Robust Stability Condition and Analysis on Steady-State Tracking Errors of Repetitive Control Systems

Tae-Yong Doh and Jung Rae Ryoo

Abstract: This paper shows that design of a robustly stable repetitive control system is equivalent to that of a feedback control system for an uncertain linear time-invariant system satisfying the well-known robust performance condition. Once a feedback controller is designed to satisfy the robust performance condition, the feedback controller and the repetitive controller using the performance weighting function robustly stabilizes the repetitive control system. It is also shown that we can obtain a steady-state tracking error described in a simple form without time-delay element if the robust stability condition is satisfied for the repetitive control system. Moreover, using this result, a sufficient condition is provided, which ensures that the least upper bound of the steady-state tracking error generated by the repetitive control system is less than or equal to the least upper bound of the steady-state tracking error only by the feedback system.

Keywords: Least upper bound, repetitive control, robust performance, robust stability, steady-state tracking error, uncertain linear time-invariant system.

1. INTRODUCTION

Repetitive control is a specialized control scheme for tracking periodic reference commands and/or attenuating periodic exogenous disturbances. Its highly accurate tracking property originates from a periodic signal generator implemented in the repetitive controller. However, the positive feedback loop to generate the periodic signal decreases the stability margin. Therefore, the tradeoff between stability and tracking performance has been considered as an important factor in the repetitive control system. Hara *et al.* [1] derived sufficient conditions for the stability of a repetitive control system and a modified repetitive control system, which sacrifices tracking performance at high frequencies for system stability. Srinivasan and Shaw [2] examined the absolute and relative stability of repetitive control systems using the regeneration spectrum and indicated that their results provide improved insights into design tradeoffs. Güvenç [3] applied the structured singular value to repetitive control systems in order to determine their stability and performance robustness in the presence of structured parametric modeling error in the plant. Weiss and Häfele [4] analyzed the stability and robustness of the MIMO repetitive control system based on regular linear system theory. Li and Tsao [5] addressed the analysis and synthesis of robust stability and robust performance repetitive control systems. Doh and Chung [6] proposed a method to design a repetitive control system ensuring robust stability for linear systems with time-varying uncertainties. M.-C. Tsai and W.-S. Yao derived upper and lower bounds of the repetitive controller parameters ensuring both stability and desired performance [7], as well as an upper bound for the square integral of the tracking error over a one time period of periodic input signals based on Fourier analysis [8]. R. C Costa-Castell *et al.* proposed a new repetitive controller with passivity. Therefore, when applied to passive plants, closed-loop stable behavior is guaranteed [9]. Dang and Owens presented a simple adaptive multi-periodic repetitive control scheme and analyzed system stability by the Lyapunov second method [10]. Zhou *et al.* derived a robust stability criterion for dual-mode structure repetitive control in terms of odd-harmonic and even-harmonic repetitive control gain [11]. Stenbuch *et al.* presented a design method of high-order repetitive controllers that is obtained by solving a convex optimization problem [12].

In most previous researches, a repetitive controller is accompanied with a feedback controller. In general, the repetitive controller is added to the existing feedback control system to improve the tracking performance, and the design problems of the feedback

Manuscript received July 25, 2007; revised April 21, 2008; accepted June 20, 2008. Recommended by Editorial Board member Guang-Hong Yang under the direction of Editor Jae Weon Choi.

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controller and the repetitive controller have been considered as two totally separate problems. Moreover, the cutoff frequency of the q-filter in the repetitive controller should be found by many trials and errors. In this paper, we show that once a feedback controller is designed to satisfy the well known robust performance condition, no effort is required for the design of a repetitive controller, and the repetitive controller including the performance weighting function robustly stabilizes the uncertain repetitive control system. This effect can be accomplished by replacing the q-filter in the repetitive controller with the performance weighting function that is used in the design of the feedback controller. Thus, both the feedback controller and the repetitive controller can be simultaneously designed using several tools from robust control theory such as $\mathcal{H}_\infty$, μ-synthesis and model matching [13,14]. It is also shown that if the robust stability condition is satisfied, the steady-state tracking error can be described in a simple form without the time-delay element. Based on the description of the steady-state tracking error, a sufficient condition is provided, which ensures that the least upper bound of the steady-state tracking error generated by the repetitive control system is less than or equal to the least upper bound of the steady-state tracking error only by the feedback control system without repetitive controller. Although single-input and single-output plants are considered for the sake of simplicity, the results of the paper can be extended to multivariable systems. Finally, two illustrative examples are provided to show the validity of the proposed method.

Throughout this paper, signals in the time domain are denoted by lower-case letters and their capitals denote their own Laplace transforms, for example, $\mathcal{L}\{f(t)\}=F(s)$. If there is no other definition, capitals such as $G(s)$ or G stand for transfer functions. The Laplace variable s and the angular frequency ω will be omitted when these do not lead to any confusion.

