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ON THE BAYES ESTIMATOR OF PARAMETER AND RELIABILITY FUNCTION OF THE ZERO-TRUNCATED POISSON DISTRIBUTION

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ABSTRACT: In this paper Bayes estimator of the parameter and reliability function of the zero-truncated Poisson distribution are obtained. Furthermore, recurrence relations for the estimator of the parameter are also derived. Monte Carlo simulation technique has been made for comparing the Bayes estimator and reliability function with the corresponding maximum likelihood estimator (MLE) of zero-truncated Poisson distribution.

1. INTRODUCTION

The Poisson distribution is defined by

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad ; x = \quad 0,1,2,\ldots \tag{1}$$

The distribution (1) has been obtained by S.D Poisson [1] as a limiting case of the Binomial distribution, for some reasons Newbold [2], Jensen [3] and David [4] preferred to give credit to De Moivre [5] rather than to S.D. Poisson for discovering of Poisson distribution. The distribution is so important among the discrete distributions that even Fisher, once remarked 'Among discontinuous distributions', the Poisson series is of the first importance. Johnson, Kotz and Kemp [6] have discussed the genesis of Poisson distribution in detail. Ahmad and Roohi [7] have discussed the characterization of the Poisson distribution. Roohi and Ahmad [8] studied the inverse ascending factorial moments and estimation of the parameter of hyper-Poisson distribution using negative moments. The Poisson distribution has been described as playing a "similar role with respect to discrete distribution to that of the normal for absolutely continuous distribution".

The commonest form of truncation is the omission of the zero class, because the observational apparatus becomes active only when at least one event occurs. The distribution (1) can be truncated at x = 0 and is defined

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$$P_1(X=x)=\frac{e^{-\lambda}\lambda^x}{x!\left(1-e^{-\lambda}\right)}; \quad x=1,2,\ldots \tag{2}$$

This is usually called the positive Poisson distribution. Cohen [9] called it a conditional Poisson distribution. The truncated distribution (2) has been first considered by David and Johnson [10] . In particular, they derived the maximum likelihood estimate (MLE) of λ and its asymptotic variance, and discussed the efficiency of the estimation by moments. Plackett [11] provided a similar estimate of λ for distribution (2), to show that it is highly efficient, and to estimate its sampling variance. Murakami [12] also discussed the maximum likelihood estimators based on censored samples from truncated Poisson distributions. Tate and Goen [13] obtained minimum variance unbiased estimation. Cohen ([14] [15]) provided the estimation of the model (2) from the sample that are truncated on the right. A brief list of authors and their works can be seen in Johnson and Kotz [16], Johnson, Kotz and Kemp [6] and Consul [17].

Bayesian estimation is a likelihood based style of inference that incorporates prior information on the unknown variables. ML estimates are equivalent to the nodes of the Bayesian posterior distribution, when the prior distribution for the unknown variables is flat. However, the goal of a Bayesian analysis is generally not just a point estimate like the posterior mode (mean or median), but a representation of the entire distribution for the unknown parameter(s) (Gelman, Carlin, Stern, Rubin, [18] 1995, page 301).

Kyriakoussis and Papadopoulos [19] (1993) derived the Bayes estimators of the probability of success and reliability function of the zero-truncated binomial and negative binomial distributions. In this paper we have made an attempt to obtain Bayes estimator of the parameter and reliability function of the zero-truncated Poisson distribution. Furthermore, recurrence relations for the estimator of the parameter are also derived. Monte Carlo simulation technique has been made for comparing the Bayes estimator and reliability function with the corresponding maximum likelihood estimator (MLE) of zero-truncated Poisson distribution.

2. BAYESIAN ESTIMATOR OF PARAMETER OF ZERO-TRUNCATED POISSON DISTRIBUTION

Let $x_1, x_2, \ldots, x_n$ be a random sample from (2). The likelihood function is given by

$$L=\frac{\bar{e}^{n\lambda}\lambda^{\Sigma x_i}(1-\bar{e}^{\lambda})^{-n}}{\prod_{i=1}^{n} x_i!}=c\,\bar{e}^{n\lambda}\lambda^{y}(1-\bar{e}^{\lambda})^{-n} \tag{3}$$

where $y = \Sigma x_i$ and $c = \prod_{i=1}^{n} x_i!$

Regarding the parameter λ in (2), as a random variable, a natural conjugate of its prior distribution is the gamma distribution, given as

$$g(\lambda/\alpha,\beta) = \frac{\alpha^{\beta}}{\Gamma(\beta)} \bar{e}^{\alpha\lambda} \lambda^{\beta-1}, \quad \alpha, \beta > 0, \quad \lambda > 0 \tag{4}$$

