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R-SEMI-GENERALIZED FUZZY COMPACTNESS

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Abstract. In this paper, we introduce several types of r-semi-generalized fuzzy compactness and fuzzy r-compactness in fuzzy topological spaces and investigate the relations between these compactness.

1. Introduction

R. Badard [1] introduced the concept of the fuzzy topological space which is an extension of Chang's fuzzy topological space [4]. Many mathematical structures in fuzzy topological spaces were introduced and studied. In particular, M. Demirci [6] studied several types of compactness in fuzzy topological spaces. K. C. Chattopadhyay and S. K. Samanta [5] and S. J. Lee and E. P. Lee [7] introduced the concepts of fuzzy r-closure and fuzzy r-interior in fuzzy topological spaces and obtained their properties. S. J. Lee and E. P. Lee [7] also introduced the concepts of fuzzy r-semi-open sets and fuzzy r-semi-continuous maps in fuzzy topological spaces which are generalizations of fuzzy semi-open sets and fuzzy semi-continuous maps in Chang's fuzzy topological space and obtained their properties. P. Bhattacharya and B. K. Lahiri [3] introduced the concepts of semi-generalized open sets and semi-generalized closed sets in fuzzy topological spaces. In [9] we introduced the concepts of r-semi-generalized fuzzy open sets, r-semi-generalized fuzzy closed sets and r-semi-generalized fuzzy continuous maps in fuzzy topological spaces and obtained their properties.

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In this paper, we introduce several types of r-semi-generalized fuzzy compactness and fuzzy r-compactness in fuzzy topological spaces and investigate the relations between these compactness.

2. Preliminaries

Throughout this paper, let X be a nonempty set, $I = [0,1]$ and $I_0 = (0,1]$. The family of all fuzzy sets of X will be denoted by I^X. By $\tilde{0}$ and $\tilde{1}$ we denote the characteristic functions of ϕ and X, respectively. For any $\mu \in I^X$, μ^c denotes the complement of μ, i.e., $\mu^c = \tilde{1} - \mu$.

A *fuzzy topology* [1, 10], which is also called a *smooth topology,* on X is a map $\tau : I^X \to I$ satisfying the following conditions:
(O1) $\tau(\tilde{0}) = \tau(\tilde{1}) = 1$;
(O2) $\forall \mu_1, \mu_2 \in I^X,\ \tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$;
(O3) for every subfamily $\{\mu_i : i \in \Gamma\} \subseteq I^X,\ \tau(\cup_{i\in\Gamma}\, \mu_i) \geq \wedge_{i\in\Gamma}\, \tau(\mu_i)$.
The pair (X, τ) is called a *fuzzy topological space* (for short, fts) which is also called a *smooth topological space.*

Definition 2.1[5, 7]. *Let (X, τ) be a fts. For $\mu \in I^X$ and $r \in I_0$, the fuzzy r-closure* of μ is defined by

$$cl(\mu, r) = \wedge\{\rho \in I^X \mid \mu \leq \rho,\ \tau(\rho^c) \geq r\}$$

and the *fuzzy r-interior* of μ is defined by

$$int(\mu, r) = \vee\{\rho \in I^X \mid \mu \geq \rho,\ \tau(\rho) \geq r\}.$$

For $r \in I_0$, we call μ a *fuzzy r-open set* of X if $\tau(\mu) \geq r$ and μ a *fuzzy r-closed set* of X if $\tau(\mu^c) \geq r$.

Definition 2.2[7]. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$. Then a map $f : (X, \tau) \to (Y, \sigma)$ is called*

(1) a *fuzzy r-continuous map* if $f^{-1}(\mu)$ is a fuzzy r-open set of X for each fuzzy r-open set μ of Y, or equivalently, $f^{-1}(\mu)$ is a fuzzy r-closed set of X for each fuzzy r-closed set μ of Y.
(2) a *fuzzy r-open map* if $f(\mu)$ is a fuzzy r-open set of Y for each fuzzy r-open set μ of X.
(3) a *fuzzy r-closed map* if $f(\mu)$ is a fuzzy r-closed set of Y for each fuzzy r-closed set μ of X.

Definition 2.3[7]. *Let* (X, τ) *be a fts,* $\mu \in I^X$ *and* $r \in I_0$.

(1) *A fuzzy set* μ *is called fuzzy r-semi-open* if there is a fuzzy r-open set ρ of X such that $\rho \leq \mu \leq cl(\rho, r)$.
(2) A fuzzy set μ is called *fuzzy r-semi-closed* if there is a fuzzy r-closed set ρ of X such that $int(\rho, r) \leq \mu \leq \rho$.

