# Optimal Waveform Design for Ultra-Wideband Communication Based on Gaussian Derivatives

Yong Guo

***Abstract:*** **Ultra-wideband (UWB) radios have attracted great interest for their potential application in short-range high-data-rate wireless communications. High received signal to noise ratio and compliance with the Federal Communications Commissions (FCC) spectral mask call for judicious design of UWB pulse shapers. In this paper, even and odd order derivatives of Gaussian pulse are used respectively as base waveforms to produce two synthesized pulses. Our method can realize high efficiency of spectral utilization in terms of normalized effective signal power (NESP). The waveform design problem can be converted into linear programming problem, which can be efficiently solved. The waveform based on even order derivatives is orthogonal to the one based on odd order derivatives.**

***Index Terms:*** **Derivatives of Gaussian monocylce, linear programming, optimal, ultra-wideband (UWB), waveform design.**

## I. INTRODUCTION

With the release of the U.S. Federal Communications Commission (FCC) spectral mask in 2002 [1], ultra-wideband (UWB) radios have attracted great interest for their potential application in short range high-data-rate wireless communications [2]. However, UWB radio must respect the FCC regulations, which imposed a spectral mask that strictly constrains the transmission of a UWB signal to be well below the noise floor in all bands. The transmission reliability of a UWB system is determined by the received signal-to-noise ratio (SNR). Therefore, efficient utilization of the bandwidth and power allowed by the FCC mask is vital to maximize the received SNR. Since the spectrum of the transmitted signal is effectively determined by that of the underlying UWB pulse, the choice of the pulse shape is a key design decision in UWB system. The Gaussian monocycle pulse, commonly used in UWB impulse radio [3]–[6], poorly fits the spectral mask. Recently, [7] proposed that the pulses be based on the dominant eigenvectors of a channel matrix that is constructed by sampling the spectral mask. Pulses generated from different eigenvectors are mutually orthogonal, and conform to the FCC spectral mask. However, they do not achieve the optimal spectral utilization, and requires a high sampling rate (64 GHz) that could lead to implementation difficulties. A single higher order derivative of Gaussian pulse has been proposed as a candidate [8]. However, a single higher order derivative of Gaussian pulse cannot exploit the allowable bandwidth sufficiently. A synthesized waveform design method by linearly combining multiple higher order derivatives of general Gaussian pulse has been introduced in [9]. This method can realize better spectral utilization efficiency. However, because the optimization is based on least squares estimator (LSE) principle [9], this method needs many times of iterations and cannot promise respecting the spectral mask at all frequencies. In this paper, we adopt the optimal method to design two synthesized waveforms: One is based on even order Gaussian derivatives; the other is based on odd order derivatives. This method optimally exploits the bandwidth and power under the constraint of the FCC mask. The design problem is equivalent to a linear programming problem. The problem can be efficiently solved and the solution is globally optimal. The two waveforms are orthogonal to each other. Section II describes the characteristics of Gaussian pulse and its derivatives. In Section III, the optimal pulse design is addressed. Concluding remarks are provided in Section IV.

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## II. GAUSSIAN PULSE AND SPECTRUM

A general Gaussian pulse is given by

$$g(t) = \frac{A}{\sqrt{2\pi}\sigma} e^{(-\frac{t^2}{2\sigma^2})} = \frac{\sqrt{2}A}{\alpha} e^{-\frac{2\pi t^2}{\alpha^2}} \quad (1)$$

where $\alpha^2 = 4\pi\sigma^2$ is the time-scaling factor, and $\sigma^2$ is the standard deviation [8]. Using the general Gaussian pulse in (1), its $n$th derivative can be determined recursively from

$$g^{(n)}(t) = -\frac{n-1}{\sigma^2} g^{(n-2)}(t) - \frac{t}{\sigma^2} g^{(n-1)}(t) \quad (2)$$

where the superscript $n$ denotes the $n$th order derivative. The derivatives of Gaussian pulse are defined as base functions or base waveforms, which are used to produce synthesized waveform through linear combination. The Fourier transform of the $n$th order derivative of Gaussian pulse is

$$G_n(f) = A(j2\pi f)^n e^{-\frac{(2\pi f\sigma)^2}{2}}. \quad (3)$$

If $n$ is even integer, $G_n(f)$ is real. If $n$ is odd integer, $G_n(f)$ is imaginary.

