1B[j]; bB[[n]]B = gB2B[j];
For[i=2, i ≤ n-1, i++,
bB[[i]]B =(0.5-((R*k)/(4*h)))*uB[[i-1,j-1]]B + uB[[i,j-1]]B +(0.5+((R*k)/(4*h)))* uB[[i+1,j-1]]B +
((R*k)/(4*h))* (uB[[i+1,j]]B - uB[[i-1,j]]B); ];
uB[[All,j]]B = TriDiagonal [Va, Vd, Vc, b];

Print[NumberForm[ TableForm[ N[ Transpose[ Chop[u] ]], TableSpacing->{0,2} ]]];
Comparison[n,m];
Print[ TableForm[ result , TableSpacing ->{0,2} ]]];

```

Output:

TABLE 1. Comparison of Temperature profiles for $PB_{rB}=1$ for the case of suction

t	y	analytical	numerical	%error
0.0175	0	1	1	0
0.0175	0.05	0.769144	0.746934	0.000222
0.0175	0.1	0.563228	0.521945	0.000413
0.0175	0.15	0.391358	0.340726	0.000506
0.0175	0.2	0.257307	0.207989	0.000493
0.0175	0.25	0.159694	0.119069	0.000406
0.0175	0.3	0.093375	0.064201	0.000292
0.0175	0.35	0.051353	0.032769	0.000186
0.0175	0.4	0.026528	0.015915	0.000106
0.0175	0.45	0.012857	0.007387	5.47E-05
0.0175	0.5	0.00584	0.003276	2.56E-05
0.0175	0.55	0.002485	0.001358	1.13E-05
0.0175	0.6	0.000989	0.000451	5.39E-06
0.0175	0.65	0.000369	5.52E-05	3.13E-06

0.0175	0.7	0.000128	0.000482	3.53E-06
0.0175	0.75	4.18E-05	0.001022	9.81E-06
0.0175	0.8	1.27E-05	0.001814	1.8E-05
0.0175	0.85	3.60E-06	0.002948	2.94E-05
0.0175	0.9	9.55E-07	0.004455	4.45E-05
0.0175	0.95	2.36E-07	0.006276	6.28E-05
0.0175	1	5.45E-08	0	5.45E-10

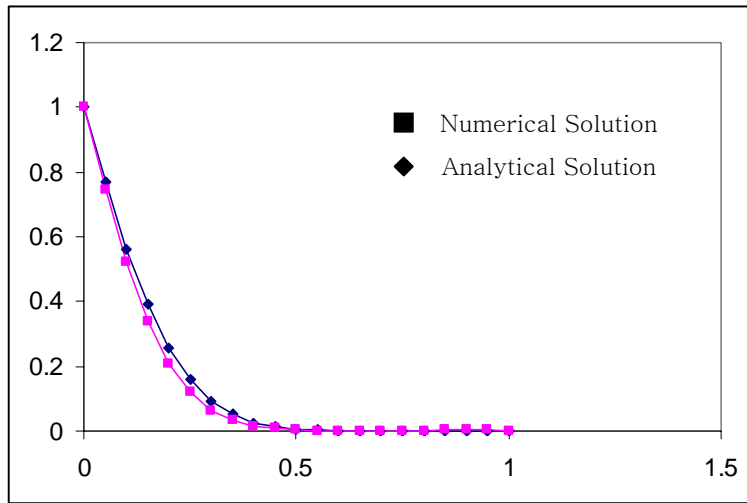


FIGURE 2: Comparison of Temperature for $P_r=1$

Code for comparison of velocity profiles for $PB_{rB} = 1$

```

CNgrid[n_,m_]:=
Module[{i,j},
u=Table[1,{n},{m}]; su= Table[1,{n},{m}];
For[i=1,i≤n,i++, uB[[i,1]]B=f[i]; suB[[i,1]]B=f[i];];
For[j=1,j ≤ m,j++, uB[[1,j]]B=gB1B[j]; suB[[1,j]]B=gB1B[j]; uB[[n,j]]B=gB2B[j];
suB[[n,j]]B=gB2B[j];];

TriDiagonal[a0_,d0_,c0_,b0_]:=
Module[{a=a0,b=b0,c=c0,d=d0,k,m,n=Length[b0],x},
For[k=2,k ≤ n,k++, dB[[k]]B= dB[[k-1]]B - aB[[k-1]]B / dB[[k-1]]B * cB[[k-1]]B;
dB[[k]]B= bB[[k-1]]B - aB[[k-1]]B / dB[[k-1]]B * bB[[k-1]]B;
x=Table[0,{n}]; xB[[n]]B= bB[[n]]B / dB[[n]]B;
For[k=n-1,1 ≤ k,k--, xB[[k]]B= (bB[[k]]B - (cB[[k]]B * xB[[k+1]]B)) / dB[[k]]B;];
Return[x]; ];
    
```

```

Comparison[n_,m_]:=
Module[ {},
  Print["Complete Table"];
  Print[" t          y          Exact          Numerical
Error"];
  Print["          solution          Solution          %
"];
  Print["====="];
