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Asset Allocation Strategies for Long-Term Investments

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<abstract>

As the life expectancy increases resulting in the aged society, the post-retirement life became one of the most important concerns of people. The long-term investment vehicles such as retirement savings and pension plans have been introduced to meet such demand of society. This paper examines the impact of asset allocation strategies on the long-term investment performance. Because of the unusually long investment horizon and the compounding effect, a suboptimal asset mix in a retirement plan can be a very costly and irreversible mistake. Instead of relying on anecdotal evidence to evaluate the merits of different allocation strategies, this paper performs various tests including stochastic dominance tests using both actual data and Monte Carlo simulated data that best fit the historical experience. The results indicate 1) the long-term investments perform better than the short-term investments, 2) the optimal asset allocation strategy for the long-term investments should be highly equity dominated.

Keywords : Asset Allocation, Long-Term Investments, Monte Carlo Simulation, Stochastic Dominance Tests, Index of Terminal Wealth

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Ⅰ. Introduction

Recently, the importance of long-term investments for a sound capital market growth is getting more attention in the market. Market participants also seem to become more and more interested in the long-term investments to prepare for their retirement. Due to the change in the family relationship, socio-economic environment and life expectancy, people are more concerned about their consumption power during their post-retirement period.

In order to meet the change in people's needs, financial institutions introduced a variety of long-term investment vehicles such as personal pension plans, retirement plans and accumulation-type funds. According to the data published by the Korean Asset Management Association, the total market size of the accumulation-type funds was 14.03 trillion won as of the end of 2005. But it has sharply grown to 64.4 trillion won as of January 2008. This increase in the market size of accumulation-type funds has been propelled by the dramatic increase in the size of stock funds. According to the Korea Exchange the size of stock funds was 9.8 trillion won at the end of 2005, but it has more than quadrupled to 46.5 trillion won by the end of 2006. It has kept growing and the size of stock funds was 106.6 trillion won as of November 2007.

The long-term investment vehicles such as pension and retirement plans are characterized by the unusually long investment horizon and the associated compounding effect. Therefore, a suboptimal asset mix in these plans can be a very costly and irreversible mistake. For example, if one dollar were invested and reinvested in stocks since 1802, it would have accumulated to nearly $8.8 million by the end of 2001. However, if the same dollar were invested in bonds, it would have grown to $13,975, still big but a mere drop compared to $8.8 millions (Siegel, 2002).

The purpose of this paper is two-fold. First, we would like to see the effectiveness of long-term investment strategy in Korea. Until recently, the investment horizon of Korean investors was very short and the turnover was very high. However, the increased concern on the post-retirement life and the longer life expectancy has induced people to consider more seriously about the long-term investment alternatives. The performance of the long-term investment is naturally of utmost importance to these investors. Second, once we show the performance of long-term investment is better, we attempt to identify the optimal asset allocation strategies for long-term investors. For this we perform a large number of simulations and stochastic dominance tests using historical asset returns in Korea from 1981 to 2007. In addition, we also perform a similar analysis based on the Monte Carlo simulated data that best fit the historical data, since using actual return data has a weakness of limited sample size and dependency across observations.

Most individuals and professional financial planners, following the practice of traditional pension plans, anecdotal evidence, and the desire for fiduciary prudence, have argued for diversification with different types of financial instruments according to the client's age and economic circumstances. A typical suggestion may be, say, 40 percent in stocks and 60 percent in bonds and money market funds.[1] Apparently, these allocation suggestions are influenced primarily by the allocation approaches employed by pension funds and balanced mutual funds without any theoretical justification or empirical support. There are, however, fundamental differences between a fund that is facing periodic cash distribution requirements and an individual who is prohibited by law from withdrawing money from the investment plan until he reaches at least 60 years of age.

Traditional wisdom suggests that an individual should pursue an aggressive investment strategy during his younger years, and then switch to a more conservative investment strategy in the older age to protect the accumulated gains from a sudden shift of capital market conditions. As will be seen later, the evidence supports the argument that even during the period close to retirement, one should not primarily invest in fixed income securities.

The main result of this study is that the optimal asset allocation for a long-term investment strategy should be equity dominated until he is close to retirement. After the aggressive investment period, the investment problems faced by the individual are logically similar to those faced by an asset management company. Factors such as mortality rates, cash distribution requirements, inflation protection and risk tolerance must be considered. These factors have been widely discussed in the retirement or pension fund literature and are not the main subjects of this investigation. In the present

1) See, for example, Lee (2008) for a list of allocation suggestions based on the investor's financial situation.

study, we are only concerned with the asset allocation decision leading up to retirement.

This paper is organized as follows. In section Ⅱ, we discuss the factors that need to be considered for asset allocation strategies by long-term investors and briefly present the literature review. In section Ⅲ, the long-term investment instruments in Korea are explained. Section Ⅳ explains the methodology, followed by a discussion of the findings in section Ⅴ. Section Ⅵ discusses the robustness of the results reported in section Ⅴ using Monte Carlo simulation method. The conclusions are in section Ⅶ.

Ⅱ. Asset Allocation for Long-Term Investments

Asset allocation has been an important issue in the investment area and there have been papers that focus on asset allocation strategies in anticipation of future economic conditions. For example, Benari (1988) proposes a relative valuation approach to determine whether a portfolio should be weighted in favor of stocks or bonds under different economic forecasts. Chen and Reichenstein (1992) examine the impact of taxation on investment allocation and show that the tax codes favor an investment in stocks for pension funds. They theorize that since pension contributions are tax deferred, stock maximizes expected tax benefits per pension dollar.

The most important debate on asset allocation has centered on asset allocation for long-term investments. Should a long-horizon investor allocate his wealth differently from a short-horizon investor? Samuelson (1969) and Merton (1969) show that if asset returns are i.i.d., an investor with power utility should choose the same asset allocation regardless of investment horizon. However, the actual asset returns are not i.i.d as evidenced by Keim and Stambaugh (1986) and Fama and French (1988, 1989). If asset returns are not i.i.d and predictable, the investment horizon may no longer be irrelevant to asset allocation decisions.

On the one hand, there have been a number of discussions that argue for the optimality of higher portfolio weights on stocks, citing proprietary studies by Ibbotson & Associates, Sanford Bernstein, Inc., etc. It is well known that the average returns over the period from 1926 to 2003 have been approximately 17.5% for small company stocks, 12.4% for large company stocks, 6.2% for long-term bonds, and 3.8% for Treasury bills

(Ibbotson and Sinquefield, 2004). Meanwhile, inflation has averaged 3.1%. Such performance pattern strongly argues in favor of investing on common stocks for investors with a long-term investment horizon. One dollar invested on the portfolio of small company stocks would have grown up to $10,954, while one dollar invested on long-term bonds would have become only $61.

This evidence supports the argument that the issue of price risk or volatility in the long-term investment plans should be different from that of an ordinary investment. First, the unusually long investment horizon makes the volatility of temporal market returns less of a concern to investors. Siegel (2002) points out that the holding period becomes an important issue in portfolio theory when the security returns do not follow the random walk process. He argues for the heavier investment on stocks for long-term investors since the relative risk of various securities changes for different time frames and the risk of stock investment declines as the investment horizon becomes longer. Second, because contributions are made regularly (usually on a monthly or quarterly basis), the investor is essentially practicing a form of forced dollar cost averaging investment approach (Rozeff, 1994). Conventional wisdom suggests that market-timing risk would be significantly reduced under this approach.

In a similar vein, Barberis (2000) examined the long-term asset allocation strategies under different economic situations depending on whether returns are predictable or not. He finds that even after incorporating parameter uncertainty, there is enough predictability in returns to make investors allocate substantially more to stocks as the investment horizon gets longer. Recently, Alestalo and Puttonen (2006) empirically investigated the strategic asset allocation in the Finnish defined benefit pension funds. They show that there is a relation between age structures and the strategic asset allocations of pension funds. In other words, the younger employees seem to invest proportionally more on equity instruments. These studies strongly suggest the optimality of using common stocks aggressively in retirement investment.

The optimality of heavier investment on stocks over safe assets is also supported by the difference in liquidity requirement between retirement funds and individuals. While it is important for a long-term investment vehicle such as a pension fund and a retirement fund to diversify its assets across a spectrum of choices among stocks, bonds, and cash equivalents, the same basic allocation strategy may not be optimal for individual investors who participate in such long-term investment plan. A long-term investment fund must meet its own current funding liabilities because of the distribution requirements of its existing retirees. To enable the retirement funds to achieve the necessary liquidity and to stabilize the value of the portfolio, investments in bonds and money market securities are necessary.

However, individuals investing in the long-term plans would not have to face these liquidity considerations for a very long while. After all, current regulations prohibit participants from withdrawing any money until the age 60 except under extenuating circumstances. Therefore, unlike typical financial planning, maintaining a measure of liquidity in investments should not be a concern for retirement plan participants before they are close to retirement. In other words, liquidity risk should not be a matter of consideration during the pre-retirement stage. In effect, the individual must therefore structure separate investment strategies to meet the needs and conditions in two distinct sub-periods, pre-retirement and post-retirement.

Differently from the above studies and arguments, there are other papers that warn against the too much emphasis on stocks in asset allocation for long-term investment. They argue that although liquidity risk and price risk are largely irrelevant to the long-term plan investors until they are close to retirement, the exposure of the portfolio to excessive amounts of default risk via non-diversified, speculative investments may result in excessive losses even in the long run. The need to control default risk exposure thus justifies diversification among asset classes.

For example, Leibowitz and Langetieg (1989) find that on average there is a 36% chance stocks may underperform bonds over a 5-year horizon applying Monte Carlo simulations on actual portfolio returns. This risk is so persistent that even over a 20-year horizon the probability of stocks underperforming remains at 24%. Their results thus question the traditional wisdom of investing in stocks for long-term growth.

Similarly, Bhide (1994) shows that a pure stock portfolio will not necessarily outperform a pure bond portfolio in the long run if the investor can lever the bond portfolio so that both portfolios have identical risk level. Modern portfolio theory suggests that a portfolio with both *levered bonds* and stocks will outperform the single asset portfolios.

Rather than relying on a static allocation scheme, fund managers must allocate their assets (levered bonds and stocks) based on their judgment about the future. While the findings are interesting for professional investment advisors, the issues raised by Bhide are largely moot for the average salaried employees, because individuals saving for retirement cannot lever their positions in the pension plans to enhance bond returns.

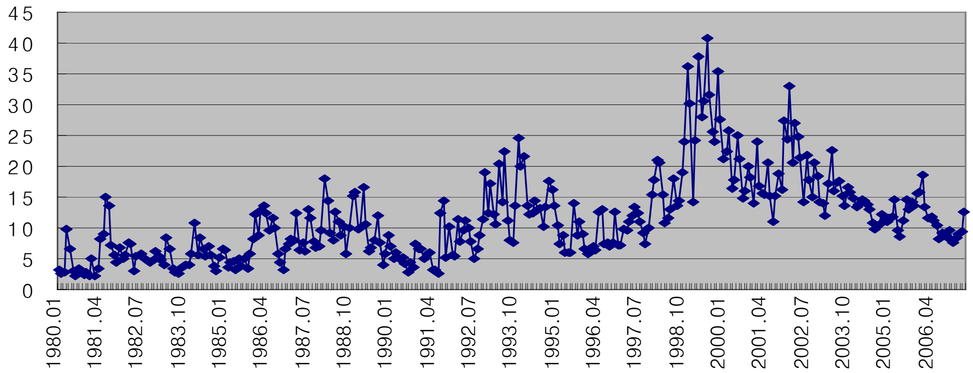
All in all, the above discussion demonstrates that the empirical results on asset allocation for long-term investments are mixed at best. Different results are obtained depending on the assumption on return generating process and the model. In this paper, we examine the performance of long-term asset allocation strategies using both actual asset return data and simulated return data that best describe the actual returns. Using actual data eliminates the problem of misspecification of the model and return generating process and the simulation makes us avoid the sample size problem.

