Low-Complexity Network Coding Algorithms for Energy Efficient Information Exchange

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Abstract: **The use of network coding in wireless networks has been proposed in the literature for energy efficient broadcast. However, the decoding complexity of existing algorithms is too high for low-complexity devices. In this work we formalize the all-to-all information exchange problem and shows how to optimize the transmission scheme in terms of energy efficiency. Furthermore, we prove by construction that there exists $O(1)$-complexity network coding algorithms for grid networks which can achieve such optimality. We also present low-complexity heuristics for random-topology networks. Simulation results show that network coding algorithms outperforms forwarding algorithms in most cases.**

Index Terms: **Broadcast, information exchange, low-complexity algorithms, network coding, wireless network.**

I. INTRODUCTION

In the area of wireless networks, energy efficiency becomes very important due to the energy constraints on mobile battery-powered devices. In certain situations, e.g., wireless sensor networks, the batteries in these devices are not easily recharged or replenished because of the low cost of device and the difficulty of maintenance. As pointed out in [1]–[4], radio communication accounts for a large portion of the total power consumption in wireless *ad hoc* and sensor networks. Thus, reducing unnecessary radio transmissions could be an effective approach to achieve energy efficiency in wireless networks. In this work, we study the all-to-all information exchange problem in wireless networks. More specifically, in a wireless network where each node can only talk to its direct neighbors, we are interested in how to exchange information among all nodes in the network with minimum number of transmissions.

Traditionally this kind of information exchange is done by forwarding the packet hop by hop. Due to the broadcast nature of wireless networks, the transmission of a source node is naturally overheard by its neighbors, hence theoretically each transmission can bring one unit of new information (or one packet) to all in-range recipients. However, in the scenario of forwarding if a node is broadcasting a neighbor's packet, the packet is already known to that neighbor, so the transmission is useless to that specific neighbor. In other words, the wireless channel is not fully utilized when broadcast is done by forwarding. In [5], Widmer *et al.* proposed a network coding approach to improve on this, and the same approach was later adopted in [6] and [7] for further investigations on energy-efficient broadcast. Please note the term *broadcast* used in these works refers to both network-wise information broadcast and physical layer broadcast. To avoid confusion, *broadcast* in this paper refers to physical layer broadcast only, and we use *all-to-all information exchange* to denote the problem of network-wise information broadcast for all nodes.

Although network coding shows promising results in [5]–[7], the decoding complexity on the node was not much considered. We believe that in wireless networks especially sensor networks, requirements for low complexity are also very important due to the limited computation capability of the nodes. In this paper we focus on low-complexity network coding algorithms for the all-to-all information exchange problem. We first examine grid networks and prove that the wireless channel can be fully utilized by a network coding algorithm with $O(1)$ decoding complexity (against $O(n^3)$-complexity algorithms in the literature). Furthermore, in the constructive proof, we provide a practical implementation of such an algorithm for grid networks. Then, we extend our algorithm to random-topology networks. Developing low-complexity network coding algorithms for random-topology networks is a non-trivial task. In this paper, we propose network coding heuristics that balance between decoding complexity and the coding gain. The performance of our algorithms for random-topology networks is verified by simulation experiments.

The rest of this paper is organized as follows. Section II provides a brief overview of some related research work in this area. In Section III, we formalize the problem and show how network coding can help on this matter. The existence of $O(1)$-complexity algorithms in grid networks is proved in Section IV with implementation details. Section V develops low-complexity heuristics for random-topology networks accompanied with simulation results and Section VI concludes the paper.

II. BACKGROUND AND RELATED WORK

Before network coding was introduced into wireless networks, energy efficiency of wireless communication was mainly investigated in the domain of transmission range related optimisation. Such optimization was based on the assumption that the transmission power needed for the effective reception at distance r is proportional to r^α ($2 \leq \alpha \leq 4$) [8]. In [9], Kirousis *et al.* studied the problem of finding minimum cost transmission range assignment for strongly connected packet radio networks. The authors gave a tight asymptotic bound for the minimum cost of a range assignment in the case of one-dimensional unit chain, and proved the case of three-dimensional Euclidean space is NP-hard. Energy-efficient broadcast in wireless networks is also studied in [8], where the authors proposed several centralized algorithms for constructing minimum-energy broadcast trees based on the aforementioned transmission power/range assumption. However, the approaches used in these works may not be suitable for use in wireless networks with low-complexity de-

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vices, e.g., sensor nodes, because those nodes may not be able to vary their transmission power, and distributed algorithms are required instead of centralized ones. We also notice that early research on energy efficient broadcast was mainly focused on the optimization of transmission power, leaving the problem of reducing the total number of transmissions needed for wireless broadcast almost untouched. This is a research direction where the latest in-network processing techniques like network coding could possibly fit in.

