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Multi-Intuitionistic Fuzzy Sets and Intuitionistic Fuzzy P Systems

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Abstract

In this paper, we introduce multi-intuitionistic fuzzy sets and intuitionistic fuzzy hybrid sets. The basic operations between such structures are defined. The use of these structures in the definition of intuitionistic fuzzy variants of P systems and their properties are presented.

Key Words : Multi-intuitionistic fuzzy sets, intuitionistic fuzzy hybrid sets, intuitionistic fuzzy-theoretic operators, fuzzy P systems

1. Introduction

Intuitionistic fuzzy sets are defined by Atanassov in 1983 [1] and form an extension of fuzzy sets. While fuzzy sets only give a membership degree to each element of the universe, and the non-membership degree equals one minus the membership degree, in intuitionistic fuzzy set theory the two degrees are more or less independent, the only constraint is that the sum of the two degrees must not exceed one. Considering the increasing world-wide interest in intuitionistic fuzzy sets – after 20 years of existence already more than 600 papers have been published on intuitionistic fuzzy sets (an overview can be found in [6]), a book has been written [2], since 1997 a conference devoted to intuitionistic fuzzy sets is yearly organized as well as several special sessions on international conferences, Deschrijver et al. [4] investigated the position of intuitionistic fuzzy set theory in the framework of the different theories modelling imprecision.

In the crisp set, an element belongs to this set or not. The number of copies of this element in the set was ignored. Multi-sets considered the number of copies for each element in the universal set. Multi-sets are really useful structures and they have found numerous application in mathematics and computer science. If we allow elements of a multi-set to occur an integer number of times (positive and negative number of times), the produced structure will be called hybrid set (see Loeb [5]).

In [10], Yager fuzzified the notion of multi-sets (fuzzy *multi-sets*). Fuzzy multi-sets are just multi-sets of pairs, where the first part of each pair is an element of the universal set and the second part the degree to which the first part belongs to the fuzzy multi-set. This means that fuzzy multi-sets are not fuzzy enough. In order to have a fuzzier structure, Syropoulos [7] fuzzified the number of occurrences of repeated elements. Clearly, this can be achieved by generalizing of the membership function. Syropoulos [7] decided to call this structure by "multi-*fuzzy set*".

A **P** system consists of membranes that are populated with multi-sets of objects, which are usually materialized as strings of symbols. In addition, there are rules that are used to change the configuration of the system. A **P** system behaves more or less like a parser, which is clearly hard-wired to a particular grammar. Thus, a **P** system stops when no rule can be applied to the system. The result of the computation is always equal to the cardinality of the multi-set that is contained in a designated compartment.

In this paper, we will define multi-intuitionistic fuzzy sets and intuitionistic fuzzy hybrid sets. Next, we will define the basic operations between such structures (e.g., union, sum, etc.). Also, we will give the definition of certain standard intuitionistic fuzzy-theoretic operators. The use of these structures in the definition of intuitionistic fuzzy variants of P systems and their properties will be presented next. The paper ends with the conclusion.

2. Preliminaries

A crisp multi-set F is the function $F: A \to N_0$ where A is a set and $N_0 = \{0,1,2,3,......\}$. It is clear that the crisp multi-set could be written in the form $F(a) = (n),\ n \in N_0$ which means that there exists n copies from the element a in the set A. Syropoulos [8] fuzzified crisp multi-sets (multi-L-fuzzy sets) as follows:

$$\mathcal{F}: A \to L \times N_0 \tag{1}$$

Where L is a fram. The value of multi-L-fuzzy set $\mathcal{F}$ at $a \in A$ is $\mathcal{F}(a) = (r,n)$, where $r \in L$ and $n \in N_0$. When $L = \{0,1\}$, Equation (1) will be reduced to that of crisp multi-set. If we substitute N_0 with Z (negative, zero and positive integer numbers) in Equation (1), then the resulting structures will be called L-fuzzy hybrid set.

For any L-fuzzy hybrid (multi-fuzzy) set $\mathcal{A}$, if $\mathcal{A}(x) = (l,n)$, then the multiplicity function $\mathcal{A}_M : X \to Z$

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(resp. $\mathcal{A}_M : X \to \mathrm{N}_0$) and the membership function $\mathcal{A}_\mu : X \to \mathrm{L}$ will be $\mathcal{A}_M(x) = n$ and $\mathcal{A}_\mu(x) = l$.