Definition 2.4[7]. *Let* (X, τ) *be a fts. For* $\mu \in I^X$ *and* $r \in I_0$*, the fuzzy r-semi-closure of* μ is defined by

$$scl(\mu, r) = \wedge\{\rho \in I^X |\ \mu \leq \rho,\ \rho \text{ is fuzzy r-semi-closed}\}.$$

and the *fuzzy r-semi-interior of* μ is defined by

$$sint(\mu, r) = \vee\{\rho \in I^X |\ \mu \geq \rho,\ \rho \text{ is fuzzy r-semi-open}\}.$$

Definition 2.5[7]. *Let* (X, τ) *and* (Y, σ) *be fts's and* $r \in I_0$*. Then a map* $f : (X, \tau) \to (Y, \sigma)$ *is called*

(1) *a fuzzy r-semi-continuous map* if $f^{-1}(\mu)$ is a fuzzy r-semi-open set of X for each fuzzy r-open set μ of Y, or equivalently, $f^{-1}(\mu)$ is a fuzzy r-semi-closed set of X for each fuzzy r-closed set μ of Y.
(2) a *fuzzy r-semi-open map* if $f(\mu)$ is a fuzzy r-semi-open set of Y for each fuzzy r-open set μ of X.
(3) a *fuzzy r-semi-closed map* if $f(\mu)$ is a fuzzy r-semi-closed set of Y for each fuzzy r-closed set μ of X.

Definition 2.6[9]. *Let* (X, τ) *be a fts,* $\mu, \rho \in I^X$ *and* $r \in I_0$.

(1) *A fuzzy set* μ *is called r-semi-generalized fuzzy closed* (for short, r-sgfc) if $scl(\mu, r) \leq \rho$ whenever $\mu \leq \rho$ and ρ is r-semi-open.
(2) A fuzzy set μ is called *r-semi-generalized fuzzy open* (for short, r-sgfo) if μ^c is r-sgfc.

Definition 2.7[9]. *Let* (X, τ) *be a fts. For* $\mu \in I^X$ *and* $r \in I_0$*, the r-semi-generalized fuzzy closure of* μ is defined by

$$sgcl(\mu, r) = \wedge\{\rho \in I^X |\ \mu \leq \rho,\ \rho \text{ is r-sgfc}\}.$$

and the *r-semi-generalized fuzzy interior of* μ is defined by

$$sgint(\mu, r) = \vee\{\rho \in I^X |\ \mu \geq \rho,\ \rho \text{ is r-sgfo}\}.$$

THEOREM 2.8[9]. *Let (X, τ) be a fts. Then for $\mu, \lambda \in I^X$ and $r, s \in I_0$,*

(1) $sgcl(\tilde{0}, r) = \tilde{0}$,
(2) $\mu \leq sgcl(\mu, r)$,
(3) $sgcl(\mu, r) \leq sgcl(\mu, s)$ *if* $r \leq s$,
(4) $sgcl(\mu, r) \leq sgcl(\lambda, r)$ *if* $\mu \leq \lambda$,
(5) $sgcl(\mu \vee \lambda, r) \geq sgcl(\mu, r) \vee sgcl(\lambda, r)$,
(6) $sgcl(sgcl(\mu, r), r) = sgcl(\mu, r)$.

THEOREM 2.9[9]. *Let (X, τ) be a fts. Then for $\mu, \lambda \in I^X$ and $r, s \in I_0$,*

(1) $sgint(\tilde{1}, r) = \tilde{1}$,
(2) $sgint(\mu, r) \leq \mu$,
(3) $sgint(\mu, r) \geq sgint(\mu, s)$ *if* $r \leq s$,
(4) $sgint(\mu, r) \leq sgint(\lambda, r)$ *if* $\mu \leq \lambda$,
(5) $sgint(\mu \wedge \lambda, r) \leq sgint(\mu, r) \wedge sgint(\lambda, r)$,
(6) $sgint(sgint(\mu, r), r) = sgint(\mu, r)$.