2. ROBUST STABILITY CONDITION OF REPETITIVE Control Systems

Consider the feedback control system in Fig. 1. In this figure, $y_r(t)$ is the reference trajectory and is assumed to be periodic and bounded within the period T, $y(t)$ is the plant output, and $u(t)$ is the feedback control input. $C(s)$ is the feedback controller that stabilizes the feedback control system. The plant $G(s)$ is described in the following multiplicative uncertain form:

$$G(s)=(1+\Delta(s)W_u(s))G_n(s), \tag{1}$$

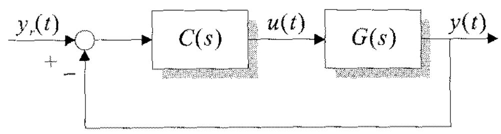

Fig. 1. Feedback control system.

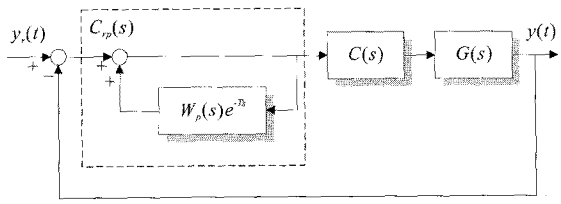

Fig. 2. Repetitive control system.

where $G_n(s)$ is the nominal plant model, $W_u(s)$ is a known stable uncertainty weighting function, and $\Delta(s)$ is an unknown stable function satisfying $\|\Delta\|_\infty\le 1$.

The following lemma, which is widely known as the robust performance condition in the robust control theory, will be used to derive our results.

Lemma 1 [13]: Consider the feedback control system in Fig. 1 with the plant $G(s)$ described in (1). Then a necessary and sufficient condition for robust performance is

$$\|W_uT_n\|_\infty<1 \quad \text{and} \quad \left\|\frac{W_pS_n}{1+\Delta W_uT_n}\right\|_\infty<1$$

which is equivalent to

$$\left\| |W_pS_n|+|W_uT_n| \right\|_\infty<1, \tag{2}$$

where $W_p(s)$ is a known stable performance weighting function, $S_n(s)=1/(1+G_n(s)C(s))$ is the nominal sensitivity function, and $T_n(s)=1-S_n(s)$ is the nominal complementary sensitivity function.

In order to effectively track the periodic reference signal, the repetitive controller $C_{rp}(s)$ is added to the existing feedback control system as the add-on module given in Fig. 2.

Theorem 1: Consider the repetitive control system in Fig. 2. Then the repetitive control system is robustly stable if the condition

$$|W_p(j\omega)|\cos(\theta(\omega)-\omega T)<1,\ \forall\omega, \tag{3}$$

where $\theta(\omega)=\arg[W_p(j\omega)]$ and the robust performance condition (2) are satisfied.

Proof: Since the repetitive controller is added on to the existing feedback control system, the sign of the

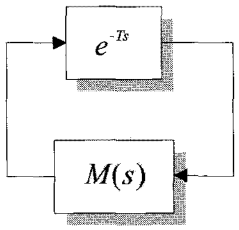

Fig. 3. An equivalent system.

repetitive controller should be positive to preserve the repetitive control system shown in Fig. 2 to be a negative feedback system. The transfer function of the repetitive controller is given by

$$C_{rp}(s) = \frac{1}{1 - W_p(s)e^{-Ts}}. \tag{4}$$

By replacing s with $j\omega$ to work with frequency response, the denominator of (4) becomes

$$\begin{aligned} &1 - |W_p(j\omega)|\cos(\theta(\omega) - \omega T) \\ &-j|W_p(j\omega)|\sin(\theta(\omega) - \omega T). \end{aligned} \tag{5}$$

To maintain the sign to be positive, the real part of (5) should be positive, i.e.,

$$1 - |W_p(j\omega)|\cos(\theta(\omega) - \omega T) > 0, \ \forall\omega,$$

which leads to (3).

To prove robust stability, we consider the repetitive control system provided in Fig. 2, but ignoring inputs. The transfer function from the output of e^{-Ts} around to the input of e^{-Ts} equals to $M(s) = W_pS_n/(1 + \Delta W_uT_n)$, so the repetitive control system shown in Fig. 2 can be converted as an equivalent system composed of e^{-Ts} with $M(s)$ as shown in Fig. 3. Since the time delay e^{-Ts} is stable and its norm is 1, the maximum loop gain equals $\| M(s)e^{-Ts} \|_\infty$, which is less than 1 if and only if the small-gain condition [13,14]

$$\left\| \frac{W_pS_n}{1 + \Delta W_uT_n} \right\|_\infty < 1 \tag{6}$$

holds. Hence, it is clear that (6) is a sufficient condition for robust stability of the repetitive control system by the small-gain theorem. Finally, according to Lemma 3, (6) is guaranteed under the robust performance condition (2). □

According to Theorem 1, the feedback controller satisfying the robust performance condition can directly guarantee the robust stability of the repetitive control system if the performance weighting function $W_p(s)$ is utilized in the repetitive controller. Therefore, there is no need to design a repetitive controller ensuring robust stability in comparison with other methods [2,3,5,7,8].