Network coding was first proposed in [10] by Ahlswede *et al.* and quickly attracted much research interest in communication networks. By combining packets from different incoming data streams on intermediate nodes, network coding allows the communication to achieve better throughput [10]. Li *et al.* showed that linear codes suffices to achieve the multicast capacity [11]. Ho *et al.* then proposed a randomized approach for selecting linear codes with bounded probability [12], and Jaggi *et al.* gave polynomial time algorithms for multicast code construction [13]. Although these early works were mainly focused on multicast, network coding is quickly adopted in wireless networks where broadcast is more widely used. A practical architecture called *COPE* was proposed in [14] for wireless mesh networks to integrate network coding into the existing network stack. By utilizing XOR operations, the throughput gains of COPE vary from a few percent to several folds depending on different network configurations. Another XOR-based coding scheme named *Growth Codes* was proposed in [15] to maximize sensor network data persistence. Generally XOR-based coding schemes can provide improved performance without complicated decoding processes. However, they do not fully exploit the benefit of network coding. A simple example is that two source packets cannot be decoded by two XOR-ed packets (they are actually the same), while they can be decoded by two linear combinations if the coding vectors are linearly independent.

A network coding approach for energy-efficient broadcast using linear combinations was proposed in [5], in which Widmer *et al.* presented network coding algorithms for circular, grid and random-topology networks. Then, an improved version of the same approach with a more rigid proof was presented in [6]. The results of this work was extended in [7] from wrap-around grid to non-wrap-around grid. We notice that in all these works the encoding process generates linear combinations from all existing source packets on the node which means the decoding matrix could be as large as $n \times n$ (n is the number of nodes in the network). Similar to [6], in this paper we define the decoding complexity of a network coding algorithm as the number of operations needed to solve the largest set of linear equations by performing Gaussian elimination. Then, most algorithms in [5]–[7] have the decoding complexity of $O(n^3)$, except that algorithms for circular/linear networks are of $O(1)$ complexity and algorithms for grid networks in [5] are of $O(n^{\frac{3}{2}})$ complexity. Obviously such decoding complexity is inappropriate for some devices, e.g., sensor nodes in a high-density sensor network where the number of nodes could be very high. In this paper, we propose $O(1)$-complexity network coding algorithms for grid networks and random-topology networks, which we believe are suitable for low-complexity devices even in large-scale networks. The idea in this paper was also inspired by the algorithms proposed in [16], where low-complexity algorithms were used to solve the two-hop local information exchange problem in sensor networks.

III. PROBLEM FORMULATION

In this section we formalize the all-to-all information exchange problem and show that how network coding can help on this matter. Consider a wireless *ad hoc* network with n nodes (node id from 1 to n) and the same transmission range for all nodes. Such a network can also be modeled as an undirected graph $G = (V, E)$ with $|V| = n$ vertices and $|E|$ edges. The graph density is defined as $\rho = 2|E|/(n(n-1))$. The degree d of a node is the number of its neighbors. The degree D of a network is the maximum degree of the nodes in the network. Each node can broadcast one unit of information to all its neighbors with one transmission. Furthermore, the operations of the network are divided into rounds and in every round each node will be scheduled for one transmission. Here the order of the transmissions in the same round from different nodes does not matter because in each round nodes will only make use of the packets received in previous rounds. In other words, the operation of the network is fully distributed and parallel.

Now we consider the all-to-all information exchange scenario in which every node intends to deliver its own information unit to all other nodes. As energy efficiency or reducing unnecessary transmissions is our main concern, in this paper we are interested in how to minimize the number of transmissions needed for such all-to-all information exchange. Since each round consists of a fixed number of n transmissions, this is equal to minimizing the number of transmission rounds needed.

