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THE PROCESS OF NEGOTIATION OF PROOFS ACCEPTABLE TO MATHEMATICS CLASSROOM

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We need to reflect the establishment of meaning and level of 'proof and argumentation in middle school mathematics'. It should be considered as human activity through communication in community. Thus, we should design instruction from this standpoint. From this point of view, we had been operated 'Geometry Inquiry Class' aimed at middle school students in eighth grade for two years to improve current geometry class in middle school. In this study, we will observe how individual students' original proof schemes are developed and accepted to the class through the process of mutual negotiation between the teacher and students. The episode with four phases begins with the initial proof schemes students have offered. Through the negotiation of class participants, it gives birth to the proof scheme unique to the current geometry classroom. Why do we pay attention to the process? It is because we think that the value of this type of instruction lies in the process of communication and mutual understanding and mutual reference, not in the completeness of the final product. This is the very appropriate proof in the middle school mathematics classroom.

I. INTRODUCTION

Proof is the fundamental feature of mathematics and a valuable tool for enhancing the understanding of the subject in that it functions as a principal method of establishing. Thus it takes a pivotal status in the mathematics curriculum (Hanna, 2007). In addition, its types can be categorized into the six multiple roles of verification, explanation, discovery, systematization, intellectual challenge, and communication, all of which are not mutually exclusive (Harel & Sowder, 2006). On the other hand, most of the students have negative attitude towards proof, thinking that mathematics is extremely difficult due to proof: Proof is the very thing to be avoided. Then what is the source of this situation? What makes the process of proof inaccessible to students? What makes the process of proof inaccessible to students? Are the reasons and evidence obvious to a teacher also deemed to be obvious to students? Does a teacher not impose a set of established methods of proof and rules upon students, rather than elaborating on the issues which are not readily accepted by students? Does a teacher only focus on providing students with already known proofs based on self-evident

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propositions rather than offering them opportunities to guess and explore about the problem? In other words, is students' negative attitude toward mathematics not due to the fact that they have been deprived of the opportunities to explore and guess the outcomes given a set of conditions? Does the third issue not lead to students' failure to appreciate proof as the crux of mathematics and rejection of it? Does this not lead consequently to their conceptualization of proof as a hard and dry, thus abstract subject, which is not related to their lives? Do all these not lead to students' difficulty in understanding the characteristics of mathematics?

Many researchers have tried to address these series of questions in the above. As discussed in the outset, it goes without saying that proof is a fundamental nature and one of the pivotal components of mathematics (Hanna & Jhanke, 1996; Ball, Hoyles, Jhanke & Movshovitz-Hadar, 2002). Ball et al.'s (2002) argument is also legitimate that proof must be the center of mathematical instruction across all school years. As their position indicates, problems in proof education may have risen from the instruction without deep consideration of the role of proof in mathematics. It is also possible that this practice has resulted in students' negative conceptualization of proof as a mere rite rather than a meaningful activity. Consequently, many students have come to view proof as no more than an established procedure to be followed, involving a set of pre-defined symbols.

Accordingly it is necessary to consider whether the roles of proof proposed by Hanna and Jhanke can be maintained even after it has been introduced to the actual mathematical curriculum in school. Some adjustment of content is inevitable when some topics in mathematicians' community are introduced to the classroom. This makes it impossible to expect proofs in the classroom to be identical to original ones. Rather, proofs in the school context may be closer to the logic of verification in the empirical science. Therefore, the contribution of proof to the school mathematics is closely related to communication for understanding crucial concepts and proof itself can be understood as a product of mutual negotiation. In other words, the school curriculum deals with proof within the pedagogical contract relationship (Brousseau, 1997) mediated by mathematics, from the perspective of its contribution to students' mathematical understanding through facilitating stakeholders' communication. It also focuses on what steps the class takes to acquire a proof as a product of mutual negotiation.

Traditionally, proof in school mathematics has been covered in the domain of geometry. This sounds very natural considering that geometry has long used deductive methods based on Euclidean Elements. Harel and Sowder(1998) classifies the research on proof education originated from the tradition into several themes. First, they reviewed how students understand the concept of proof and achieve in the related performance and found that the former concept-based activity is completed mostly by advanced students. They also reviewed pedagogical approach to proof instruction. They emphasize two things: (1) Current methodology in proof instruction views learners as passive recipient, and (2) There is a problem in the belief that proof should be taught in its complete form. This confirms that a new method in proof instruction should focus on giving students the opportunity to appreciate the nature of proof. Lastly, they propose a new pedagogy, which focuses on communication and socialization in the classroom rather than follows a set of strict formats.