DEFINITION 2.10[9]. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$ and let $f : (X, \tau) \to (Y, \sigma)$ be a map.*

(1) f *is called r-semi-generalized fuzzy continuous* (for short, r-semi-gf-continuous) if $f^{-1}(\mu)$ is a r-sgfc set of X for each fuzzy r-closed set μ of Y.
(2) f is called *strongly r-semi-generalized fuzzy continuous* (for short, strongly r-semi-gf-continuous) if $f^{-1}(\mu)$ is a fuzzy r-closed set of X for each r-sgfc set μ of Y.
(3) f is called *r-semi-generalized fuzzy irresolute* (for short, r-semi-gf-irresolute) if $f^{-1}(\mu)$ is a r-sgfc set of X for each r-sgfc set μ of Y.
(4) f is called *r-semi-generalized fuzzy open* (for short, r-semi-gf-open) if $f(\mu)$ is a r-sgfo set of Y for each fuzzy r-open set μ of X.
(5) f is called *strongly r-semi-generalized fuzzy open* (for short, strongly r-semi-gf-open) if $f(\mu)$ is a r-sgfo set of Y for each r-sgfo set μ of X.

THEOREM 2.11[9]. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$ and let $f : (X, \tau) \to (Y, \sigma)$ be a map. Then the following are equivalent:*

(1) f *is r-semi-gf-continuous.*

(2) $f^{-1}(\mu)$ *is a r-sgfo set of* X *for each fuzzy r-open set* μ *of* Y.

Theorem 2.12[9]. *Let* (X, τ) *and* (Y, σ) *be fts's and* $r \in I_0$. *If* $f : (X, \tau) \to (Y, \sigma)$ *is a r-semi-gf-continuous map, then* $f(sgcl(\mu, r)) \leq cl(f(\mu), r)$ *for each* $\mu \in I^X$.

Theorem 2.13[9]. *Let* (X, τ) *and* (Y, σ) *be fts's and* $r \in I_0$. *Then* $f : (X, \tau) \to (Y, \sigma)$ *is a r-semi-gf-irresolute map if and only if* $f^{-1}(\mu)$ *is a r-sgfo set of* X *for each r-sgfo set* μ *of* Y.

Theorem 2.14[9]. *Let* (X, τ) *and* (Y, σ) *be fts's and* $r \in I_0$. *If* $f : (X, \tau) \to (Y, \sigma)$ *is a r-semi-gf-irresolute map, then*

(1) $f(sgcl(\mu, r)) \leq sgcl(f(\mu), r)$ *for each* $\mu \in I^X$,
(2) $sgcl(f^{-1}(\mu), r) \leq f^{-1}(sgcl(\mu, r))$ *for each* $\mu \in I^Y$,
(3) $f^{-1}(sgint(\mu, r)) \leq sgint(f^{-1}(\mu), r)$ *for each* $\mu \in I^Y$.

Theorem 2.15[9]. *Let* (X, τ) *and* (Y, σ) *be fts's and* $r \in I_0$. *If* $f : (X, \tau) \to (Y, \sigma)$ *is a strongly r-semi-gf-open map, then* $f(sgint(\mu, r)) \leq sgint(f(\mu), r)$ *for each* $\mu \in I^X$.

3. Results

A collection $\{\mu_i | \ i \in \Gamma\}$ of fuzzy r-open sets of X is called a fuzzy r-open cover of X if $\vee_{i \in \Gamma} \mu_i = \tilde{1}$.

A collection $\{\mu_i | \ i \in \Gamma\}$ of r-sgfo sets of X is called a r-sgfo cover of X if $\vee_{i \in \Gamma} \mu_i = \tilde{1}$.

Definition 3.1. *Let* (X, τ) *be a fts and* $r \in I_0$.

(1) (X, τ) *is called fuzzy r-compact* if for every fuzzy r-open cover $\{\mu_i | \ i \in \Gamma\}$ of X, there exists a finite subset Γ_0 of Γ such that $\vee_{i \in \Gamma_0} \mu_i = \tilde{1}$.
(2) (X, τ) is called *nearly fuzzy r-compact* if for every fuzzy r-open cover $\{\mu_i | \ i \in \Gamma\}$ of X, there exists a finite subset Γ_0 of Γ such that $\vee_{i \in \Gamma_0} int(cl(\mu_i, r), r) = \tilde{1}$.
(3) (X, τ) is called *almost fuzzy r-compact* if for every fuzzy r-open cover $\{\mu_i | \ i \in \Gamma\}$ of X, there exists a finite subset Γ_0 of Γ such that $\vee_{i \in \Gamma_0} cl(\mu_i, r) = \tilde{1}$.

DEFINITION 3.2. *Let* (X,τ) *be a fts and* $r \in I_0$.