  result=Table["-----",{(m*n)+m-20},{5}];
  row=1;t=0;
  For[i=2,i ≤ m,i++,
    t=t+k; y=-0.05;
    For[j=1,j ≤ n,j++,
      y=y+h; eta=(y/(2*√t));
      z=√(((1/4)*R*R)+M);
      expr1=1-(GBrB/M)+eP2R*eta*M*√t;
      expr2=eP2*eta*P*√tPz*Erfc[eta-z*√t]PP;
      expr3=eP2*eta*P*√tPz*Erfc[eta+z*√t]PP;
      expr4=0.5*(GBrB/M)*eP2*eta*P*√tPPErfc[eta-
((R/2)*√t)];
      expr5=0.5*(GBrB/M)*eP2*eta*P*√tP*
PErfc[eta+((R/2)*√t)];
      expr6=1-eP(-M)*t;
      answer=expr1*(expr2+expr3)+expr4+expr5;
      resultB[[row,1]]B=t;resultB[[row,2]]B=y; resultB[[row,3]]B=answer;
      resultB[[row,4]]B=uB[[j,i]]; resultB[[row,5]]B=Abs[(answer-uB[[j,i]]B)/100];
      row=row+1;];
      resultB[[row,1]]B="-----"; resultB[[row,2]]B="-----";
resultB[[row,3]]B="-----";
      resultB[[row,4]]B="-----";resultB[[row,5]]B="-----"; row=row+1;
]; ];
a=1.0; b=0.1; c=1; sc=1; n=21; m=41; GBrB=2; M=3; R=0.5;
F[x_]=0; GB1B[t]=1.0; GB2B[t]=0.0;
h= a/(n-1); k=b/(m-1);
f[i_]=F[h(i-1)]; gB1B[j_]=GB1B[k(j-1)]; g2[j_]=GB2B[k(j-1)]; CNgrid[n,m];
r=(cP2P*k)/hP2P; sr=(scP2P*k)/hP2P; tr=0.5;

Va=Vc=Table[-1,{n-1}]; VaB[[n-1]]B=VcB[[1]]B=0; Vd=Table[2+(2/r),{n}];
sVd=Table[2+(2/sr),{n}]; VdB[[1]]B=VdB[[n]]B=1; sVdB[[1]]B=sVdB[[n]]B=1;

b=Table[0,{n}];
For[j=2,j ≤ m,j++, bB[[1]]B=gB1B[j]; bB[[n]]B=gB2B[j];

```

```

For[i=2, i ≤ n-1, i++,  bB[[i]]B=uB[[i-1,j-1]]B+ ((2/sr)-2)* uB[[i,j-1]]B+ uB[[i+1,j-1]]B;
uB[[All,j]]B = TriDiagonal [VBaB,VBd,BVBc,Bb];

sb=Table[0,{n}]
For[j=2,j ≤ m,j++,      sbB[[1]]B = gB1B[j];   sbB[[n]]B = gB2B[j];
  For[i=2,i ≤ n-1,i++,
    SbB[[i]]B=suB[[i-1,j-1]]B+((2/sr)-2)*suB[[i,j-1]]B + suB[[i+1,j-1]]B + (R/4)*h*(( suB[[i+1,j-1]]B-
suB[[i-1,j]]B))+
    (R/4)*h*(( suB[[i+1,j-1]]B- suB[[i-1,j-1]]B))+ (GBrB*h*h*(uB[[i,j-1]]B+uB[[i,j]]B))-
(2*M*h*h* suB[[i+1,j-1]]B)];
suB[[All,j]]B = TriDiagonal [VBaB,sVBd,BVBcB,sb];

Print[NumberForm[ TableForm[ N[ Transpose[ Chop[u] ]],TableSpacing->{0,2} ]]];
Print[NumberForm[ TableForm[ N[ Transpose[ Chop[su] ]],TableSpacing->{0,2} ]]];
Comparison[n,m];
Print[ TableForm[ result ,TableSpacing->{0,2} ]]];

```

Output:

TABLE 2. Comparison of velocity for $P_r = 1, M = 2, R = 0.5$

t	y	analytical	numerical	%error
0.005	0	1	1	0
0.005	0.05	0.647309	0.618802	0.000285
0.005	0.1	0.348037	0.331615	0.000164
0.005	0.15	0.152804	0.133284	0.000195
