Performance Analysis of Transmit Diversity in Multiuser Data Networks With Fading Correlation

Kai Zhang and Zhisheng Niu

Abstract: **This paper studies the performance of multiuser data networks with transmit diversity under correlated fading channels. Previous work shows that correlated fading reduces the link performance of multiple antenna systems, but how correlated fading affects the throughput of multiuser data networks is still unknown since the throughput depends not only on the link performance but also on the multiuser diversity. We derive the throughput of the multiuser data networks with various transmit diversity schemes under correlated fading channels. The impact of correlated fading on the throughput is investigated. Analytical and simulation results show that, although correlated fading is harmful for link performance, it increases the throughput of the multiuser data networks if the transmit scheme is appropriately selected.**

Index Terms: **Correlated fading, multiuser diversity, transmit diversity.**

I. INTRODUCTION

Transmit diversity is one of the key technologies to combat channel fading and increase the system capacity when multiple antennas are available at the transmitter [1]. In the near future, base stations (BSs) are expected to be equipped with at least two antennas for downlink transmit diversity, which has already been incorporated in 3GPP standard [2]. Recent work [3]–[5] points out another form of diversity, known as multiuser diversity, in a wireless data network with multiple users. The diversity gain is from the fact that different users in the system usually have vastly different channel states. Overall network throughput is maximized by allocating the whole resource to the user with the best channel at any one time.

The combination of multiuser diversity and transmit diversity has attracted significant interests [6]–[10] recently. For slow fading systems, opportunistic beamforming [6] is desirable since it converts a static channel to a time-variant one. However, for fast fading Rayleigh channels, opportunistic beamforming does not provide any performance gain. In [7], the impact of space-time block coding (STBC) on multiuser diversity with greedy scheduler is discussed. The authors conclude that since STBC eliminates the probability of constructive peaks as well as the deep fades of the fading channel, a scheduler based on single-input single-output (SISO) outperforms a STBC-based scheduler. Similar results are also given by [8] and [9]. However, [10] illuminates that transmit diversity does not reduce system capacity when spatial diversity is exploited in an efficient manner, which requires at least partial channel state information (CSI) at the transmitter. On the other hand, it has recently been shown that a BS with multiple antennas can transmit to multiple users simultaneously to achieve higher rate when full CSI is available at the transmitter [11]. Dirty-paper coding (DPC) allows the BS to transmit to multiple users at the same time avoiding multiuser interference by jointly encoding the transmitted signals [12], but it is difficult to implement in practical systems due to high computational burden when the number of users is large. Therefore, this paper assumes that the BS transmits to one user at any one time.

Most previous work assumes independent fading among the transmit antennas, but antenna correlation is inevitable in practical systems. For instance, BS antennas in cellular system are placed high above the ground and close to each other. Therefore, the BS antennas are unobstructed and see no local scatterers, resulting in high correlation among the BS antennas [13]. It is well known that fading correlation reduces the link performance of multiple antenna systems [14]. However, how correlated fading affects system capacity for multiuser data networks is still unknown since network throughput depends not only on link performance, but also on multiuser diversity. Although recent work [15] has presented the concept of multiuser diversity in correlated channel, their work mostly relies on simulations and is limited to one transmit diversity scheme known as eigenbeamforming.

It is well known that multiple antenna systems provide array gain which increases the average receive signal-to-noise ratio (SNR), and diversity gain which reduces the impact of the fluctuation of fading channels [16]. Previous work has shown that with larger dynamic fluctuations of the channels, the more multiuser diversity gain can be achieved. Thus, the diversity gain of the multiple antenna systems leads to the degradation of the system capacity of multiuser data networks, while the array gain does not conflict with the multiuser diversity, since increasing the average SNR of each user also means increasing the average SNR of the multiuser data networks. Therefore, if an appropriate transmit diversity scheme keeps the array gain and only reduces the diversity gain under correlated fading channels, the system capacity may benefit from the correlated fading.

In this paper, we analyze of the impact of fading correlation on the throughput of multiuser data networks with open-loop and close-loop transmit diversity schemes. In Section II, the channel model is given and five transmit schemes are presented. After that, we derive the throughput of the multiuser data networks with different transmit schemes under correlated fading channels in Section III. We analytically evaluate the impact of

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fading correlation on the system capacity and show how to combine the multiuser diversity and transmit diversity under correlated fading channels. After that, simulation results are given in Section IV, while Section V contains our conclusions.