Here we assume there is no or little correlation among the information from different nodes, so data compressing techniques are not considered. Then, the all-to-all information exchange needs to deliver a total of $n(n-1)$ new information units ($(n-1)$ new information units to each of the n nodes). Next we define the *optimal* transmission scheme as following:

Definition 1: A transmission scheme that delivers m new information units with R rounds is *optimal* if no other transmission schemes can deliver the same amount of new information units in less than R rounds.

The above definition indicates that an *optimal* transmission scheme achieves energy efficiency in terms of transmission rounds. So our task is to find such an optimal transmission scheme for the all-to-all information exchange problem. Regarding the optimality of the transmission scheme, we have the following theorem:

Theorem 1: For a network with the degree D, if a transmission scheme averagely delivers DR new information units per node in R rounds, it is an *optimal* transmission scheme.

Proof: We prove Theorem 1 by contradiction. Suppose there is a transmission scheme that delivers DR new information units per node in R' rounds ($R' < R$). Let u_k be the total number of new information units that node n_k brings to all its neighbors in transmission k. As a transmission of node i can at most bring a total of d_i new information units to all its neighbors (d_i is the degree of node i), we have

$$u_k \leq d_{n_k} \leq D$$

$$\frac{1}{n}\sum_{k=1}^{nR'} u_k \leq DR' < DR. \quad (1)$$

The above formula (1) means less than DR new information units can be delivered per node in R' rounds, which leads to contradiction. □

Theorem 1 indicates a possible approach to an optimal transmission scheme: Just ensure each transmission averagely delivers D new information units. If a node uses a flooding approach, i.e., forwarding the information it receives to its neighbors by broadcasting, even when the transmitting node has the degree of D, this condition cannot always be satisfied. When a node is forwarding a neighbor's information, that piece of information is already known to that neighbor and thus the total number of new information units delivered in that transmission is less than D. However, if a node uses a network coding approach, i.e., broadcast a linear combination of the packets it receives to its neighbors, it is possible for each transmission to always bring some new information to all neighbors with a small packet overhead.

To implement this idea, a straightforward network coding algorithm for wrap-around grid networks was proposed in [6]. The algorithm operates in iterations and for each iteration every node transmits a linear combination of the source packets that belongs in the span of the previously received coding vectors. The authors of [6] also proved that for each iteration four transmissions from a node's four neighbors increase the size of the node's vector space by four. Considering each node is a neighbor of four nodes, we can say each transmission in this algorithm delivers D new information units (here $D = 4$) because each iteration increases the size of each node's vector space by four which is equal to bringing four units of new information when source packets are decoded. This result was extended to non-wrap-around grid networks using the same algorithm in [7].

However, although network coding is possible to make an *optimal* transmission scheme, it comes with the price of encoding and decoding. While usually the encoding process just randomly forms a linear combination of source packets over a finite field, the decoding process is much more complicated. Because the algorithms in [6] and [7] simply form linear combinations of all source packets, the ease of encoding adds the complexity on the decoder. As pointed out in [6], decoding n linearly independent equations by Gaussian elimination has the complexity of $O(n^3)$. Reducing decoding complexity was considered in [6] by choosing "sparse" linear combinations, but the resulted decoding complexity is still as high as $O(n^2\log(n))$. A scheduler for wrap-around grid networks was proposed in [5] by forming linear combinations of the newly decoded information units from sources at the same distance. Thus, the size of linearly independent equations is reduced to $O(\sqrt{n})$ and the decoding complexity is reduced to $O(n^{\frac{3}{2}})$.

In the next section, we propose a network coding algorithm with the decoding complexity of $O(1)$ for all-to-all information exchange in grid networks. More precisely, by using a carefully-designed transmission scheme in our algorithm, the size of linearly independent equations to be decoded is always equal to or less than 8, no matter how large is the network size.

IV. A LOW-COMPLEXITY ALGORITHM FOR GRID NETWORKS

In this paper, two topologies of networks are considered. One is grid topology, where nodes are placed on the vertices of a square grid and each node can only reach its four nearest neighbors. The other is random topology, where nodes are randomly placed in a square area with their X and Y coordinates uniformly distributed.