Ⅲ. Long-Term Investments in Korea

Traditionally, many Korean investors preferred short-term investments rather than long-term investments as can be seen in [Figure 1]. The monthly turnover measured in market value is on average 11.4 times the total market capitalization over the period from January 1980 to April 2007. The tendency of short-term investment increased sharply after the IMF bailout reaching the maximum of over 40 times.

[Figure 1] Turnover Measured in Stock Market Capitalization

Source : the Bureau of Statistics.



However, the trend seems to decline after year 2000. Part of it can be explained by the increased interest in the long-term investments by individual investors. People became more concerned about their purchasing power after the retirement as the life expectancy is getting longer. Also, the rapid aging of population and very low fertility rate is pushing old generation to take care of themselves. According to the Bureau of Statistics Korea will turn into the aged society from 2018 and the super-aged society from 2026.[2] The fast aging process has made people pay more attention to the long-term investment vehicles such as pension/retirement plans.

Until recently, national pension provision in Korea has been fairly ineffective, which lead the Korean Government to submit a bill proposing a new regulatory framework for retirement schemes. The bill called 'the Employee Retirement Security Act (ERSA)' was passed by the National Assembly in late 2004 and became effective on December 1st, 2005.

According to ERSA there are three different retirement plans; defined benefit plans (DB plans), defined contribution plans (DC plans), and individual retirement accounts (IRAs).[3] In a defined benefit plan, the company promises to pay at least one month of 'final pay' for each year of service when he retires. The actual amount depends on the length of service and the pre-retirement salary. Usually, the employee is also required to contribute, on a pre-tax basis, a small percentage of the salary. Stricter guidelines apply to the funding and disclosure of information that employers must provide, and restrictions apply on withdrawing funds before age 60.

The defined contribution plan works like a savings plan, so that regular contributions are added to an account by the employer and invested in accordance with specific instructions. Employers must contribute at least 8.3 percent of the monthly payment into these accounts and additional voluntary employee contributions are allowable, with the eventual payment being linked to the value of the account. Guidelines covering investment choices, disclosure of information and withdrawal of funds before age 60

2) If the population of people aged 65 years or older is bigger than 14% (20%) of total population, it is called the 'aged (super-aged) society'.

3) There are similar retirement plans in the U.S., i.e., a defined benefit plan, a defined contribution plan, and the Keogh retirement plan for self-employed individuals.

apply. Unlike the DB plan, the company has minimal fiduciary responsibilities under the DC plan. Individuals are solely responsible for their own investment selections and the portfolio performance. As more companies seek to avoid the responsibilities and costs of managing a company pension plan, the defined contribution plan has become an increasingly popular choice for companies.[4)]

As another alternative, employers with a workforce lower than 10 employees or individuals wishing to secure additional retirement benefits can establish the Individual Retirement Accounts (IRA).

As more companies adopt the defined contribution plans than the defined benefit plans, the burden of achieving a reasonable and adequate return performance for retirement investments falls squarely on the average individuals. In light of the fact that for most people the pension plan probably represents the single largest long-term investment in the financial market, there is an urgent need for more studies on the performance of long-term investment strategies. Besides the amount of contributions, which is limited by regulations and salary, the most important decision for long-term investors is the asset allocation regarding the proportions of stocks, bonds, and/or money market funds in the plan.[5)]

Ⅳ. Research Methodology

This paper uses monthly returns for the Korean Composite Stock Price Index (KOSPI) and the average call rate during the period of February 1981 through March 2007.[6)] All data are obtained from the Bureau of Statistics. Besides, in order to overcome the problems of using actual calendar returns, this study also employs Monte Carlo simulation to sample from populations based on the observed means and variances

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4) Fenner (1992) reports that more than 80% of all pension programs in the U.S. are defined contribution plans.

5) The influence of money market funds on the long-term performance is minimal. For simplicity, this study only considers the performance of two most popular generic investments, i.e. the bond portfolio and the stock portfolio.

6) This is the longest time series we can get in Korea. In addition, since our main concern is on asset allocation between risky and safe assets rather than asset allocation among different asset classes, the call rate can serve as a proxy for the return on safe asset, i.e. bonds.

without limiting the observations to the historical data itself.[7)] The simulation approach provides generalized probabilistic analysis of asset allocation strategies based on repeated sampling from probability distributions rather than using historical data exclusively.

We form five portfolios by allocating different percentage weights between stocks and bonds. For each portfolio, two measures of portfolio performance, the average holding period return and the index of terminal wealth, are derived over various investment horizons. Given the historical performance of common stocks, an investment strategy with a greater weight on stocks may appear to naturally lead to a higher long-term return. However, because an average investor would not have a holding period that starts in 1981 and spans close to 27 years, the real issue that needs to be addressed empirically is whether, for any starting date and any specific long-term investment horizon, the expected return of a portfolio heavily weighted in stocks would be significantly higher than that of a portfolio more heavily invested in bonds. In order to examine this issue, this study computes the frequency of positive holding period equity risk premia for each allocation strategy and also evaluates the portfolios in terms of first and second-degree stochastic dominance criteria.

The specifics of the methodology are discussed in the following. The notation $S_{i,j}(w_s, w_b)$ is used for each investment strategy where i represents the total investment period. This study considers four different investment horizons, i.e., i = 5, 10, 15, and 20 years. Each investment horizon is divided into two sub-periods, an aggressive and a conservative investment period. The subscript j denotes the length of conservative investment period for a given investment horizon, i. To illustrate, if the total investment horizon, i, is 10 years and the conservative investment period, j, is 2 years, the aggressive investment period would be 8 years. For simplicity, we assume that during the conservative investment period an individual simply allocates 100% of his wealth accumulated over the aggressive investment period to a very safe bond. We consider various conservative investment periods up to 2.5 years. Historically, none of the bear

7) The populations of Monte Carlo simulation are based on the data for the period from January 1999 to March 2007. This is to reflect the impact of the Fourth Liberalization of Interest Rate (1997) and eliminate the abnormality caused by the IMF bailout (1997~1998) in the Korean financial market.

markets (defined as a 20 percent decline from the peak level) lasted longer than 2.5 years.[8)]

The portfolio weight on stocks and bonds for the aggressive investment period (i.e., i-j years) is denoted as w_s and w_b, respectively. For this analysis we consider the following five portfolio weights; $S_{i,j}(100, 0)$, $S_{i,j}(75, 25)$, $S_{i,j}(50, 50)$, $S_{i,j}(25, 75)$, and $S_{i,j}(0, 100)$. $S_{i,j}(100, 0)$ is the most aggressive portfolio strategy with a 100% investment in stocks and 0% in bonds during the aggressive investment period, while $S_{i,j}(0, 100)$ is the most conservative strategy with a 100% in bonds.

The average holding period return and the index of terminal wealth for each investment strategy $S_{i,j}(w_s, w_b)$ are calculated as follows :

1. Average Holding Period Returns

Let $HPR_{i,j}(w_s, w_b)$ be the annualized monthly holding period return of strategy $S_{i,j}(w_s, w_b)$. Then,

$$HPR_{i,j}(w_s, w_b) = \left[\left(\prod_{n=1}^{12(i-j)} [1 + r_{p,n}(w_s, w_b)] \prod_{z=12(i-j)+1}^{12i} (1 + r_{b,z}) \right)^{\frac{1}{12i}} - 1 \right] * 12 \qquad (1)$$

where

i = 5, 10, 15, 20 ;

j = 0, 0.5, 1, 1.5, 2, 2.5 ;

(w_s, w_b) = (100, 0), (75, 25), (50, 50), (25, 75), (0, 100) ;

$r_{p,n}(w_s, w_b)$ = the portfolio return in the n-th month of the first sub-period (aggressive investment period) ; and

$r_{b,z}$ = the bond yield in the z-th month of the second sub-period (conservative investment period).

For example, consider a strategy $S_{10,2}(75, 25)$. Using the monthly data of 10 years

8) According to the Bureau of Statistics, the average business cycle of Korea is 53 months that are composed of 33-month expansion period and 19-month contraction period.

(February 1981 to January 1991), we compute the annualized 10-year holding period (120 months) return resulting from 8 years (February 1981 to January 1989) of investment on a portfolio with 75% in stocks and 25% in bonds and the remaining 2 years (February 1989 to January 1991) with 100% in bonds. Formally, rewriting equation (1), we obtain

$$HPR_{10,2}(75,25)=\left[\left(\prod_{n=1}^{96}[1+r_{p,n}(75,25)]\prod_{z=97}^{120}(1+r_{b,z})\right)^{\frac{1}{120}}-1\right]*12 \tag{2}$$

We then roll over this 10-year window forward by one month and recalculate the portfolio return. That is, we compute the second 10-year holding period return using the data of the subsequent 120 months (March 1981 to February 1991), maintaining the same investment horizons with the same durations for the first and second sub-periods (8 and 2 years) and the same portfolio weights (75, 25) for the first sub-period. The process is repeated month by month until the last 10-year period return through March 2007 is calculated. Then the historical average of these 218 annualized 10-year holding period returns on the strategy $S_{10,2}(75, 25)$ is computed. The average holding period returns for other strategies are computed in the same manner.

2. Index of Terminal Wealth

The average holding period returns do not account for the fact that accumulation-type funds typically require regular contributions which increase the portfolio size throughout the holding period. To allow for the compounding effect on each periodic contribution during the holding period, we derive a second measure of portfolio performance as follows. First, we assume that the periodic contributions would grow at the rate of g% annually. Then, the amount of accumulation up to the end of t-th period, $AC_t(w_s, w_b)$, can be shown as

$$\begin{aligned} AC_t(w_s,w_b) &= w_s[AC_{t-1}(w_s,w_b)+(1+g)^{t-1}](1+r_{s,t}) \\ &\quad + w_b[AC_{t-1}(w_s,w_b)+(1+g)^{t-1}](1+r_{b,t}) \\ &\qquad \forall\, t \in \text{aggressive investment period, and} \end{aligned} \tag{3}$$

$$AC_t(w_s, w_b) = [AC_{t-1}(w_s, w_b) + (1+g)^{t-1}](1+r_{b,t}) \quad (4)$$
$$\forall\, t \in \text{conservative investment period,}$$

where $r_{s,t}$ is the stock return over the t-th period, $r_{b,t}$ is the bond yield over the t-th period, and $(1+g)^{t-1}$ is the amount of contribution in the t-th period. Note that the portfolio is rebalanced at the beginning of each period to maintain the fixed portfolio weight (w_s, w_b). Solving recursively Equations (3) and (4), we compute the total compounded accumulation at the end of the holding period for monthly contributions as follows :

$$TAC_{i,j}(w_s, w_b) = \sum_{m=1}^{12(i-j)} \left((1+g)^{m-1} \prod_{n=m}^{12(i-j)} [1 + r_{p,n}(w_s, w_b)] \prod_{z=12(i-j)+1}^{12i} (1 + r_{b,z}) \right) + \sum_{m=12(i-j)+1}^{12i} \left((1+g)^{m-1} \prod_{z=m}^{12i} (1 + r_{b,z}) \right) \quad (5)$$

where $TAC_{i,j}(w_s, w_b)$ is the total accumulation following portfolio strategy $S_{i,j}(w_s, w_b)$, $r_{p,n}(w_s, w_b)$ is the portfolio return in the n-th month of the first sub-period, and $r_{b,z}$ is the bond yield in the z-th month of the second sub-period.