The peak frequency of $G_n(f)$ is given by [9]

$$f_{\text{peak}} = \sqrt{n}\frac{1}{\alpha\sqrt{\pi}}. \quad (4)$$

The derivative operation shifts the pulse energy to higher spectrum. The $f_{\text{peak}}$ increases with the derivation order. We can see this in Fig. 1, in which the normalized power spectral densities (PSDs) of 1st to 15th derivatives of Gaussian pulse are illustrated. The $f_{\text{peak}}$ also depends on $\alpha$. We can adjust the peak

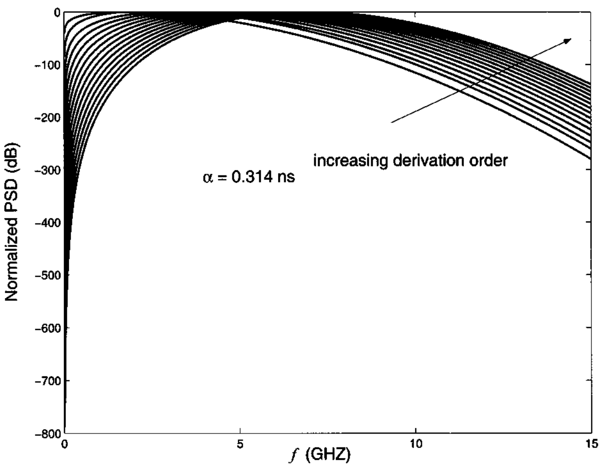


Fig. 1. Normalized PSDs of 1st to 15th derivatives of Gaussian pulse.

frequency by adjusting $\alpha$. In addition $\alpha$ affects the bandwidth of every derivative of general Gaussian pulse.

In a UWB impulse radio, the basic pulses are modulated to carry information. The basic pulse shaping is crucial because it affects the PSDs of the transmitted signal. For efficient radiation, the pulse should have zero dc offset. The general Gaussian pulse does not meet this requirement. Its higher derivatives meet this requirement. By choosing appropriate $n$ and $\alpha$, a single higher order derivative of general Gaussian pulse can meet the spectral mask. In [8], fifth order derivative is proposed as a candidate. But a single pulse of $n$th order derivative of general Gaussian pulse does not exploit the allowable bandwidth efficiently. In [9], 1st–15th derivatives of general Gaussian pulse are combined together to generate a synthesized pulse in order to make use of the allowable bandwidth more efficiently. The $\alpha$s can be different for different order derivatives of Gaussian pulse. Standard procedure for error estimation such as the LSE can be used to acquire the desired solution. Therefore, many times iterations are needed in the process. Moreover, respecting spectral mask at all frequencies cannot be guaranteed because the optimization criterion is based on the root mean square distance. There are no restrictions on PSD at all frequencies.

## III. OPTIMAL PULSE DESIGN

In this section, we present our design of the waveform. The desired waveforms are produced by linearly combining even and odd order base waveforms derived from the general Gaussian pulse respectively. The synthesized waveforms should maximize the spectral utilization efficiency, while at the same time respect the spectral mask. The spectral utilization efficiency is measured in terms of the normalized effective signal power (NESP), which is the ratio of the power transmitted in the designated passband of the spectral mask over the total power that is permissible under the given mask. Formally stated, if $F_p$ denotes the band (or collection of bands) that constitute the passband, then the NESP is defined as $\overline{\phi} = \int_{F_p} S_p(f)df / \int_{F_p} S(f)df$ where $S_p(f)$ is the power spectrum of the designed pulse $p(t)$ and $S(f)$ is the power spectrum of the spectral mask. Because $S(f)$ is independent of design parameters, maximizing $\overline{\phi}$ is to maximize

$$\phi = \int_{f_p} S_p(f)df. \tag{5}$$

In our design, even and odd order derivatives of Gaussian pulse are adopted respectively to form the synthesized pulses. The highest order derivative of Gaussian pulse is 14th. From (3), the normalized Fourier transform of the $n$th order derivative pulse is

$$\overline{G}_n(f) = j^n f^n e^{-\frac{\pi\alpha^2 f^2}{2}} / (f_{\text{peak}}^n e^{-\frac{\pi\alpha^2 f_{\text{peak}}^2}{2}}). \tag{6}$$