The partial order $\ll$ over Z defined as follows:

$$\begin{aligned} n \ll m \equiv\ & (n=0) \vee \\ & \big((n>0)\wedge(m>0)\wedge(n\le m)\big) \vee \\ & \big((n<0)\wedge(m>0)\big) \vee \\ & \big((n<0)\wedge(m<0)\wedge(|n|\le|m|)\big) \end{aligned}$$

where $\wedge$ and $\vee$ denote the classical logical conjunction and disjunction operators, $|n|$ is the absolute value of n and $\le$, $<$ denote the usual integer ordering relations.

Proposition 2.1 [9]. The relation $\ll$ is a partial order.

3. Multi-Intuitionistic Fuzzy Sets

Definition 3.1 A multi-intuitionistic fuzzy set $\tilde{A}$ is the function $\tilde{A} : X \to I \times I \times \mathrm{N}_0$ such that $\tilde{A}(x) = (l, m, n)$, where l and m are gradation of membership and gradation of non-membership of n ($n \in \mathrm{N}_0$) copies of the element x in X and $0 < l + m \le 1$.

Every multi-fuzzy set $\mathcal{A}$ on a non-empty set X is obviously a multi-intuitionistic fuzzy set having the form:

$$\widetilde{\mathcal{A}}(x) = (l, 1-l, n), \text{ for each } l \in I \text{ and } x \in X.$$

Assuming that $\tilde{A}$ is a multi-intuitionistic fuzzy set, then one can define the following two functions: the multiplicity function $\tilde{A}_M : X \to \mathrm{N}_0$, the membership function $\tilde{A}_\mu : X \to I$ and the non-membership function $\tilde{A}_\nu : X \to I$. Clearly, if $\tilde{A}(x) = (l, m, n)$, then $\tilde{A}_M(x) = n$, $\tilde{A}_\mu(x) = l$ and $\tilde{A}_\nu(x) = m$. Note that $\widetilde{\mathcal{A}}_\mu$ and $\widetilde{\mathcal{A}}_\nu$ should satisfy the condition $0 < \widetilde{\mathcal{A}}_\mu(x) + \widetilde{\mathcal{A}}_\nu(x) \le 1$ for each $x \in X$

Definition 3.2. Assume that $\widetilde{\mathcal{A}}, \widetilde{\mathcal{B}} : X \to I \times I \times \mathrm{N}_0$ are two multi-intuitionistic fuzzy sets, then $\widetilde{\mathcal{A}} \subseteq \widetilde{\mathcal{B}}$ if and only if $\widetilde{\mathcal{A}}_\mu(x) \le \widetilde{\mathcal{B}}_\mu(x)$, $\widetilde{\mathcal{A}}_\nu(x) \ge \widetilde{\mathcal{B}}_\nu(x)$ and $\widetilde{\mathcal{A}}_M(x) \le \widetilde{\mathcal{B}}_M(x)$, for each $x \in X$.

Definition 3.3. Assume that $\widetilde{\mathcal{A}}, \widetilde{\mathcal{B}} : X \to I \times I \times \mathrm{N}_0$ are two multi-intuitionistic fuzzy sets, then their union denoted by $\widetilde{\mathcal{A}} \cup \widetilde{\mathcal{B}}$, is defined as follows:

$$(\widetilde{\mathcal{A}} \cup \widetilde{\mathcal{B}})(x) = \big(\widetilde{\mathcal{A}}_\mu(x) \vee \widetilde{\mathcal{B}}_\mu(x),\ \widetilde{\mathcal{A}}_\nu(x) \wedge \widetilde{\mathcal{B}}_\nu(x),\ \max\{\widetilde{\mathcal{A}}_M(x), \widetilde{\mathcal{B}}_M(x)\}\big).$$

Definition 3.4. Assume that $\widetilde{\mathcal{A}}, \widetilde{\mathcal{B}} : X \to I \times I \times \mathrm{N}_0$ are two multi-intuitionistic fuzzy sets, then their intersection denoted by $\widetilde{\mathcal{A}} \cap \widetilde{\mathcal{B}}$, is defined as follows:

$$(\widetilde{\mathcal{A}} \cap \widetilde{\mathcal{B}})(x) = \big(\widetilde{\mathcal{A}}_\mu(x) \wedge \widetilde{\mathcal{B}}_\mu(x),\ \widetilde{\mathcal{A}}_\nu(x) \vee \widetilde{\mathcal{B}}_\nu(x),\ \min\{\widetilde{\mathcal{A}}_M(x), \widetilde{\mathcal{B}}_M(x)\}\big).$$