While it is difficult to find the optimal transmission scheme in random-topology networks because different nodes may have different node degrees, it is possible to find the optimal scheme for grid networks. In a grid network, each node has four neighbors except those nodes on the edge or the corner of the network. To eliminate this edge effect, the square grid could be assumed to envelope the surface of a torus as in [5], [6]. For simplicity, in this paper we assume that the grid network is infinitely large so that each node has the degree of 4. Under the above "infinitely-large grid network" assumption, the number of transmission rounds needed for all-to-all information exchange is also infinite. Nevertheless, our next theorem shows that there exists a transmission scheme that remains *optimal* in terms of rounds. Moreover, in the proof of the theorem, we construct a network coding algorithm that implements such a transmission scheme with $O(1)$ decoding complexity. As mentioned in Section II, the decoding complexity in this paper refers to the number of operations needed to solve the largest set of linear equations by performing Gaussian elimination.

Theorem 2: For an infinitely-large grid network, there exists a distributed transmission scheme that for any large number N there exists a number M that this transmission scheme is *optimal* for the first M rounds and $M > N$.

For the proof of Theorem 2, we first prove the following lemma:

Lemma 1: For an infinitely-large grid network, there exists an $O(1)$-complexity network coding algorithm that each node can decode $4j$ source packets at the end of round j for any $j = k(k+1)/2, k \in \mathbb{N}$.

Proof of Lemma 1: To simplify the notation, we place the grid network on an XY-plane as shown in Fig. 1 and the nodes are denoted by their coordinates (p, q), where $p, q \in \mathbb{Z}$. The source packet on node (p, q) is denoted by $x_{p,q}$. Here, we further restrict the proposed network coding algorithm to be non-directional and the same for all nodes. Thus, we can analyze node $(0, 0)$ only and the results should apply to all other nodes in the network.

Noticing that a node has $4j$ neighbor nodes within k hops (here $j = 1+2+3+\cdots+k = k(k+1)/2$), Lemma 1 is equivalent to finding an $O(1)$-complexity network coding algorithm that when k increases, each node can consecutively decode $4k$ source packets from all its k-hop neighbors in k rounds. We prove this by induction.

In round 1, let each node (p, q) broadcast $x_{p,q}$ to its 4 1-hop neighbors. Then, node $(0, 0)$ will receive $x_{0,1}$, $x_{1,0}$, $x_{0,-1}$, and $x_{-1,0}$. Hence, Lemma 1 holds for $k = 1$.

In rounds 2 to 3, let each node (p, q) broadcast a linear combination of the source packets of its 4 1-hop neighbors in each round. For example, node $(0, 1)$ will broadcast two linear combinations of $\{x_{1,1}, x_{0,2}, x_{-1,1}, x_{0,0}\}$ (circle-shaped nodes

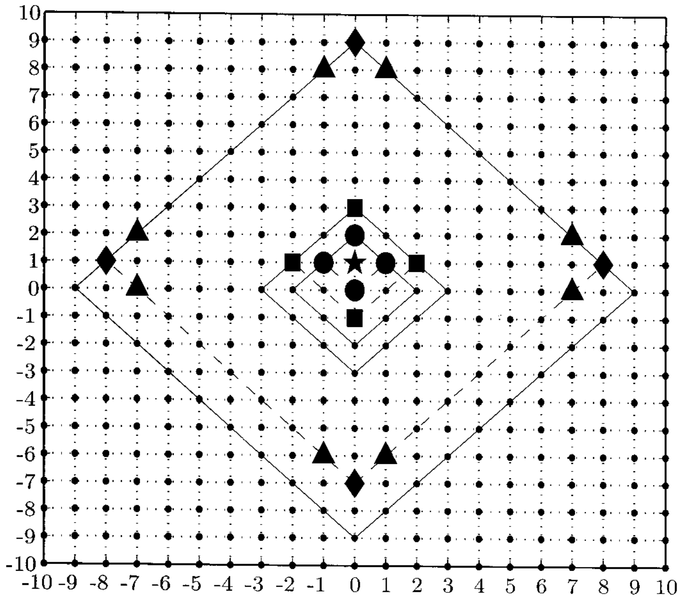

Fig. 1. A grid network placed on XY-plane with nodes denoted by dots. In each round, node $(0, 1)$ (marked as a star) will broadcast a linear combination of the nodes marked in the same shape, e.g., circle-shaped nodes in round 2 and square-shaped nodes in round 4.