To concisely describe the components of the derivatives and weight factor, we define

$$\overline{\boldsymbol{G}}_{\text{odd}} = [\overline{\boldsymbol{G}}_1(f), \overline{\boldsymbol{G}}_3(f), \overline{\boldsymbol{G}}_5(f), \overline{\boldsymbol{G}}_7(f), \overline{\boldsymbol{G}}_9(f), \overline{\boldsymbol{G}}_{11}(f), \overline{\boldsymbol{G}}_{13}(f)]^T$$

and $\boldsymbol{X}_{\text{odd}} = [x1, x3, x5, x7, x9, x11, x13]^T$,

$$\overline{\boldsymbol{G}}_{\text{even}} = [\overline{\boldsymbol{G}}_2(f), \overline{\boldsymbol{G}}_4(f), \overline{\boldsymbol{G}}_6(f), \overline{\boldsymbol{G}}_8(f), \overline{\boldsymbol{G}}_{10}(f), \overline{\boldsymbol{G}}_{12}(f), \overline{\boldsymbol{G}}_{14}(f)]^T$$

and $\boldsymbol{X}_{\text{even}}$=$[x2, x4, x6, x8, x10, x12, x14]^T$ where $\boldsymbol{X}$ is weight factor vector.

For even order case, $S_p(f)$ can be expressed as $S_p(f) = |\overline{\boldsymbol{G}}_{\text{even}}^T \boldsymbol{X}_{\text{even}}|^2$ and $\phi$ can be expressed as $\phi = \int_{F_p} |\overline{\boldsymbol{G}}_{\text{even}}^T \boldsymbol{X}_{\text{even}}|^2 df$. The optimal problem is: Given $\overline{\boldsymbol{G}}_{\text{even}}$, find $\boldsymbol{X}_{\text{even}}$ that maximize the $\phi$, subject to spectral constraints $S_p(f) \leq S(f)$, for all f, or show that none exits. It can be formulated as

$$\max_X \; \phi = \int_{F_p} |\overline{\boldsymbol{G}}_{\text{even}}^T \boldsymbol{X}_{\text{even}}|^2 df \tag{7}$$

$$\text{subject to } |\overline{\boldsymbol{G}}_{\text{even}}^T \boldsymbol{X}_{\text{even}}|^2 \leq S(f). \tag{8}$$

This optimization problem is equivalent to the following two optimization problems:

$$\max_X \; \phi_+ = (\int_{F_p} \overline{\boldsymbol{G}}_{\text{even}} df)^T \boldsymbol{X}_{\text{even}} \tag{9}$$

$$\text{subject to } -\sqrt{S(f)} \leq \overline{\boldsymbol{G}}_{\text{even}}^T \boldsymbol{X}_{\text{even}} \leq \sqrt{S(f)}, \tag{10}$$

$$\min_X \; \phi_- = (\int_{F_p} \overline{\boldsymbol{G}}_{\text{even}} df)^T \boldsymbol{X}_{\text{even}} \tag{11}$$

$$\text{subject to } -\sqrt{S(f)} \leq \overline{\boldsymbol{G}}_{\text{even}}^T \boldsymbol{X}_{\text{even}} \leq \sqrt{S(f)}. \tag{12}$$

If we use $\boldsymbol{C}$ to substitute for $\int_{F_p} \overline{\boldsymbol{G}}_{\text{even}}^T df$, then the latter problem turns into the standard form of linear programming:

$$\min_X \; \phi_- = \boldsymbol{C}^T \boldsymbol{X}_{\text{even}} \tag{13}$$

$$\text{subject to } -\sqrt{S(f)} \leq \overline{\boldsymbol{G}}_{\text{even}}^T \boldsymbol{X}_{\text{even}} \leq \sqrt{S(f)}. \tag{14}$$

The above two problems are the same. The latter problem is a convex optimization problem. It is a standard form of linear programming. We use the optimization toolbox of MATLAB to solve the problem. There are many other softwares that can be used to solve this problem.