We will now define the sum of two multi-intuitionistic fuzzy sets:

Definition 3.5. Assume that $\widetilde{\mathcal{A}}, \widetilde{\mathcal{B}} : X \to I \times I \times \mathrm{N}_0$ are two multi-intuitionistic fuzzy sets, then their sum denoted by $\widetilde{\mathcal{A}} \oplus \widetilde{\mathcal{B}}$, is defined as follows:

$$(\widetilde{\mathcal{A}} \oplus \widetilde{\mathcal{B}})(x) = \big(\widetilde{\mathcal{A}}_\mu(x) \vee \widetilde{\mathcal{B}}_\mu(x),\ \widetilde{\mathcal{A}}_\nu(x) \wedge \widetilde{\mathcal{B}}_\nu(x),\ \widetilde{\mathcal{A}}_M(x) + \widetilde{\mathcal{B}}_M(x)\big).$$

Theorem 3.1. The operations $\cup$, $\cap$ and $\oplus$ defined above have the following properties:
(1) Commutative
(2) Associative
(3) Idempotent
(4) Distributive
(5) The sum operation $\oplus$ is distributive over $\cap$ and $\cup$.

Proof. Trivial.

The α-cut of multi-fuzzy set is just crisp multi-set. Similarly, the (α, β)-cut of multi-intuitionistic fuzzy set has to be a crisp multi-set. By $[x]_n$, we denote the crisp multi-set that consists of only n copies of x (singleton multi-set).

Definition 3.6. Suppose that $\widetilde{\mathcal{A}}$ is a multi-intuitionistic fuzzy set with the universe set X. For $\alpha, \beta \in I$ such that $0 \le \alpha + \beta \le 1$, the (α, β)-cut of $\widetilde{\mathcal{A}}$, denoted by $\widetilde{\mathcal{A}}^{\alpha,\beta}$ is the crisp multi-set defined by:

$$\widetilde{\mathcal{A}}^{\alpha,\beta} = \bigcup_{\substack{x \in X \\ \alpha \le \widetilde{\mathcal{A}}_\mu(x),\ \beta \ge \widetilde{\mathcal{A}}_\nu(x)}} [x]_{\widetilde{\mathcal{A}}_m(x)}.$$

Theorem 3.2. For any two multi-intuitionistic fuzzy sets $\widetilde{\mathcal{A}}$ and $\widetilde{\mathcal{B}}$ in X, the following properties are true:
(1) If $\alpha_1 \le \alpha_2$ and $\beta_1 \ge \beta_2$, then $\widetilde{\mathcal{B}}^{\alpha_2,\beta_2} \subseteq \widetilde{\mathcal{A}}^{\alpha_1,\beta_1}$.
(2) $(\widetilde{\mathcal{A}} \cap \widetilde{\mathcal{B}})^{\alpha,\beta} = \widetilde{\mathcal{A}}^{\alpha,\beta} \cap \widetilde{\mathcal{B}}^{\alpha,\beta}$, $(\widetilde{\mathcal{A}} \cup \widetilde{\mathcal{B}})^{\alpha,\beta} = \widetilde{\mathcal{A}}^{\alpha,\beta} \cup \widetilde{\mathcal{B}}^{\alpha,\beta}$ and $(\widetilde{\mathcal{A}} \oplus \widetilde{\mathcal{B}})^{\alpha,\beta} = \widetilde{\mathcal{A}}^{\alpha,\beta} \oplus \widetilde{\mathcal{B}}^{\alpha,\beta}$.