in Fig. 1). Considering that node $(0, 0)$ has already received the source packets of its 4 1-hop neighbors in round 1, node $(0, 0)$ will receive 8 linear combinations of the source packets of its 8 2-hop neighbors $x_{p,q}, 0 \leq p, q \leq 2, p + q = 2$. It is easy to verify that these packets can be decoded given that coding vectors are linearly independent (in the rest of the proof, we assume coding vectors are all linearly independent). Hence, Lemma 1 holds for $k = 2$.

In rounds 4 to 6, let each node (p, q) broadcast a linear combination of the source packets of its 4 furthest 2-hop neighbors in each round. For example, node $(0, 1)$ will broadcast three linear combinations of $\{x_{2,1}, x_{0,3}, x_{-2,1}, x_{0,-1}\}$ (square-shaped nodes in Fig. 1). Since node $(0, 0)$ already knows its 1-hop neighbor's source packet $x_{0,-1}$, it can decode the other three 3-hop neighbors. As each of node $(0, 1)$, $(1, 0)$, $(0, -1)$, and $(-1, 0)$ contributes the source packets of 3 different 3-hop neighbors of node $(0, 0)$, node $(0, 0)$ can decode the source packets of all its 12 3-hop neighbors. Hence, Lemma 1 holds for $k = 3$.

Suppose Lemma 1 holds for $k = i - 1, i \in \mathbb{N}, i \geq 4$. Then, node $(0, 0)$ can decode $2i(i-1)$ source packets from all $2i(i-1)$ neighbor nodes within $i - 1$ hops at the end of round $i(i - 1)/2$. We schedule i rounds of transmissions as following:

In rounds $i(i - 1)/2 + 1$ to $i(i - 1)/2 + 3$, let each node broadcast a linear combination of the source packets of its 4 furthest $(i - 1)$-hop neighbors in each round. For example, when $i = 9$, node $(0, 1)$ will broadcast three linear combinations of $\{x_{8,1}, x_{0,9}, x_{-8,1}, x_{0,-7}\}$ (diamond-shaped nodes in Fig. 1). Similar to rounds 4 to 6, node $(0, 0)$ will decode 12 packets $x_{u,v}, |u| + |v| = i, ||u| - |v|| \geq i - 1$.

If i is an odd number, then for each of the remaining $i - 3$ rounds let node (p, q) broadcast a linear combination of 8 packets $x_{u,v}$ where $|u - p| + |v - q| = i - 1$ and $||u - p| - |v - q|| = i - 1 - 2r$, where $r = 1, 2, \cdots, (i - 3)/2$ and r stays the same for every two rounds. For example, when $i = 9$ and $r = 1$, node $(0, 1)$ will broadcast two linear combinations of $\{x_{1,8}, x_{-1,8}, x_{1,-6}, x_{-1,-6}, x_{7,2}, x_{7,0}, x_{-7,2}, x_{-7,0}\}$ (triangle-shaped nodes in Fig. 1). Thus, after every two rounds, node $(0, 0)$ will receive 2 linear combinations of 8 packets $x_{u,v}$ where $|u| + |v - 1| = i - 1$ and $||u| - |v - 1|| = i - 1 - 2r$ from node $(0, 1)$. Among these 8 packets only $x_{i-r-1,r+1}$ and $x_{r+1-i,r+1}$ are previously unknown packets and they can be decoded. Considering the packets from node $(0, -1)$, $(1, 0)$, and $(-1, 0)$, node $(0, 0)$ will decode 8 new packets for every two rounds.