0.005	0.2	0.054119	0.047618	6.5E-05
0.005	0.25	0.015329	0.015949	6.2E-06
0.005	0.3	0.003451	0.005128	1.68E-05
0.005	0.35	0.000615	0.001603	9.88E-06
0.005	0.4	8.64E-05	0.000491	4.05E-06
0.005	0.45	9.54E-06	0.000148	1.38E-06
0.005	0.5	8.28E-07	4.41E-05	4.32E-07
0.005	0.55	5.64E-08	1.3E-05	1.29E-07
0.005	0.6	3.01E-09	3.80E-06	3.79E-08
0.005	0.65	1.25E-10	1.10E-06	1.1E-08
0.005	0.7	4.09E-12	3.18E-07	3.18E-09
0.005	0.75	1.04E-13	9.13E-08	9.13E-10
0.005	0.8	2.08E-15	2.61E-08	2.61E-10
0.005	0.85	3.24E-17	7.42E-09	7.42E-11
0.005	0.9	3.93E-19	2.09E-09	2.09E-11
0.005	0.95	3.73E-21	5.49E-10	5.49E-12
0.005	1	2.76E-23	0	2.76E-25

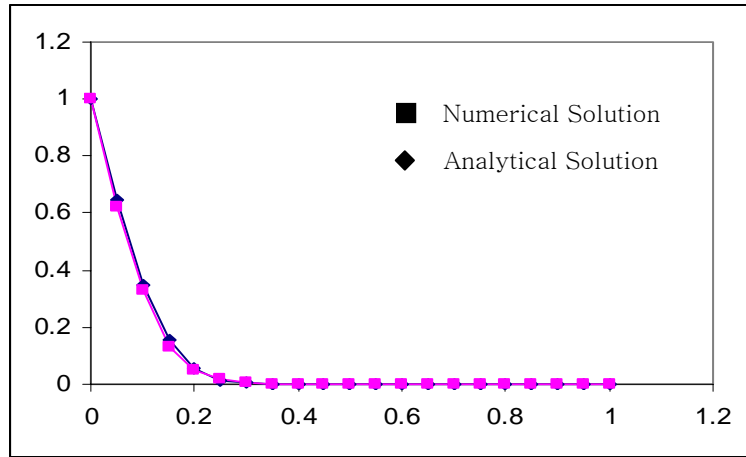


FIGURE 3: Comparison of velocity for $Pr = 1, M=2, R=0.5$

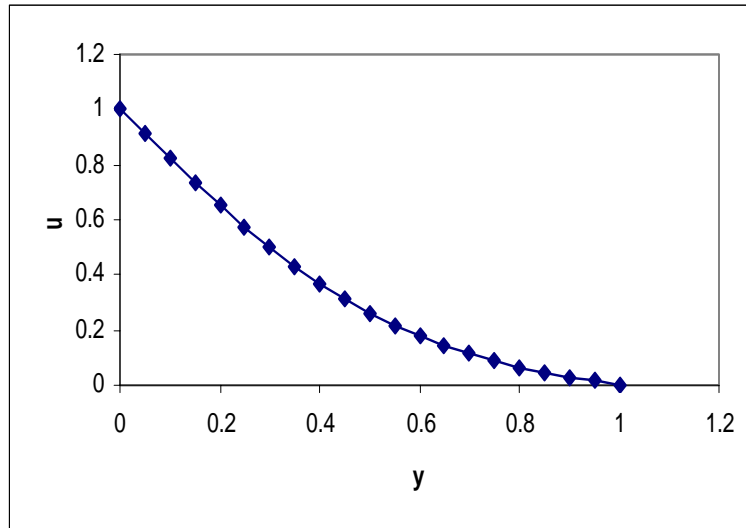


FIGURE 4a. Numerical solution for velocity in case of suction for water($R=0.5, M=2$)

TABLE 3: Numerical solution for velocity in case of suction for water($R=0.5, M=2$)

U	u	i
0.1	0.05	0.910932
0.1	0.1	0.822971
0.1	0.15	0.737182
0.1	0.2	0.654549
0.1	0.25	0.575941
0.1	0.3	0.502082
0.1	0.35	0.433536
0.1	0.4	0.370691
0.1	0.45	0.313758
0.1	0.5	0.262779
0.1	0.55	0.217635
0.1	0.6	0.178064
0.1	0.65	0.143685
0.1	0.7	0.114013
0.1	0.75	0.088485
0.1	0.8	0.06648
0.1	0.85	0.047331
0.1	0.9	0.030345
0.1	0.95	0.014809
0.1	1	0

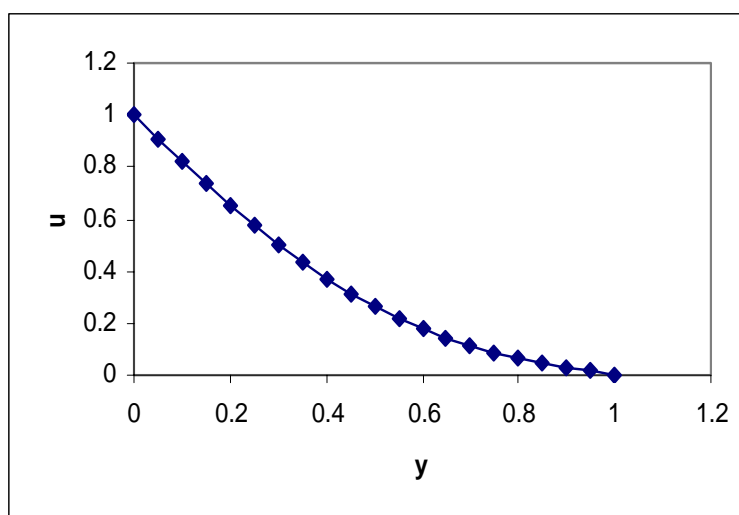


FIGURE 4b. Numerical solution for the case of injection for water($R=-0.5, M=2$)

TABLE 4. Numerical solution for the case of suction
for water ($R=-0.5$, $M=2$)

t	y	Numerical Solutions($R=-$ 0.5)
0.1	0	1
0.1	0.05	0.910932
0.1	0.1	0.822971
0.1	0.15	0.737182
0.1	0.2	0.654549
0.1	0.25	0.575941
0.1	0.3	0.502082
0.1	0.35	0.433536
0.1	0.4	0.370691
0.1	0.45	0.313758
0.1	0.5	0.262779
0.1	0.55	0.217635
0.1	0.6	0.178064
0.1	0.65	0.143685
0.1	0.7	0.114013
0.1	0.75	0.088485
0.1	0.8	0.06648
0.1	0.85	0.047331
0.1	0.9	0.030345
0.1	0.95	0.014809
0.1	1	0

5. RESULTS AND DISCUSSION

For the purpose of discussing the results some numerical calculations are carried out for non-dimensional temperature T , velocity u , skin friction τ and the Nusselt number N_u , by making use of it, the rate of heat transfer was studied.