(1) (X,τ) *is called r-semi-generalized fuzzy compact* (for short, r-semi-gf-compact) if for every r-sgfo cover $\{\mu_i |\ i \in \Gamma\}$ of X, there exists a finite subset Γ_0 of Γ such that $\vee_{i\in\Gamma_0}\mu_i = \tilde{1}$.
(2) (X,τ) is called *nearly r-semi-generalized fuzzy compact* (for short, nearly r-semi-gf-compact) if for every r-sgfo cover $\{\mu_i |\ i \in \Gamma\}$ of X, there exists a finite subset Γ_0 of Γ such that
$\vee_{i\in\Gamma_0} sgint(sgcl(\mu_i, r), r) = \tilde{1}$.
(3) (X,τ) is called *almost r-semi-generalized fuzzy compact* (for short, almost r-semi-gf-compact) if for every r-sgfo cover $\{\mu_i |\ i \in \Gamma\}$ of X, there exists a finite subset Γ_0 of Γ such that
$\vee_{i\in\Gamma_0} sgcl(\mu_i, r) = \tilde{1}$.
(4) (X,τ) is called *r-semi-generalized fuzzy regular* (for short, r-semi-gf-regular) if each r-sgfo set μ of X can be written as $\mu = \vee\{\rho \in I^X | \rho \text{ is r-sgfo}, sgcl(\rho,\ r) \le \mu\}$.

THEOREM 3.3. *Let* (X,τ) *be a fts and* $r \in I_0$. *Then* (X,τ) *is r-semi-gf-compact* $\Rightarrow$ (X,τ) *is nearly r-semi-gf-compact* $\Rightarrow$ (X,τ) *is almost r-semi-gf-compact.*

Proof. Let (X,τ) be r-semi-gf-compact. Then for every r-sgfo cover $\{\mu_i |\ i \in \Gamma\}$ of X, there exists a finite subset Γ_0 of Γ such that $\vee_{i\in\Gamma_0}\mu_i = \tilde{1}$. Since $\mu_i = sgint(\mu_i, r)$ for each $i \in \Gamma$,

$\mu_i = sgint(\mu_i, r) \le sgint(sgcl(\mu_i, r), r)$ for each $i \in \Gamma$.

Hence $\tilde{1} = \vee_{i\in\Gamma_0}\mu_i \le \vee_{i\in\Gamma_0} sgint(sgcl(\mu_i, r), r)$, i.e., $\vee_{i\in\Gamma_0} sgint(sgcl(\mu_i, r), r) = \tilde{1}$. Thus (X,τ) is nearly r-semi-gf-compact.

Now let (X,τ) be nearly r-semi-gf-compact. Then for every r-sgfo cover $\{\mu_i |\ i \in \Gamma\}$ of X, there exists a finite subset Γ_0 of Γ such that $\vee_{i\in\Gamma_0} sgint(sgcl\ (\mu_i, r), r) = \tilde{1}$. Since $sgint(sgcl(\mu_i, r), r) \le sgcl(\mu_i, r)$ for each $i \in \Gamma$, $\tilde{1} = \vee_{i\in\Gamma_0} sgint(sgcl\ (\mu_i, r), r) \le \vee_{i\in\Gamma_0} sgcl(\mu_i, r)$, i.e., $\vee_{i\in\Gamma_0} sgcl(\mu_i, r) = \tilde{1}$. Hence (X,τ) is almost r-semi-gf-compact. □

THEOREM 3.4. *Let* (X,τ) *be a fts and* $r \in I_0$. *If* (X,τ) *is almost r-semi-gf-compact and r-semi-gf-regular, then* (X,τ) *is r-semi-gf-compact.*

Poof. Let $\{\mu_i | \ i \in \Gamma\}$ be a r-sgfo cover of X. Since (X,τ) is r-semi-gf-regular, $\mu_i = \vee_{j_i \in J_i}\{\rho_{j_i} \in I^X | \ \rho_{j_i}$ is r-sgfo, $sgcl(\rho_{j_i}, r) \leq \mu_i\}$ for each $i \in \Gamma$. Since $\vee_{i\in\Gamma}\mu_i = \vee_{i\in\Gamma}(\vee_{j_i\in J_i}\rho_{j_i}) = \tilde{1}$ and (X,τ) is almost r-semi-gf-compact, there exists a finite subfamily $\{\rho_j \in I^X | \ \rho_j$ is r-sgfo, $j \in J\}$ such that $\vee_{j\in J} sgcl(\rho_j, r) = \tilde{1}$. Since for each $j \in J$ there exists $i \in \Gamma$ such that $sgcl(\rho_j, r) \leq \mu_i$, we have $\vee_{i\in\Gamma_0}\mu_i = \tilde{1}$, where Γ_0 is a finite subset of Γ. Hence (X,τ) is r-semi-gf-compact.