II. SYSTEM MODEL

A. Channel Model

Consider a downlink model of a time division multiple access (TDMA)-like cellular data network, where the BS equipped with M_t antennas sends packets to K mobile users equipped with single antenna. The channels are assumed to be quasi-static[1] flat fading and denoted by $\mathbf{H}_k(t) = (h_{1,k}(t), \cdots, h_{M_t,k}(t))$, where $h_{i,k}(t)$ is the channel gain from the ith transmit antenna to the kth user at time slot t. Given that the signal transmitted from the ith transmit antenna to the kth user at time slot t is $s_{i,k}(t)$, the transmit signal vector $\mathbf{s}_k(t)$ is $(s_{1,k}(t), \cdots, s_{M_t,k}(t))'$. We use $(\cdot)'$ and $(\cdot)^\dagger$ to denote the transpose and conjugate transpose operation respectively, and $(\cdot)^*$ to denote the conjugate of a number. Thus, the received signal of the kth user at time slot t is

$$y_k(t) = \mathbf{H}_k(t)\mathbf{s}_k(t) + n(t) \tag{1}$$

where $n(t)$ denotes the AWGN term which are modeled as independent, identically distributed (i.i.d.) zero mean complex Gaussian random variables with unitary variance. The long-term average SNR of the kth user is denoted by ρ_{k0} and $\mathrm{E}[\mathbf{s}_k^*(t)\mathbf{s}_k(t)] = \rho_{k0}$, where $\mathrm{E}[\cdot]$ is the expectation operator.

Denote the covariance between $h_{i,k}(t)$ and $h_{j,k}(t)$ by $r_k(i,j)$. Then the transmit correlation matrix of the kth user $\mathbf{R}_k$ is $(r_k(i,j))_{M_t \times M_t}$, which is assumed to be time-invariant since it varies much slower than the fading. The channel model of the kth user at time slot t is [14]

$$\mathbf{H}_k(t) = \mathbf{H}_k^{(w)}(t)\mathbf{B}_k \tag{2}$$

where the vector, $\mathbf{H}_k^{(w)}(t)$, contains i.i.d. complex Gaussian entries with zero mean and unitary variance, and the $M_t \times M_t$ matrix $\mathbf{B}_k^\dagger$ is $\mathbf{R}_k^{\frac{1}{2}}$ and satisfies $\mathbf{B}_k^\dagger \mathbf{B}_k = \mathbf{R}_k$.

B. Transmit Diversity Schemes

B.1 One Antenna Scheme

The baseline case is the SISO case, in which the BS has a single antenna. The SNR of the kth user at time slot t is

$$\rho_{\mathrm{SISO},k}(t) = \rho_{k0}|h_{1,k}(t)|^2. \tag{3}$$

The average of $\rho_{\mathrm{SISO},k}(t)$ over time is

$$\mathrm{E}[\rho_{\mathrm{SISO,k}}(\mathrm{t})] = \rho_{\mathrm{k0}}. \tag{4}$$

[1]Quasi-static means that the channel are constant over a frame length and changed independently between different frames.

B.2 STBC Scheme

STBC was proposed in [17] to exploit full diversity gain without CSI at the transmitter. The SNR of the kth user in the STBC scheme is

$$\begin{aligned}\rho_{\mathrm{STBC,k}}(t) &= \frac{\rho_{k0}}{M_t}\|\mathbf{H}_k(t)\|_F^2 \\ &= \frac{\rho_{k0}}{M_t}\mathbf{H}_k^{(w)}(t)\mathbf{B}_k\mathbf{B}_k^\dagger\mathbf{H}_k^{(w)\dagger}(t)\end{aligned} \tag{5}$$

where $\|\mathbf{H}_k(t)\|_F$ is the Frobenius norm of $\mathbf{H}_k(t)$. Since $\mathbf{B}_k\mathbf{B}_k^\dagger$ is non-negative definite Hermitian, it can be decomposed into

$$\mathbf{B}_k\mathbf{B}_k^\dagger = \mathbf{U}_k\mathbf{D}_k\mathbf{U}_k^\dagger \tag{6}$$

where $\mathbf{U}_k$ is a unitary matrix and $\mathbf{D}_k$ is a diagonal matrix with the diagonal elements, $d_{i,k}$ $(1 \le i \le M_t)$, which are not only the eigenvalues of $\mathbf{B}_k\mathbf{B}_k^\dagger$ but also the eigenvalues of $\mathbf{R}_k$ (Theorem 1.3.20 [18]). Without loss of generality, we assume that $d_{1,k} \ge d_{2,k} \ge \cdots \ge d_{M_t,k} \ge 0$.

Since $\mathbf{U}_k$ is a unitary matrix, $\mathbf{H}_k^{(w)}(t)\mathbf{U}_k$ still contains i.i.d. complex Gaussian entries with zero mean and unitary variance (Lemma 5, [19]). Representing $\mathbf{H}_k^{(w)}(t)\mathbf{U}_k$ by $\tilde{\mathbf{H}}_k^{(w)}(t)$, we have

$$\begin{aligned}\rho_{\mathrm{STBC},k}(t) &= \frac{\rho_{k0}}{M_t}\tilde{\mathbf{H}}_k^{(w)}(t)\mathbf{D}_k\tilde{\mathbf{H}}_k^{(w)\dagger}(t) \\ &= \frac{\rho_{k0}}{M_t}\sum_{i=1}^{M_t} d_{i,k}|\tilde{h}_{i,k}^{(w)}(t)|^2\end{aligned} \tag{7}$$