We also need to review the meaning of "proof and demonstration of the middle school mathematics" and set the difficulty level for them. Proof and demonstration at the middle school level should be understood as human activity through communication within a learning group and we need to orient the class based on this premise. In an effort to improve the middle school proof instruction in geometry, we have run a 'Geometry Inquiry Class' for middle school students for two years, which has its foundation on Harel and Sowder(1998, 2006). The class focuses on proving the properties of figure within the middle school curriculum and involves students' individual activities, followed by their presentation and discussion. In this study, we will observe how individual students' original proof schemes are developed and accepted to the class through the process of mutual negotiation between the teacher and students. We aim to discuss the level and method of proof, which can be integrated into the middle school mathematics classroom.

II. MAIN STUDY

We introduce an episode in a Geometry Inquiry Class aimed to give mathematical training to middle school students on a long-term basis. The teacher, Mr. Park who has reform-oriented disposition invited 8th grade students from three middle schools neighboring his place of employment. The tasks handled in the class were mainly proving problems based on middle school geometry curriculum. Through 25 years of teaching experience, Mr. Park conceived several problems of current proof teaching in middle school. For the purpose of overcoming problems and drawing up a well-formed teaching method, he designed and proceeded the class with students whose aim is learning mathematics meaningfully. Like this, Geometry Inquiry Class is experimental class designed for understanding students' trajectory of proof learning.

The episode occurred in the process of solving a "Surprise Proof Quiz," whose objective is to offer students the opportunities to have the awareness of mathematical proof. It involves defining odd and even numbers, exploring properties of them and providing proof for them. In the previous lessons, students had learned the meaning of mathematical proposition, definition, theorem and proof and proved the three properties of an isosceles triangle. And the task in this episode designed for enriching students' sense of mathematical proof.

It seems relevant to briefly describe the social context of the class to enhance the understanding of the following episode. In the class, students as presenters are required to solve the problems on their own and give a detailed explanation of the results to other classmates. In the position of the audience, they are expected to understand the presenter's explanation, point out issues of the presentation, and discover possible discrepancy between the presented problem solving strategy and their own approach. All these expectations are at work as social norm in the classroom. The typical socio-mathematical norms state that a group should approach the best proof possible by identifying, modifying and complementing the members' different approaches to the proof (Cobb & Bauersfeld, 1995; Yackel & Cobb, 1996; Cobb & Yackel, 1996). The current episode epitomizes the sociocultural characteristics of classroom contexts except for situations of the individual presentation.

1. Mutual Defining Activity

The first activity in the episode can be termed mutual defining activity (Blumer, 1969). The teacher proposes the class that they begin with the definition of even number and asks students to describe what even number is. Dae-ho and Tea-hoon answer the question by saying "2, 4, 6, 8, …." The teacher points out those are examples of even number. He goes on to ask about odd number. Students answer the question by saying, "1, 3, 5, 7, 9, …," in the same way they addressed the question about even number. The teacher, in response to this, says that those are also examples of odd number and that students failed in giving definitions to two kinds of numbers. Assuming that asking further questions may not get the class further, he decides to review the previous class in which students studied the definition of isosceles triangle. He asks students to define the base of an isosceles triangle, emphasizing that it is not proper to define it as "the side at the bottom". While Tae-hoon struggles to give the answer for a moment, Dae-ho says, "the opposite side of vertical angle". The teacher puts a strong emphasis on the concept of definition by repeating the questions as many as six times, as noted in the following excerpts.

Teacher: (At the same time of Chang-hoon's completion of his utterance) You said "the opposite side of vertical angle." Then what is the opposite side of vertical angle in relation to the base? Is it a property of the base?

Tae-hoon: Definition.

Teacher: A definition of base?

Dae-ho: Definition.

Teacher: Can you say that (it is a definition of base)?

Tae-hoon: Yes.

Teacher: Do you think it would be right to say that it is a definition?

Teacher: A base is the opposite side of vertical angle. Is this the definition of base?