□

THEOREM 3.5. *Let (X,τ) and (Y,σ) be fts's and $r \in I_0$ and let $f : (X,\tau) \to (Y,\sigma)$ be a surjective r-semi-gf-continuous map.If (X,τ) is r-semi-gf-compact, then (Y,σ) is fuzzy r-compact.*

Proof. Let $\{\mu_i | \ i \in \Gamma\}$ be a fuzzy r-open cover of Y. Since f is r-semi-gf-continuous, by Theorem 2.11 $\{f^{-1}(\mu_i) | \ i \in \Gamma\}$ is a r-sgfo cover of X. Since (X,τ) is r-semi-gf-compact, there exists a finite subset Γ_0 of Γ such that $\vee_{i\in\Gamma_0} f^{-1}\ (\mu_i) = \tilde{1}_X$. Since f is surjective, $\tilde{1}_Y = f(\tilde{1}_X) = f(\vee_{i\in\Gamma_0} f^{-1}(\mu_i)) = \vee_{i\in\Gamma_0} f(f^{-1}(\mu_i)) = \vee_{i\in\Gamma_0}\mu_i$, i.e., $\vee_{i\in\Gamma_0}\mu_i = \tilde{1}_Y$. Hence (Y,σ) is fuzzy r-compact.

□

THEOREM 3.6. *Let (X,τ) and (Y,σ) be fts's and $r \in I_0$ and let $f : (X,\tau) \to (Y,\sigma)$ be a surjective r-semi-gf-continuous map. If (X,τ) is almost r-semi-gf-compact, then (Y,σ) is almost fuzzy r-compact.*

Proof. Let $\{\mu_i | \ i \in \Gamma\}$ be a fuzzy r-open cover of Y. Since f is r-semi-gf-continuous, by Theorem 2.11 $\{f^{-1}(\mu_i) | \ i \in \Gamma\}$ is a r-sgfo cover of X. Since (X,τ) is almost r-semi-gf-compact, there exists a finite subset Γ_0 of Γ such that $\vee_{i\in\Gamma_0} sgcl(f^{-1}(\mu_i), r) = \tilde{1}_X$. Since f is surjective, $\tilde{1}_Y = f(\tilde{1}_X) = f(\vee_{i\in\Gamma_0}\ sgcl(f^{-1}(\mu_i), r)) = \vee_{i\in\Gamma_0}\ f(sgcl(f^{-1}(\mu_i), r))$. Since f is r-semi-gf-continuous, by Theorem 2.12 $f(sgcl(f^{-1}(\mu_i), r)) \leq cl(f(f^{-1}(\mu_i)), r)$. Hence $\tilde{1}_Y = \vee_{i\in\Gamma_0} f(sgcl(f^{-1}(\mu_i), r)) \leq \vee_{i\in\Gamma_0} cl(f(f^{-1}(\mu_i)), r) = \vee_{i\in\Gamma_0} cl(\mu_i, r)$. Thus $\vee_{i\in\Gamma_0}\ cl(\mu_i, r) = \tilde{1}_Y$. Hence (Y,σ) is almost fuzzy r-compact.

□

THEOREM 3.7. *Let (X,τ) and (Y,σ) be fts's and $r \in I_0$ and let $f : (X,\tau) \to (Y,\sigma)$ be a surjective r-semi-gf-irresolute map. Then*

(1) *If (X,τ) is r-semi-gf-compact, then (Y,σ) is r-semi-gf-compact.*

(2) *If* (X,τ) *is almost r-semi-gf-compact, then* (Y,σ) *is almost r-semi-gf-compact.*

Proof. (1) Let $\{\mu_i|\ i\in\Gamma\}$ be a r-sgfo cover of Y. Since f is r-semi-gf-irresolute, by Theorem 2.13 $\{f^{-1}(\mu_i)|\ i\in\Gamma\}$ is a r-sgfo cover of X. Since (X,τ) is r-semi-gf-compact, there exists a finite subset Γ_0 of Γ such that $\vee_{i\in\Gamma_0}f^{-1}(\mu_i)=\tilde{1}_X$. Since f is surjective, $\tilde{1}_Y=f(\tilde{1}_X)=f(\vee_{i\in\Gamma_0}f^{-1}(\mu_i))=\vee_{i\in\Gamma_0}f(f^{-1}(\mu_i))=\vee_{i\in\Gamma_0}\mu_i$, i.e., $\vee_{i\in\Gamma_0}\mu_i=\tilde{1}_Y$. Hence (Y,σ) is r-semi-gf-compact.