Using Bayes theorem, the posterior distribution of λ from (3) and (4) can be shown to be

$$\prod(\lambda/y) = \frac{L\, g(\lambda/\alpha,\beta)}{\int_0^{\infty} L\, g(\lambda/\alpha,\beta) d\lambda}$$

$$= \frac{\bar{e}^{(\alpha+n)\lambda}\, \lambda^{(y+\beta)-1} \left(1-\bar{e}^{\lambda}\right)^{-n}}{\int_0^{\infty} \bar{e}^{(\alpha+n)\lambda}\, \lambda^{(y+\beta)-1} \left(1-\bar{e}^{\lambda}\right)^{-n} d\lambda} \tag{5}$$

Under squared error loss function, the Bayes estimator of λ is the posterior mean

$$p^*(y,n) = \int_0^{\infty} \lambda \prod(\lambda/y) d\lambda$$

$$= \frac{\int_0^{\infty} \bar{e}^{(n+\alpha)\lambda}\, \lambda^{(y+\beta)} \left(1-\bar{e}^{\lambda}\right)^{-n} d\lambda}{\int_0^{\infty} \bar{e}^{(n+\alpha)\lambda}\, \lambda^{(y+\beta)-1} \left(1-\bar{e}^{\lambda}\right)^{-n} d\lambda} \tag{6}$$

Using identity

$$(1-z)^{-n} = \sum_{k=0}^{\infty} \binom{n+k-1}{k} z^k, \qquad |z| < 1$$

and the relation

$$\int_0^{\infty} \bar{e}^{at} t^{b-1} dt = \Gamma(b)/a^b, \quad a, b > 0, \; t > 0$$

Where

$$\Gamma(b) = \int_0^{\infty} \bar{e}^{t} t^{b-1} dt$$

we obtain,

$$\int_0^{\infty} \bar{e}^{(n+\alpha)\lambda} \lambda^{(y+\beta)} \left(1-\bar{e}^{\lambda}\right)^{-n} d\lambda = \int_0^{\infty} \bar{e}^{(n+\alpha)} \lambda^{(y+\beta)} \sum_{k=0}^{\infty} \binom{n+k-1}{k} \left(\bar{e}^{\lambda}\right)^k d\lambda$$

$$= \sum_{k=0}^{\infty} \binom{n+k-1}{k} \int_0^{\infty} \bar{e}^{(n+\alpha+k)\lambda} \lambda^{(y+\beta)} d\lambda$$

$$= \Gamma(y+\beta+1) \sum_{k=0}^{\infty} \binom{n+k-1}{k} \frac{1}{(\alpha+n+k)^{y+\beta+1}} \tag{7}$$

$$= \Gamma(y+\beta+1) \sum_{k=n}^{\infty} \binom{k-1}{n-1} \frac{1}{(\alpha+k)^{y+\beta+1}} \tag{8}$$

and similarly,

$$\int_0^{\infty} \bar{e}^{(n+\alpha)\lambda} \lambda^{y+\beta-1} \left(1-\bar{e}^{\lambda}\right)^{-n} d\lambda = \Gamma(y+\beta) \sum_{k=n}^{\infty} \binom{k-1}{n-1} \frac{1}{(\alpha+k)^{y+\beta}} \tag{9}$$

Substituting (8) and (9) into (6) and using relations $\Gamma(b+1) = b\Gamma b$ and

$$k\binom{k-1}{n-1} = n\binom{k-1}{n} + n\binom{k-1}{n-1}, \tag{10}$$

we get

$$p^*(y,n,\alpha,\beta) = \frac{(y+\beta)\sum_{k=n}^{\infty} \binom{k-1}{n-1} \frac{1}{(\alpha+k)^{y+\beta+1}}}{\sum_{k=n}^{\infty} \binom{k-1}{n-1} \frac{(k+\alpha)}{(\alpha+k)^{y+\beta+1}}}$$