Proof. (1) Let $x \in X$ and assume that $\alpha_1 \le \alpha_2$ and $\beta_1 \ge \beta_2$. If $\widetilde{\mathcal{A}}_\mu(x) \not\le \alpha_2$ and $\widetilde{\mathcal{A}}_\nu(x) \not\le \beta_2$, then $\widetilde{\mathcal{A}}^{\alpha_1,\beta_1}(x) = \widetilde{\mathcal{A}}^{\alpha_2,\beta_2}(x)$. If $\alpha_1 \le \widetilde{\mathcal{A}}_\mu(x) \le \alpha_2$ and $\beta_1 \ge \widetilde{\mathcal{A}}_\nu(x) \ge \beta_2$ then $\widetilde{\mathcal{A}}^{\alpha_1,\beta_1}(x) \ge \widetilde{\mathcal{A}}^{\alpha_2,\beta_2}(x)$. If $\alpha_1 \not\le \widetilde{\mathcal{A}}_\mu(x)$ and $\beta_1 \not\ge \widetilde{\mathcal{A}}_\nu(x)$, then $\widetilde{\mathcal{A}}^{\alpha_1,\beta_1}(x) = \widetilde{\mathcal{A}}^{\alpha_2,\beta_2}(x) = 0$. Thus, for all possible cases $\widetilde{\mathcal{A}}^{\alpha_1,\beta_1}(x) \ge \widetilde{\mathcal{A}}^{\alpha_2,\beta_2}(x)$, which means that $\widetilde{\mathcal{A}}^{\alpha_2,\beta_2} \subseteq \widetilde{\mathcal{A}}^{\alpha_1,\beta_1}$.
(2) Suppose that $(\widetilde{\mathcal{A}} \cap \widetilde{\mathcal{B}})^{\alpha,\beta}(x) > 0$, then $(\widetilde{\mathcal{A}} \cap \widetilde{\mathcal{B}})_\mu(x) \ge \alpha$, $(\widetilde{\mathcal{A}} \cap \widetilde{\mathcal{B}})_\nu(x) \le \beta$ and hence $\widetilde{\mathcal{A}}_\mu(x) \wedge \widetilde{\mathcal{B}}_\mu(x) \ge \alpha$, $\widetilde{\mathcal{A}}_\nu(x) \vee \widetilde{\mathcal{B}}_\nu(x) \le \beta$. This means that $\widetilde{\mathcal{A}}_\mu(x) \ge \alpha$, $\widetilde{\mathcal{B}}_\mu(x) \ge \alpha$ and $\widetilde{\mathcal{A}}_\nu(x) \le \beta$, $\widetilde{\mathcal{B}}_\nu(x) \le \beta$, which implies that $(\widetilde{\mathcal{A}}^{\alpha,\beta} \cap \widetilde{\mathcal{B}}^{\alpha,\beta})(x) > 0$. In other words,

$$\min\{\widetilde{A}_M(x),\widetilde{B}_M(x)\}\le \min\{\widetilde{A}^{\alpha,\beta}(x),\widetilde{B}^{\alpha,\beta}(x)\}. \quad (2)$$

Conversely, if we suppose that $(\widetilde{A}^{\alpha,\beta}\cap\widetilde{B}^{\alpha,\beta})(x)>0$, then $\widetilde{A}^{\alpha,\beta}(x)>0$ and $\widetilde{B}^{\alpha,\beta}(x)>0$; this means that $\widetilde{A}_\mu(x)\ge\alpha$, $\widetilde{B}_\mu(x)\ge\alpha$ and $\widetilde{A}_\nu(x)\le\beta$, $\widetilde{B}_\nu(x)\le\beta$, hence $\widetilde{A}_\mu(x)\wedge\widetilde{B}_\mu(x)\ge\alpha$, $\widetilde{A}_\nu(x)\vee\widetilde{B}_\nu(x)\le\beta$. This means that $(\widetilde{A}\cap\widetilde{B})_\mu(x)\ge\alpha$, $(\widetilde{A}\cap\widetilde{B})_\nu(x)\le\beta$, that is $(\widetilde{A}\cap\widetilde{B})^{\alpha,\beta}(x)>0$. In other words,

$$\min\{\widetilde{A}_M(x),\widetilde{B}_M(x)\}\ge \min\{\widetilde{A}^{\alpha,\beta}(x),\widetilde{B}^{\alpha,\beta}(x)\}. \quad (3)$$

From (2.1) and (2.2) we conclude that

$$\min\{\widetilde{A}_M(x),\widetilde{B}_M(x)\}= \min\{\widetilde{A}^{\alpha,\beta}(x),\widetilde{B}^{\alpha,\beta}(x)\},$$

this proves the first equality. Similarly, we can prove the other equalities.

4. Intuitionistic Fuzzy Hybrid Sets

The intuitionistic fuzzy hybrid set is just multi-intuitionistic fuzzy set. The deference is that we can assign a negative occurrence to its elements. If we substitute N_0 with Z (the set of integer numbers) in Definition 3.1, the resulting definition will be the definition of intuitionistic fuzzy hybrid sets.

Definition 4.1. Let $\widetilde{A}$ and $\widetilde{B}$ be two intuitionistic fuzzy hybrid sets, then $\widetilde{A}\subseteq\widetilde{B}$ if and only if $\widetilde{A}_\mu(x)\le\widetilde{B}_\mu(x)$, $\widetilde{A}_\nu(x)\ge\widetilde{B}_\nu(x)$ and $\widetilde{A}_M(x)\ll\widetilde{B}_M(x)$, for each $x\in X$.