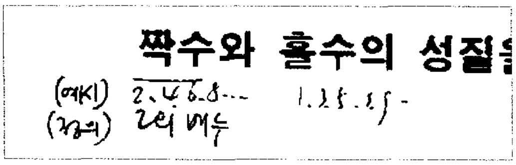
Tae-hoon: Yes.

Teacher: OK?

Tae-hoon: Yes.

In this way, the teacher encourages students to look at the meaning of definition by revisiting the base of isosceles triangle. Now he comes back to the definition of odd and even numbers. They have reached an agreement that they should come up with definition when they are asked to explain what odd and even numbers are. Again, the teacher asks the students to define even number. Tae-hoon says he doesn't know how he should do it and the teacher encourages students to seek the definition without giving up. At the moment, Hak-min says "multiple of 2". The instructor, rather than accepting Hak-min's answer immediately, secures some time for students to ponder upon and come up with their own definition by raising another question, "Then the odd number is multiple of what?" He emphasizes that they are not looking for examples and need to propose definition.

Having emphasized the importance of "defining activity," the teacher asks the class how they can define odd number if even number is defined as "multiple of 2" as Hak-min proposed. In no time, Il-young answers, "even number minus 1". After repeating "even number minus 1" for several times, the teacher asks the class to come up with a better idea, if any. Now he pushes the students into answering his question, saying that they will run out of time for proving the properties of even and odd numbers if they spend too much time defining them. He turns to reviewing Il-young's suggestion. He takes the strategy to compare it with other definitions like those of figure, divisor, multiple, which are fresh in students' memory. The criterion of a good definition is not in its content, but in the way of expression. Is it expressed in words or in formulas? The teacher says that Il-young's expression is not wrong and can be a definition but there is something to be desired. He moves on to ask students to elaborate Il-young's suggestion, proposing that they express the definitions of odd and even numbers in the fashion as they defined figure.



[Figure Ⅱ-1] exemplification and definition of even number

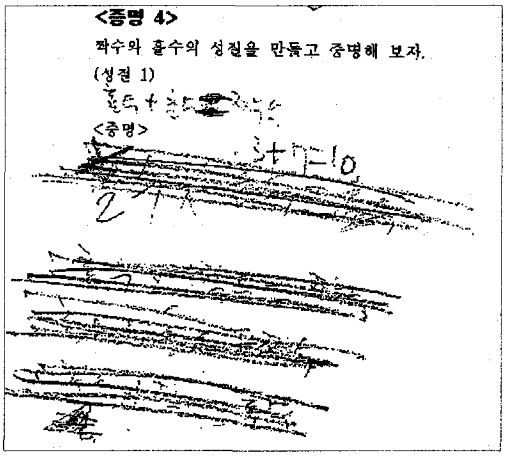
The teacher temporarily defines even number as multiple of 2 and clarifies the difference of exemplification and definition. He asks students whether defining even number as "multiple of 2" rather than as "2, 4, 6, 8" is motivated by mere preference or by any substantial reason. Tae-hoon answers that the case of "2, 4, 6, 8" prevents them from knowing that 10 is also an even number. The teacher, in response to this, takes issue of his opinion by saying that they can add 10 to the definition. Tae-hoon opposes to the argument by stating that they need to keep adding numbers to the infinity to offer that kind of definition. The teacher adds to this argument that the case of "multiple of 2" is free from that kind of problem as well as effective. He puts forward his own opinion that definition must be accurate and simple to properly explain a concept.

In the same vein, the teacher asks students to give the definition of odd number. Hak-min again proposes his definition of "numbers other than even number". Without hesitation, the teacher asks Hak-min whether 1/2 is an odd number or not. Dae-ho says "No" immediately. While Hak-min is about to give his answer, Il-young joins the conversation, saying, "Among natural numbers." Hak-min also says, "Natural number other than even number." In this context, the teacher introduces concepts related to set theory. He says "numbers other than even number" involves the concept of complement, which necessitates the existence of a whole set. He explains that he introduced 1/2 as counter-evidence because Hak-min failed to mention the whole set. He now narrows down the whole set to natural number and defines odd number as "numbers other than even number among natural number". When the teacher is about to conclude the conversation, Dae-ho raises a question of whether it is also possible to define even number as "numbers other than odd number among natural number". The teacher gives a quick comment that it can be also a definition and the conversation is closed.