Finally, we derive the index of terminal wealth, $IDX_{i,j}(w_s, w_b)$, by dividing the total accumulation by the total contributions made during the holding period as follows:[9]

$$IDX_{i,j}(w_s, w_b) = \frac{TAC_{i,j}(w_s, w_b)}{\dfrac{(1+g)^{12i} - 1}{g}} \quad (6)$$

As an illustration, the index of terminal wealth of strategy $S_{10,2}(75, 25)$, $IDX_{10,2}(75, 25)$, is :

$$IDX_{10,2}(75,25) = \left(\frac{g}{(1+g)^{120} - 1} \right) \sum_{m=1}^{96} \left((1+g)^{m-1} \prod_{n=m}^{96} [1 + r_{p,n}(75,25)] \prod_{z=97}^{120} (1 + r_{b,z}) \right) + \left(\frac{g}{(1+g)^{120} - 1} \right) \sum_{m=97}^{120} \left((1+g)^{m-1} \prod_{z=m}^{120} (1 + r_{b,z}) \right) \quad (7)$$

9) Direct comparison of TACs with different investment periods would not be appropriate, since the sizes of portfolios using different strategies are varied. For example, the TAC of a portfolio with a 15-year investment horizon will necessarily be grater than that with a 10-year investment period.

We compute the index of terminal wealth using monthly data for each 10-year horizon within the sample period. Then the historical average of these 218 indexes of terminal wealth from strategy $S_{10,2}(75, 25)$ is obtained. Indexes of terminal wealth for other strategies are calculated in the same manner. Mathematically, $IDX_{1,j}(w_s,w_b)$ in Equation (6) measures the performance of each asset allocation alternative relative to a strategy of putting money in the savings account mechanically every month whereby the amount of deposit increases by g% every period. In this study, we simply assume g% to be 6% since generally most annuities yield between 5% and 7%.

3. Monte Carlo Simulation

We calculate the holding period return and the index of terminal wealth using the actual return data in the previous analysis. The use of actual calendar returns produces a limited number of non-overlapping observations. One way to overcome this limitation is to simulate the future by assuming that future returns come from a stable probability distribution. This approach implicitly assumes that returns are independent across periods. In order to check the robustness of the results derived from the actual data we employed the second approach assuming independency of the holding period returns. For this purpose, we use Monte Carlo simulation to generate sample return distribution for different portfolio strategies of holding periods of 5 to 20 years.

The detailed procedure is as follows. First of all, we try to determine theoretical return distributions for stocks and bonds that most closely describe our empirical distributions. In order to measure how well the sample data fit a hypothesized probability density function, we use the Chi-Square goodness-of-fit tests. The Chi-Square test is the most common goodness-of-fit test. It can be used with any sample input data and any type of distribution function (discrete or continuous). The null hypothesis of the Chi-Square test is that the data follow a specified distribution. The fit statistics of Chi-Square test are calculated using @RISK software so as to assess the fitted distribution of bond and stock returns. <Table 1> shows the results of the Chi-Square test. The results indicate that our empirical return data seem to be generated from lognormal distribution for bonds and normal distribution for stocks.

<Table 1> The Estimation of Probability Density Function

In order to measure how well the sample data fit a hypothesized probability density function, we use the Chi-Square goodness-of-fit tests. The null hypothesis of the Chi-Square test is that the data follow a specified distribution. The fit statistics of Chi-Square test are calculated using @RISK software so as to assess the fitted distribution of bond and stock returns. *** $p < 0.01$.

Distribution function type	Chi-Square Test	
	Bond	Stock
Lognormal	17.56(0.063)	210.7(0.000)***
Normal	24.67(0.006)***	8.667(0.564)
Uniform	48.44(0.000)***	62.89(0.000)***
Exponential	48.67(0.000)***	84.44(0.000)***

In the next step, Monte Carlo simulations have been performed to determine the posterior model probabilities as a function of the number of samples (n = 60 (months) * 1,000 (times), 120 (months) * 1,000 (times), 180 (months) * 1,000(times), 240 (months) * 1,000 (times)), and the distribution type (normal and lognormal) from which the samples were generated.

Specifically, Monte Carlo simulations have been performed to generate monthly returns for bonds and stocks from the above distributions for each holding period, i.e. i = 5, 10, 15 and 20 years. For the sake of explanation, let's assume i = 10 years. Then, the computer will generate 120 monthly return series for bonds and stocks. Based on these simulated 10-year returns on bonds and stocks, the annualized monthly holding period returns of strategy $S_{10,j}(w_s, w_b)$ are calculated for each j and portfolio weights. After calculating the first set of holding period returns, we generate another 10-year return sequence for bonds and stocks and compute the second set of holding period returns. This process is repeated 1,000 times to get the distribution of holding period returns for strategy $S_{10j}(w_s, w_b)$. For other holding periods, we follow the same procedure to obtain the simulated probability distribution of holding period returns for each j and portfolio weights.

We follow a similar procedure to generate the sample IDX distribution for different portfolio strategies $S_{i,j}(w_s, w_b)$ so as to check the robustness of the IDX results generated by historical return data. Specifically, we first generate monthly returns for bonds and stocks from lognormal and normal distribution for each investment horizon. If i = 10, the computer will generate 120 monthly return series for bonds and stocks. Based on these simulated 10-year returns on bonds and stocks, the IDX of strategy $S_{10,j}(w_s, w_b)$ are calculated for each j and portfolio weights. After calculating the first set of IDXs, we generate another 10-year return sequence for bonds and stocks and compute the second set of IDXs. This process is repeated 1,000 times to get the distribution of IDXs for strategy $S_{10,j}(w_s, w_b)$. For other holding periods, we follow the same procedure to obtain the simulated probability distribution of IDXs for each j and portfolio weights.

V. Empirical Results Using Actual Data

1. Long-term vs. Short-term Investment

<Table 2> reports the annualized average holding period returns (HPRs) and the associated standard deviations for the various strategies and horizons using actual data. There are four total investment periods ; 5, 10, 15, and 20 years. The first column indicates the conservative investment period j. The second column shows the investment strategies with different portfolio weights on stocks and bonds over the aggressive investment sub-period (i.e., i-j). Strategy A has the largest (smallest) weight on stocks (bonds), i.e., (100, 0) and strategy E has the smallest (largest) weight on stocks (bonds), i.e., (0, 100).

Our first concern is whether the performance of the long-term investment strategy is better than the performance of the short-term strategy. If we compare only the means of 5-, 10-, 15-, and 20-year strategies for the same portfolio weights, we cannot say the HPR of long-term investment strategy is always higher than the HPR of short-term investment strategy. However, if we consider both the average returns and the standard deviations, we may find that the long-term investment strategy is superior to the short-term investment strategy. In fact, the results in <Table 3> show that the long-term investment is better than the short-term investment. The Sharpe Index of long-term strategy is bigger than that of short-term strategy for the same portfolio weight strategies. As the investment horizon becomes longer the standard deviation becomes smaller, as argued by Siegel (2002). This induces the Sharpe Index of the long-term investment to be higher.

<Table 2> Annualized Monthly Average Holding Period Returns

This table shows the annualized monthly average holding period returns of different portfolio strategies. The figures are calculated using the KOSPI index and bond yield data over the period of February 1981 to March 2007. [a] i = the total investment period, [b] j = the length of the conservative investment period, [c] $S_{i,j}$ (w_s, w_b) = the investment strategy where w_s (w_b) represents the portfolio weight on stocks (bonds), A = $S_{i,j}$ (100, 0), B = $S_{i,j}$ (75, 25), C = $S_{i,j}$ (50, 50), D = $S_{i,j}$ (25, 75), E = $S_{i,j}$ (0, 100).

		5 years [a]		10 years		15 years		20 years	
		Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
j = 0 [b]	A[c]	0.1438	0.1180	0.1244	0.0461	0.1240	0.0398	0.1438	0.0120
	B	0.1373	0.0876	0.1246	0.0354	0.1243	0.0315	0.1374	0.0079
	C	0.1276	0.0589	0.1216	0.0265	0.1214	0.0236	0.1278	0.0042
	D	0.1148	0.0359	0.1153	0.0212	0.1153	0.0166	0.1150	0.0035
	E	0.0987	0.0339	0.1058	0.0223	0.1059	0.0122	0.0989	0.0069
j = 0.5	A	0.1370	0.1193	0.1232	0.0497	0.1217	0.0413	0.1403	0.0109
	B	0.1316	0.0890	0.1234	0.0384	0.1224	0.0326	0.1347	0.0069
	C	0.1235	0.0601	0.1206	0.0286	0.1200	0.0243	0.1260	0.0033
	D	0.1126	0.0368	0.1148	0.0223	0.1146	0.0169	0.1141	0.0032
	E	0.0987	0.0339	0.1058	0.0223	0.1059	0.0122	0.0989	0.0069
j = 1.0	A	0.1303	0.1208	0.1230	0.0558	0.1196	0.0426	0.1372	0.0107
	B	0.1261	0.0906	0.1230	0.0432	0.1206	0.0336	0.1323	0.0070
	C	0.1195	0.0618	0.1202	0.0320	0.1187	0.0250	0.1243	0.0038
	D	0.1104	0.0381	0.1144	0.0238	0.1138	0.0174	0.1132	0.0036
	E	0.0987	0.0339	0.1058	0.0223	0.1059	0.0122	0.0989	0.0069
j = 1.5	A	0.1238	0.1198	0.1235	0.0621	0.1181	0.0443	0.1347	0.0092
	B	0.1208	0.0904	0.1231	0.0482	0.1193	0.0349	0.1303	0.0063
	C	0.1156	0.0624	0.1200	0.0355	0.1177	0.0260	0.1229	0.0041
	D	0.1083	0.0391	0.1143	0.0255	0.1133	0.0180	0.1124	0.0043
	E	0.0987	0.0339	0.1058	0.0223	0.1059	0.0122	0.0989	0.0069
j = 2.0	A	0.1185	0.1153	0.1250	0.0677	0.1172	0.0449	0.1348	0.0091
	B	0.1163	0.0876	0.1239	0.0526	0.1184	0.0354	0.1303	0.0070
	C	0.1123	0.0612	0.1204	0.0385	0.1170	0.0265	0.1228	0.0055
	D	0.1065	0.0394	0.1144	0.0269	0.1128	0.0184	0.1123	0.0054
	E	0.0987	0.0339	0.1058	0.0223	0.1059	0.0122	0.0989	0.0069
j = 2.5	A	0.1138	0.1096	0.1263	0.0711	0.1163	0.0445	0.1362	0.0102
	B	0.1123	0.0838	0.1246	0.0552	0.1175	0.0351	0.1311	0.0084
	C	0.1093	0.0594	0.1207	0.0403	0.1162	0.0264	0.1233	0.0069
	D	0.1048	0.0394	0.1144	0.0279	0.1124	0.0184	0.1125	0.0064
	E	0.0987	0.0339	0.1058	0.0223	0.1059	0.0122	0.0989	0.0069

<Table 3> The Sharpe Ratios for Holding Period Returns of Different Portfolio Strategies

The numbers in the table are generated by using the following definition of excess holding period return, $D_{i,j}$.

$$D_{i,j}(w_s, w_b) = \left[\left(\prod_{n=1}^{12(i-j)} [1 + (r_{p,n}(w_s, w_b) - r_{b,z})] \prod_{z=12(i-j)+1}^{12i} (1+0) \right)^{\frac{1}{12i}} - 1 \right] *12$$

The difference between $HPR_{i,j}$ and $D_{i,j}$ lies in differencing the portfolio return based on the bench-mark return, $r_{b,z}$. The other procedure is identical to that of HPR. $\overline{D}$ is the average of $D_{i,j}$ and σ_{D_t} is the standard deviation of $D_{i,j}$.