Remark 1: By replacing a q-filter with a performance weighting function $W_p(s)$, the robust stability of the repetitive control system is automatically satisfied. In general, the bandwidth of the q-filter does not have to be wider than the control bandwidth since the frequencies of references and disturbances are much lower than the cutoff frequency of the open loop. The performance weighting function plays a role to determine the open loop properties of the feedback system. In Fig. 1, if the robust performance condition is satisfied, the following condition is also met.

$$\| W_p(s)S(s) \|_\infty < 1, \tag{7}$$

where $S(s)$ is the sensitivity function and described as

$$S(s) = \frac{1}{1 + G(s)C(s)} = \frac{1}{1 + L(s)}. \tag{8}$$

Equations (7) and (8) lead to

$$|W_p(j\omega)| < |1 + L(j\omega)|, \ \forall\omega. \tag{9}$$

A sufficient condition for (9) is written as

$$|W_p(j\omega)| < |L(j\omega)|, \ \forall\omega.$$

Therefore, by selecting $W_p(s)$ as a q-filter, the q-filter has a proper bandwidth considering that of the open loop $L(s)$ and then the repetitive controller can track or attenuate references or disturbances in the control bandwidth effectively.

3. ESTIMATION OF THE STEADY-STATE TRACKING ERROR

The following theorem indicates that the steady-state tracking error of the repetitive control system in Fig. 2 can be obtained irrespective of the time-delay element if the robust stability condition of the repetitive control system is satisfied, i.e., the robust performance condition is ensured in the feedback control system in Fig. 1.

Theorem 2: Consider the repetitive control system in Fig. 2. The tracking error $e(t)$ approaches to

$$\begin{aligned} e_{ss} &= \lim_{t\to\infty} \varepsilon(t) \\ &= \lim_{t\to\infty} \mathscr{L}^{-1}\left\{ \frac{S_n(1 - W_p)}{1 + \Delta W_uT_n - W_pS_n} Y_r(s) \right\} \end{aligned} \tag{10}$$

as $t \to \infty$ if the repetitive control system satisfies the conditions (2), (3) where

$$\varepsilon(t) = \mathscr{L}^{-1}\left\{\frac{S_n(1-W_p)}{1+\Delta W_u T_n - W_p S_n} Y_r(s)\right\}. \quad (11)$$

Proof: From Fig. 2, the tracking error is given by

$$\begin{aligned} E(s) &= Y_r(s) - Y(s) \\ &= Y_r(s) - \frac{G(s)C(s)}{1 - W_p(s)e^{-Ts}} E(s). \end{aligned}$$

From the above equation, we obtain

$$\begin{aligned} E(s) - W_p(s)e^{-Ts}E(s) = Y_r(s) - W_p(s)Y_r(s)e^{-Ts} \\ -G(s)C(s)E(s). \end{aligned}$$

Using $W_p/(1+G_n(1+\Delta W_u)C) = W_p S_n/(1+\Delta W_u T_n)$, the tracking error becomes

$$\begin{aligned} E(s) &= \frac{S_n(1-W_p e^{-Ts})}{1+\Delta W_u T_n - W_p S_n e^{-Ts}} Y_r(s) \\ &= \frac{\frac{S_n(1-W_p e^{-Ts})}{1+\Delta W_u T_n}}{1 - \left(\frac{W_p S_n}{1+\Delta W_u T_n}\right) e^{-Ts}} Y_r(s). \end{aligned} \quad (12)$$

Since $\| W_p S_n/(1+\Delta W_u T_n) \|_\infty < 1$, the tracking error can be obtained by expanding the denominator in terms of a power series as follows:

$$\begin{aligned} E(s) &= \frac{S_n(1-W_p e^{-Ts})}{1+\Delta W_u T_n} Y_r(s) \left\{1 + \frac{W_p S_n}{1+\Delta W_u T_n} e^{-Ts} \right. \\ &\quad \left. + \left(\frac{W_p S_n}{1+\Delta W_u T_n}\right)^2 e^{-2Ts} + \cdots \right\} \\ &= \frac{S_n}{1+\Delta W_u T_n} Y_r(s) - \frac{T_n(1+\Delta W_u)}{1+\Delta W_u T_n} \cdot \\ &\quad \left(\frac{W_p S_n}{1+\Delta W_u T_n}\right) e^{-Ts} Y_r(s) \\ &\quad - \frac{T_n(1+\Delta W_u)}{1+\Delta W_u T_n}\left(\frac{W_p S_n}{1+\Delta W_u T_n}\right)^2 e^{-2Ts} Y_r(s) - \cdots \\ &= E_0(s) + \sum_{k=1}^{\infty} E_k(s) e^{-kTs} \end{aligned} \quad (13)$$