The teacher proposes they summarize the discussion and starts talking about the definition of odd and even numbers. He states that mathematicians have devised the most convenient definition of them: Numbers divisible by 2 is even number and numbers whose remainder is 1 when divided by 2 is odd number. Then he points out that students covered only natural number in their elementary school, however odd and even numbers are applied to integer. He adds that 0 is also an even number based on the fact that 0 is divided by 2. This concludes the first phase - defining activity of odd and even number.

2. Individual Activity: finding properties and proving

The second activity of the episode is the process of identifying and proving the properties of odd and even number, defined through the previous activity. The teacher explains that 'properties we have located', formula, law, right proposition are all called theorem and says that it is students' task to discover the theorem. He goes on to say that it is a true proposition and characteristic as well as a fact that the sum of even numbers is an even number. Now students are asked to identify about two properties and prove them. As the task begins, Dae-ho asks the teacher whether he can use the characteristic they have just discussed. The teacher replies in the positive and advises students to focus more on proving the found properties than on identifying them. To hear this, Yong-chan, who has focused on identifying characteristics, erases what he has found. Observing students' activity, the teacher once again emphasizes that students should focus on doing proof than on finding many properties.

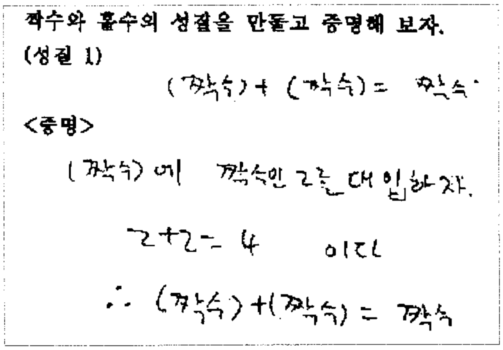


[Figure Ⅱ-2] Yong-chan erases what he has found.

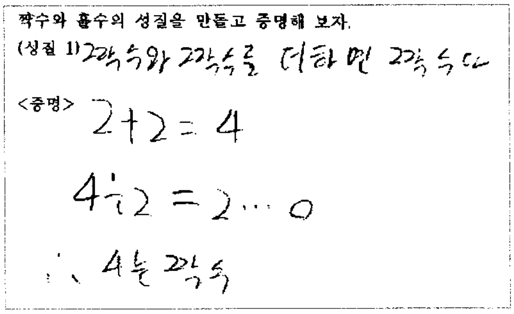
3. Decision Making

The third activity continues after the individual task is finished. The teacher collects students' worksheets and writes down the properties of odd and even numbers students have identified. In doing this, he asks students to judge whether each property is true or false. As there is no response from students, he tells them to jot down the things on their notebooks. He puts down noteworthy properties on the board while excluding overlapping items. Meanwhile, he draws students' attention to the difference between students' and his expressions and explains the protocol for describing properties. Through all these steps, he adopts three items to be reviewed.

<Property 1> (Even number) + (Even number) = (Even number)
[(Odd number) + (Odd number) = (Even number)]
<Property 2> (Odd number) × (Odd number) = (Odd number)
<Property 3> (Odd number) - (Even number) = (Odd number)



[Figure Ⅱ-3] Tae-hoon's worksheet



[Figure Ⅱ-4] Dae-ho's worksheet

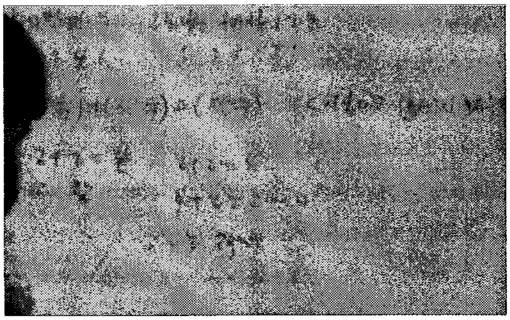
The teacher copies the students' proofs on worksheets onto the board, talking like the following.

Let me write down several people's proofs here. Let's judge, discuss, and compare them. I am going to just copy the proofs.

Even though identity of the writers is revealed later, the teacher guarantees anonymity and copy the written proofs while reading them, saying "One student did the proof like this." For <Property 1>, Tae-hoon and Dae-ho's proofs are adopted. However, the teacher copies Dae-ho's while modifying Tae-hoon's in the process of transcription.