		5 years			10 years			15 years			20 years		
		$\overline{D}$ (Mean)	σ_{D_t} (Std.)	Sharpe Ratio	$\overline{D}$ (Mean)	σ_{D_t} (Std.)	Sharpe Ratio	$\overline{D}$ (Mean)	σ_{D_t} (Std.)	Sharpe Ratio	$\overline{D}$ (Mean)	σ_{D_t} (Std.)	Sharpe Ratio
j = 0	A	0.0451	0.1257	0.36	0.0186	0.0468	0.40	0.0181	0.0340	0.53	0.0449	0.0178	2.53
	B	0.0385	0.0943	0.41	0.0188	0.0349	0.54	0.0184	0.0255	0.72	0.0385	0.0134	2.87
	C	0.0289	0.0629	0.46	0.0158	0.0231	0.68	0.0155	0.0170	0.91	0.0289	0.0090	3.21
	D	0.0160	0.0315	0.51	0.0095	0.0115	0.83	0.0094	0.0085	1.10	0.0161	0.0045	3.56
	E	-	-	-	-	-	-	-	-	-	-	-	-
j = 0.5	A	0.0382	0.1262	0.30	0.0174	0.0490	0.36	0.0158	0.0352	0.45	0.0415	0.0170	2.44
	B	0.0329	0.0948	0.35	0.0176	0.0366	0.48	0.0165	0.0263	0.63	0.0359	0.0129	2.78
	C	0.0248	0.0633	0.39	0.0148	0.0243	0.61	0.0141	0.0175	0.81	0.0271	0.0087	3.12
	D	0.0138	0.0317	0.44	0.0090	0.0121	0.74	0.0086	0.0087	0.99	0.0152	0.0044	3.47
	E	-	-	-	-	-	-	-	-	-	-	-	-
j = 1.0	A	0.0315	0.1263	0.25	0.0173	0.0535	0.32	0.0137	0.0359	0.38	0.0383	0.0163	2.35
	B	0.0273	0.0949	0.29	0.0172	0.0401	0.43	0.0147	0.0267	0.55	0.0334	0.0124	2.69
	C	0.0207	0.0635	0.33	0.0144	0.0267	0.54	0.0128	0.0177	0.72	0.0254	0.0084	3.02
	D	0.0116	0.0319	0.36	0.0087	0.0133	0.65	0.0079	0.0088	0.90	0.0143	0.0043	3.36
	E	-	-	-	-	-	-	-	-	-	-	-	-
j = 1.5	A	0.0251	0.1235	0.20	0.0177	0.0583	0.30	0.0122	0.0366	0.33	0.0358	0.0139	2.57
	B	0.0220	0.0930	0.24	0.0173	0.0436	0.40	0.0134	0.0271	0.49	0.0314	0.0107	2.94
	C	0.0169	0.0622	0.27	0.0143	0.0290	0.49	0.0118	0.0179	0.66	0.0240	0.0073	3.31
	D	0.0095	0.0313	0.30	0.0085	0.0145	0.59	0.0073	0.0089	0.83	0.0135	0.0037	3.66
	E	-	-	-	-	-	-	-	-	-	-	-	-
j = 2.0	A	0.0198	0.1174	0.17	0.0192	0.0630	0.30	0.0113	0.0366	0.31	0.0360	0.0114	3.16
	B	0.0176	0.0884	0.20	0.0182	0.0472	0.38	0.0125	0.0270	0.46	0.0314	0.0087	3.59
	C	0.0136	0.0592	0.23	0.0146	0.0314	0.47	0.0110	0.0177	0.62	0.0239	0.0060	4.00
	D	0.0077	0.0298	0.26	0.0086	0.0157	0.55	0.0069	0.0087	0.79	0.0135	0.0031	4.40
	E	-	-	-	-	-	-	-	-	-	-	-	-
j = 2.5	A	0.0150	0.1100	0.14	0.0205	0.0658	0.31	0.0103	0.0357	0.29	0.0373	0.0095	3.94
	B	0.0136	0.0829	0.16	0.0188	0.0492	0.38	0.0116	0.0262	0.44	0.0323	0.0072	4.46
	C	0.0106	0.0556	0.19	0.0149	0.0328	0.46	0.0103	0.0172	0.60	0.0244	0.0049	4.95
	D	0.0061	0.0280	0.22	0.0087	0.0164	0.53	0.0065	0.0084	0.77	0.0136	0.0025	5.43
	E	-	-	-	-	-	-	-	-	-	-	-	-

<Table 6>, <Table 7> present the results using the index of terminal wealth (IDX) from Equation (6). The better performance of the long-term strategy over the short-term strategy given same portfolio weights is more strongly supported by the result in <Table 6>, <Table 7>. The average IDXs of long-term strategies are bigger than those of short-term strategies as shown in <Table 6>. The same conclusion can be drawn when the average IDX is normalized by the standard deviation as in <Table 7>. Therefore, we can conjecture that the long-term investment is preferable to the short-term investment.

2. Asset Allocation for Long-Term Investments

Since we found that the long-term investment is better than the short-term investment, our next question is what the optimal portfolio weights between risky and safe assets are for the long-term investment strategies. The results in <Table 2> show that the average returns of strategies with the same investment horizon tend to increase monotonically with higher weights on stocks. To further corroborate this observation, we perform the pairwise t-tests on the holding period returns of the five different investment strategies for each investment horizon.

Results are presented in <Table 4>. Evidently, returns for investment strategies with a greater portfolio weight on stocks tend to be significantly higher than those with a lower equity weight for all investment horizons. The t-values become higher as we compare two portfolio strategies with a larger difference in the weights on stocks. Therefore, it appears that long-term investors should adopt a very aggressive investment strategy with a high proportion of equity securities.

A more critical issue is whether aggressive investment strategies are superior to conservative ones even when we compare strategies in light of both return and risk factors. <Table 5> shows the frequency of a portfolio with a larger stock weight outperforming another portfolio with a smaller stock weight. The results demonstrate that the aggressive investment strategies perform better than the conservative ones even when we consider both return and risk factors. For example, in case of the 20-year holding period with $j = 2$, a strategy with a 100% stock allocation outperforms the (75, 25) strategy more than 95 percent of the time (D1). Similarly, returns from a portfolio

<Table 4> Pairwise t-tests on Holding Period Returns of Different Portfolio Strategies

The figures stand for the t-values of the mean tests between the holding period returns of two different portfolio strategies. They are calculated using the KOSPI index and call rate data over the period from February 1981 to March 2007. A-B = $HPR_{i,j}$ (100, 0)-$HPR_{i,j}$ (75, 25); B-C = $HPR_{i,j}$ (75, 25) -$HPR_{i,j}$ (50, 50); C-D = $HPR_{i,j}$ (50, 50)-$HPR_{i,j}$ (25, 75); D-E = $HPR_{i,j}$ (25, 75)-$HPR_{i,j}$ (0, 100). [a] i = the total investment period, *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

		j = 0	j = 0.5	j = 1.0	j = 1.5	j = 2.0	j = 2.5
5[a] Years	A-B	3.235***	2.635***	2.060**	1.543	1.177	0.848
	A-C	4.007***	3.320***	2.663***	2.078**	1.658*	1.273
	A-D	4.784***	4.009***	3.268***	2.615***	2.139**	1.699*
	A-E	5.569***	4.702***	3.875***	3.152***	2.620***	2.123**
	B-C	4.778***	4.004***	3.263***	2.610**	2.135**	1.695*
	B-D	5.558***	4.693***	3.866***	3.144***	2.613**	2.117**
	B-E	6.343***	5.385***	4.472***	3.679***	3.091***	2.539**
	C-D	6.335***	5.378***	4.466***	3.674***	3.086***	2.535**
	C-E	7.121***	6.069***	5.069***	4.205***	3.561***	2.953***
	D-E	7.901***	6.754***	5.665***	4.730***	4.030***	3.366***
10 Years	A-B	-0.189	-0.221	0.020	0.372	0.884	1.326
	A-C	1.614	1.416	1.436	1.594	1.942*	2.269**
	A-D	3.466***	3.089***	2.879***	2.838***	3.017***	3.227***
	A-E	5.367***	4.797***	4.347***	4.103***	4.110***	4.201***
	B-C	3.447***	3.072***	2.864***	2.825***	3.006***	3.217***
	B-D	5.334***	4.768***	4.322***	4.081***	4.092***	4.184***
	B-E	7.267***	6.496***	5.803***	5.355***	5.193***	5.166***
	C-D	7.242***	6.474***	5.784***	5.340***	5.180***	5.154***
	C-E	9.203***	8.218***	7.275***	6.621***	6.288***	6.143***
	D-E	11.174***	9.961***	8.761***	7.899***	7.394***	7.130***
15 Years	A-B	-0.387	-0.826	-1.196	-1.356	-1.376	-1.447
	A-C	1.670*	1.060	0.553	0.262	0.151	0.026
	A-D	3.758***	2.988***	2.355**	1.942*	1.747*	1.574
	A-E	5.873***	4.957***	4.210***	3.686***	3.415***	3.202***
	B-C	3.739***	2.970***	2.338**	1.925*	1.730*	1.558
	B-D	5.842***	4.927***	4.181***	3.657***	3.386***	3.174***
	B-E	7.963***	6.917***	6.071***	5.450***	5.114***	4.872***
	C-D	7.941***	6.895***	6.049***	5.429***	5.092***	4.850***
	C-E	10.062***	8.901***	7.971***	7.269***	6.879***	6.620***
	D-E	12.162***	10.903 ***	9.905***	9.141***	8.711***	8.451***
20 Years	A-B	11.547***	10.700***	9.915***	10.579***	13.581***	17.498***
	A-C	14.339***	13.555***	12.817***	13.887***	17.494***	22.124***
	A-D	17.119***	16.381***	15.666***	17.094***	21.257***	26.632***
	A-E	19.897***	19.188***	18.472***	20.215***	24.881***	31.002***
	B-C	17.088***	16.349***	15.633***	17.057***	21.211***	26.580***
	B-D	19.843***	19.133***	18.417***	20.153***	24.806***	30.915***
	B-E	22.600***	21.903***	21.164***	23.169***	28.267***	35.094***
	C-D	22.560***	21.862***	21.123***	23.123***	28.213***	35.030***
	C-E	25.300***	24.599***	23.814***	26.043***	31.520***	39.007***
	D-E	28.005***	27.284***	26.432***	28.847***	34.651***	42.741***

strategy with 75% in stocks and 25% in bonds are *always* greater than those of (50, 50) strategy (D2). In case of the 20-year holding period with j = 2.5, during the entire 26 year span from 1981 to 2007 with its seventy six 20-year moving average periods, returns for portfolios with a larger equity position are *always higher* than those with a lower weight on stocks in each moving average period. Therefore, the higher standard deviations associated with aggressive portfolio strategies do not *in reality* translate into additional risk for the long-term investors.

