New Design for Linear Complex Precoding over ABBA Quasi-Orthogonal Space-Time Block Codes

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ABSTRACT

ABBA codes, a class of quasi-orthognal space-time block codes (QoSTBC) proposed by Tirkkonen and others, allow full rate and a fast maximum likelihood (ML) decoding, but do not have full diversity. In this paper, a linear complex precoder is proposed for ABBA codes to achieve full rate and full diversity. Moreover, the same diversity produce as that of orthogonal space-time block code with linear complex precoder (OSTBC-LCP) is achieved. Meanwhile, the size of the linear complex precoder can be reduced by half without affecting performance, which means the same complexity of decoding as that of the conventional ABBA code is guaranteed.

Key Words : Space-time coding, Precoder, Full Diversity

Ⅰ. Introduction

Space-time block code design has recently attracted considerable attentions. One attractive approach of space-time block codes (STBC) is the orthogonal designs as proposed by Alamouti[1], Tarokn, Jafarkhani and Calderbank[2]. The codes can achieve full diversity and have fast maximum likelihood (ML) decoding at the receiver. However, full-rate orthogonal STBC (OSTBC) for general quadrature amplitude modulation (QAM) does not exist when the number of transmit antennas is larger than two.

Recently, ABBA codes[3], a class of space-time block code from quasi-orthogonal designs, have been proposed by *Tirkkonen et al.*, to increase the signal rate. With the quasi-orthogonal structure, the ML decoding at the receiver can be done by searching pairs of symbols. However, these codes do not have full diversity. The performance of these codes is better than OSTBC at low SNR, but worse at high SNR.

It is desired to have ABBA codes with full diversity to ensure good performance at high SNR. A quasi-orthogonal space time block code with signal constellation (SC-ABBA) in [4] achieve this goal by properly choosing the signal constellations. In this paper, we achieve this goal by properly design a linear complex precoder for ABBA codes. The resulting codes guarantee both full rate and full diversity. Moreover, they achieve the same diversity advantage defined in [2] as that of the orthogonal space-time block code design with a linear complex precoder (OSTBC-LCP) in [5]. Finally, the linear complex precoder (LCP) is simplified. Thus the decoding complexity is the same as that of the conventional ABBA codes.

Ⅱ. Linear Complex Precoder for ABBA Codes with Full Diversity

2.1 ABBA Codes

ABBA codes with full rate and partial diversity

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for a system with four-transmit antennas were proposed in [3]. In that scheme, four symbols are arranged as follows:

$$C=\begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_3 & s_4 & s_1 & s_2 \\ -s_4^* & s_3^* & -s_2^* & s_1^* \end{pmatrix} \tag{1}$$

where the matrix has two copies of the 2×2 Alamouti block code with symbols s_1 and s_2 on the block diagonal, and two copies of the Alamouti code with symbols s_3 and s_4 on the anti-diagonal block, ie. in the form

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix} \tag{2}$$

The scheme is thus called "ABBA Codes".

For any pair of distinct transmitted matrices C and $\hat{C}$, the difference matrix is $\Delta C= C(\tilde{s}_1,\dots,\tilde{s}_4)$, $\tilde{s}_i=s_i-\hat{s}_i$. Then from (1), we have the property matrix

$$\Delta C\Delta C^H=\begin{pmatrix} \Delta a I_2 & \Delta b I_2 \\ \Delta b I_2 & \Delta a I_2 \end{pmatrix} \tag{3}$$

where

$$\Delta a=\sum_{i=1}^{4}|s_i|^2 \quad \text{and}$$

$$\Delta b=\sum_{i=1}^{2}\left[(\tilde{s}_i)(\widetilde{s_{i+2}})^* + (\tilde{s}_i)^*(\widetilde{s_{i+2}})\right].$$

Hence, the determinant of (3) is given as

$$\begin{aligned}\det(\Delta C\Delta C^H)^{1/2} &= \left[(\Delta a)^2-(\Delta b)^2\right] \\ &= \left(\sum_{i=1}^{2}|\tilde{s}_i-\widetilde{s_{i+2}}|^2\right)\left(\sum_{i=1}^{2}|\tilde{s}_i+\widetilde{s_{i+2}}|^2\right)\end{aligned} \tag{4}$$

Note that (4) sometimes could be zero, for example when $\tilde{s}_i=\widetilde{s_{i+2}}$, which means the space-time signals can not have the full diversity. Hence, it is desired to find a way to ensure the determinant is always nonzero.