$$= \frac{(y+\beta)\sum_{k=n}^{\infty} \binom{k-1}{n-1} \frac{1}{(\alpha+k)^{y+\beta+1}}}{n\sum_{k=n+1}^{\infty} \binom{k-1}{n} \frac{1}{(\alpha+k)^{y+\beta+1}} + (n+\alpha)\sum_{k=n}^{\infty} \binom{k-1}{n-1} \frac{1}{(\alpha+k)^{y+\beta+1}}} \tag{11}$$

or

$$p^*(y,n) = \frac{(y+\beta)M(y,n)}{n\,M(y,n+1) + (n+\alpha)M(y,n)} \tag{12}$$

$y = n, n+1\ldots,$ $n = 1, 2, \ldots$

where

$$M(y,n) = \sum_{k=n}^{\infty} \binom{k-1}{n-1} \frac{1}{(\alpha+k)^{y+\beta+1}} \tag{13}$$

and

$$M(y,n+1) = \sum_{k=n+1}^{\infty} \binom{k-1}{n} \frac{1}{(\alpha+k)^{y+\beta+1}} \tag{14}$$

After simplification (12) becomes

$$p^{*}(y,n)=\frac{(y+\beta)}{\{(n+\alpha)+n\,B(y,n)\}} \tag{15}$$

y = n, n + 1, …, n = 1, 2,…
where

$$B(y,n)=\frac{M(y,n+1)}{M(y,n)} \tag{16}$$

3. RECURRENCE RELATIONS

In order to obtain a recurrence relation for $p^*(y, n)$, first we need recurrence relations for the numbers M(y,n) and B(y,n), which are obtained by following two lemmas:

LEMMA1: The numbers M(y,n), satisfy the recurrence relations:

$$M(y,n+1)=\frac{1}{n}M(y-1,n)-\frac{(n+\alpha)}{n}M(y,n) \tag{17}$$

y = n, n + 1, n + 2, … , n = 1, 2, …
with initial condition

$$M(y,1)=\sum_{k=1}^{\infty}\frac{1}{(\alpha+k)^{y+\beta+1}}, \quad y = 1,2,3\ldots \tag{18}$$

Proof: From the relation (13), we have

$$M(y-1,n)=\sum_{k=n}^{\infty}\binom{k-1}{n-1}\frac{1}{(\alpha+k)^{(y-1)+\beta+1}}$$
$$=\sum_{k=n}^{\infty}\binom{k-1}{n-1}\frac{(\alpha+k)}{(\alpha+k)^{y+\beta+1}} \tag{19}$$

Using the relation (10) then (3.3) becomes

$$M(y-1,n)=n\sum_{k=n+1}^{\infty}\binom{k-1}{n}\frac{1}{(\alpha+k)^{y+\beta+1}}+(n+\alpha)\sum_{k=n}^{\infty}\binom{k-1}{n-1}\frac{1}{(\alpha+k)^{y+\beta+1}}$$

From (13) and (14) we have

$$M(y-1,n) = n\,M(y,n+1) + (n+\alpha)\,M(y,n) \tag{20}$$

from which , we have (17).Also from(13) for n=1 we have the relation (18)

Remark 1: Since, α is a positive integer and $\beta > 0$, from (13) we have

$$M(y,1)=\sum_{K=1}^{\infty}\frac{1}{(\alpha+K)^{y+\beta+1}}$$

$$= \frac{1}{y+\beta}\sum_{K=1}^{\infty}\left\{\frac{1}{(\alpha+K)^{y+\beta}} - \frac{1}{(\alpha+K)^{y+\beta}}\right\}$$

$$\leq \frac{1}{y+\beta}\sum_{m=\alpha+1}^{\infty}\left\{\frac{1}{(m-1)^{y+\beta}} - \frac{1}{(m)^{y+\beta}}\right\}$$

$$= \frac{1}{y+\beta}\cdot\frac{1}{(\alpha+1-1)^{y+\beta}}$$

We also have

$$M(y,1) \geq \frac{1}{(\alpha+1)^{y+\beta+1}},$$

consequently the series M(y, 1) exists and from (17) by mathematical induction we conclude that the series M(y, n) also exists.