The temperature profiles for water ($P_r = 6.75$) for the case of suction are shown in FIG.(5.5) and for air ($P_r = 0.733$) for the case of injection are drawn in Fig (5.6). When the suction parameter r increases and t is kept fixed, the temperature increasing in the case of water which can be seen from FIG. (5.5). Also from the same Figure, when r is kept fixed and t is increased, the temperature still increases. FIG.(5.6) reveals that for air ($P_r = 0.733$) for the case of injection, a decrease in r for fixed t , temperature decreases and when r is kept fixed and t increases, the temperature also increases. The effect of time t , magnetic field parameter M and the injection parameter r , for heating of the plate on the velocity profiles are predicted in FIG.(5.7). As time increases the velocity profiles increases. While the injection parameter r decreases, velocity also decreases. Similarly the effect of t , M and r for cooling of the plate on velocity profiles are

predicted as shown in FIG.(5.12). as the suction parameter ‘r’ increased by keeping t, M, G_r fixed, the velocity increases.

The Skin friction at the plate for the case of suction and injection at different G_r are predicted as can be seen from FIG.S. (5.8) and (5.9). From FIG. (5.8), it follows when G_r is kept fixed and r is increased, then the skin friction increases and as r is kept fixed and G_r is increased, the skin friction decreases. The skin-friction for the case of injection is depicted in FIG. (5.9). When r is decreased and G_r is kept fixed, an increase in skin friction is noticed and when r is fixed and G_r is decreased, a decrease in skin friction is noticed.

From the technological point of view, it is important to know the rate of heat transfer between the plate and the fluid. This can be found by using the non-dimensional quantity, the Nusselt number N_u . The numerical values of the Nusselt number against time t are shown in FIGS. (5.10) and (5.11). FIG. (5.10) shows the heat transfer for different suction parameter r. As t increases, the rate of heat transfer increases too. As r increases for the same t, the Nusselt number increases. FIG. (5.11) shows the rate of heat transfer for the case of injection. Here too, as time increases, the heat transfer increases gradually.

The conclusions of the present chapter have already been stated under the above section of Results and Discussion.