2+2=4. True. (Tae-hoon)

2+2=4, 4/2=2, therefore 4 is even. (Dae-ho)

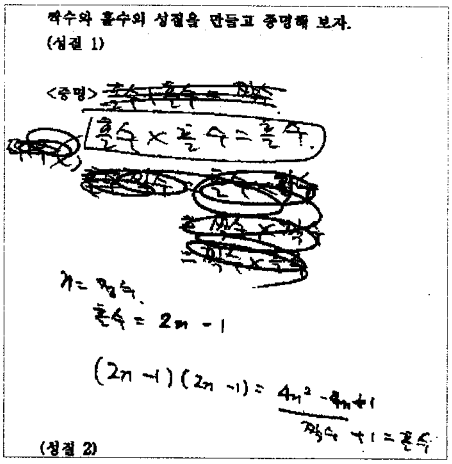

[Figure Ⅱ-5] teacher's copy

For <Property 2>, Il-young's proof is adopted.

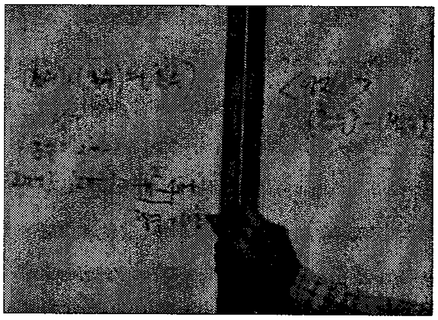
n:integer

odd=2n-1

(2n-1)(2n+1)=4n2-4n+1 =(even)+1



[Figure Ⅱ-6] Il-young's worksheet



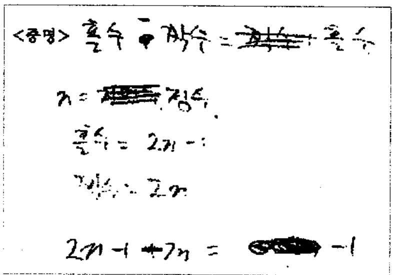
[Figure Ⅱ-7> teacher's copy

Il-young's proof is selected again for <Characteristic 3>.

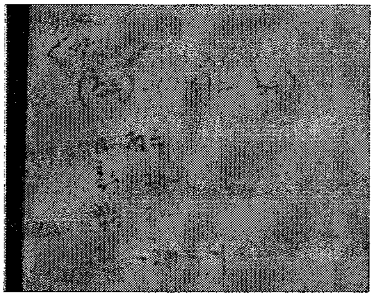
(odd)-(even)=(odd)

n:integer

odd=2n-1, even=2n, 2n-1-2n=-1.



[Figure Ⅱ-8] Il-young's worksheet

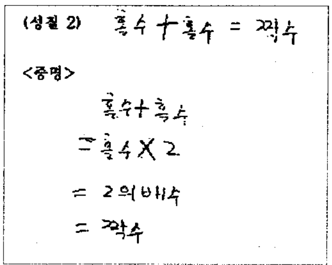


[Figure Ⅱ-9] teacher's copy

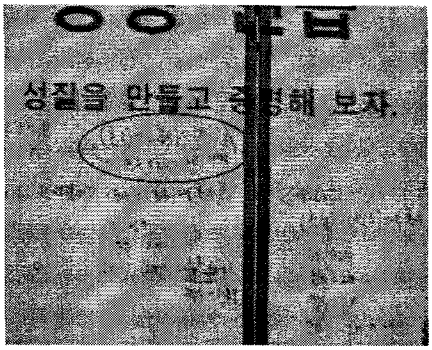
The teacher develops his explanation by asking students about their opinion about the transcribed proofs. First, Il-young says, "Proof is partial" when asked to give comments on the two proofs on <Property 1>. The teacher asks him what he means by "partial," and he replies, "It should be able to show all the cases." The teacher, adding to his comment, explains that the proof is incomplete, reminding students of the difference between definition and exemplification, which they discussed in the process of defining even number.

Due to Il-young's contribution, it is readily agreed that two proofs on <Property 1> have some problems. Class discussion continues as the teacher introduces Hak-min's proof. Hak-min has proved that (Odd number) + (Odd number) = (Even number).