where $\tilde{h}_{i,k}^{(w)}(t)$ is the ith element of $\tilde{\mathbf{H}}_k^{(w)}(t)$. Denoting $\mathrm{Tr}(\cdot)$ as the trace of a matrix, we get the average of $\rho_{\mathrm{STBC},k}(t)$ over time as

$$\begin{aligned}\mathrm{E}[\rho_{\mathrm{STBC,k}}(\mathrm{t})] &= \frac{\rho_{k0}}{M_t}\sum_{i=1}^{M_t} d_{i,k} \\ &= \frac{\rho_{k0}}{M_t}\mathrm{Tr}(\mathbf{R}_\mathrm{k}) \\ &= \rho_{k0}.\end{aligned} \tag{8}$$

B.3 Optimal Transmit Beamforming With Perfect CSI Feedback

This scheme assumes that the BS has perfect knowledge of the CSI for all the users. Given perfect CSI at the transmitter, it is well known that the SNR of the kth user is maximized by optimal transmit beamforming (TB), in which the transmit signal is transmitted via all the antennas and weighted by $h_{i,k}^*(t)/\|\mathbf{H}_k(t)\|_F$ for the ith antenna. The SNR of the kth user at time slot t is

$$\rho_{\mathrm{TB},k}(t) = \rho_{k0}\|\mathbf{H}_k(t)\|_F^2, \tag{9}$$

which is rewritten as

$$\rho_{\mathrm{TB},k}(t) = \rho_{k0}\sum_{i=1}^{M_t} d_{i,k}|\tilde{h}_{i,k}^{(w)}(t)|^2 \tag{10}$$

with similar notations for the STBC scheme. Therefore, the average of $\rho_{\mathrm{TB,k}}(t)$ over time is

$$\mathrm{E}[\rho_{\mathrm{TB},k}(t)] = M_t\rho_{k0}. \tag{11}$$

B.4 Selected Transmit Diversity With Partial CSI Feedback

The optimal TB scheme requires perfect CSI feedback, which is infeasible in practical systems due to large feedback bandwidth requirement[2]. Selected transmit diversity (STD) scheme [1] lets each user inform the BS about which antenna has the largest gain for each time slot. Then the BS transmits a signal only via the antenna with the largest gain. The SNR of the kth user at time slot t is given by

$$\rho_{\mathrm{STD},k}(t) = \rho_{k0} \max_{1\leq i\leq M_t} |h_{i,k}(t)|^2. \tag{12}$$

Actually, the system with STD scheme is equivalent to the SISO system with $M_t K$ users. However, in the highly correlated fading case, the gains of different antennas tend to be the same. Therefore, selecting antennas does not obtain performance enhancement. Obviously, the STD scheme is not good for the multiuser system under correlated fading channels.

B.5 Eignenbeamforming With Long-Term CSI Feedback

The other transmit diversity scheme with low feedback channel bandwidth requirement is the eigenbeamforming (EBF) scheme [20], which requires the slow-varying transmit antenna covariance matrix of each user. The signal for the kth user is transmitted via all of the transmit antennas and weighted by the dominant eigenvector corresponding to the largest eigenvalue of $\mathbf{R}_k$. The EBF scheme maximizes the average SNR at the receiver. Given that the dominant eigenvector is $\mathbf{W}_k$, the SNR of the kth user at time slot t is

$$\begin{aligned}\rho_{\mathrm{EBF},k}(t) &= \rho_{k0}|\mathbf{H}_k(t)\mathbf{W}_k|^2\\ &= \rho_{k0}\mathbf{H}_k^{(w)}(t)\mathbf{B}_k\mathbf{W}_k\mathbf{W}_k^\dagger\mathbf{B}_k^\dagger\mathbf{H}_k^{(w)\dagger}(t).\end{aligned} \tag{13}$$

Since $\mathbf{B}_k\mathbf{W}_k\mathbf{W}_k^\dagger\mathbf{B}_k^\dagger$ is Hermitian, it can be decomposed as

$$\mathbf{B}_k\mathbf{W}_k\mathbf{W}_k^\dagger\mathbf{B}_k^\dagger = \mathbf{V}_k\mathbf{\Lambda}_k\mathbf{V}_k^\dagger \tag{14}$$

where $\mathbf{V}_k$ is a unitary matrix and $\mathbf{\Lambda}_k$ is a diagonal matrix. Moreover, the rank of $\mathbf{B}_k\mathbf{W}_k\mathbf{W}_k^\dagger\mathbf{B}_k^\dagger$ is 1 and the only nonzero eigenvalue is $d_{1,k}$, which corresponds to the eigenvector $\mathbf{B}_k\mathbf{W}_k$. Denoting $\mathbf{H}_k^{(w)}(t)\mathbf{V}_k$, which contains i.i.d. complex Gaussian entries with zero mean and unitary variance, by $\hat{\mathbf{H}}_k^{(w)}(t)$, we have