For each intuitionistic fuzzy hybrid set $\widetilde{A}$ in X, we will call $\pi_{\widetilde{A}}(x)=1-\widetilde{A}_\mu(x)-\widetilde{A}_\nu(x)$, the *intuitionistic index* of $\widetilde{A}$.

Definition 4.2. Let $\widetilde{A}$ be an intuitionistic fuzzy hybrid set on X . First, we define the following two cardinalities of $\widetilde{A}$:

The least cardinality of $\widetilde{A}$, and is called here $LC(\widetilde{A})$:

$$LC(\widetilde{A})=\sum_{x\in X}\widetilde{A}_\mu(x)\odot\widetilde{A}_M(x)$$

where $\odot$ is a binary relation maps $l\in I$ and $n\in Z$ to some real number.

The biggest cardinality of $\widetilde{A}$, which is possible due to $\pi_{\widetilde{A}}$, is called $BC(\widetilde{A})$:

$$BC(\widetilde{A})=\sum_{x\in X}\left(\widetilde{A}_\mu(x)+\pi_{\widetilde{A}}(x)\right)\odot\widetilde{A}_M(x)$$

Then the cardinality of intuitionistic fuzzy hybrid set $\widetilde{A}$ is defined as the interval:

$$\mathrm{card}(\widetilde{A})=[LC(\widetilde{A}),BC(\widetilde{A})].$$

If we substitute $\widetilde{A}_M(x)$ with $|\widetilde{A}_M(x)|$ in the previous definition, then the resulting structure will be called *weak* cardinality of $\widetilde{A}$.

If A and B are two crisp hybrid sets, one can not define their union and their intersection. The reason being the fact that the set of all subsets of a given hybrid set with the subsethood relation do not form a lattice (see [5]). But the sum of hybrid sets is a well defined operation. Therefore, we can easily extend its definition as follows:

Definition 4.3. Let $\widetilde{A}$ and $\widetilde{B}$ be two intuitionistic fuzzy hybrid sets on X . The sum of $\widetilde{A}$ and $\widetilde{B}$, denoted by, $\widetilde{A}\oplus\widetilde{B}$, is defined as follows:

$$(\widetilde{A}\oplus\widetilde{B})(x)=\left(\widetilde{A}_\mu(x)\vee\widetilde{B}_\mu(x),\ \widetilde{A}_\nu(x)\wedge\widetilde{B}_\nu(x),\ \widetilde{A}_M(x)+\widetilde{B}_M(x)\right).$$

Let $\{A_i\mid i\in\Gamma\}$ be a finite collection of crisp hybrid sets with a common universal set X such that each of this crisp hybrid sets contains repeated occurrence of only one element $x_i\in X$ and no two crisp hybrid set will have common elements. By $\oplus_{i\in\Gamma}A_i$, we denote the unique crisp hybrid set that is the sum of all A_i .

Definition 4.4. Let $\widetilde{A}$ be an intuitionistic fuzzy hybrid set on X . The (α,β) -cut for $\widetilde{A}$, denoted by $\widetilde{A}^{\alpha,\beta}$, is the crisp hybrid set $\oplus_{i\in\Gamma}A_i$, $A_i(x_i)=\widetilde{A}_M(x)$ if and only if $\alpha\le\widetilde{A}_\mu(x)$ and $\beta\ge\widetilde{A}_\nu(x)$, for each $x_i\in X$.

Theorem 4.1. Let $\widetilde{A}$ and $\widetilde{B}$ be two intuitionistic fuzzy hybrid sets, then the following properties are true:

(1) If $\alpha_1\le\alpha_2$ and $\beta_1\ge\beta_2$, then $\widetilde{B}^{\alpha_2,\beta_2}\subseteq\widetilde{A}^{\alpha_1,\beta_1}$.

(2) $(\widetilde{A}\oplus\widetilde{B})^{\alpha,\beta}=\widetilde{A}^{\alpha,\beta}\oplus\widetilde{B}^{\alpha,\beta}$.