Remark 2: Combining the relations (12), (13) and (14) we get that

$$B(y,n) = \frac{\frac{1}{n}M(y-1,n) - \frac{1}{n}(n+\alpha)M(y,n)}{M(y,n)}$$

$$= \left[\frac{M(y-1,n)}{M(y,n)} - (\alpha+n)\right]\Big/ n \tag{21}$$

and

$$\overset{*}{p}(y,n) = (y+\beta)M(y,n)/\left[(n+\alpha)M(y,n) + n\left(\tfrac{1}{n}(y-1,n) - \tfrac{1}{n}(n+\alpha)M(y,n)\right)\right]$$

$$= \frac{(y+\beta)M(y,n)}{M(y-1,n)} \tag{22}$$

LEMMA 2: The numbers B(y, n), satisfy the recurrence relations:

$$B(y,n+1) = \frac{[nB(y,n) + (n+\alpha)]B(y-1,n)}{(n+1)B(y,n)} - \frac{[(n+1)+\alpha]}{(n+1)} \tag{23}$$

n = 1, 2, …, y = n, n + 1, …

with initial conditions

$$B(y,1) = \frac{M(y-1,1)}{M(y,1)} - (1+\alpha) \tag{24}$$

Proof: From the relation (14) and the recurrence relation (17), we get

$$B(y,n+1) = \frac{M(y,n+2)}{M(y,n+1)}$$

$$= \frac{\frac{1}{(n+1)}M(y-1,n+1) - \frac{1}{n+1}(n+1+\alpha)M(y,n+1)}{M(y,n+1)}$$

$$= \frac{1}{(n+1)} \frac{M(y-1,n+1)/M(y,n) - [(n+1)+\alpha]B(y,n)}{B(y,n)} \tag{25}$$

We also have,

$$B(y-1,n) = \frac{M(y-1,n+1)}{M(y-1,n)} = \frac{M(y-1,n+1)/M(y,n)}{M(y-1,n)/M(y,n)}$$

$$= \frac{M(y-1,n+1)/M(y,n)}{n\,M(y,n+1)/M(y,n) + (n+\alpha)}$$

From (16) and (20) we have

$$B(y-1,n) = \frac{M(y-1,n+1)/M(y,n)}{n\,B(y,n) + (n+\alpha)} \tag{26}$$

or

$$M(y-1,n+1)/M(y,n) = [n\,B(y,n) + (n+\alpha)]B(y-1,n) \tag{27}$$

Substituting (26) into (25) we obtain (23).Using the relation (22) for n=1 we easily obtain the initial conditions (23).

Theorem 1: The Bayes estimator of the parameter λ satisfies the recurrence relation:

$$p^*(y,n+1) = \frac{[(y+\beta)-(n+\alpha)p^*(y,n)]p^*(y-1,n)}{[(y-1+\beta)-(n+\alpha)p^*(y-1,n)]} \tag{28}$$

y= n,n+1,….. , n=1,2,3….

with initial conditions

$$p^*(y,1) = \frac{(y+\beta)M(y,1)}{M(y-1,1)} \tag{29}$$

Proof: From the relation (15) we have

$$p^*(y,n+1) = \frac{(y+\beta)}{(n+1+\alpha)+(n+1)B(y,n+1)} \tag{30}$$

Substituting (23) into (30) and using (15) we obtain (28), after some algebraic manipulations. From the relation (22) for n = 1 we easily get (29).

4. BAYES ESTIMATOR OF THE RELIABILITY FUNCTION OF ZERO-TRUNCATED POISSON DISTRIBUTION

The Bayes estimator $R^*(t)$, for R(t) = P(X > t), where the random variable X has the distribution (2), is given by

$$R^*(t) = E[R(t)/x_1, x_2, ..., x_n]$$

$$= \frac{\int_0^\infty R(t)\bar{e}^{(n+\alpha)\lambda}\lambda^{(y+\beta)-1}(1-\bar{e}^{\lambda})^{-n}d\lambda}{\int_0^\infty \bar{e}^{(n+\alpha)\lambda}\lambda^{(y+\beta)-1}(1-\bar{e}^{\lambda})^{-n}d\lambda} \qquad (31)$$

where

$$R(t) = \sum_{x=[t]+1}^{\infty} \frac{\bar{e}^{\lambda}\lambda^{x}}{x!}\left(1-\bar{e}^{\lambda}\right)^{-1}$$,[t], the integer part of t. (32)