<Table 5> Differences in Annualized Monthly Average Holding Period Returns

The figures are calculated using domestic stock index and call rate over the period from February 1981 to March 2007. D1 = $HPR_{i,j}$ (100, 0)−$HPR_{i,j}$ (75, 25); D2 = $HPR_{i,j}$ (75, 25)−$HPR_{i,j}$ (50, 50); D3 = $HPR_{i,j}$ (50, 50)−$HPR_{i,j}$ (25, 75); and D4 = $HPR_{i,j}$ (25, 75)−$HPR_{i,j}$ (0, 100). [a] i = the total investment period, [b] j = the length of the conservative investment period, [c] The percentage of times when the value of each variable is greater than zero.

i [a]		j = 0 [b]		j = 0.5		j = 1.0		j = 1.5		j = 2.0		j = 2.5	
		Mean	% [c]	Mean	%	Mean	%	Mean	%	Mean	%	Mean	%
5	D1	0.66	52.7	0.53	48.6	0.42	47.7	0.30	50.6	0.22	46.9	0.15	44.9
	D2	0.97	56.8	0.81	53.1	0.66	49.8	0.52	53.1	0.40	51.0	0.30	45.3
	D3	1.28	60.5	1.09	58.4	0.91	53.1	0.73	53.9	0.59	53.9	0.45	48.1
	D4	1.60	63.4	1.38	60.5	1.16	56.8	0.95	55.1	0.77	55.1	0.61	50.2
10	D1	−0.02	54.9	−0.02	48.9	0.00	51.1	0.04	51.6	0.10	57.1	0.16	57.1
	D2	0.30	63.7	0.28	65.4	0.29	60.4	0.31	61.0	0.35	65.4	0.39	66.5
	D3	0.62	70.9	0.59	70.9	0.57	69.8	0.58	73.1	0.60	74.2	0.63	74.7
	D4	0.95	76.4	0.90	73.1	0.87	78.0	0.85	81.3	0.86	79.1	0.87	77.5
15	D1	−0.03	46.7	−0.07	46.7	−0.10	46.7	−0.12	45.1	−0.12	50.8	−0.12	55.7
	D2	0.29	55.7	0.24	59.8	0.19	64.8	0.16	61.5	0.15	60.7	0.13	61.5
	D3	0.61	68.0	0.55	73.0	0.49	72.1	0.44	73.0	0.41	65.6	0.38	64.8
	D4	0.94	88.5	0.86	78.7	0.79	78.7	0.73	78.7	0.69	77.0	0.65	75.4
20	D1	0.64	88.7	0.56	91.9	0.49	88.7	0.44	85.5	0.46	95.2	0.51	100.0
	D2	0.96	100.0	0.87	100.0	0.80	96.8	0.74	100.0	0.75	100.0	0.79	100.0
	D3	1.28	100.0	1.19	100.0	1.11	100.0	1.04	100.0	1.04	100.0	1.07	100.0
	D4	1.61	100.0	1.52	100.0	1.43	100.0	1.35	100.0	1.35	100.0	1.36	100.0

Theoretically, because there are no cash distribution requirements/possibilities for long-term investors such as the pension/retirement plan participants, the standard deviation of portfolio returns may not be an appropriate measure of risk during the applicable holding period horizon. Volatility only matters *after* retirement when there are regular and occasionally unexpected cash requirements.[10] Before retirement, the real risk faced by the investor lies in selecting a strategy that, in terms of portfolio return or terminal wealth accumulation, underperforms other strategies for a significant portion of the time during the pre-retirement period.

Then, the optimal portfolio strategy is the one that generates the highest expected return or terminal wealth with a very high degree of certainty. <Table 4>, <Table 5> indicate that a portfolio dominated by equity securities is empirically such an optimal strategy in the long run. In sum, although a strategy with a higher proportion of stocks usually generates a higher standard deviation, such additional volatility does not really matter to the long-term investors. Barberis (2000) got similar result when asset returns are predictable. In his model, a buy-and-hold investor invests substantially more in risky equities in the presence of asset return predictability, the longer his horizon. Time variation in expected returns induces mean-reversion in returns, slowing the growth of conditional variances of long-horizon returns (Fama and French, 1988; Poterba and Summers, 1988). This makes equities appear less risky at long horizons, and hence more attractive to the investor.

Of course, this observation is made based on the analysis that was performed with one sequence of historical returns on stocks and bonds, and we know history may not repeat itself.[11] But the odds are very much in the long-term investor's favor as can be shown later in the Monte Carlo simulation results that assume independency of the return generating process.

10) Obviously, one would want the expected dollar return from the pension portfolio to be as close as possible to the cash requirements after retirement. That is why volatility of the portfolio would be a concern after retirement.

11) The limitation of research in this area is that we only observe one realized sequence of returns without knowing the true return generating process. We could obtain more reliable results if we can get a longer time series data due to the law of large numbers. Unfortunately, the time series we used is so far the longest in Korea. Alternatively, we may theoretically assume a particular return generating process, but this option is subject to the model misspecification problem.

<Table 6> Indexes of Terminal Wealth for Different Portfolio Strategies

The indexes are calculated by comparing a specific asset allocation strategy with a base strategy that assumes the annualized growth rate of periodic contributions of 6% per period (see Equation (6)). The sample period is from February 1981 to March 2007. [a] i = the total investment period, [b] j = the length of the conservative investment period, [c] $S_{i,j}$ (w_s, w_b) = the investment strategy where w_s (w_b) represents the portfolio weight on stocks (bonds), A = $S_{i,j}$ (100, 0), B = $S_{i,j}$ (75, 25), C = $S_{i,j}$ (50, 50), D = $S_{i,j}$ (25, 75), E = $S_{i,j}$ (0, 100).

		5 years [a]		10 years		15 years		20 years	
		Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
j = 0 [b]	A[c]	3.0225	1.2216	3.0712	1.0063	3.4092	1.0591	4.0793	0.5880
	B	2.8275	0.9661	2.9720	0.7981	3.3111	0.7746	3.8546	0.4092
	C	2.6638	0.7899	2.8558	0.5981	3.1626	0.4993	3.5239	0.2445
	D	2.5261	0.6847	2.7283	0.4226	2.9723	0.2458	3.1218	0.1047
	E	2.4106	0.6442	2.5955	0.3182	2.7510	0.0568	2.6854	0.0249
j = 0.5	A	2.9009	1.1183	3.0137	0.9179	3.2763	0.9578	3.8158	0.5645
	B	2.7415	0.9017	2.9234	0.7267	3.2073	0.7021	3.6599	0.4002
	C	2.6094	0.7557	2.8202	0.5458	3.0919	0.4526	3.4002	0.2438
	D	2.5002	0.6722	2.7092	0.3930	2.9369	0.2201	3.0647	0.1066
	E	2.4106	0.6442	2.5955	0.3182	2.7510	0.0568	2.6854	0.0249
j = 1.0	A	2.7883	1.0118	2.9952	0.8609	3.1573	0.8441	3.5852	0.4871
	B	2.6632	0.8375	2.8998	0.6621	3.1140	0.6224	3.4876	0.3517
	C	2.5607	0.7231	2.7992	0.4888	3.0282	0.4036	3.2893	0.2190
	D	2.4773	0.6612	2.6965	0.3581	2.9048	0.1966	3.0128	0.0994
	E	2.4106	0.6442	2.5955	0.3182	2.7510	0.0568	2.6854	0.0249
j = 1.5	A	2.6855	0.9029	3.0055	0.8564	3.0744	0.7267	3.4114	0.4318
	B	2.5928	0.7724	2.8943	0.6147	3.0468	0.5387	3.3538	0.3159
	C	2.5175	0.6904	2.7883	0.4303	2.9809	0.3512	3.2008	0.2004
	D	2.4574	0.6502	2.6883	0.3187	2.8804	0.1707	2.9703	0.0954
	E	2.4106	0.6442	2.5955	0.3182	2.7510	0.0568	2.6854	0.0249
j = 2.0	A	2.6103	0.8337	3.0599	0.9399	3.0294	0.6685	3.4534	0.6733
	B	2.5418	0.7361	2.9175	0.6243	3.0065	0.4981	3.3698	0.4789
	C	2.4867	0.6755	2.7944	0.4003	2.9501	0.3266	3.2021	0.2978
	D	2.4433	0.6470	2.6876	0.2882	2.8632	0.1593	2.9670	0.1414
	E	2.4106	0.6442	2.5955	0.3182	2.7510	0.0568	2.6854	0.0249
j = 2.5	A	2.5497	0.7826	3.0999	0.9794	2.9867	0.6313	3.5948	0.9517
	B	2.5011	0.7115	2.9347	0.6264	2.9678	0.4744	3.4531	0.6649
	C	2.4624	0.6675	2.7988	0.3787	2.9202	0.3148	3.2428	0.4068
	D	2.4325	0.6465	2.6870	0.2670	2.8465	0.1568	2.9806	0.1898
	E	2.4106	0.6442	2.5955	0.3182	2.7510	0.0568	2.6854	0.0249

<Table 6> reports the results based on the terminal wealth index, which reinforces the observation from the above analysis. Similar to the results of average holding period returns, the terminal wealth indexes increase monotonically with the weights on stocks. The IDX in <Table 6> compares each allocation strategy's terminal wealth with a base

case strategy of periodic contribution that grows at the rate of 6%. For example, the strategy $S_{15,0}(100, 0)$, that is, the strategy with 15 years of 100% equity investment with no conservative investment horizon, generates payoff that is on average 3.41 times the total investment amount. We can easily see that the strategies with more weights on stocks generate higher IDXs with no exception.

<Table 7> The Sharpe Ratios for Terminal Wealth Indexes of Different Portfolio Strategies

The numbers in the table are calculated by the difference of IDX defined as follows : $D_{i,j}$ (w_s, w_b) = $IDX_{i,j}$ (w_s, w_b)−$IDX_{i,j}$ (0, 100). $\overline{D}$ is the average of $D_{i,j}$ and σ_{D_t} is the standard deviation of $D_{i,j}$.

		5 years			10 years			15 years			20 years		
		$\overline{D}$ (Mean)	σ_{D_t} (Std.)	Sharpe Ratio	$\overline{D}$ (Mean)	σ_{D_t} (Std.)	Sharpe Ratio	$\overline{D}$ (Mean)	σ_{D_t} (Std.)	Sharpe Ratio	$\overline{D}$ (Mean)	σ_{D_t} (Std.)	Sharpe Ratio
	A	0.6119	1.1379	0.54	0.4756	0.9259	0.51	0.6582	1.0512	0.63	1.3939	0.5988	2.33
	B	0.4169	0.7844	0.53	0.3765	0.6961	0.54	0.5601	0.7658	0.73	1.1692	0.4193	2.79
j = 0	C	0.2532	0.4867	0.52	0.2603	0.4645	0.56	0.4116	0.4891	0.84	0.8385	0.2538	3.30
	D	0.1155	0.2291	0.50	0.1328	0.2323	0.57	0.2213	0.2317	0.96	0.4364	0.1121	3.89
	E	−	−	−	−	−	−	−	−	−	−	−	−
	A	0.4736	0.9725	0.49	0.4061	0.8515	0.48	0.5149	0.9354	0.55	1.1085	0.5619	1.97
	B	0.3205	0.6741	0.48	0.3189	0.6413	0.50	0.4470	0.6859	0.65	0.9557	0.4005	2.39
j = 0.5	C	0.1930	0.4202	0.46	0.2190	0.4294	0.51	0.3339	0.4415	0.76	0.7011	0.2469	2.84
	D	0.0872	0.1986	0.44	0.1110	0.2157	0.51	0.1821	0.2112	0.86	0.3720	0.1111	3.35
	E	−	−	−	−	−	−	−	−	−	−	−	−
	A	0.3511	0.8098	0.43	0.3704	0.8244	0.45	0.3875	0.8076	0.48	0.8654	0.4715	1.84
	B	0.2361	0.5651	0.42	0.2837	0.6107	0.46	0.3463	0.5971	0.58	0.7715	0.3414	2.26
j = 1.0	C	0.1410	0.3542	0.40	0.1909	0.4046	0.47	0.2645	0.3885	0.68	0.5809	0.2137	2.72
	D	0.0631	0.1681	0.38	0.0952	0.2020	0.47	0.1469	0.1883	0.78	0.3149	0.0976	3.23
	E	−	−	−	−	−	−	−	−	−	−	−	−
	A	0.2450	0.6549	0.37	0.3551	0.8352	0.43	0.2942	0.6786	0.43	0.6841	0.4012	1.71
	B	0.1635	0.4598	0.36	0.2618	0.5985	0.44	0.2705	0.5068	0.53	0.6299	0.2924	2.15
j = 1.5	C	0.0967	0.2893	0.33	0.1709	0.3869	0.44	0.2111	0.3343	0.63	0.4858	0.1839	2.64
	D	0.0427	0.1376	0.31	0.0831	0.1898	0.44	0.1191	0.1647	0.72	0.2685	0.0844	3.18
	E	−	−	−	−	−	−	−	−	−	−	−	−
	A	0.1708	0.5270	0.32	0.3724	0.8839	0.42	0.2382	0.6023	0.40	0.7063	0.5989	1.18
	B	0.1131	0.3719	0.30	0.2623	0.6123	0.43	0.2214	0.4535	0.49	0.6301	0.4217	1.49
j = 2.0	C	0.0661	0.2346	0.28	0.1646	0.3839	0.43	0.1742	0.3023	0.58	0.4760	0.2568	1.85
	D	0.0287	0.1116	0.26	0.0773	0.1835	0.42	0.0990	0.1508	0.66	0.2595	0.1143	2.27
	E	−	−	−	−	−	−	−	−	−	−	−	−
	A	0.1145	0.4147	0.28	0.3761	0.8726	0.43	0.1886	0.5435	0.35	0.8154	0.8277	0.99
	B	0.0751	0.2934	0.26	0.2572	0.5967	0.43	0.1775	0.4129	0.43	0.6893	0.5723	1.20
j = 2.5	C	0.0433	0.1852	0.23	0.1569	0.3688	0.43	0.1407	0.2782	0.51	0.5011	0.3429	1.46
	D	0.0185	0.0881	0.21	0.0718	0.1737	0.41	0.0804	0.1405	0.57	0.2657	0.1503	1.77
	E	−	−	−	−	−	−	−	−	−	−	−	−