$$\begin{aligned}\rho_{\mathrm{EBF},k}(t) &= \rho_{k0}\hat{\mathbf{H}}_k^{(w)}(t)\mathbf{\Lambda}_k\hat{\mathbf{H}}_k^{(w)\dagger}(t)\\ &= \rho_{k0}d_{1,k}|\hat{h}_{1,k}^{(w)}(t)|^2\end{aligned} \tag{15}$$

where $\hat{h}_{1,k}^{(w)}(t)$ represents the first element of $\hat{\mathbf{H}}_k^{(w)}(t)$. The average of $\rho_{\mathrm{EBF},k}(t)$ over time is given by

$$\mathrm{E}[\rho_{\mathrm{EBF},k}(t)] = \rho_{k0}d_{1,k}. \tag{16}$$

[2]Most of the practical systems are frequency division duplex (FDD) and use feedback channel to provide CSI at the transmitter.

III. ANALYSIS OF NETWORK THROUGHPUT

In this section, we derive the throughput of the multiuser data networks with the transmit diversity schemes of Section II and a fair scheduler [21]. We also evaluate the impact of fading correlation on the system capacity.

A. Throughput of Multiuser Data Networks

We assume that in each time slot t all users track their CSI and perfectly feed back the received SNR and some other CSI, which depends on the particular transmit diversity scheme being used. The BS selects the user $\tilde{k}$ to send data in the next slot according to the following fair scheduling rule

$$\tilde{k} = \arg\max_k \frac{\rho_k(t)}{\bar{\rho}_k} \tag{17}$$

where $\rho_k(t)$ is the SNR of the kth user in time slot t and $\bar{\rho}_k$ is the long-term average SNR of the kth user. Let $\alpha_k(t)$ be $\rho_k(t)/\bar{\rho}_k$, which is the instantaneous small-scale-fading channel gain.

Since the users are scheduled according to $\alpha_k(t)$, the SNR of the selected user in time slot t, $\tilde{\rho}(t)$, can be expressed as a multiplication process of two independent random variables of the long-term average SNR of the scheduled user, $\tilde{\bar{\rho}}(t)$, and the instantaneous small-scale-fading channel gain of the scheduled user, $\tilde{\alpha}(t)$, as follows

$$\tilde{\rho}(t) = \tilde{\bar{\rho}}(t)\tilde{\alpha}(t). \tag{18}$$

On the assumption of equal access time for each user, the probability density function (PDF) of $\tilde{\bar{\rho}}(t)$ is [21]

$$f_{\tilde{\bar{\rho}}}(x) = \frac{1}{K}\sum_{k=1}^{K}\delta(x-\bar{\rho}_k) \tag{19}$$

where $\delta(x)$ is Dirac delta function. The cumulative distribution function (CDF) of $\tilde{\alpha}(t)$ can be computed on the assumption of independent distribution of all $\alpha_k(t)$ [22] as

$$F_{\tilde{\alpha}}(x) = \prod_{k=1}^{K} F_{\alpha_k}(x) \tag{20}$$

where $F_{\alpha_k}(x)$ is the CDF of $\alpha_k(t)$. Then, the PDF the $\tilde{\alpha}(t)$ is

$$f_{\tilde{\alpha}}(x) = \frac{dF_{\tilde{\alpha}}(x)}{dx}. \tag{21}$$

Thus, the PDF of $\tilde{\rho}(t)$ is

$$\begin{aligned}f_{\tilde{\rho}}(x) &= \int_{-\infty}^{\infty}\frac{1}{|u|}f_{\tilde{\bar{\rho}}}(u)f_{\tilde{\alpha}}\left(\frac{x}{u}\right)dx\\ &= \frac{1}{K}\sum_{k=1}^{K}\frac{1}{\bar{\rho}_k}f_{\tilde{\alpha}}\left(\frac{x}{\bar{\rho}_k}\right).\end{aligned} \tag{22}$$

The network throughput is then given by

$$C = \int_0^{\infty}\log_2(1+x)f_{\tilde{\rho}}(x)\,dx. \tag{23}$$

For particular transmit diversity schemes, the $\bar{\rho}_k$'s are

$$\bar{\rho}_{k,\mathrm{SISO}} = \rho_{k0}, \tag{24}$$
$$\bar{\rho}_{k,\mathrm{STBC}} = \rho_{k0}, \tag{25}$$
$$\bar{\rho}_{k,\mathrm{TB}} = M_t\rho_{k0}, \tag{26}$$
$$\bar{\rho}_{k,\mathrm{EBF}} = d_{1,k}\rho_{k0}. \tag{27}$$