Making similar computations, as for $p^*(y, n)$ we get

$$R^*(t) = \sum_{x=[t]+1}^{\infty} \frac{1}{x!} \frac{\int_0^\infty R(t)\bar{e}^{(\alpha+n+1)\lambda}\lambda^{(y+\beta+x)-1}(1-\bar{e}^{\lambda})^{-(n+1)}d\lambda}{\int_0^\infty \bar{e}^{(n+\alpha)\lambda}\lambda^{(y+\beta)-1}(1-\bar{e}^{\lambda})^{-n}d\lambda} \qquad (33)$$

Using the identity

$$(1-z)^{-n} = \sum_{k=0}^{\infty}\binom{n+k-1}{k}z^{k}, \qquad z| < 1$$

we obtain

$$\int_0^\infty \bar{e}^{(\alpha+n+1)\lambda}\lambda^{(y+\beta+n)-1}\left(1-\bar{e}^{\lambda}\right)^{-(n+1)}d\lambda = \Gamma(y+\beta+x)\sum_{k=n+1}^{\infty}\binom{k-1}{n}\frac{1}{(\alpha+k)^{(y+\beta+x)}} \qquad (34)$$

Similarly,

$$\int_0^\infty \bar{e}^{(n+\alpha)\lambda}\lambda^{(y+\beta)-1}\left(1-\bar{e}^{\lambda}\right)^{-n}d\lambda = \Gamma(y+\beta)\sum_{k=n}^{\infty}\binom{k-1}{n-1}\frac{1}{(\alpha+k)^{y+\beta}} \qquad (35)$$

Using (35) and (34) in (33), we get

$$R^*(t) = \sum_{x=[t]+1}^{\infty}\frac{\Gamma(y+\beta+x)}{x!\Gamma(y+\beta)}\frac{\sum_{k=n+1}^{\infty}\binom{k-1}{n}\frac{1}{(\alpha+k)^{y+\beta+x}}}{\sum_{k=n}^{\infty}\binom{k-1}{n-1}\frac{1}{(\alpha+k)^{y+\beta}}} \qquad (36)$$

Using the relation (10), we get

$$R^*(t) = \sum_{x=[t]+1}^{\infty}\frac{\Gamma(y+\beta+x)}{x!\Gamma(y+\beta)}\frac{\sum_{k=n+1}^{\infty}\binom{k-1}{n}\frac{(\alpha+k)}{(\alpha+k)^{y+\beta+x+1}}}{\sum_{k=n}^{\infty}\binom{k-1}{n-1}\frac{(\alpha+k)}{(\alpha+k)^{y+\beta+1}}}$$

$$= \sum_{x=[t]+1}^{\infty} \frac{\Gamma(y+\beta+x)}{x!\Gamma(y+\beta)} \left[\frac{(n+1)\sum_{k=n+2}^{\infty}\binom{k-1}{n+1}\frac{1}{(\alpha+k)^{y+x+\beta+1}} + (n+1+\alpha)\sum_{k=n+1}^{\infty}\binom{k-1}{n}\frac{1}{(\alpha+k)^{y+x+\beta+1}}}{n\sum_{k=n+1}^{\infty}\binom{k-1}{n}\frac{1}{(\alpha+k)^{y+\beta+1}} + (n+\alpha)\sum_{k=n}^{\infty}\binom{k-1}{n-1}\frac{1}{(\alpha+k)^{y+\beta+1}}} \right]$$

$$= \sum_{x=[t]+1}^{\infty} \frac{\Gamma(y+\beta+x)}{\Gamma(y+\beta)} \left[\frac{(n+1)M(y+x,n+2)+(n+1+\alpha)M(y+x,n+1)}{(n+\alpha)M(y,n)+nM(y,n+1)} \right] \quad (37)$$

Where

$$M(y,n) = \sum_{k=n}^{\infty}\binom{k-1}{n-1}\frac{1}{(\alpha+k)^{y+\beta+1}}$$

Or

$$R^*(t) = 1 - \sum_{x=0}^{[t]} \frac{1}{x!} \frac{\Gamma(y+\beta+x)}{\Gamma(y+\beta)} \left[\frac{(n+1)M(y+x,n+2)+(n+1+\alpha)M(y+x,n+1)}{(n+\alpha)M(y,n)+nM(y,n+1)} \right] \quad (38)$$