6. STABILITY AND CONVERGENCE FOR THE FINITE DIFFERENCE SCHEME

The stability criterion of the present implicit finite difference scheme for constant mesh sizes are examined by using Von Neumann analysis as explained by Carnahan et al[11]. The general terms of the fourier expansions for u and T at a time arbitrarily called t=0 are both $\exp(i\alpha x) \exp(i\beta y)$ (where, $i = \sqrt{-1}$). At a later time t, these terms will become

$$\begin{aligned} u &= F(t) \exp(i\alpha x) \exp(i\beta y) \\ T &= G(t) \exp(i\alpha x) \exp(i\beta y) \end{aligned} \tag{14}$$

Now the implicit finite difference scheme, the equations (6) and (7) respectively become

$$\begin{aligned} \frac{u_{i,j}^{k+1} - u_{i,j}^k}{\Delta t} &= \frac{-r[u_{i,j+1}^{k+1} - u_{i,j-1}^{k+1} + u_{i,j+1}^k - u_{i,j-1}^k]}{4(\Delta y)} + \\ &\quad \frac{[u_{i,j+1}^{k+1} + u_{i,j-1}^{k+1} - 2u_{i,j}^{k+1} + u_{i,j+1}^k + u_{i,j-1}^k - 2u_{i,j}^k]}{2(\Delta y)^2} \\ &\quad + \frac{G_r(T_{i,j}^{k+1} + T_{i,j}^k)}{2} - \frac{M(u_{i,j}^{k+1} + u_{i,j}^k)}{2}, \end{aligned} \tag{15}$$

$$\frac{T_{i,j}^{k+1} - T_{i,j}^k}{\Delta t} = \frac{-r[T_{i,j+1}^{k+1} - T_{i,j-1}^{k+1} + T_{i,j+1}^k - T_{i,j-1}^k]}{4(\Delta y)} + \left(\frac{1}{P_r}\right) \frac{[T_{i,j+1}^{k+1} + T_{i,j-1}^{k+1} - 2T_{i,j}^{k+1} + T_{i,j+1}^k + T_{i,j-1}^k - 2T_{i,j}^k]}{2(\Delta y)^2}. \quad (16)$$

Now substituting (14) in (15) and (16) we have,

$$\frac{F' - F}{\Delta t} = \frac{-r[(F' + F)i \sin \beta \Delta y]}{2(\Delta y)} + \frac{(F' + F)(\cos \beta \Delta y - 1)}{(\Delta y)^2} + \frac{G_r(G' + G)}{2} - \frac{M(F' + F)}{2}, \quad (17)$$

$$\frac{G' - G}{\Delta t} = \frac{-r[(G' + G)i \sin \beta \Delta y]}{2(\Delta y)} + \frac{1}{P_r} \left[\frac{(G' + G)(\cos \beta \Delta y - 1)}{(\Delta y)^2} \right] \quad (18)$$

On simplifying and rearranging the terms in the above equations, we get

$$F' - F = (F' + F) \left[\frac{-r(\Delta t)i(\sin \beta \Delta y)}{2(\Delta y)} - \frac{((1 - \cos \beta \Delta y)(\Delta t))}{(\Delta y)^2} - \frac{M(\Delta t)}{2} \right] + \frac{G_r(G' + G)\Delta t}{2}, \quad (19)$$

$$G' - G = (G' + G) \left[\frac{-r(\Delta t)i(\sin \beta \Delta y)}{2(\Delta y)} - \frac{((1 - \cos \beta \Delta y)(\Delta t))}{P_r(\Delta y)^2} \right]. \quad (20)$$

The above equations can be written as follows:

$$(1+A) F' = (1-A)F + \frac{G_r(G' + G)\Delta t}{2}, \quad (21)$$

$$(1+B)G' = (1-B)G. \quad (22)$$

where

$$-A = \left[\frac{r(\Delta t)i(\sin \beta \Delta y)}{2(\Delta y)} + \frac{((1 - \cos \beta \Delta y)(\Delta t))}{(\Delta y)^2} + \frac{M(\Delta t)}{2} \right],$$

$$B = \left[\frac{-r(\Delta t)i(\sin \beta \Delta y)}{2(\Delta y)} - \frac{((1 - \cos \beta \Delta y)(\Delta t))}{P_r(\Delta y)^2} \right].$$

Using equation (21), equation (22) becomes,

$$F' = \frac{(1-A)}{(1+A)}F + D_1G, \quad (23)$$

$$G' = 0 + \frac{(1-B)}{(1+B)}G, \quad (24)$$

where $D_1 = \frac{G_r \Delta t}{(1+A)(1+B)}$.