2.2 A complex precoder for ABBA codes with full diversity

A new linear complex precoder is proposed as follows:

$$\Phi_n = D\cdot\Phi = diag\left(e^{j\psi_1},e^{j\psi_2},e^{j\psi_3},e^{j\psi_4}\right)\cdot\Phi \tag{5}$$

which is a product of a diagonal matrix D and the unitary matrix Φ with a Vandermonde structure given as

$$\Phi=\frac{1}{\lambda}\begin{pmatrix} 1 & \alpha_1^1 & \cdots & \alpha_1^{M-1} \\ 1 & \alpha_2^1 & \cdots & \alpha_2^{M-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha_M^1 & \cdots & \alpha_M^{M-1} \end{pmatrix} \tag{6}$$

where $\alpha_i=\exp(j\pi(4i-1)/2M)$ and $1/\lambda$ is the normalizing factor ensuring that $tr(\Phi\Phi^H)=M$.

Hence, the transmission matrix for the new ABBA codes with LCP (5) is changed into

$$C(\Phi_n S)=C(X)=\frac{1}{2}\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ x_3 & x_4 & x_1 & x_2 \\ -x_4^* & x_3^* & -x_2^* & x_1^* \end{pmatrix} \tag{7}$$

where $1/2$ is the normalizing factor so that $tr\left[C(X)C(X)^H\right]=4$ and $S=\left[s_1,s_2,s_{3,}s_4\right]^T$. The determinant of the property matrix is given as

$$\begin{aligned}\det(\Delta C\Delta C^H)^{\frac{1}{2}} &= \left(\frac{1}{4}\sum_{i=1}^{2}\left|e^{j\psi_i}\theta_i\bar{S}-e^{j\psi_{i+2}}\theta_{i+2}\bar{S}\right|^2\right) \\ &\quad\cdot\left(\frac{1}{4}\sum_{i=1}^{2}\left|e^{j\psi_i}\theta_i\bar{S}+e^{j\psi_{i+2}}\theta_{i+2}\bar{S}\right|^2\right)\end{aligned} \tag{8}$$

where θ_i is the ith row vector of Φ and

$$\bar{S}=\left[s_1-\hat{s}_1,\cdots,s_4-\hat{s}_4\right]^T.$$

Theorem: *when* $|\psi_i-\psi_{i+2}|=\pi/2$, $C(\Phi_n S)$ *can guaranntee the full diversity and achieve the same diversity product as that of OSTBC-LCP for a signal constellation drawn from a square lattice.*

Proof: if the determinant (8) is assumed to be zero, we have

$$e^{j\psi_i}\theta_i\bar{S}=e^{j\psi_{i+2}}\theta_{i+2}\bar{S} \sqcup e^{j\psi_i}\theta_i\bar{S}=-e^{j\psi_{i+2}}\theta_{i+2}\bar{S}$$

From properties of the precoder (6), we observe that

$$\theta_i=\left[1,\alpha_i,\alpha_i^2,\alpha_i^3\right] \text{ and } \theta_{i+2}=\left[1,-\alpha_i,\alpha_i^2,-\alpha_i^3\right],$$

thus

$$\begin{aligned}\Delta &= e^{j\psi_i}\theta_i\bar{S}-e^{j\psi_{i+2}}\theta_{i+2}\bar{S} \\ &= \begin{pmatrix} e^{j\psi_i}-e^{j\psi_{i+2}} \\ \left(e^{j\psi_i}+e^{j\psi_{i+2}}\right)\alpha_i \\ \left(e^{j\psi_i}-e^{j\psi_{i+2}}\right)\alpha_i^2 \\ \left(e^{j\psi_i}+e^{j\psi_{i+2}}\right)\alpha_i^3 \end{pmatrix}^T\cdot\bar{S}=0 \\ &\Rightarrow\left(1,\alpha_i,\alpha_i^2,\alpha_i^3\right)\begin{pmatrix} \bar{\bar{s}}_1 \\ \frac{e^{j\psi_i}+e^{j\psi_{i+2}}}{e^{j\psi_i}-e^{j\psi_{i+2}}}\bar{s}_2 \\ \bar{s}_3 \\ \frac{e^{j\psi_i}+e^{j\psi_{i+2}}}{e^{j\psi_i}-e^{j\psi_{i+2}}}\bar{s}_4 \end{pmatrix}=0 \\ &\Rightarrow\theta_i\overline{S_\Phi}\end{aligned} \tag{9}$$

where $\overline{S_\Phi}=\left[\overline{s_1},\frac{e^{j\psi_i}+e^{j\psi_{i+2}}}{e^{j\psi_i}-e^{j\psi_{i+2}}}\overline{s_2},\overline{s_3},\frac{e^{j\psi_i}+e^{j\psi_{i+2}}}{e^{j\psi_i}-e^{j\psi_{i+2}}}\overline{s_4}\right]^T$.