<Table 8> shows the results of pairwise t-tests on the difference between the IDXs of the two portfolio strategies, one with higher weight on stocks and the other with

<Table 8> Pairwise t-tests on Terminal Wealth Indexes of Different Portfolio Strategies

The figures stand for the t-values of the mean tests between the terminal wealth indexes of two different portfolio strategies. The indexes are calculated by comparing a specific assetallocation strategy with a base strategy that assumes the annualized growth rate of periodic contributions of 6% per period constantly (see Equation (6)). A-B = $IDX_{i,j}$ (100, 0) - $IDX_{i,j}$ (75, 25); B-C = $IDX_{i,j}$ (75, 25)-$IDX_{i,j}$ (50, 50); C-D = $IDX_{i,j}$ (50, 50)-$IDX_{i,j}$ (25, 75); D-E = $IDX_{i,j}$ (25, 75)-$IDX_{i,j}$ (0, 100). [a] i = the total investment period, *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

		j = 0	j = 0.5	j = 1.0	j = 1.5	j = 2.0	j = 2.5
5 Years [a]	A-B	8.350***	7.707***	7.041***	6.291***	5.694***	5.077***
	A-C	8.401***	7.677***	6.938***	6.126***	5.478***	4.837***
	A-D	8.401***	7.597***	6.789***	5.923***	5.233***	4.571***
	A-E	8.347***	7.466***	6.593***	5.680***	4.957***	4.282***
	B-C	8.418***	7.605***	6.790***	5.916***	5.221***	4.558***
	B-D	8.365***	7.470***	6.587***	5.666***	4.936***	4.260***
	B-E	8.251***	7.279***	6.335***	5.374***	4.620***	3.935***
	C-D	8.254***	7.276***	6.326***	5.361***	4.604***	3.918***
	C-E	8.074***	7.022***	6.016***	5.020***	4.247***	3.559***
	D-E	7.825***	6.701***	5.644***	4.622***	3.840***	3.155***
10 Years	A-B	5.627***	5.382***	5.200***	5.016***	5.214***	5.596***
	A-C	6.152***	5.812***	5.548***	5.285***	5.382***	5.680***
	A-D	6.592***	6.152***	5.816***	5.499***	5.515***	5.735***
	A-E	6.930***	6.387***	5.985***	5.636***	5.594***	5.748***
	B-C	6.648***	6.212***	5.875***	5.553***	5.553***	5.757***
	B-D	7.027***	6.486***	6.081***	5.721***	5.654***	5.780***
	B-E	7.296***	6.65***	6.179***	5.794***	5.684***	5.748***
	C-D	7.366***	6.721***	6.245***	5.851***	5.723***	5.766***
	C-E	7.560***	6.809***	6.266***	5.846***	5.687***	5.675***
	D-E	7.709***	6.857***	6.242***	5.787***	5.589***	5.519***
15 Years	A-B	3.738***	2.929***	2.111**	1.569	1.432	1.277
	A-C	4.788***	3.968***	3.180***	2.680***	2.489**	2.252**
	A-D	5.848***	5.010***	4.240***	3.771***	3.515***	3.183***
	A-E	6.916***	6.050***	5.282***	4.825***	4.491***	4.049***
	B-C	5.864***	5.026***	4.257***	3.791***	3.537***	3.210***
	B-D	6.963***	6.095***	5.330***	4.877***	4.544***	4.107***
	B-E	8.078***	7.167***	6.384***	5.918***	5.489***	4.927***
	C-D	8.128***	7.213***	6.431***	5.968***	5.539***	4.978***
	C-E	9.296***	8.318***	7.493***	6.983***	6.438***	5.736***
	D-E	10.551***	9.481***	8.58***	7.986***	7.303***	6.448***
20 Years	A-B	9.818***	7.427***	5.654***	3.902***	3.370***	3.879***
	A-C	12.644***	10.156***	8.656***	7.146***	5.257***	5.076***
	A-D	15.473***	12.838***	11.554***	10.272***	7.188***	6.340***
	A-E	18.328***	15.512***	14.408***	13.363***	9.192***	7.685***
	B-C	15.704***	13.017***	11.712***	10.395***	7.280***	6.405***
	B-D	18.768***	15.846***	14.699***	13.597***	9.380***	7.822***
	B-E	21.954***	18.756***	17.737***	16.872***	11.624***	9.370***
	C-D	22.338***	19.041***	17.981***	17.081***	11.801***	9.502***
	C-E	26.013***	22.316***	21.33***	20.68***	14.393***	11.339***
	D-E	30.641***	26.31***	25.303***	24.906***	17.605***	13.674***

lower weight on stocks. The results indicate that the IDX of a strategy with higher weight on stocks is bigger than that of another strategy with lower weight on stocks, and the results are statistically very significant. As can be seen from the t-values, this effect is more pronounced when the time horizon becomes longer.

<Table 9> Differences in Indexes of Terminal Wealth for Different Portfolio Strategies

The indexes are calculated by comparing a specific asset allocation strategy with the base strategy that assumes the annualized growth rate of periodic contributions of6% per period (see Equation (6)). The sample period is from February 1981 to March 2007. [a] D1 = $IDX_{i,j}$ (100, 0)−$IDX_{i,j}$ (75, 25); D2 = $IDX_{i,j}$ (75, 25)−$IDX_{i,j}$ (50, 50); D3 = $IDX_{i,j}$ (50, 50)−$IDX_{i,j}$ (25, 75); and D4 = $IDX_{i,j}$ (25, 75)−$IDX_{i,j}$ (0, 100), [b] i = the total investment horizon, [c] j = the length of the conservative investment period, [d] The percentage of times when the value of each variable is greater than zero.

i[b]		j = 0 [c]		j = 0.5		j = 1.0		j = 1.5		j = 2.0		j = 2.5	
		Mean	%[d]	Mean	%	Mean	%	Mean	%	Mean	%	Mean	%
5	D1[a]	0.19	56.4	0.15	53.1	0.12	52.3	0.08	53.1	0.06	48.6	0.04	45.3
	D2	0.16	59.7	0.13	56.8	0.10	56.4	0.07	55.1	0.05	49.4	0.03	46.9
	D3	0.14	62.6	0.11	60.1	0.08	58.4	0.05	57.2	0.04	50.6	0.02	48.1
	D4	0.12	63.8	0.09	61.3	0.06	59.7	0.04	56.8	0.03	52.3	0.02	48.6
10	D1	0.10	63.2	0.09	62.6	0.09	62.6	0.09	57.7	0.11	51.1	0.12	51.1
	D2	0.12	69.2	0.10	68.7	0.09	69.2	0.09	63.7	0.10	56.6	0.10	54.9
	D3	0.13	74.2	0.11	73.1	0.10	72.5	0.09	67.6	0.09	60.4	0.09	59.3
	D4	0.13	78.6	0.11	77.5	0.10	74.7	0.08	70.3	0.08	63.2	0.07	61.0
15	D1	0.10	45.9	0.07	47.5	0.04	50.8	0.02	49.2	0.02	50.0	0.01	49.2
	D2	0.15	59.8	0.11	58.2	0.08	60.7	0.06	60.7	0.05	61.5	0.04	60.7
	D3	0.19	75.4	0.15	73.0	0.12	68.0	0.09	68.0	0.08	68.0	0.06	66.4
	D4	0.22	82.8	0.18	81.1	0.15	78.7	0.12	76.2	0.10	76.2	0.08	74.6
20	D1	0.22	90.3	0.15	82.3	0.09	75.8	0.05	67.7	0.08	54.8	0.13	56.5
	D2	0.33	98.4	0.25	96.8	0.19	95.2	0.14	93.5	0.15	90.3	0.19	87.1
	D3	0.40	100.0	0.33	100.0	0.27	100.0	0.22	100.0	0.22	100.0	0.24	98.4
	D4	0.44	100.0	0.37	100.0	0.31	100.0	0.27	100.0	0.26	100.0	0.27	100.0

<Table 9> shows the frequency of one investment strategy outperforming another strategy. For longer retirement horizons, the differences in the terminal wealth are fairly substantial. To illustrate, the strategy $S_{20,0}$(100, 0) generates a terminal wealth that is greater than $S_{20,0}$(75, 25) by 0.22 times of the total original investment amounts. Similarly, $S_{20,0}$(75, 25) creates terminal wealth that is higher than that of $S_{20,0}$(50, 50) by 0.33 times of the total investment amounts. Besides, $S_{20,0}$(75, 25) outperforms $S_{20,0}$(50, 50) more

than 98 percent of the times. Therefore, we can conclude that long-term investors should adopt a very aggressive investment strategy with a very high proportion of equity securities.[12)]

3. Stochastic Dominance Tests

While the mean-variance framework works well for managing short-term investments, it may fail to address the concerns of long-term investments such as pension plans. As the investment horizon lengthens, the distribution of portfolio values becomes increasingly asymmetric, and the portfolio return variance loses its intuitive appeal as a measure of risk. A stochastic dominance test is a good alternative to evaluate portfolios under this circumstance. Unlike the mean/variance framework that requires return distributions be identically and independently distributed, stochastic dominance approach makes no assumptions about the return distributions. Also, it makes minimal assumptions on investors' utility function. In return for putting weak constraints on investors' preferences, the stochastic dominance criterion requires stringent conditions on the relative realized returns between assets to establish preference ordering on risky assets. Since the index of terminal wealth is more reflective of the cash contribution pattern of retirement pension plans, we only report here the stochastic dominance tests based on the distribution of terminal wealth indexes.[13)]

Formally, given U(IDX) 0, a portfolio strategy X with F_1 dominates portfolio strategy Y with G_1 under first degree stochastic dominance (FSD) if and only if:

$$F_1(IDX) \leq G_1(IDX) \qquad \forall IDX \tag{8}$$

(with strict inequality for at least one value of IDX), where U(IDX) is the first derivative of the utility function, and F_1 and G_1 are the cumulative distributions of the index of

12) The call rate was unusually high, i.e. over 10 percent at the beginning of our sample period and then has continuously decreased to a single digit. Since the market efficiency has improved over time, more recent data reflect the market situation better. If we took this point into account, our argument would be even more strongly supported.