Note that we do not analyze the system with the STD scheme since it is obviously not good under correlated fading channels as mentioned above. From (3), (7), (10), and (15), the $\alpha_k(t)$ of the transmit diversity schemes are

$$\alpha_{k,\mathrm{SISO}}(t) = |h_{1,k}(t)|^2, \tag{28}$$
$$\alpha_{k,\mathrm{STBC}}(t) = \frac{1}{M_t}\sum_{i=1}^{M_t} d_{i,k}|\tilde{h}_{i,k}^{(w)}(t)|^2, \tag{29}$$
$$\alpha_{k,\mathrm{TB}}(t) = \frac{1}{M_t}\sum_{i=1}^{M_t} d_{i,k}|\tilde{h}_{i,k}^{(w)}(t)|^2, \tag{30}$$
$$\alpha_{k,\mathrm{EBF}}(t) = |\hat{h}_{1,k}^{(w)}(t)|^2. \tag{31}$$

Thus, the CDFs of $\alpha_{k,SISO}(t)$ and $\alpha_{k,EBF}(t)$ are

$$F_{\alpha_{k,\mathrm{SISO}}}(x) = F_{\alpha_{k,EBF}}(x) = 1 - e^{-x},\ (x \geq 0). \tag{32}$$

The distributions of $\alpha_{k,\mathrm{STBC}}(t)$ and $\alpha_{k,\mathrm{TB}}(t)$ are given by generalized Erlang distribution, which is a kind of phase-type (PH) distribution [23]. Let

$$\mathbf{e} = (\underbrace{1, 1, \cdots, 1}_{M_t})', \tag{33}$$

$$\beta = (\underbrace{1, 0, \cdots, 0}_{M_t}), \tag{34}$$

and

$$\mathbf{T}_k = \begin{pmatrix} -\frac{1}{d_{1,k}} & \frac{1}{d_{1,k}} & & & \mathbf{0} \\ & -\frac{1}{d_{2,k}} & \frac{1}{d_{2,k}} & & \\ & & \ddots & \ddots & \\ & & & -\frac{1}{d_{M_t-1,k}} & \frac{1}{d_{M_t-1,k}} \\ \mathbf{0} & & & & -\frac{1}{d_{M_t,k}} \end{pmatrix}. \tag{35}$$

The CDFs of $\alpha_{k,\mathrm{STBC}}(t)$ and $\alpha_{k,\mathrm{TB}}(t)$ are

$$F_{\alpha_{k,\mathrm{STBC}}}(x) = F_{\alpha_{k,\mathrm{TB}}}(x) = 1 - \beta \exp(\mathbf{T}_k x)\,\mathbf{e},\ (x \geq 0) \tag{36}$$

where $\exp(\mathbf{T}_k x)$ is $\sum_{n=1}^{\infty} (\mathbf{T}_k x)^n / n!$. Moreover, the PDFs of $\alpha_{k,\mathrm{STBC}}(t)$ and $\alpha_{k,\mathrm{TB}}(t)$ are

$$f_{\alpha_{k,\mathrm{STBC}}}(x) = f_{\alpha_{k,\mathrm{TB}}}(x) = \beta \exp(\mathbf{T}_k x)(-\mathbf{T}_k\mathbf{e}),\ (x \geq 0). \tag{37}$$

Then, by (19)–(23) we obtain the network throughput with the transmit diversity schemes under correlated fading channels.

B. Impact of Fading Correlation on System Capacity

In order to directly show the impact of fading correlation, we simplify the transmit correlation matrix of all the users as

$$\mathbf{R}_t = \begin{pmatrix} 1 & r & \cdots & r^{(M_t-1)} \\ r & 1 & \cdots & r^{(M_t-2)} \\ \vdots & \vdots & \ddots & \vdots \\ r^{(M_t-1)} & r^{(M_t-2)} & \cdots & 1 \end{pmatrix} \tag{38}$$

where r $(0 \leq r \leq 1)$ is the correlation coefficient. We denote the eigenvalues of $\mathbf{R}_t$ by $d_1, d_2, \cdots, d_{M_t}$ and assume $(d_1 \geq d_2 \geq \cdots \geq d_{M_t} \geq 0)$ without loss of generality. Thus, by (19)-(23), we obtain the PDFs of the $\tilde{\alpha}(t)$'s with the transmit diversity schemes as

$$f_{\tilde{\alpha},\mathrm{SISO}}(x) = f_{\tilde{\alpha},\mathrm{EBF}}(x) = Ke^{-x}\left(1 - e^{-x}\right)^{(K-1)} \tag{39}$$

and

$$f_{\tilde{\alpha},\mathrm{TB}}(x) = f_{\tilde{\alpha},\mathrm{STBC}}(x) = K\beta \exp(\mathbf{T}x)(-\mathbf{T}\mathbf{e})\left\{1 - \beta\exp(\mathbf{T}x)\,\mathbf{e}\right\}^{(K-1)} \tag{40}$$

where

$$\mathbf{T} = \begin{pmatrix} -\frac{1}{d_1} & \frac{1}{d_1} & & & \mathbf{0} \\ & -\frac{1}{d_2} & \frac{1}{d_2} & & \\ & & \ddots & \ddots & \\ & & & -\frac{1}{d_{M_t-1}} & \frac{1}{d_{M_t-1}} \\ \mathbf{0} & & & & -\frac{1}{d_{M_t}} \end{pmatrix}. \tag{41}$$