If $\frac{e^{j\psi_i}+e^{j\psi_{i+2}}}{e^{j\psi_i}-e^{j\psi_{i+2}}}\overline{s_k}, k=2,4,$ belongs to a signal constellations drawn from a square lattice (4QAM, 16QAM), $\theta_i\overline{S_\Phi}$ is equivalent to zero if and only if $\overline{S_\Phi}$ is a zero vector [6], which means $\overline{S}=0$. However, the result conflicts with the assumption that $\overline{S}\neq 0$. So $e^{j\psi_i}\theta_i\overline{S}\neq e^{j\psi_{i+2}}\theta_{i+2}\overline{S}$. Similarly, $e^{j\psi_i}\theta_i\overline{S}\neq -e^{j\psi_{i+2}}\theta_{i+2}\overline{S}$.

Now, angles ψ_i and ψ_{i+2} should be selected so that $\frac{e^{j\psi_i}+e^{j\psi_{i+2}}}{e^{j\psi_i}-e^{j\psi_{i+2}}}\overline{s_k}$ are included by a signal constellation drawn from a square lattice. One of necessary condition is

$$\left|\frac{e^{j\psi_i}+e^{j\psi_{i+2}}}{e^{j\psi_i}-e^{j\psi_{i+2}}}\right|=1\Rightarrow\left|\frac{\cos[(\psi_i-\psi_{i+2})/2]}{\sin[(\psi_i-\psi_{i+2})/2]}(-j)\right| \\ \Rightarrow|\cot[(\psi_i-\psi_{i+2})/2]|=1 \tag{10}$$

and

$$\left|\frac{e^{j\psi_i}-e^{j\psi_{i+2}}}{e^{j\psi_i}+e^{j\psi_{i+2}}}\right|=1\Rightarrow\left|\frac{\sin[(\psi_i-\psi_{i+2})/2]}{\cos[(\psi_i-\psi_{i+2})/2]}(j)\right| \\ \Rightarrow|\tan[(\psi_i-\psi_{i+2})/2]|=1 \tag{11}$$

Hence $|\psi_i-\psi_{i+2}|=\pi/2$.

Since $\psi_i-\psi_{i+2}=\pm\pi/2$, the diversity product of the ABBA-LCP is given as follows:

$$\begin{aligned}\zeta&=\min_{S\neq\hat{S}}\det(\Delta C\Delta C^H)^{\frac{1}{2M}}\\&=\min\left(\frac{1}{4}\sum_{i=1}^{2}\left|e^{j\psi_i}\theta_i\overline{S}\mp e^{j\psi_{i+2}}\theta_{i+2}\overline{S}\right|^2\right)^{1/M}\\&\cdot\left(\frac{1}{4}\sum_{i=1}^{2}\left|e^{j\psi_i}\theta_i\overline{S}\pm e^{j\psi_{i+2}}\theta_{i+2}\overline{S}\right|^2\right)^{1/M}\end{aligned} \tag{12}$$

where M is the number of transmit antennas and we define that

$$\begin{aligned}\Gamma_1&=\left|e^{j\psi_i}\theta_i\overline{S}\mp e^{j\left(\psi_i+\frac{\pi}{2}\right)}\theta_{i+2}\overline{S}\right|^2\\&=|1\mp j|^2\left|(1,\alpha_i,\alpha_i^2,\alpha_i^3)\begin{pmatrix}\overline{s_1}\\\pm j\overline{s_2}\\\overline{s_3}\\\pm\overline{s_4}\end{pmatrix}\right|^2=2|\theta_i\overline{S_n}|\end{aligned} \tag{13}$$

$$\begin{aligned}\Gamma_2&=\left|e^{j\psi_i}\theta_i\overline{S}\pm e^{j\left(\psi_i+\frac{\pi}{2}\right)}\theta_{i+2}\overline{S}\right|^2\\&=|1\pm j|^2\left|(1,-\alpha_i,\alpha_i^2,-\alpha_i^3)\begin{pmatrix}\overline{s_1}\\\pm j\overline{s_2}\\\overline{s_3}\\\pm\overline{s_4}\end{pmatrix}\right|^2=2|\theta_{i+2}\overline{S_n}|\end{aligned} \tag{14}$$

where $\overline{S_n}=(s_1-\hat{s}_1,\pm j(s_2-\hat{s}_2),s_3-\hat{s}_3,\pm j(s_4-\hat{s}_4))$.