13) The results of stochastic dominance tests based on average holding period return are qualitatively very similar to those based on the indexes of terminal wealth.

terminal wealth from portfolio X and portfolio Y, respectively. In other words, X dominates Y under FSD if the cumulative distribution of X lies completely to the right of the cumulative distribution of Y. When the above condition is satisfied, the probability of realizing an index of terminal wealth less than or equal to IDX is greater for strategy Y than for strategy X.

The FSD test places no restrictions on the form of the utility function beyond the usual requirement that it be non-decreasing. Thus, this criterion is appropriate for risk averters and risk lovers alike since the utility function may contain concave as well as convex segments. Owing to its generality, FSD permits a preliminary screening of portfolios to eliminate those sets that no rational investor (regardless of the attitude toward risk) will ever choose.

The second-degree stochastic dominance (SSD) is defined as follows. Given U(IDX) 0 and U(IDX) 0, X dominates Y under SSD if and only if

$$F_2(IDX) \leq G_2(IDX) \qquad \forall IDX \tag{9}$$

(with strict inequality for at least one IDX), where U(IDX) is the second derivative of the utility function and

$$F_2(IDX) = \int_{-\infty}^{IDX} F_1(t)dt \text{ and } G_2(IDX) = \int_{-\infty}^{IDX} G_1(t)dt \tag{10}$$

Since F_2 and G_2 denote respectively the area under F_1 and G_1, SSD allows the cumulative distributions to cross by small amounts as long as the area under the cumulative distribution of X is always less than the area under the cumulative distribution of Y. In other words, portfolio X can dominate portfolio Y under SSD, although X does not dominate Y under FSD. Because the utility function is assumed to be concave under SSD, it is an appropriate efficiency criterion for all risk averters. With its stronger assumptions, SSD permits a more sensitive selection of investments than the FSD criterion.

<Table 10> shows the results of stochastic dominance tests based on the index of

terminal wealth. The letters A, B, C, D, and E represent $IDX_{i,j}(100, 0)$, $IDX_{i,j}(75, 25)$, $IDX_{i,j}(50, 50)$, $IDX_{i,j}(25, 75)$, and $IDX_{i,j}(0, 100)$. The evidence indicates that during the sample period, the terminal wealth of portfolios with higher percentage of stock investments dominates the terminal wealth of portfolios with lower percentage of stocks under first degree stochastic dominance. FSD with all its generality can be so unselective that the FSD efficient set may include too many feasible portfolios. However, for investment horizons of 15 years or longer, the portfolios with more stock components clearly dominate other portfolios with smaller stock components even by this broad FSD criterion.

<Table 10> Stochastic Dominance and the Resulting Efficient Sets

The indexes are calculated by comparing a specific asset allocation strategy with a base strategy that assumes the annualized growth rate of periodic contributions to be 6% per period constantly (see Equation (6)). The sample period is from February 1981 to March 2007. $S_{i,j}$ (w_s, w_b) = the investment strategy where w_s (w_b) represents the portfolio weight on stocks (bonds). A = $S_{i,j}$ (100, 0), B = $S_{i,j}$ (75, 25), C = $S_{i,j}$ (50, 50), D = $S_{i,j}$ (25, 75), E = $S_{i,j}$ (0, 100). [a] j = the length of the conservative investment period.

Total investment period	(A : 100% KOSPI, B : Bond * 25% + KOSPI * 75%, C : Bond * 50% + KOSPI * 50%, D : Bond * 75% + KOSPI * 25%, E : 100% Bond)					
	j = 0 [a]	j = 0.5	j = 1.0	j = 1.5	j = 2.0	j = 2.5
5	FSD: (A, B, C, D, E) SSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E	FSD: (A, B, C, D, E) SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: (A, B, C, D, E) SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: (A, B, C, D, E) SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: (A, B, C, D, E) SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: (A, B, C, D, E) SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E
10	FSD: (A, B, C, D, E) SSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E	FSD: (A, B, C, D, E) SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: (A, B, C, D, E) SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: (A, B, C, D, E) SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: (A, B, C, D, E) SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: (A, B, C, D, E) SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E
15	FSD: (A, B, C, D, E) SSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E	FSD: D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: D>E SSD: A>(B, C, D, E), B>(C, D, E), >(D, E), D>E	FSD: (A, B, C, D, E) SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: (A, B, C, D, E) SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: (A, B, C, D, E) SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E
20	FSD: A>E, B>(D, E), C>(D, E), D>E SSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E	FSD: A>E, B>(D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>E, B>(D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: B>(D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: B>E, C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: C>D, D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E

On the other hand, under the more restrictive SSD criterion, the all equity strategy dominates all other strategies regardless of their investment horizons. The clear implication is that any risk-averse investor with a long retirement horizon who makes a portfolio choice on the basis of expected utility should select an aggressive equity portfolio strategy over a safe but less profitable portfolio strategy.

Ⅵ. Monte Carlo Simulation Results

One of the important drawbacks of the previous analysis is that the holding period returns or the IDXs are not independent due to overlapping periods. Also, the use of actual calendar returns produces a limited number of observations, which causes a small sample problem. In order to overcome this limitation we use Monte Carlo simulation to generate sample return distribution for different portfolio strategies for holding periods of 5 to 20 years and repeat the previous analysis. To save the space we only report the results based on the index of terminal wealth that better reflects the investment on the accumulation type funds.[14)]

<Table 11> shows the indexes of terminal wealth calculated by Monte Carlo simulated data on stocks and bonds. For given portfolio weights the longer-term strategies generate higher terminal wealth. <Table 12> shows the results of pairwise comparison between the IDXs of two portfolios with a larger stock weight and a smaller stock weight for a given investment horizon. The mean IDX of a portfolio strategy with a high weight on stocks is bigger than that of a strategy with a low weight on stocks and the results are statistically significant at the 1% level. The t-values are much higher than those reported in <Table 8> and increase with the length of investment horizon.

We show the frequency of one portfolio strategy outperforming another portfolio strategy in <Table 13>. Except the case of $i = 5$ a portfolio strategy with a high weight on stocks always outperforms another strategy with a low weight on stocks. The results in <Table 13> are much stronger than those reported in <Table 9> that are generated with the actual data.

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14) The results of the holding period returns are quite similar to those of IDXs. The results are available on request.

<Table 11> Monte Carlo Simulated Indexes of Terminal Wealth for Different Portfolio Strategies

The indexes are calculated by comparing a specific asset allocation strategy with a base strategy that assumes the annualized growth rate of periodic contributions of 6% per period constantly (see Equation (6)). The Monte Carlo simulation generates sample data for stocks and bonds that best fit the actual data over the period from January 1999 to March 2007 (n = 60(months) * 1,000(times), 120 (months) * 1,000(times), 180(months) * 1,000(times), 240(months) * 1,000(times)). [a] i = the total investment period, [b] j = the length of the conservative investment period, [c] $S_{i,j}$ (w_s, w_b) = the investment strategy where w_s (w_b) represents the portfolio weight on stocks (bonds), A = $S_{i,j}$ (100, 0), B = $S_{i,j}$ (75, 25), C = $S_{i,j}$ (50, 50), D = $S_{i,j}$ (25, 75), E = $S_{i,j}$ (0, 100).

		5 years [a]		10 years		15 years		20 years	
		Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
j = 0 [b]	A [c]	1.4835	0.1002	2.2141	0.0848	3.3854	0.1344	5.0444	0.2014
	B	1.3546	0.0263	1.8099	0.0318	2.4012	0.0417	3.1896	0.0981
	C	1.2691	0.0112	1.5938	0.0167	1.9947	0.0162	2.5136	0.0335
	D	1.1881	0.0095	1.3961	0.0068	1.6278	0.0063	1.8878	0.0111
	E	1.1102	0.0002	1.2219	0.0002	1.3360	0.0002	1.4512	0.0002
j = 0.5	A	1.3776	0.0847	2.0574	0.0886	3.1339	0.1431	4.7492	0.2147
	B	1.2917	0.0261	1.7437	0.0476	2.3258	0.0745	3.1011	0.1532
	C	1.2193	0.0168	1.5322	0.0202	1.9205	0.0253	2.4027	0.0298
	D	1.1518	0.0083	1.3532	0.0068	1.5785	0.0108	1.8293	0.0076
	E	1.0893	0.0003	1.1977	0.0003	1.3090	0.0002	1.4216	0.0002
j = 1.0	A	1.2919	0.0714	1.9197	0.0827	2.8845	0.1306	4.4061	0.1836
	B	1.2209	0.0221	1.6192	0.0254	2.1455	0.0350	2.8736	0.1014
	C	1.1693	0.0103	1.4584	0.0109	1.8251	0.0157	2.2801	0.0257
	D	1.1210	0.0070	1.3102	0.0057	1.5255	0.0156	1.7672	0.0061
	E	1.0736	0.0003	1.1766	0.0004	1.2846	0.0002	1.3944	0.0002
j = 1.5	A	1.2241	0.0600	1.7898	0.0633	2.6905	0.1012	3.9829	0.1729
	B	1.1776	0.0215	1.5649	0.0376	2.0860	0.0650	2.7860	0.0822
	C	1.1370	0.0083	1.4103	0.0168	1.7575	0.0211	2.1968	0.0249
	D	1.0991	0.0062	1.2767	0.0072	1.4839	0.0174	1.7177	0.0067
	E	1.0628	0.0004	1.1587	0.0003	1.2628	0.0002	1.3695	0.0001
j = 2.0	A	1.1700	0.0489	1.6772	0.0477	2.4947	0.0873	3.7253	0.1640
	B	1.1364	0.0168	1.4696	0.0213	1.9348	0.0233	2.5827	0.0419
	C	1.1085	0.0117	1.3520	0.0125	1.6760	0.0149	2.0820	0.0164
	D	1.0824	0.0049	1.2434	0.0058	1.4399	0.0120	1.6642	0.0057
	E	1.0569	0.0003	1.1438	0.0002	1.2434	0.0001	1.3468	0.0002
j = 2.5	A	1.1308	0.0358	1.5787	0.0381	2.3392	0.0861	3.4706	0.1542
	B	1.1102	0.0147	1.4268	0.0277	1.8877	0.0542	2.5105	0.0685
	C	1.0911	0.0113	1.3169	0.0114	1.6234	0.0188	2.0121	0.0255
	D	1.0731	0.0040	1.2186	0.0056	1.4057	0.0120	1.6210	0.0064
	E	1.0555	0.0004	1.1317	0.0001	1.2264	0.0002	1.3262	0.0002

<Table 12> Pairwise t-tests on Monte Carlo Simulated Terminal Wealth Indexes of Different Portfolio Strategies

The figures stand for the t-values of the mean tests between the terminal wealth indexes of two portfolio strategies with different weights on stocks. They are calculated using Monte Carlo simulated domestic stock index and call rate over the period January 1999 to March 2007. A-B = $IDX_{i,j}$ (100, 0)-$IDX_{i,j}$ (75, 25); B-C = $IDX_{i,j}$ (75, 25)-$IDX_{i,j}$ (50, 50); C-D = $IDX_{i,j}$ (50, 50)-$IDX_{i,j}$ (25, 75); D-E = $IDX_{i,j}$ (25, 75)-$IDX_{i,j}$ (0, 100). [a] i = the total investment period, [b] j = the length of the conservative investment period, *** $p < 0.01$.