Moreover, the PDFs of $\tilde{\rho}(t)$'s with the transmit diversity schemes are

$$f_{\tilde{\rho},\mathrm{SISO}}(x) = \frac{1}{K}\sum_{k=1}^{K}\frac{1}{\rho_{k0}} f_{\tilde{\alpha},\mathrm{SISO}}\left(\frac{x}{\rho_{k0}}\right), \tag{42}$$
$$f_{\tilde{\rho},\mathrm{STBC}}(x) = \frac{1}{K}\sum_{k=1}^{K}\frac{1}{\rho_{k0}} f_{\tilde{\alpha},\mathrm{STBC}}\left(\frac{x}{\rho_{k0}}\right), \tag{43}$$
$$f_{\tilde{\rho},\mathrm{TB}}(x) = \frac{1}{K}\sum_{k=1}^{K}\frac{1}{M_t\rho_{k0}} f_{\tilde{\alpha},\mathrm{TB}}\left(\frac{x}{M_t\rho_{k0}}\right), \tag{44}$$
$$f_{\tilde{\rho},\mathrm{EBF}}(x) = \frac{1}{K}\sum_{k=1}^{K}\frac{1}{d_1\rho_{k0}} f_{\tilde{\alpha},\mathrm{EBF}}\left(\frac{x}{d_1\rho_{k0}}\right). \tag{45}$$

Therefore, by (23), (42)–(45) we obtain the network throughput in this case. The network throughput with the transmit diversity schemes are shown in (46)–(49) in the top of next page, where $f_{\tilde{\alpha},\mathrm{SISO}}(x)$, $f_{\tilde{\alpha},\mathrm{STBC}}(x)$, $f_{\tilde{\alpha},\mathrm{TB}}(x)$, and $f_{\tilde{\alpha},\mathrm{EBF}}(x)$ are shown in (39) and (40).

Assuming $\rho_{k0} = 0$ dB $(k = 1, 2, \cdots, K)$ and $M_t = 2$, we plot the network throughput with $K = 1$, i.e., the network throughput is equal to the link performance, in Fig. 1. Since the fading correlation decreases the diversity gain of the TB and STBC schemes, the link performance dereases as r increases. On the other hand, as r increases the EBF scheme achieves more

$$C_{\rm SISO} = \int_0^\infty \log(1+x)\frac{1}{K}\sum_{k=1}^{K}\frac{1}{\rho_{k0}}f_{\tilde{\alpha},{\rm SISO}}\left(\frac{x}{\rho_{k0}}\right)dx, \tag{46}$$

$$C_{\rm STBC}(x) = \int_0^\infty \log(1+x)\frac{1}{K}\sum_{k=1}^{K}\frac{1}{\rho_{k0}}f_{\tilde{\alpha},{\rm STBC}}\left(\frac{x}{\rho_{k0}}\right)dx, \tag{47}$$

$$C_{\rm TB}(x) = \int_0^\infty \log(1+x)\frac{1}{K}\sum_{k=1}^{K}\frac{1}{M_t\rho_{k0}}f_{\tilde{\alpha},{\rm TB}}\left(\frac{x}{M_t\rho_{k0}}\right)dx, \tag{48}$$

$$C_{\rm EBF}(x) = \int_0^\infty \log(1+x)\frac{1}{K}\sum_{k=1}^{K}\frac{1}{d_1\rho_{k0}}f_{\tilde{\alpha},{\rm EBF}}\left(\frac{x}{d_1\rho_{k0}}\right)dx \tag{49}$$

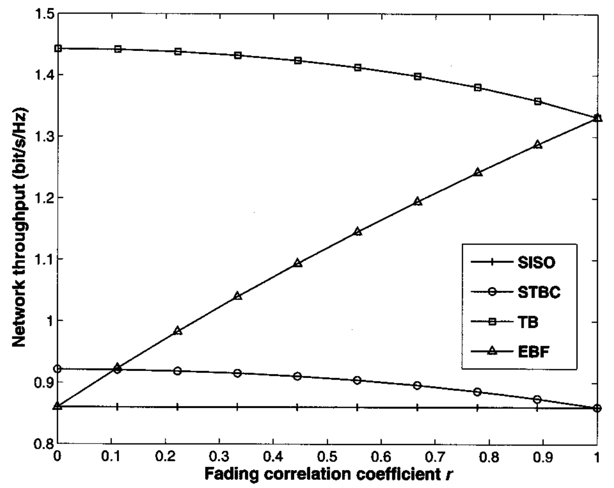

Fig. 1. System capacity under different fading correlation coefficient with single user.