Therefore,

$$\zeta=\min_{\overline{S_n}\neq 0}\left(|\theta_1\overline{S_n}|^2+|\theta_2\overline{S_n}|^2\right)^{1/M}\left(|\theta_3\overline{S_n}|^2+|\theta_4\overline{S_n}|^2\right)^{1/M}$$

from [5], we know the diversity product of OSTBC-LCP is

$$\begin{aligned}\zeta_{OSTBC-LCP}&=\frac{1}{\sqrt[M]{4}}min_{\overline{S}\neq 0}\left(|\theta_1\overline{S}|^2+|\theta_2\overline{S}|^2\right)^{1/M}\left(|\theta_3\overline{S}|^2+|\theta_4\overline{S}|^2\right)^{1/M}\\&\geq\min_{\overline{S}\neq 0}\left(\prod_{i=1}^{4}|\theta_i\overline{S}|\right)^{1/M}=\zeta_{DSTBC-LCP}\end{aligned}$$

It is easy to observe that OSTBC-LCP and the proposed scheme have the same diversity product equations with different symbol vector notations, so both of them have the same diversity product. We also observe that the diversity product of OSTBC-LCP is greater than or equal to that of DSTBC-LCP [6], then the diversity product of the proposed code should be greater than or equal to that of DSTBC-LCP too.

2.3 A simplified LCP for a fast ML decoding

From (13) and (14), we observe that

$$\min_{\overline{S}\neq 0}|\theta_i\overline{S}|\leq\min_{\overline{S}\neq 0}\left|(1,\alpha_i^2)\begin{pmatrix}\overline{s_1}\\\overline{s_3}\end{pmatrix}\right|+\left|(1,\alpha_i^2)\begin{pmatrix}\overline{s_2}\\\overline{s_4}\end{pmatrix}\right|$$

Equality exists when one of symbol pairs are zeros. It is possible to simplify the LCP (5) for the reduction of decoding complexity with negligible performance effect. Therefore, the LCP (5) is simplified as

$$\Phi_n^1=\frac{1}{\sqrt{2}}\begin{pmatrix}e^{j\psi_1}&0&e^{j\psi_1}\alpha_1^2&0\\0&e^{j\psi_2}&0&e^{j\psi_2}\alpha_2^2\\e^{j\psi_3}&0&e^{j\psi_3}\alpha_3^2&0\\0&e^{j\psi_4}&0&e^{j\psi_4}\alpha_4^2\end{pmatrix} \tag{15}$$

which can be looked as a shifted and punctured matrix from (5).

If we define $V_i, i=1,\cdots,4$, as the ith column of $C(X)$, from the property of ABBA codes it is easy to see that

$$\langle V_1,V_2\rangle=\langle V_1,V_4\rangle=\langle V_3,V_2\rangle=\langle V_3,V_4\rangle=0$$

where $\langle V_i,V_j\rangle=\sum_{l=1}^{4}(V_i)_l(V_j)_l^*$ is the inner product of vector V_i and V_j. Therefore, the subspace created by V_1 and V_3 is orthogonal to the subspace created by V_2 and V_4. For this orthogonality, the maximum-likelihood decision metric can be calculated as the sum of two terms $f_{13}(x_1,x_3)+f_{24}(x_2,x_4)$ [7], where f_{13} is a function of x_1 and x_3 and it is independent of x_2 and x_4.

Similarly, f_{24} is another function of x_2 and x_4 and independent of x_1 and x_3. If the full-size LCP (5) is used, using two independent functions f_{13} and f_{24} is helpless to reduce the complexity of decoding because each transmitted symbol x_i includes all original symbols s_i. But if the simplified LCP (15) is used, both x_1 and x_3 only include s_1 and s_3, meanwhile x_2 and x_4 only include s_2 and s_4. In other words, first the decoder finds the (s_1,s_3) that minimizes $f_{13}(x_1,x_3)$, in parallel, the decoder selects the pair (s_2,s_4) which minimizes $f_{24}(x_2,x_4)$. So ABBA codes with LCP (15) not only achieve fulll diversity gain but also have the same decoding complexity as those of the conventional ABBA codes.