		j = 0 [b]	j = 1	j = 2	j = 3	j = 4	j = 5
5 Years [a]	A-B	38.968***	30.533***	29.765***	22.743***	20.730***	16.628***
	A-C	67.339***	58.058***	53.755***	45.646***	38.553***	33.868***
	A-D	93.471***	83.934***	75.456***	65.196***	56.496***	50.703***
	A-E	117.779***	107.650***	96.768***	85.025***	73.142***	66.531***
	B-C	95.293***	81.920***	66.810***	56.850***	42.656***	32.611***
	B-D	188.263***	167.271***	136.153***	110.135***	97.680***	77.467***
	B-E	293.811***	245.392***	211.532***	168.517***	150.075***	117.560***
	C-D	178.632***	117.475***	122.491***	116.514***	65.410***	46.941***
	C-E	450.340***	244.127***	292.750***	283.072***	139.188***	99.590***
	D-E	260.208***	236.962***	212.261***	184.552***	162.882***	138.155***
10 Years	A-B	141.516***	100.004***	109.666***	96.071***	127.140***	99.483***
	A-C	226.168***	182.043***	174.367***	183.572***	209.964***	209.750***
	A-D	304.557***	251.436***	232.360***	254.274***	285.804***	295.882***
	A-E	370.084***	307.005***	284.188***	315.391***	353.965***	371.098***
	B-C	189.330***	147.731***	181.813***	135.596***	152.291***	141.582***
	B-D	409.585***	267.375***	367.150***	247.131***	321.677***	244.549***
	B-E	584.311***	362.749***	550.703***	342.021***	483.177***	336.622***
	C-D	344.396***	281.070***	383.277***	242.838***	252.561***	268.024***
	C-E	702.475***	525.029***	819.597***	474.745***	528.532***	513.977***
	D-E	810.611***	717.172***	742.554***	519.932***	542.233***	489.092***
15 Years	A-B	221.355***	161.821***	172.983***	160.478***	197.240***	140.237***
	A-C	324.956***	265.335***	254.278***	287.839***	290.941***	258.679***
	A-D	412.869***	342.380***	327.231***	368.997***	375.532***	339.578***
	A-E	482.083***	403.281***	387.449***	445.939***	453.105***	409.018***
	B-C	288.410***	190.913***	262.369***	175.798***	290.202***	173.579***
	B-D	580.009***	328.316***	510.573***	297.696***	600.113***	285.130***
	B-E	807.072***	431.574***	777.103***	400.686***	937.922***	385.909***
	C-D	665.259***	419.741***	430.250***	336.142***	397.739***	326.665***
	C-E	1289.350***	763.801***	1086.076***	741.575***	920.277***	668.353***
	D-E	1471.472***	786.562***	488.239***	402.761***	515.790***	471.693***
20 Years	A-B	258.037***	203.622***	228.817***	198.574***	213.699***	180.977***
	A-C	391.418***	343.666***	362.966***	324.761***	313.997***	295.668***
	A-D	495.387***	430.426***	452.545***	413.362***	396.011***	378.480***
	A-E	564.234***	490.122***	518.769***	477.998***	458.568***	439.805***
	B-C	210.043***	152.083***	177.981***	257.575***	346.660***	247.635***
	B-D	416.180***	265.190***	344.594***	425.034***	686.997***	424.843***
	B-E	560.113***	346.594***	461.214***	545.034***	933.201***	546.707***
	C-D	565.231***	656.058***	617.135***	656.553***	760.159***	511.965***
	C-E	1,002.311***	1,039.983***	1,090.008***	1,049.534***	1,421.28***	849.529***
	D-E	1,240.948***	1,695.505***	1,944.879***	1,650.138***	1,747.806***	1,467.726***

<Table 13> Differences in Monte Carlo Simulated Indexes of Terminal Wealth from Different Portfolio Strategies

The indexes are calculated by comparing a specific asset allocation strategy with a base strategy that assumes the annualized growth rate of periodic contributions of6% per period constantly (see Equation (6)). The Monte Carlo simulation generates sample data that best fit the actual data on stocks and bonds over the period from January 1999 to March 2007 (n = 60(months) * 1,000(times), 120(months) * 1,000(times), 180(months) * 1,000(times), 240(months) * 1,000(times)). D1 = $IDX_{i,j}$ (100, 0) $-IDX_{i,j}$ (75, 25); D2 = $IDXi_{j}$ (75, 25)$-IDX_{i,j}$ (50, 50); D3 = $IDX_{i,j}$ (50, 50)$-IDX_{i,j}$ (25, 75); and D4 = $IDX_{i,j}$ (25, 75)$-IDX_{i,j}$ (0, 100). [a] i = the total investment horizon, [b] j = the length of the conservative investment period, [c] The percentage of times when the value of each variable is greater than zero.

i [a]		j = 0 [b]		j = 0.5		J = 1.0		j =1.5		j = 2.0		j = 2.5	
		Mean	% [c]	Mean	%	Mean	%	Mean	%	Mean	%	Mean	%
5	D1	0.129	87.7	0.086	79.6	0.071	79.6	0.047	74.5	0.034	71.8	0.021	68.2
	D2	0.085	99.7	0.072	99.7	0.052	98.8	0.041	96.1	0.028	90.1	0.019	84.1
	D3	0.081	100.0	0.067	100.0	0.048	100.0	0.038	100.0	0.026	99.3	0.018	92.1
	D4	0.078	100.0	0.063	100.0	0.047	100.0	0.036	100.0	0.026	100.0	0.018	100.0
10	D1	0.404	100.0	0.314	100.0	0.301	100.0	0.225	100.0	0.208	100.0	0.152	100.0
	D2	0.216	100.0	0.211	100.0	0.161	100.0	0.155	100.0	0.118	100.0	0.110	100.0
	D3	0.198	100.0	0.179	100.0	0.148	100.0	0.134	100.0	0.109	100.0	0.098	100.0
	D4	0.174	100.0	0.156	100.0	0.134	100.0	0.118	100.0	0.100	100.0	0.087	100.0
15	D1	0.984	100.0	0.808	100.0	0.739	100.0	0.605	100.0	0.560	100.0	0.452	100.0
	D2	0.406	100.0	0.405	100.0	0.320	100.0	0.329	100.0	0.259	100.0	0.264	100.0
	D3	0.367	100.0	0.342	100.0	0.300	100.0	0.274	100.0	0.236	100.0	0.218	100.0
	D4	0.292	100.0	0.270	100.0	0.241	100.0	0.221	100.0	0.196	100.0	0.179	100.0
20	D1	1.855	100.0	1.648	100.0	1.533	100.0	1.197	100.0	1.143	100.0	0.960	100.0
	D2	0.676	100.0	0.698	100.0	0.593	100.0	0.589	100.0	0.501	100.0	0.498	100.0
	D3	0.626	100.0	0.573	100.0	0.513	100.0	0.479	100.0	0.418	100.0	0.391	100.0
	D4	0.437	100.0	0.408	100.0	0.373	100.0	0.348	100.0	0.317	100.0	0.295	100.0

Finally, the stochastic dominance tests on IDXs of different portfolio strategies are reported in <Table 14>. The results are much stronger than the results in <Table 10> that are generated based on the actual data. For investment horizons i = 10, 15, and 20, a portfolio strategy with a higher weight on stocks dominates another strategy with a lower weight on stocks by the first order stochastic dominance without any exception. If a random variable dominates another random variable by the first degree stochastic dominance, it also dominates another variable by the second order stochastic dominance, too. So, every portfolio strategy with a higher weight on stocks for investment horizons i = 10, 15, and 20 also stochastically dominates another strategy with a lower weight on stocks by the SSD. Even in the case of i = 5 all portfolio strategies with higher weights on stocks dominate other strategies with lower weights on stocks by the SSD.

<Table 14> Stochastic Dominance and the Resulting Efficient Sets Using Monte Carlo Simulation

The indexes are calculated by comparing a specific asset allocation strategy with a base strategy that assumes the annualized growth rate of periodic contributions of6% per period constantly (see Equation (6)). The Monte Carlo simulation generates sample data that best fit the actual data on stocks and bonds over the period from January 1999 to March 2007 (n = 60(months) * 1,000(times), 120 (months) * 1,000(times), 180(months) * 1,000(times), 240(months) * 1,000(times)). $S_{i,j}$ (w_s, w_b) = the investment strategy where w_s (w_b) represents the portfolio weight on stocks (bonds). A = $S_{i,j}$ (100, 0), B = $S_{i,j}$ (75, 25), C = $S_{i,j}$ (50, 50), D = $S_{i,j}$ (25, 75), E = $S_{i,j}$ (0, 100). [a] j = the length of the conservative investment period.

Total investment period	(A : 100% KOSPI, B : Bond * 25% + KOSPI * 75%, C : Bond * 50% + KOSPI * 50%, D : Bond * 75% + KOSPI * 25%, E : 100% Bond)					
	j = 0 [a]	j = 0.5	j = 1.0	j = 1.5	j = 2.0	j = 2.5
5	FSD: A>(C, D, E), B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E	FSD: A>(C, D, E), B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>(D, E), B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>(D, E), B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>(D, E), B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>E, B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E
10	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E
15	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E
20	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E	FSD: A>(B, C, D, E) B>(C, D, E), C>(D, E), D>E SSD: A>(B, C, D, E), B>(C, D, E), C>(D, E), D>E

All in all, the results strongly support the idea that the long-term investment must put higher weights on stocks. And this idea is even more strongly supported as the investment horizon becomes longer. The results hold for the case of the actual data and the Monte Carlo simulated data as well.

Ⅶ. Conclusions

Due to the change in the family relationship, socio-economic environment and life expectancy, people became more and more interested in the long-term investments. Such change in people's needs has induced financial institutions to introduce a variety of long-term investment vehicles such as personal pension plans, retirement plans and accumulation-type funds. In order to invest on these long-term financial instruments inexperienced individuals must face with the difficult task of selecting a proper asset allocation scheme. Following the asset allocation strategies of large retirement and pension funds, most financial planning advisors have been advocating a dual approach emphasizing both growth and safety. However, the precise proportions of stock and bond investment are often controversial.

Theoretically, an individual's retirement planning horizon can be segmented into two periods, i.e. pre-retirement and post-retirement. During the pre-retirement period, the goal should be to maximize expected returns or terminal wealth, and the usual measure of portfolio risk, the standard deviation, may not be appropriate measure of risk. It is because individuals cannot get the money until their retirement. Portfolio risk or standard deviation only becomes a real issue after retirement. Alternative techniques to measure risk during the pre-retirement period then would be to compute the frequency that a particular strategy *outperforms* another strategy or to rely on stochastic dominance tests using empirical data.

In this paper, we performed various tests including stochastic dominance tests to see whether a portfolio strategy with a higher weight on stocks outperforms another strategy with a lower weight on stocks. The tests were carried out based on the actual data on stocks and bonds since 1981. And the same test procedures were employed for the Monte Carlo simulated data that best fit the actual data to see the robustness of the results.

The results indicate that the optimal allocation strategy should be a very heavy emphasis on stocks until the individuals are close to retirement. Such strategy generates the highest portfolio return and also maximizes the terminal wealth for investors. Moreover, for the very long retirement horizons, equity-dominated portfolios outperform other portfolios in *all* moving average periods from 1981 to 2007. This result is reinforced by the result from the analysis using Monte Carlo simulated data. Therefore, the optimal asset mix for the long-term investors should be more equity dominated and the weights on stocks must be higher as the investment horizon becomes longer.

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