Table 1. Peak frequencies, $\alpha$s and weight factors of even order derivatives of Gaussian pulse.

| Derivation order | $\alpha$ (ns) | $f_{\text{peak}}$ (GHz) | $X$ |
|---|---|---|---|
| 2nd | 0.40 | 1.994 | 0.12E − 5 |
| 4th | 0.35 | 3.324 | 0.65E − 5 |
| 6th | 0.27 | 5.081 | 0.27E − 3 |
| 8th | 0.25 | 6.383 | 0.69E − 4 |
| 10th | 0.23 | 7.757 | 0.13E − 3 |
| 12th | 0.19 | 10.286 | −0.54E − 3 |
| 14th | 0.19 | 11.110 | −0.49E − 3 |

Table 2. Peak frequencies, $\alpha$s and weight factors of odd order derivatives of Gaussian pulse.

| Derivation order | $\alpha$ (ns) | $f_{\text{peak}}$ (GHz) | $X$ |
|---|---|---|---|
| 1st | 0.40 | 1.411 | −7.149E − 5 |
| 3rd | 0.35 | 2.792 | −1.219E − 4 |
| 5th | 0.27 | 4.672 | −0.86E − 2 |
| 7th | 0.25 | 5.71 | −0.26E − 2 |
| 9th | 0.23 | 7.359 | −0.42E − 2 |
| 11th | 0.19 | 9.848 | 0.0164 |
| 13th | 0.19 | 10.706 | 0.0152 |

For odd order derivatives case, $S_p(f)$ can be similarly expressed as

$$S_p(f) = |j|^2 \left| \frac{\overline{G}^T_{\text{odd}}}{j} X_{\text{odd}} \right|^2 = \left| \frac{\overline{G}^T_{\text{odd}}}{j} X_{\text{odd}} \right|^2 \quad (15)$$

where $\frac{\overline{G}^T_{\text{odd}}}{j} X_{\text{odd}}$ is real, then we can solve the problem as the even order derivatives case.

$\overline{G}$ is dependent on time-scaling factors of derivatives of Gaussian pulse. The $\alpha$s and corresponding peak frequencies are listed in Tables. Peak frequencies of Gaussian derivatives are chosen roughly distributed in the frequency range from 1 GHz–11 GHz. The choice of peak frequencies is not rigorous. As long as the peak frequencies are roughly evenly distributed in the above frequency range, the optimization results are very good and very close to one another. The $\alpha$s is decided by (4). The solution of $X$ is also listed in Tables.

The PSDs of the synthesized waveforms do not violate the FCC spectrum mask, as illustrated in Fig. 2. The NESP $\phi_{\text{even}}$ is as large as 74.57%, and $\phi_{\text{odd}}$ is as large as 77.86%. The pulse shape based on even order Gaussian derivatives is given in Fig.3. This method needs very short time to complete optimization.

The two synthesized waveforms are orthogonal to each other. We define the waveform based on even order derivatives as $s_{\text{even}}(t)$, the waveform based on odd order derivatives as $s_{\text{odd}}(t)$. The correlation of the two waveforms is $R(\tau) = \int_{-\infty}^{+\infty} s_{\text{even}}(t)s_{\text{odd}}(t-\tau)dt = s_{\text{even}}(t) * s_{\text{odd}}(-t)$, where * denotes the convolution operation. $R(\tau)$ is real. Note that $R(\tau) = s_{\text{even}}(t) * s_{\text{odd}}(-t) \Longleftrightarrow S_{\text{even}}(\omega)S_{\text{odd}}(-\omega)$, where $\Longleftrightarrow$ denotes FFT and IFFT operation, and $S_{\text{even}}(\omega)$ and $S_{\text{odd}}(-\omega)$ are the FFT of $s_{\text{even}}(t)$ and $s_{\text{odd}}(-t)$, respectively. Then, we have $R(0) = \int_{-\infty}^{+\infty} S_{\text{even}}(\omega)S_{\text{odd}}(-\omega)d\omega$. As we know $S_{\text{even}}(\omega)$ is real and $S_{\text{odd}}(-\omega)$ is imaginary, so $R(0)$ will be imaginary. It is

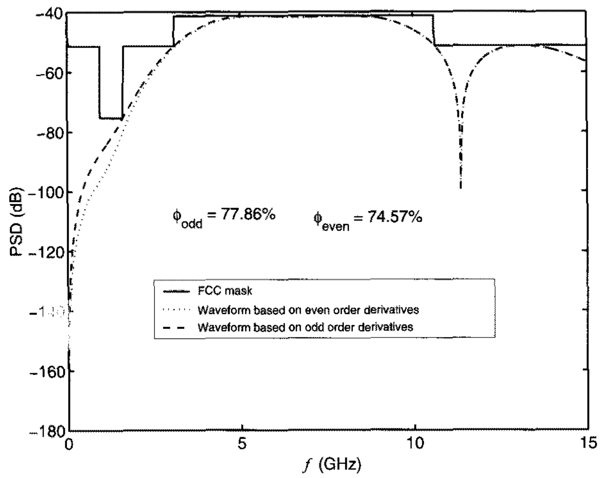


Fig. 2. PSD of the optimal synthesized waveforms under the FCC mask.