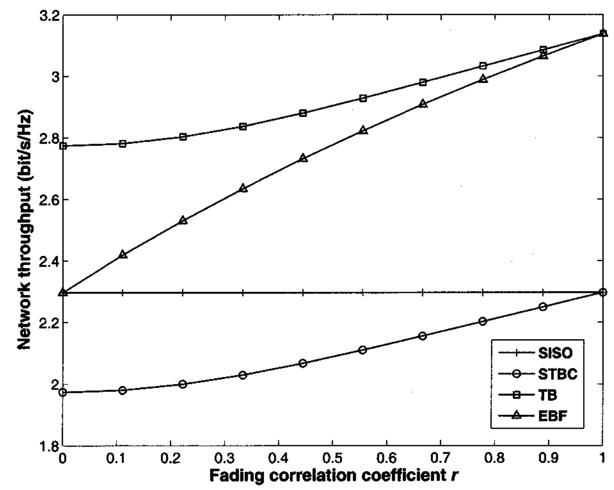

Fig. 2. System capacity under different fading correlation coefficient with 32 users.

array gain leading to better link performance. Fig. 2 plots the network throughput with $K = 32$. The results illustrate that the fading correlation improves the performance of the multiuser networks with the transmit diversity schemes. Furthermore, the EBF scheme with long-term CSI feedback achieves a performance close to that of the optimal TB scheme when r is high, e.g., $r \geq 0.7$.

Fig. 3 plots the network throughput versus the number of users under low and high correlation, i.e., $r = 0.1$ and $r = 0.7$. Firstly, the results show that the network throughput increases as the number of users increases. Secondly, consistent with previous observation, fading correlation improves the network throughput with transmit diversity schemes. Furthermore, the network throughput of STBC is worse than that of SISO, while the network throughput of EBF is close to that of TB under high correlation.

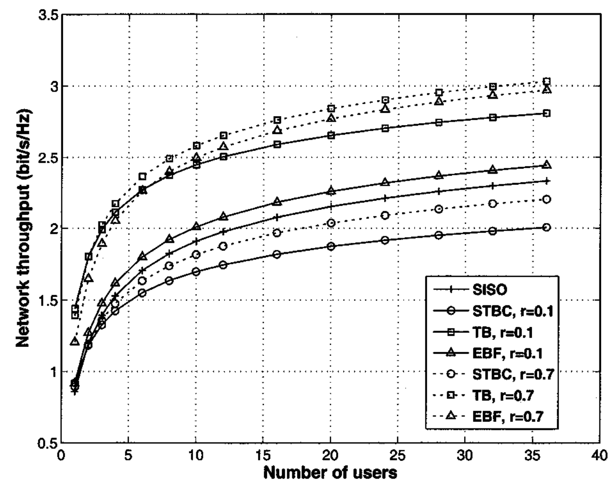

Fig. 3. System capacity vs. number of users under different fading correlation.

IV. SIMULATION RESULTS

In this section, we evaluate the system capacity of the transmit diversity schemes under correlated fading channels by simulations. In every time slot (1.67 ms), the users send back some CSI to the BS which in turn selects one of the users to receive data in the next time slot by proportional fair scheduling [5]. Assuming the supported data rate of the kth user at time slot t is $C_k(t)$, the BS selects the user $\tilde{k}$ with the largest $C_k(t)/T_k(t)$, where the average throughput of the kth user $T_k(t)$ is updated

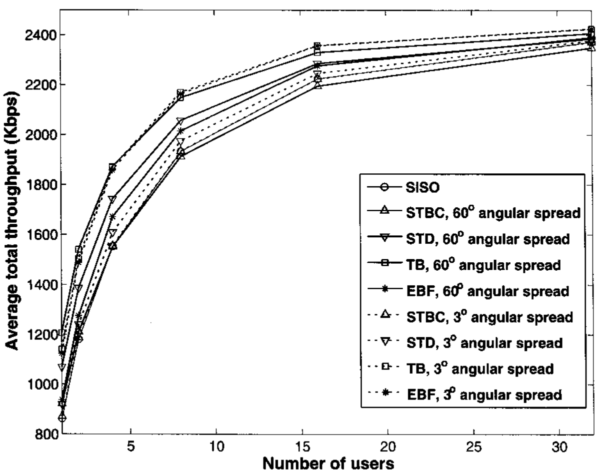

Fig. 4. Average throughput of the transmit diversity schemes with 2 λ space between BS antennas with the angular spread 3° and 60° for 50 Hz Doppler frequency.