Expressing the above equations in matrix form, we have

$$\begin{bmatrix} F' \\ G' \end{bmatrix} = \begin{bmatrix} \frac{1-A}{1+A} & D_1 \\ 0 & \frac{1-B}{1+B} \end{bmatrix} \begin{bmatrix} F \\ G \end{bmatrix} \quad (25)$$

Now for stability the modulus of each eigen value of the amplification matrix should not exceed unity. The eigen values of the amplification matrix are

$$\lambda_1 = \frac{1-A}{1+A},$$

and

$$\lambda_2 = \frac{1-B}{1+B}.$$

Now to prove that

$$|\lambda_1| \leq 1$$

and

$$|\lambda_2| \leq 1.$$

Let

$$a = \left[\frac{r}{2} \frac{(\Delta t)}{(\Delta y)} \right], \quad b = \left[\frac{(\Delta t)}{(\Delta y)^2} \right], \quad c = \left[\frac{M(\Delta t)}{2} \right].$$

We can write A as

$$A = - \left[ai \sin(\beta \Delta y) + 2b \sin^2 \left(\frac{\beta \Delta y}{2} \right) + c \right].$$

Since the real part of -A is greater than or equal to 0, the reader may note that Δt and Δy can be chosen arbitrarily,

$$\text{hence } |\lambda_1| \leq 1 \text{ always.}$$

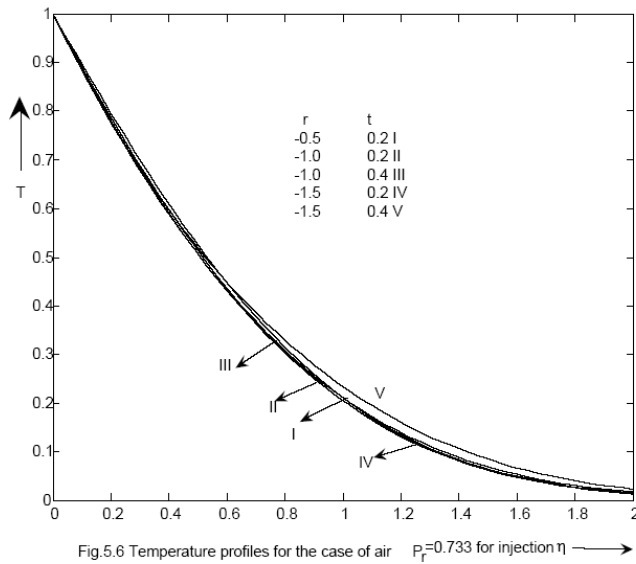
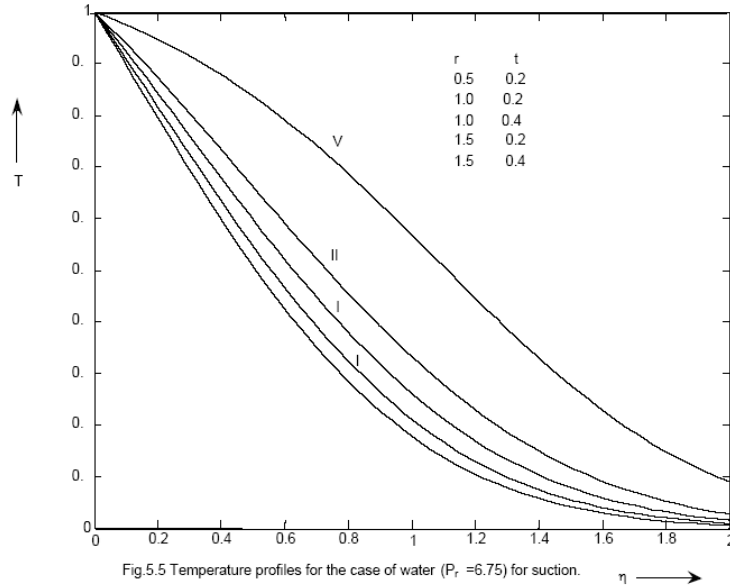
Similarly we can write B as

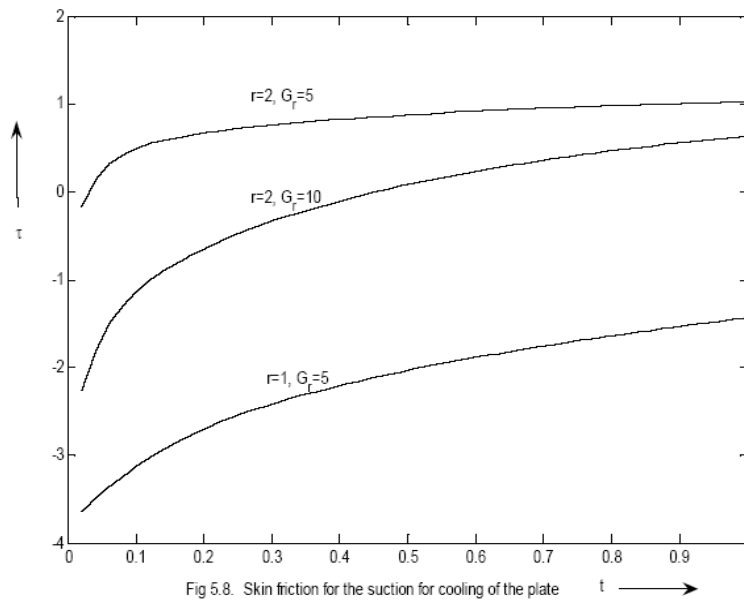
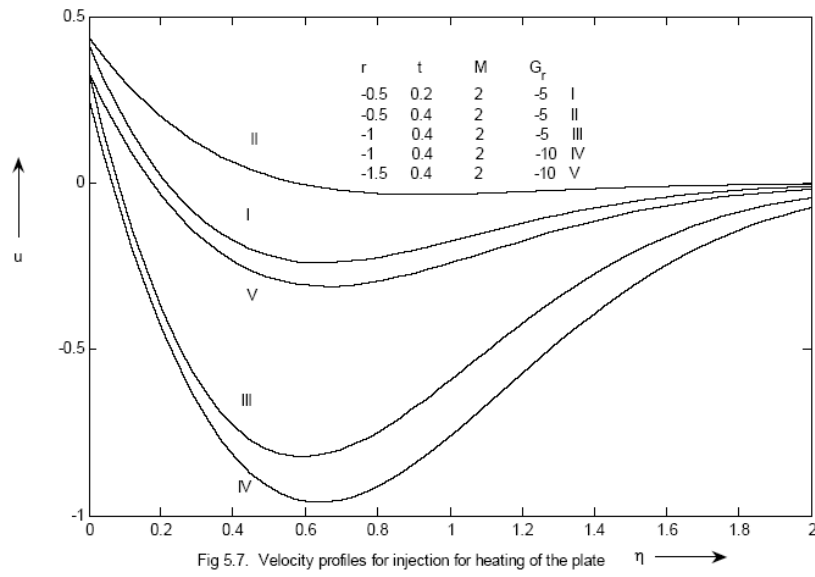
$$B = \left[-ai \sin(\beta \Delta y) + \frac{2b}{P_r} \sin^2 \left(\frac{\beta \Delta y}{2} \right) \right]$$