(odd)+(odd)=2(odd)=(even)



[Figure Ⅱ-10] Hak-min's worksheet



[Figure Ⅱ-11] teacher's copy

After listing students' proofs on the board, the teacher categorizes them into three impromptu types.

(1) Proof using examples
(2) Proof using words
(3) Proof using sign

First, the dialogue begins with the discussion of the inappropriateness of "Proof using numeric examples." When asked to comment on the first proof type, Dae-ho accurately points out the problem of the type even though he himself adopted it. According to Dae-ho, (1) is good in that it is simple. However, it cannot explain the whole because "it captures just, a certain portion." The teacher comments on this opinion by saying that this type of proof is similar to defining even number as "2, 4, 6, 8." He moves on to ask students what difference lies between Hak-min's type (2) proof and Il-young's type (3) proof. Il-young says that he likes the proof using signs. The teacher states that both of the proofs perfectly meet the criteria considering students' age level and adds that he would like students to use formula in defining as well as linguistic expression because using formula requires them to have some mathematical competence. This is the teacher's minimum expectation of students' performance.

4. Reflection

The fourth activity of the current episode happens after the categorization and evaluation of proof types, as shown above. Now the class reviews some problems in the proofs. First, they list up the points to be corrected in Hak-min's proof using words. It is agreed that it covers all the odd numbers. Thus, it does not attract the criticism made for Type (1) proof. The teacher briefly comments that it would have been better if they had used signs. He says he points out problems of the proposed proofs because he wants students to be accurate in thinking and to be able to provide proper grounds for their argument. He says:

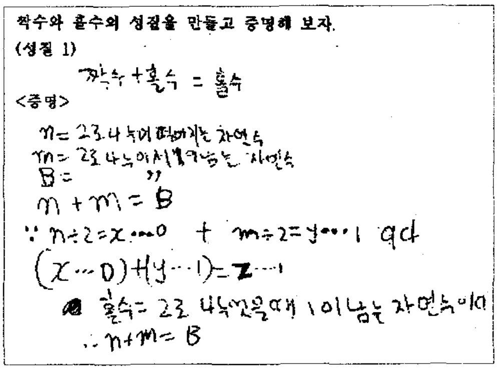
It is natural that this is difficult for people at your age. However, you need to be trained just a little (to properly do the proof). You can do this if you understand what I point out.

In the following, Hak-min takes the first turn in raising issues for his own proof. He points out that (Odd number) plus (Odd number) cannot make 2×(Odd number) because the two odd numbers are not the same. The teacher clarifies Hak-min's explanation by giving an example of 3 and 5, whose sum makes a multiple of 4: this is neither two times of 3 nor of 5. He goes makes another point about Hak-min's proof. He points out that Hak-min should say "Even number is divisible by 2" rather than "Even number is multiple of 2." However, it seems that the student's expression leaves no problem. After the discussion, the teacher gives an alternative to the proof using verbal expression: "If we use the definition of odd number, we have remainder of 1 in each odd number. The sum of these two '1' makes 2, which is divided by 2, which makes (Odd number) plus (Odd number) (Even number)."

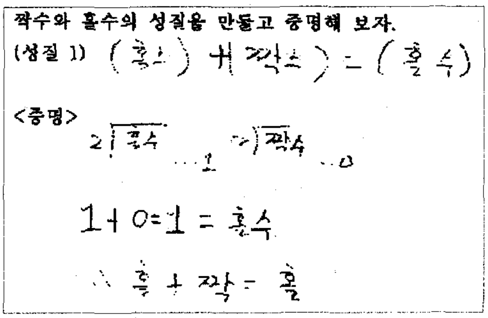
Now it is time to discuss problems with Il-young's proof. The teacher asks him why he expressed odd number as 2n-1. What is the rationale of using 2n-1 rather than 2n+1 although 2n+1 is more proper for indicating the remainder? Il-young replies that he was about to use 2n+1 but wanted to take the way which has never been used. The teacher says that "When they grow older" they will likely use 2n-1 more often. He explains the reason by inserting 1 to 2n-1 and 2n+1. Although results of two notations are the same, the teacher suggests, it would be better to use 2n+1 in the proof, which is more faithful to its definition. Then he asks whether multiplication sign can be omitted between two parentheses and states that product formula (which is introduced in the 3rd year) is needed to expand the multiplication of two odd numbers. The teacher goes on to read aloud the proofs and explains that the remainder is 1, followed by his comment that Il-young and Hak-min have made the same mistake. Il-young admits his. When the class is asked what strategy they can use to address the problem, Dae-ho says that they can use 2n-1 and 2n+1. At the moment, Il-young points out the problem of Dae-ho's suggestion and comes up with the use of 2n-1 and 2b+1 to express two odd numbers. The teacher integrates two students' opinions and demonstrates that multiplication of two odd numbers makes an odd number by using m and n and distributive law.