If i is an even number, then for the first $i - 4$ rounds of the remaining $i - 3$ rounds, use the similar schedule as above, that is, let node (p, q) broadcast a linear combination of 8 packets $x_{u,v}$ where $|u - p| + |v - q| = i - 1$ and $||u - p| - |v - q|| = i - 1 - 2r$, where $r = 1, 2, \cdots, (i - 4)/2$ and r stays the same for every two rounds. Similarly, node $(0, 0)$ will decode 8 new packets for every two rounds. Then, for the last round, let each node broadcast a linear combination of 8 packets $x_{u,v}$ where $|u - p| + |v - q| = i - 1$ and $||u - p| - |v - q| = 1|$. Then, node $(0, 0)$ will receive 4 linear combinations from 4 neighbors, among which only $x_{r/2,r/2}$, $x_{-r/2,r/2}$, $x_{-r/2,-r/2}$, and $x_{r/2,-r/2}$ are previously unknown packets. So node $(0, 0)$ can decode 4 new packets after the last round.

Given the above schedule, node $(0, 0)$ can decode another $4i$ source packets in another i rounds. Because the decoding process needs to solve no more than 8 linear equations at one time, the algorithm is of $O(1)$ complexity. Thus, Lemma 1 holds for $k = i$.

By the induction above, we have proved Lemma 1 holds for any $k \in \mathbb{N}$. □

Proof of Theorem 2: Now we prove the proposed transmission scheme in the proof of Lemma 1 also satisfies the condition in Theorem 2. Let U_j be the total number of new information units that a node delivers to all its neighbors in the first j rounds of the transmission scheme. Let V_j be the total number of new information units that a node receives from all its neighbors in the first j rounds of the transmission scheme. As our proposed transmission scheme is non-directional and identical to all nodes, all nodes should have the same U_j and V_j. Because the total number of new information units delivered in the network for j rounds should equal to the total number of new information units received, then we have $U_j = V_j$, Lemma 1 has shown that $V_j = 4j$ holds for any $j = k(k + 1)/2, k \in \mathbb{N}$. Then, $U_j = 4j$ also holds for any $j = k(k + 1)/2, k \in \mathbb{N}$. Because the degree of the network is 4, according to Theorem 1 the transmission scheme is *optimal* for j rounds. Since j could be infinitely large, Theorem 2 holds. □

Our constructive proof of Lemma 1 presents a low-complexity network coding algorithm for all-to-all information exchange in grid networks. In fact, most linearly independent equations needed to be solved in our algorithm are of size 2 (only a few are of size 3, 4, and 8). Compared with $O(n^3)$-complexity algorithms in other works, each set of linearly independent equations to be solved in our algorithm is of $O(1)$ complexity, although a node needs to solve $O(n)$ sets of such equations to decode the source packets of n nodes. Thus, the algorithm is more suitable for wireless sensor networks which are equipped with low-complexity devices.

In order to implement the algorithm in the proof of Lemma 1, we need to consider two practical issues: The packet overhead and the selection of coding coefficients. For an original source packet, the packet overhead is just the coordinates of the source node which identify the packet. For a linear combination packet, the packet overhead includes two parts: The coordinates of the source nodes and the corresponding coding coefficients. Because the number of source packets for a linear combination is equal or less than 8, the packet overhead is always bounded compared with the actual packet length. A common approach for obtaining linearly independent coding coefficients with high probability is to select coefficients randomly over a finite field [12]. However, as our algorithm depends on the successful decoding of previously received packets, even small probability of unsuccessful decoding will make the algorithm unusable. Noticing that in our algorithm linear equations to be solved are from the same packet sender except in rounds 2, 3, and $i(i+1)/2$ (when i is an even number and $i \geq 4$), here we can store deterministic coding coefficients on each node to guarantee the linear combinations generated to be solvable. For transmission rounds other than rounds 2, 3, and $i(i+1)/2$, we can simply store the same sets of coding coefficients on all nodes. For rounds 2, 3, and $i(i+1)/2$, we just need to guarantee that nodes within two hops are using different sets of coding coefficients which are solvable. In any case, if a node already knows the coding coefficients that its neighbors will use, its neighbors no longer need to include coding coefficients in the linear combination packets and this part of packet overhead is eliminated.