Since real part of B is greater or equal to 0,

$$\text{hence } |\lambda_2| \leq 1 \text{ always.}$$

Hence the Scheme is unconditionally stable. Local truncation error is $O((\Delta t)^2 + (\Delta y)^2)$ and tends to zero as Δt . Hence the Scheme is compatible. Stability and compatibility ensures **convergence**.





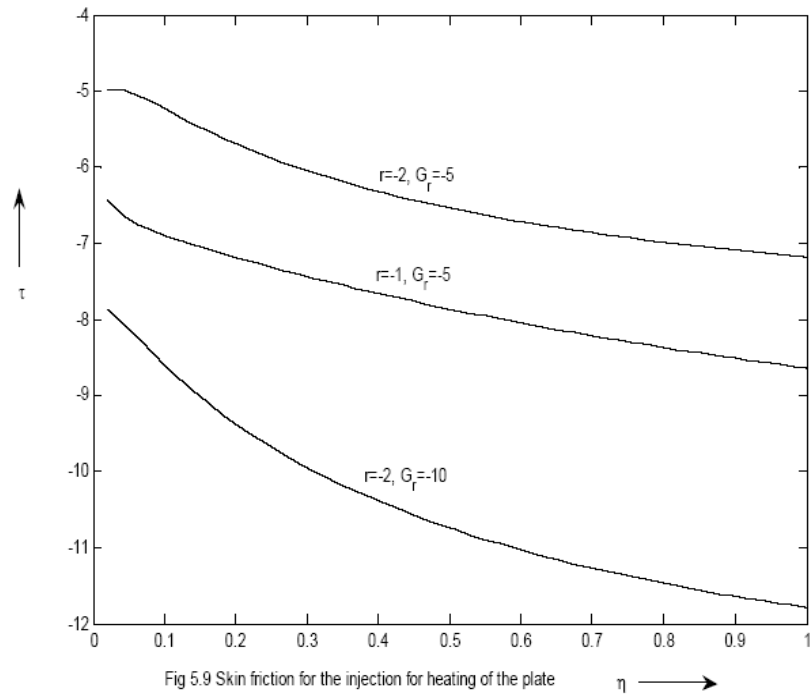


Fig 5.9 Skin friction for the injection for heating of the plate

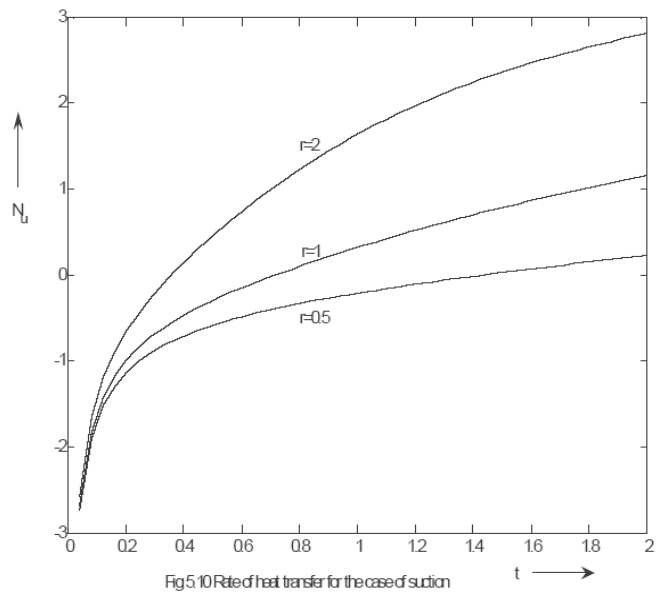
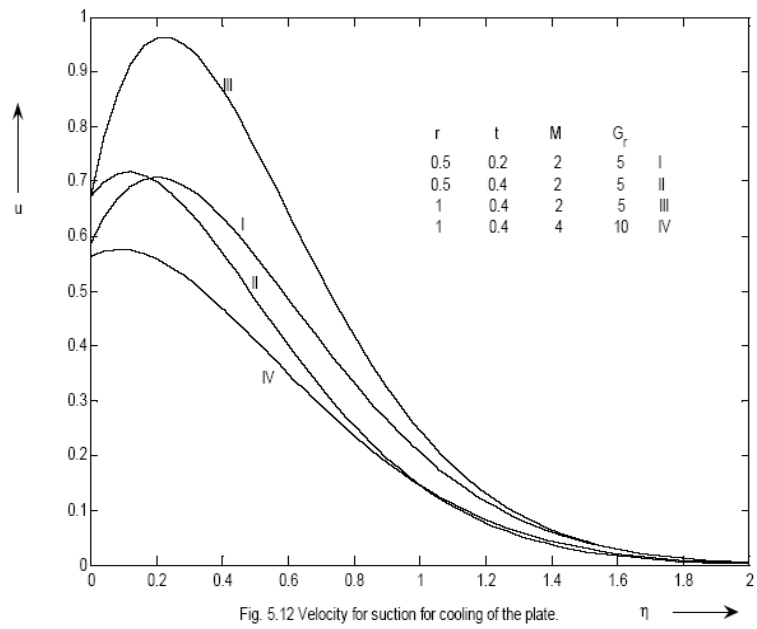
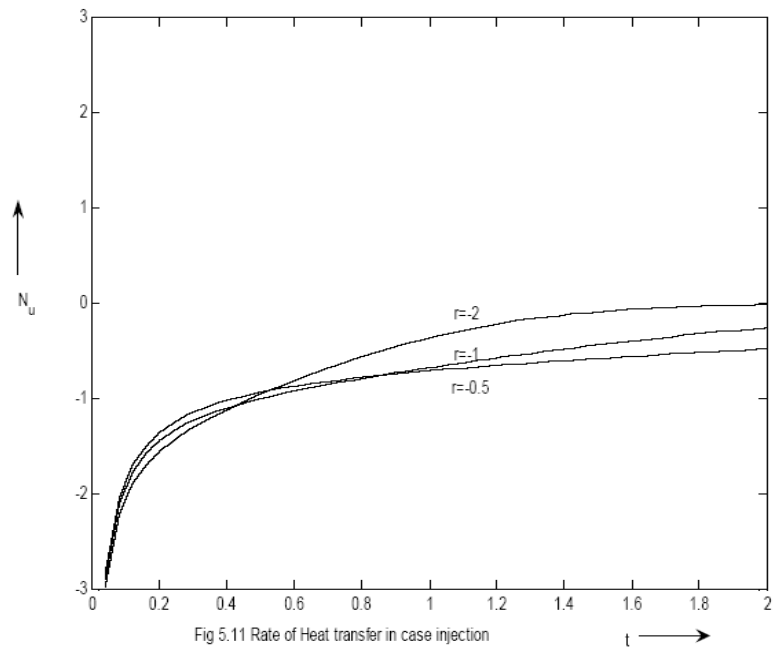


Fig 5.10 Rate of heat transfer for the case of suction



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