(2) Let $\{\mu_i|\ i\in\Gamma\}$ be a r-sgfo cover of Y. Since f is r-semi-gf-irresolute, by Theorem 2.13 $\{f^{-1}(\mu_i)|\ i\in\Gamma\}$ is a r-sgfo cover of X. Since (X,τ) is almost r-semi-gf-compact, there exists a finite subset Γ_0 of Γ such that $\vee_{i\in\Gamma_0}sgcl(f^{-1}(\mu_i),r)=\tilde{1}_X$. Since f is surjective, $\tilde{1}_Y=f(\tilde{1}_X)=f(\vee_{i\in\Gamma_0}\ sgcl\ (f^{-1}(\mu_i),r))=\vee_{i\in\Gamma_0}\ f(sgcl(f^{-1}(\mu_i),r))$. Since f is r-semi-gf-irresolute, by Theorem 2.14 $f(sgcl(f^{-1}(\mu_i),\ r))\le sgcl(f(f^{-1}(\mu_i)),r)$. Hence $\tilde{1}_Y=\vee_{i\in\Gamma_0}f(sgcl(f^{-1}(\mu_i),r))\le\vee_{i\in\Gamma_0}sgcl\ (f(f^{-1}(\mu_i)),r)=\vee_{i\in\Gamma_0}sgcl(\mu_i,r)$ and so $\vee_{i\in\Gamma_0}sgcl(\mu_i,r)=\tilde{1}_Y$. Hence (Y,σ) is almost r-semi-gf-compact.

□

THEOREM 3.8. *Let* (X,τ) *and* (Y,σ) *be fts's and* $r\in I_0$ *and let* $f:(X,\tau)\to(Y,\sigma)$ *be a surjective, r-semi-gf-irresolute and strongly r-semi-gf-open map. If* (X,τ) *is nearly r-semi-gf-compact, then* (Y,σ) *is nearly r-semi-gf-compact.*

Proof. Let $\{\mu_i|\ i\in\Gamma\}$ be a r-sgfo cover of Y. Since f is r-semi-gf-irresolute, by Theorem 2.13 $\{f^{-1}(\mu_i)|\ i\in\Gamma\}$ is a r-sgfo cover of X. Since (X,τ) is nearly r-semi-gf-compact, there exists a finite subset Γ_0 of Γ such that $\vee_{i\in\Gamma_0}sgint(sgcl(f^{-1}(\mu_i),\ r),r)=\tilde{1}_X$. Since f is surjective, $\tilde{1}_Y=f(\tilde{1}_X)=f(\vee_{i\in\Gamma_0}sgint(sgcl(f^{-1}(\mu_i),\ r),r))=\vee_{i\in\Gamma_0}f(sgint(sgcl(f^{-1}(\mu_i),r),r))$.

Since f is strongly r-semi-gf-open, by Theorem 2.15 $f(sgint(sgcl\ (f^{-1}(\mu_i),r),r))\le sgint(f(sgcl(f^{-1}(\mu_i),r)),r)$ for each $i\in\Gamma$.

Since f is r-semi-gf-irresolute, by Theorem 2.14 $f(sgcl(f^{-1}(\mu_i),r))\le$

$sgcl(f(f^{-1}(\mu_i)), r)$. Hence we have

$$\begin{aligned}\tilde{1}_Y &= \vee_{i\in\Gamma_0} f(sgint(sgcl(f^{-1}(\mu_i), r), r))\\ &\leq \vee_{i\in\Gamma_0} sgint(f(sgcl(f^{-1}(\mu_i), r)), r)\\ &\leq \vee_{i\in\Gamma_0} sgint(sgcl(f(f^{-1}(\mu_i)), r), r)\\ &= \vee_{i\in\Gamma_0} sgint(sgcl(\mu_i, r), r).\end{aligned}$$

Thus $\vee_{i\in\Gamma_0} sgint(sgcl(\mu_i, r), r) = \tilde{1}_Y$. Therefore (Y, σ) is nearly r-semi-gf-compact.

□

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