V. LOW-COMPLEXITY HEURISTICS FOR RANDOM-TOPOLOGY NETWORKS

In this section we extend our low-complexity algorithm to random-topology networks. Applying network coding on general networks is non-trivial as the coding gain is not guaranteed. Currently there exist two major approaches to utilize network coding in wireless networks. One is to use XOR-based coding as in [14] and [15] and the other is to use linear combinations of all existing packets as in [5]–[7]. XOR-based coding is usually easy for encoding and decoding. However, an XOR-ed packet of n packets is useful to a node only when $n-1$ packets are already known on that node. Therefore, to generate optimized XOR packets a node will need to collect information from its neighbors, which adds to the communication overhead. Also, XOR packets of the same set of packets but from different nodes will not help on decoding as they are identical. Compared with XOR-based coding, linear combinations of all existing packets involve much more complexity when decoding, but on the other hand also increase the possibility of successful decoding.

To balance among decoding complexity, communication overhead and coding gain, our proposed heuristics use linear combinations of selected packets instead of all packets. More precisely, we let each node broadcast a linear combination of two randomly selected existing packets in each round. In order to improve the coding gain and reduce unnecessary transmissions, we let each node cache the received packets of linear combinations for future decoding. Also, linear combinations of

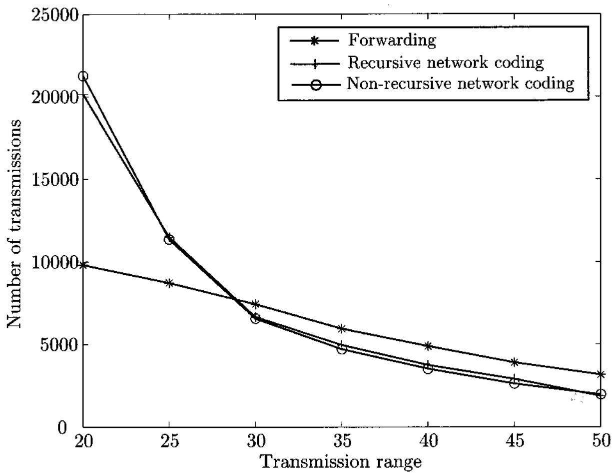

Fig. 2. Number of transmissions needed achieve all-to-all information exchange for different transmission ranges in a 100-node network.

the same two packets will be transmitted at most twice.

The coding gain in our heuristic comes from two aspects. One is the decoding opportunity from a linear combination and an already known packet, which is similar to XOR-based coding. The other is the decoding opportunity from a set of linear combinations of a group of packets, which is similar to the approach using linear combinations of all existing packets. Note that the coding scheme proposed here has a flexible decoding potential. If low-complexity algorithms are needed, then each node can just check if two linear combinations of the same two packets are received. If more complicated algorithms are affordable, then each node can check over a larger size of packet set. For example, if a node has three linear combinations of $\{x_1, x_2\}$, $\{x_2, x_3\}$, and $\{x_1, x_3\}$, respectively, then x_1, x_2, and x_3 can all be decoded.

The performance of the proposed heuristic is verified by simulation experiments. We randomly deploy 100 nodes in an area of 100×100 units. For each deployment of nodes, we check the connectivity of the network. If the topology of the generated network is not fully connected, that deployment will be discarded. For each transmission range, we generate 10 valid deployments and average the results. Note that the node deployments generated for different algorithms are the same such that the comparison of algorithms is not affected by topology. A simple TDMA scheduler is used to guarantee one transmission for each node in each round. We have implemented random forwarding and network coding algorithms under different configurations for the comparison purpose. As low complexity is our primary concern, network coding algorithms in our simulations will only consider linear combinations of the same two packets.

First, we compare network coding algorithms with a random forwarding algorithm in the scenario that nodes are not aware of neighbors' status. The transmission range of nodes varies from 20 to 50. The corresponding graph density and the minimum number of 1-hop neighbors are calculated in Table 1. The resulted graph density varies from 0.11 to 0.49, which can represent a broad range of topologies. In the forwarding algorithm, we let each node randomly select an untransmitted packet for broadcast. We have also implemented two network coding algo-

Table 1. Transmission range, corresponding graph density and minimum number of 1-hop neighbors.