At the next session, the class discusses the problems of Il-young's proof. Here it is observed again that the same character has been used to express two different numbers. Dae-ho suggests that 2n-1 be used for odd number and 2m for even number. The teacher, agreeing with Dae-ho, finishes his explanation by saying that two different characters should be used.

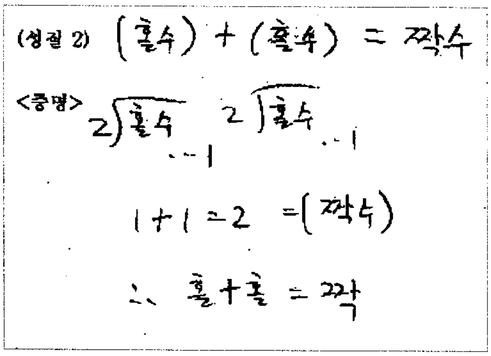
This concludes the whole episode. It seems relevant, though, that we review the students' assignments, which require them to revisit the issues raised in the class. Two cases are provided below: The first one is Tae-hoon's and the other two are Hak-min's. We can observe that these faithfully address the very issues covered in the episode.



[Figure Ⅱ-12] Tae-hoon's homework



[Figure Ⅱ-13] Hak-min's homework 1



[Figure Ⅱ-14] Hak-min's homework 2

Ⅲ. RESULTS AND DISCUSSION

The "Surprise Proof Quiz" we have discussed above can be divided into four phases. The first one is the mutual defining activity. This begins with participants' confirmation of the difference between exemplification and definition. Therefore, this step involves mutual agreement that they should come up with a definition, not examples, when asked to answer what odd number or even number is. This is followed by the process in which the class reaches more refined definitions based on the remainder when divided by 2, using their tentative definition of even number (the multiple of 2). The second activity is the process of identifying and proving the properties of odd and even number defined through negotiation between the teacher and students. Students are assigned two tasks of identifying two properties of odd or even numbers and proving them on their own. The teacher encourages students to focus more on proving the found properties than on identifying them. At the third phase, the teacher transcribes identified properties and performed proofs on the board, asks the class to comment on them, and categorizes them into three types (proofs using examples, verbal expression, and mathematical signs). He goes on to evaluate them with the class. The final phase witnesses the class pointing out some problems of the proofs which have been positively evaluated and integrating students' comments to construct the final proof, which is acceptable to the current class.

The episode with four phases begins with the initial proof schemes students have offered. Through the negotiation of class participants, it gives birth to the proof scheme unique to the current geometry classroom. Majority of the initial schemes are based on external conviction or empirical (Harel & Sowder, 1998, 2006). These schemes proposed by students are categorized into three types through the negotiation processes. The final scheme is obtained through the collective evaluation of each scheme. The following is one example proof for the second property.

(Odd number) × (Odd number) = (Odd number)
Proof:

$(2n+1)\times(2m+1)$

$=4mn+2n+2m+1$

$=2(2mn+n+m)+1$

Odd number because the remainder is 1 when divided by 2

It is crucial to note the process of negotiation from setting the point of departure to reaching the final agreement. Initially, the teacher assigns the substantial amount of time to guarantee the opportunities for students to grasp the concept of proof on their own. It is also noteworthy that the class has the atmosphere to foster students' intellectual autonomy(Yackel & Cobb, 1996). This enables the class to adopt students' own contributions as major discussion topics, rather than relegate them to objects of evaluation and criticism. Consequently, students themselves construct alternatives to the proofs through the collaborative effort to identify and resolve their problems.