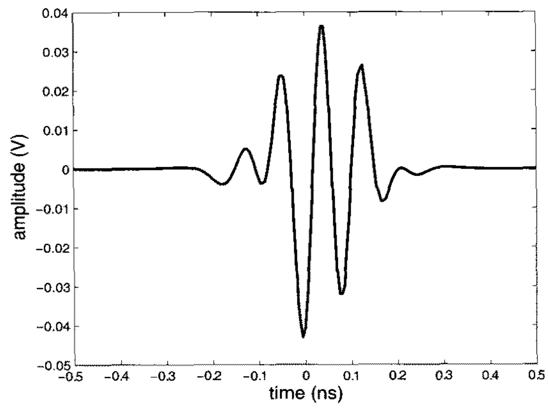


Fig. 3. Pulse shape based on even order Gaussian derivatives.

contradictory except when $R(0) = \int_{-\infty}^{+\infty} S_{\text{even}}(\omega)S_{\text{odd}}(-\omega) = 0$. That is, $s_{\text{even}}(t)$ and $s_{\text{odd}}(t)$ are orthogonal to each other.

In some cases, there are other wireless communications in 3.1 GHz to 10.6 GHz. For example, IEEE 802.11a WLAN occupies 5 GHz band. The UWB signal should avoid the emissions in the WLAN band. By using our method, we design two waveforms: one occupies the band from 3.1 GHz to 4.8 GHZ; the other occupies the band from 6 GHz to 10.6 GHz. The PSD of the two waveforms are illustrated in Figs. 4 and 5. The peak frequencies and $\alpha$s need to be adjusted accordingly.

## IV. CONCLUSION

In this paper, we develop an efficient method for the design of synthesized UWB waveforms that satisfy a spectral mask constraint by using Gaussian base functions. We use even and odd order Gaussian derivatives respectively. The waveform based on even order derivatives is orthogonal to the one based on odd order derivatives. The optimal waveform design problem is translated into a linear programming problem, from which a globally optimal solution can be efficiently found. The NESP of our method can be 74.57% and 77.86%. Our method has an advantage over the method in [9].

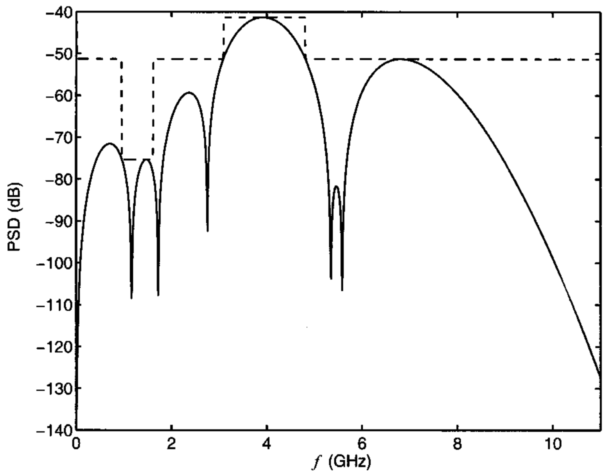


Fig. 4. PSD of the optimally synthesized waveforms for 3.1 GHz to 4.8 GHz.

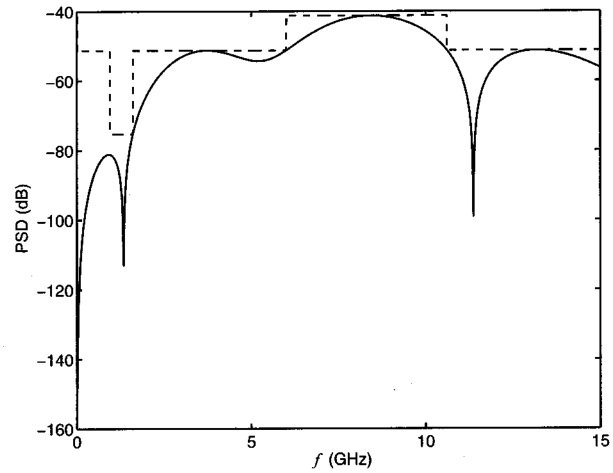


Fig. 5. PSD of the optimally synthesized waveforms for 6 GHz to 10.6 GHz.

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