Proof. The proof of part (1) is trivial. So, we will only give the proof of part (2):

Assume that $(\widetilde{A}\oplus\widetilde{B})^{\alpha,\beta}\gg 0$, then $(\widetilde{A}\oplus\widetilde{B})_\mu(x)\ge\alpha$ and $(\widetilde{A}\oplus\widetilde{B})_\nu(x)\le\beta$. Then $\widetilde{A}_\mu(x)\vee\widetilde{B}_\mu(x)\ge\alpha$ and $\widetilde{A}_\nu(x)\wedge\widetilde{B}_\nu(x)\le\beta$. This means that either $\widetilde{A}_\mu(x)\ge\alpha$ or $\widetilde{B}_\mu(x)\ge\alpha$ and either $\widetilde{A}_\nu(x)\le\beta$ or $\widetilde{B}_\nu(x)\le\beta$ or even both $\widetilde{A}_\mu(x)\ge\alpha$, $\widetilde{B}_\mu(x)\ge\alpha$ and $\widetilde{A}_\nu(x)\le\beta$, $\widetilde{B}_\nu(x)\le\beta$. Clearly, this implies that $(\widetilde{A}^{\alpha,\beta}\oplus\widetilde{B}^{\alpha,\beta})\gg 0$. In other words,

$$\widetilde{A}_M+\widetilde{B}_M(x)\ll\widetilde{A}^{\alpha,\beta}(x)+\widetilde{B}^{\alpha,\beta}(x) \quad (4)$$

Conversely, if we supposed that $(\widetilde{A}^{\alpha,\beta}\oplus\widetilde{B}^{\alpha,\beta})\gg 0$, then $\widetilde{A}^{\alpha,\beta}(x)\gg 0$ and $\widetilde{B}^{\alpha,\beta}(x)\gg 0$, this means that $\widetilde{A}_\mu(x)\ge\alpha$, $\widetilde{A}_\nu(x)\le\beta$ and $\widetilde{B}_\mu(x)\ge\alpha$, $\widetilde{B}_\nu(x)\le\beta$ and hence $\widetilde{A}_\mu(x)\vee\widetilde{B}_\mu(x)\ge\alpha$, $\widetilde{A}_\nu(x)\wedge\widetilde{B}_\nu(x)\le\beta$, which implies that

$(\widetilde{A} \oplus \widetilde{B})_{\mu}(x) \geq \alpha$, $(\widetilde{A} \oplus \widetilde{B})_{\nu}(x) \leq \beta$, that is $(\widetilde{A} \oplus \widetilde{B})^{\alpha,\beta} \gg 0$.

In other words,

$$\widetilde{A}_M + \widetilde{B}_M(x) \gg \widetilde{A}^{\alpha,\beta}(x) + \widetilde{B}^{\alpha,\beta}(x) \quad (5)$$

From the antisymmetry of $\ll$ and Equations (4) and (5), we have

$$\widetilde{A}_M + \widetilde{B}_M(x) = \widetilde{A}^{\alpha,\beta}(x) + \widetilde{B}^{\alpha,\beta}(x)$$

That is, $(\widetilde{A} \oplus \widetilde{B})^{\alpha,\beta} = \widetilde{A}^{\alpha,\beta} \oplus \widetilde{B}^{\alpha,\beta}$

5. Intuitionistic Fuzzy P Systems

Syropoulos [8, 9] has proposed fuzzified versions of **P** systems. If one replaces the multi-fuzzy sets employed in [8, 9] with the multi-intuitionistic fuzzy sets, the computational power of the resulting **P** systems will be greater and useful for modelling of living organisms.

Definition 5.1. An intuitionistic fuzzy P system is a construction:

$$IFPS = \{O, \mu, \omega^{(1)}, \ldots\ldots\ldots, \omega^{(m)}, R_1, \ldots\ldots, R_m, i_0\}$$

Where:

(1) O a set of distinct entities whose elements are called objects;
(2) μ is the membrane structure of degree $m \geq 1$; membranes are injectivelly labeled with succeeding natural numbers starting with one;
(3) $\omega^{(i)} : O \to I \times I \times Z$, $1 \leq i \leq m$, are intuitionistic fuzzy hybrid sets over O that are associated with each region i;
(4) R_i, $1 \leq i \leq m$, are finite sets of multi-set rewriting rules over O.
(5) $i_0 \in \{1,2,3,\ldots.m\}$ is the label of an elementary membrane, called the output membrane.

Theorem 5.1. An intuitionistic fuzzy P system can compute any real number.

6. Conclusion

In this paper, we have introduced multi-intuitionistic fuzzy sets and intuitionistic fuzzy hybrid sets as well as their basic operations. The new sets are an extension of multi-fuzzy sets and fuzzy hybrid sets which introduced by Yager [10]. Lastly, we used the new notions to define the intuitionistic fuzzy **P** systems. We think that the new defined systems may be quite useful for modeling of living organisms.

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