Why do we pay attention to the process? It is because we think that the value of this type of instruction lies in the process of communication and mutual understanding and mutual reference, not in the completeness of the final product. This is the very appropriate proof in the middle school mathematics classroom. This gives a valuable insight to our endeavour to remove students' negative attitude towards proof from the classroom. We have reviewed the level of proofs accepted to the community of "Geometry Inquiry Class" through students' participation in its practices (Wenger, 1998). We have also observed what negotiation processes involve establishing those proofs. In fact, they are the very processes through which students learn proof.

References

Ball, D. L., Hoyles, C., Jahnke, H. N., & Movshovitz-Hadar, N. (2002). *The teaching of proof.* Paper presented at the ICM 2002.

Blumer, H. (1969). *Symbolic Interactionism.* Berkely and Los Angeles: University of California Press.

Brousseau, G. (1997). *Theory of Didactical Situations in Mathematics.* Dordrecht: Kluwer Academic Publishers.

Cobb, P., & Bauersfeld, H. (Eds.). (1995). *The emergence of mathematical meaning: Interaction in classroom cultures.* Hillsdale, NJ: Lawrence Erlbaum Associates.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspective in the context of developmental research. *Educational Psychologist*, 31(3/4), 175-190.

Hanna, G. and Jahnke, H.N. (1996). Proof and proving, in A. Bishop, K. Clements, C. Keitel, J. Kilpatrick and C. Laborde (eds.), *International Handbook of Mathematics Education*, Kluwer Academic Publishers, Dordrecht, pp 877-908.

Hanna, G. (2007). The ongoing value of proof. In P. Boero (Ed.), *Theorems in school: From history, epistemology and cognition to classroom practice* (pp. 3-16). Rotterdam: Sense Publishers.

Harel, G., & Sowder, L. (1998). Students' proof schemes: Results from exploratory students. In A. H. Schoenfeld, J. Kaput & E. Dubinsky (Eds.), *CBMS Issues in Mathematics Education* (Vol. 3, pp. 234-283). American Mathematical Society.

Harel, G., & Sowder, L. (2006). Toward comprehensive perspectives on the learning and teaching of proof. In J. Frank K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning: a project of the national council of teachers of mathematics.* Information Age Publishing Inc.

Wenger, E. (1998). *Communities of practice:*

Learning, meaning, and identity. New York: Cambridge University Press.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. Journal for Research in Mathematics Education, 27, 458-477.

수학교실에서 수용 가능한 증명의 상호 교섭 과정

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우리는 '중학교 수학에서의 증명과 논증'의 의미와 수준의 설정을 검토할 필요가 있다. 중학생 수준에서의 증명과 논증은 모두 학습 집단 속에서의 의사소통을 통한 인간 활동으로 보아야 하며, 이에 부합하도록 수업의 방향을 잡아야 한다. 이런 노력의 일환으로, 우리는 중학교 기하수업 개선을 목적으로 중학교 2학년 학생들을 대상으로 2년 동안 기하탐구교실 수업을 진행해 왔다. 우리는 이 수업 중 벌어진 상황 중 하나를 선택하여, 최초 발현된 학생들의 증명 도식이 상호교섭의 과정을 통해 어떻게 교실에서 수용되는 증명 도식으로 형성되어 가는지 그 과정을 살펴볼 것이다. 네 단계에 걸친 활동을 통해 기하탐구교실에서의 증명은 학생들에 의해 발현된 초기 증명 도식에서 출발하여 상호교섭의 과정을 통한 결과물로서의 기하탐구교실만의 증명 도식의 생성으로 이어진다. 우리가 이 과정에 주목하는 이유는 교섭의 산출물이 갖는 수학적 완결성 때문이 아니라, 그것이 갖는 의사소통과 상호 이해, 상호 참조라는 가치에 있으며, 이것이야말로 수학교실에서 수용될 수 있는 적법한 증명이라는 것을 말하기 위해서다. 이상과 같이 우리는 기하탐구교실이라는 학생들의 참여를 통해 공동체 내에서 수용되는 증명의 수준이 어디까지이며, 그것이 어떤 협상 과정을 거쳐 수립되는지 살펴보았다. 이 과정은 동시에 학생들 스스로 증명을 학습해가는 과정이기도 하다.

* **Key Words** : proof in the middle school classroom(수학교실에서의 증명), proof scheme(증명 도식), communication(의사소통), process of negotiation(교섭의 과정)

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