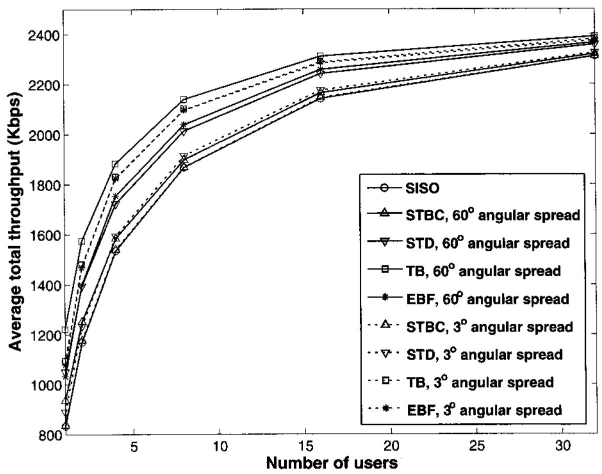

Fig. 5. Average throughput of the transmit diversity schemes with 2 λ space between BS antennas with the angular spread 3° and 60° for 1 Hz Doppler frequency.

by

$$T_k(t+1) = \begin{cases} \left(1 - \frac{1}{t_c}\right) T_k(t) + \frac{1}{t_c} C_k(t), & k = \tilde{k}, \\ \left(1 - \frac{1}{t_c}\right) T_k(t), & k \neq \tilde{k}. \end{cases} \quad (50)$$

t_c is constrained by the maximum delay tolerance and set to 3000 slots (5 seconds) in the simulation.

Path loss and Rayleigh fading are included and the Jakes model [24] is used to simulate the time-varying channels. The "one-ring" antenna correlation model [14] is used to model the situation where the BS is elevated and seldom obstructed. The users are distributed uniformly around the BS, which is equipped with 2 antennas. The distance between the BS and the users is from 100 m to 3000 m and the path loss exponent is 2. Once a random location is determined for the user as described above, the user's location and transmit correlation matrix is unchanged for 10 seconds (equivalent of 6000 time slots). After 6000 time slots, a new location is determined according to the uniform distribution. A total of 50 locations are generated for each user. The mean SNR of the users is assumed to be 0 dB. The transmission data rates used in the simulations are the same as that specified in Table 2 of [5].

In Figs. 4 and 5, the average throughput of the transmit diversity schemes with 2 wavelength (λ) space between the BS antennas with 50 Hz and 1 Hz Doppler frequency are plotted, respectively. When the Doppler frequency is 50 Hz, the channel coherent time is much less than t_c and the channels are fast fading. Thus, the simulation results in Fig. 4 are similar as the analytical results. Although multiple antennas are used at the BS, the throughput of the STBC scheme is less than the throughput of the SISO scheme because the CSI is not exploited at the BS, while the transmit schemes with partial or full CSI feedback can improve the performance to a greater extend. As the angular spread varies from 60° to 3°, i.e., from low fading correlation to high fading correlation, the throughput of the TB and STBC schemes decrease when the number of simultaneous users K is less than 4, but the throughput increases when K is more than 4. Moreover, the throughput of the EBF scheme is improved greatly as the fading correlation increases and is close to the optimal TB scheme in the 3° angular spread case. In comparison to the SISO scheme, the STD scheme achieves performance enhancement, but the enhancement decreases as the fading correlation increases. When there is only 1 user in the multiuser network, the system is equivalent to a point to point link. In this case, the performance of the system reduced as the correlation increases. The optimal TB scheme achieves the best performance in all the cases.

When the Doppler frequency is 1 Hz, t_c is not much larger than the channels coherent time, i.e., the channels vary slowly, making the proportional fair scheduling can not select the user with the best channel due to the delay constraint. As a result, the multiuser diversity can not be fully exploited and the system throughput decreases. In Fig. 5, the throughput of all the schemes is less than the throughput in Fig. 4. Moreover, the system throughput is affected more by the link performance of the transmit schemes. Thus, the throughput of the TB and STBC schemes decreases as the fading correlation increases. And the throughput of the STBC scheme in the low correlation case is even better than the SISO scheme. However, the EBF scheme still has great performance improvement than the SISO scheme in this case.

V. CONCLUSION

We have evaluated the performance of different transmit diversity schemes under correlated fading channels in this paper. Although correlated fading is harmful for link performance, it increases the average throughput of the multiuser networks with TB, STBC, and EBF schemes, which has been validated by the analytical and simulation results. Moreover, the EBF scheme has been shown to be an efficient scheme to combine multiuser diversity and multiple antennas with only long-term CSI feedback, which reduces the bandwidth requirement of the feedback channel. Therefore, in order to improve the throughput of the multiuser data networks with multiple antennas, we can use the EBF transmit scheme and reduce the BS antennas space.

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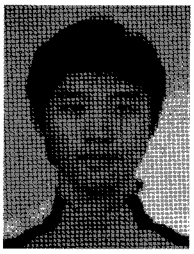

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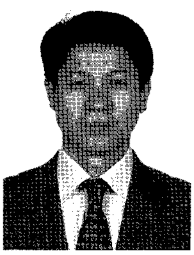

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