5. COMPUTER SIMULATION AND CONCLUSION

In order to compare the estimators, Monte Carlo Simulations were performed on 1000 samples for each simulation. The following steps summarize the simulation,

1) A value is generated from a gamma distribution with specified parameters α and β.
2) Based on the realization from the gamma distribution a sample of size n=8 or 30 is generated from the zero-truncated Poisson distribution.
3) The estimates of the parameter and reliability function are computed from the generated sample, and the estimates and their squared error losses were stored.
4) Steps 1-3 were repeated 1000 times.
5) Average values and root mean square errors (RMSE's) of the estimates are computed over the 1000 samples.

Tables 1-4 show some of the results. In comparing the estimators the root mean square error criterion will be used, namely the estimator with the smallest RMSE is the best estimator. The reliability function was evaluated arbitrarily at times 1, 2 and 3. Two sample sizes of n=8, 30 were utilized in the simulation.

TABLE 1. Average values and RMSE's for the estimators of the zero- truncated Poisson.

Gamma prior with $\alpha = 1$ and $\beta = 1$, Sample Size n=8

Parameter							
True Value		Bayes Ave.	RMSE	MLE Ave.	RMSE	RMSE ratio MLE/Bayes
4.0832		4.0776	3.6190	4.0784	3.6200	1.0003
Reliability						
Time	Exact Value	Bayes Ave.	RMSE	MLE Ave.	RSME	RMSE ratio MLE/Bayes
1	4.4816	4.4724	3.6230	4.4695	3.6264	1.0009
2	4.3879	4.3769	3.6392	4.3779	3.6408	1.0004
3	4.2785	4.2681	3.6523	4.2699	3.6542	1.0005

TABLE 2. Average values and RMSE's for the estimators of the zero- truncated Poisson.

Gamma prior with $\alpha = 2$ and $\beta = 5$, Sample Size n=8

Parameter						
True Value		Bayes Ave.	RMSE	MLE Ave.	RMSE	RMSE ratio MLE/Bayes
2.1904		2.1912	1.9665	2.1792	1.9708	1.0022
Reliability						
Time	Exact Value	Bayes Ave.	RMSE	MLE Ave.	RSME	RMSE ratio MLE/Bayes
1	2.7034	2.6915	2.0026	2.6771	2.0142	1.0058
2	2.4588	2.4526	2.0305	2.4403	2.0396	1.0045
3	2.2387	2.2435	2.0244	2.2321	2.0324	1.0040

TABLE 3. Average values and RMSE's for the estimators of the zero- truncated Poisson.

Gamma prior with $\alpha = 1$ and $\beta = 1$, Sample Size n=30

Parameter						
True Value		Bayes Ave.	RMSE	MLE Ave.	RMSE	RMSE ratioMLE/Bayes
15.1380		15.1020	14.6255	15.1020	14.6256	1.0000
Reliability						
Time	Exact Value	Bayes Ave.	RMSE	MLE Ave.	RSME	RMSE ratioMLE/Bayes

1	15.5245	15.4889	14.6310	15.4882	14.6311	1.0000
2	15.4269	15.3892	14.6370	15.3890	14.6372	1.0000
3	15.3205	15.2819	14.6420	15.2821	14.6422	1.0000

TABLE 4. Average values and RMSE's for the estimators of the zero- truncated Poisson. Gamma prior with $\alpha = 2$ and $\beta = 5$, Sample Size n=30

Parameter						
True Value		Bayes Ave.	RMSE	MLE Ave.	RMSE	RMSE ratio MLE/Bayes
8.5200		8.4870	8.2306	8.4840	8.2311	1.0001
Reliability						
Time	Exact Value	Bayes Ave.	RMSE	MLE Ave.	RSME	RMSE ratio MLE/Bayes
1	9.0363	8.9992	8.2502	8.9961	8.2520	1.0002
2	8.8006	8.7641	8.2642	8.7617	8.2655	1.0002
3	8.5798	8.5466	8.2619	8.5438	8.2630	1.0001

In comparing the estimators, the Bayes ones have the smallest RMSE and are better. This is to be expected since the Bayes estimators take advantage of the known prior parameters α and β. By examining the RMSE ratios we can conclude that the estimates are sensitive to the choice of prior parameters and to sample size.

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