Tx Range	20	25	30	35	40	45	50
Graph density	0.11	0.16	0.22	0.28	0.35	0.41	0.49
min{#nbr}	2.6	4.6	7	9.9	12.9	15.7	20.3

rithms: with and without recursive decoding. With recursive decoding, in each round the decoding process runs recursively until no more new packets can be decoded. Fig. 2 shows the number of transmissions needed by forwarding and network coding algorithms in all-to-all information exchange. We see that network coding outperforms forwarding only when the transmission range reaches a certain threshold. The reason is that when the graph density is low, some nodes may have very few neighbors so that the decoding opportunity is slim. In an extreme case, if a node has only one neighbor, the number of possible linear combinations needed from that neighbor may be far beyond the number of new packets in the network. Contrary to our intuition, network coding algorithms with and without recursive decoding do not differ much in performance. This is because although recursive decoding can decode more packets in each round, those packets are added into the pool of source packets and the decoding opportunity will be decreased when the size of the pool grows large.

Next we consider an idealized situation in which nodes have perfect knowledge of their neighbors' status. Here, we define *informative* as following:

Definition 2: A packet on a node is the most *informative* if the packet is unknown to the most number of the node's neighbors compared with other packets on the same node.

For the forwarding algorithm, instead of randomly choosing an untransmitted packet for broadcast, a node may select the most *informative* packet to broadcast to its neighbors. For the network coding algorithm, instead of forming a linear combination of two randomly selected packets, a node may select two most *informative* packets to form the linear combination. We call these modified heuristics as idealized forwarding and idealized network coding. In practice this idealized scenario can be approximated by gradually learning neighbors' status with small communication overhead, i.e., having each node add the information of the packets received in the last round to the transmission of the current round. Please note this gradual learning method has one round of delay, that is, at the end of current round nodes only have the neighbors' status up to the last round.

Fig. 3 shows the number of transmissions needed by idealized forwarding and network coding algorithms in all-to-all information exchange. We noticed network coding algorithms outperforms the forwarding algorithm for all different transmission ranges. This shows that when a heuristic based on neighbor status is used, network coding algorithms can effectively narrow the pool of source packets for transmission and thus increase the decoding opportunity even when graph density is low. Again, network coding algorithms with and without recursive decoding provide similar performance. In practice we could just pick the non-recursive version of network coding algorithm and it also fits our "low-complexity" design principle.

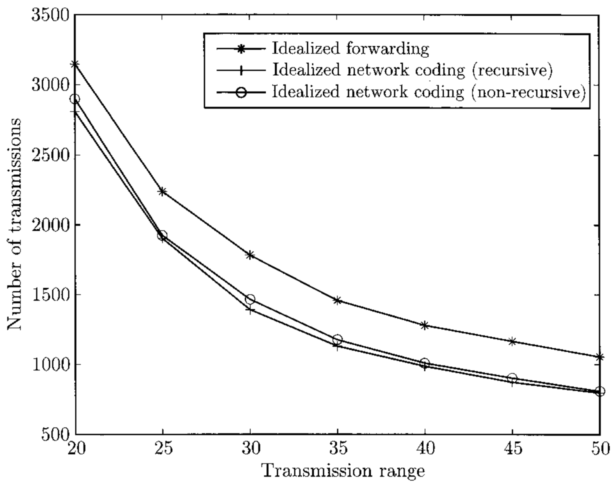

Fig. 3. Number of transmissions needed to achieve idealized all-to-all information exchange for different transmission ranges in a 100-node network.

VI. CONCLUSION

In this paper we study the optimal transmission problem of all-to-all information exchange in terms of energy efficiency and have proved there exists an $O(1)$-complexity network coding algorithm to achieve such optimality in grid networks. Furthermore, the detailed constructive proof provides an instance of such algorithm. We have also proposed low-complexity network coding heuristics for random-topology networks accompanied with simulation results. As expected, network coding algorithms can provide a performance gain over forwarding algorithms with low additional complexity. However, attention needs to be paid to some special scenarios, e.g. low-density networks, where network coding algorithm may have poor performance. We also noticed techniques like recursive decoding did not improve the overall performance of network coding in our scenarios. Our future work includes a study of how to generalize the algorithm for grid networks to arbitrary networks with low degrees, and a further analysis on how the codeword degree may affect the performance in random-topology networks.

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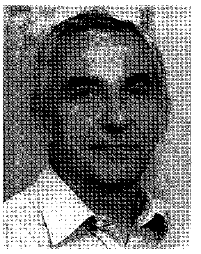

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