Ⅲ. Simulation Results

The performance of ABBA codes with different linear complex precoders (LCP) is evaluated in this section via simulations. Four transmit and one receiver antenna are assumed. 4QAM and 16QAM constellations are considered. Demodulation is performed via maximum likelihood (ML) detection. For our simulation, the channel is assumed to be uncorrelated complex Gaussian with unit variance. It is constant across a space-time block code and changes independently from block to block.

Fig.1 provides performance comparison for ABBA codes [3], SC-ABBA [4], DSTBC-LCP [6], OSTBC-LCP [5] and ABBA codes with different LCPs using 4QAM. Simulation results show that the performance of the proposed scheme is merged to that of OSTBC-LCP, which is coincident with our proof. Besides full-size LCP (5) and the simplified LCP (15) applied to

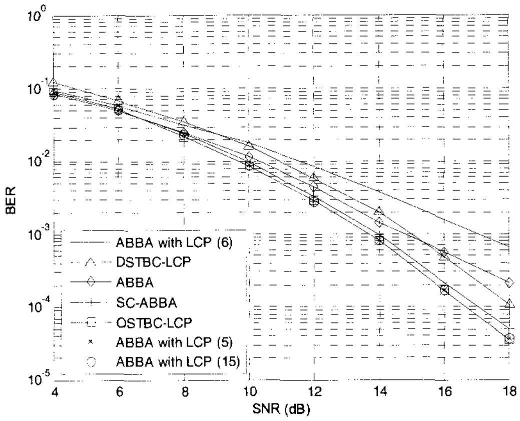

Fig. 1. BER performance Vs SNR for 4-QAM

ABBA codes achieve almost the same performance, which further prove our argument that using the simplified LCP has little effect on the performance.

Simulation results in Fig.1 also show that the performance of ABBA codes with LCP (5) or (15) is close to that of the conventional ABBA codes when the SNR is low. As the SNR increases, codes with full diversity work better and get more benefit from the SNR increase than partial diversity codes. Furthermore, the ABBA codes with LCP (5) and (15) outperform DSTBC-LCP not only in low SNR region but also in high SNR region, which is mainly due to the fact that the proposed scheme has larger diversity advantage. ABBA codes with LCP also improve performance gain compared to SC-ABBA. When only the (6) is used in ABBA codes, the performance is even worse than that of the conventional ABBA codes, which prove that the Vandermonde matrix can help OSTBC [5] to achieve full diversity gain but can not help ABBA codes.

Fig.2 provides simulation results of DSTBC-LCP, OSTBC-LCP and ABBA codes with LCP using 16QAM for 4 transmit and 1 receive antenna. We observe that ABBA codes with LCP are approximately 1.5dB better than DSTBC-LCP at BER 10^{-3}. Performance curves of OSTBC-LCP and ABBA codes with LCP are still merged and better than that of DSTBC-LCP.

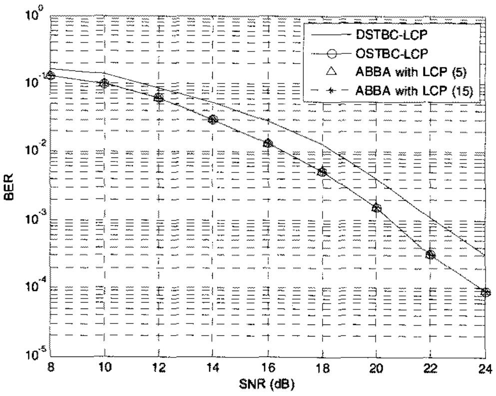

Fig. 2. BER performance Vs SNR for 16-QAM

Ⅳ. Conclusion

In this paper, a linear complex precoder is designed for ABBA codes to achieve full diversity and full rate. The proposed scheme also achieves the same diversity products as that of OSTBC-LCP in [5]. Moreover, a simplified linear complex precoder is proposed to keep a fast maximum likelihood decoding with that same performance as that of the full-size complex precoder.

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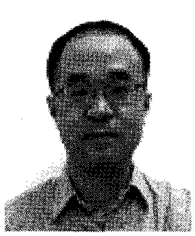

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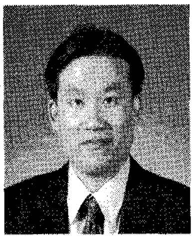

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