where

$$E_0(s) = \frac{S_n}{1+\Delta W_u T_n} Y_r(s)$$

and

$$E_k(s) = -\frac{T_n(1+\Delta W_u)}{1+\Delta W_u T_n}\left(\frac{W_p S_n}{1+\Delta W_u T_n}\right)^k Y_r(s)$$

for all $k \in \mathbb{N}$. Since transfer functions $-S_n$, T_n, Δ, W_n, and W_p – are analytic in the right half-plane and $y_r(t)$ is bounded, there exist inverse Laplace transforms for each of the terms in (13). The tracking error in the time domain is

$$e(t) = e_0(t) + \sum_{k=1}^{\infty} e_k(t - kT), \quad (14)$$

where $e_0(t) = \mathscr{L}^{-1}\{E_0(s)\}$ is the tracking error generated only by the feedback control system in Fig. 1 and $e_k(t-kT) = \mathscr{L}^{-1}\{E_k(s)e^{-kTs}\}$ for all $k \in \mathbb{N}$. Thus, the steady-state tracking error becomes

$$\begin{aligned} e_{ss} &= \lim_{t\to\infty} e(t) \\ &= \lim_{t\to\infty}\left\{e_0(t) + \sum_{k=1}^{\infty} e_k(t-kT)\right\} \\ &= \lim_{t\to\infty}\left\{e_0(t) + \sum_{k=1}^{\infty} e_k(t)\right\} \\ &= \lim_{t\to\infty} \mathscr{L}^{-1}\left\{E_0(s) + \sum_{k=1}^{\infty} E_k(s)\right\} \\ &= \lim_{t\to\infty} \varepsilon(t). \end{aligned} \quad (15)$$

Since $\| W_p S_n/(1+\Delta W_u T_n) \|_\infty < 1$, the sum of $E_0(s)$ and the infinite series $\sum_{k=1}^{\infty} E_k(s)$ is $S_n(1-W_p)/(1+\Delta W_u T_n - W_p S_n)Y_r(s)$. Finally, the steady-state tracking error can be described as (10). □

A few important facts are obtained from Theorem 2. First, compared with the following steady-state tracking error derived from (12):

$$\begin{aligned} e_{ss}^* &= \lim_{t\to\infty} e(t) \\ &= \lim_{t\to\infty} \mathscr{L}^{-1}\left\{\frac{S_n(1-W_p e^{-Ts})}{1+\Delta W_u T_n - W_p S_n e^{-Ts}} Y_r(s)\right\}, \end{aligned}$$

the steady-state tracking error of (10) has a simple form without time delay element. Second, let $W_p(s)$ be 1 for all frequencies. Then, the perfect tracking is achievable from (10) regardless of the periodicity of $y_r(t)$. Third, let $W_p(s)$ be an ideal low-pass filter with DC gain of 1 and cutoff frequency ω_c. Then, the steady-state tracking error approaches to zero even if $y_r(t)$ is composed of signals containing the frequencies lower than ω_c.

Remark 2: Not only e_{ss} but also e_{ss}^* can be used to estimate the steady-state error in the repetitive control system. From the viewpoint of the steady-state error, two equations are equivalent, although there are differences in the description and in the transient responses of $e(t)$ and $\varepsilon(t)$. e_{ss}^* are directly obtained from the tracking error (12) and e_{ss} are obtained by using the properties of limit (15).

Corollary 1: Consider the repetitive control system in Fig. 2. Then, for $k \in \mathbb{N}$, $\| e_k \|_2 \to 0$ as $k \to \infty$ if the conditions (2) and (3) are satisfied.

Proof: In (13) and (14), it can be found that

$$E_{k+1}(s) = \left(\frac{W_p S_n}{1+\Delta W_u T_n} \right) E_k(s) \tag{16}$$

for $k \in \mathbb{N}$. In the time domain, by the properties of the $\mathcal{L}_2$-norm [14],

$$\begin{aligned} \| e_{k+1} \|_2 &\le \left\| \frac{W_p S_n}{1+\Delta W_u T_n} \right\|_\infty \| e_k \|_2 \\ &\le \left\| \frac{W_p S_n}{1+\Delta W_u T_n} \right\|_\infty^k \| e_1 \|_2 . \end{aligned} \tag{17}$$

Therefore, if condition (2) is satisfied, $\| e_k \|_2$ approaches to zero as $k \to \infty$. □

The response corresponding to $e_{k+1}(t-(k+1)T)$ is superimposed on $e_1(t-T)$, $e_2(t-2T)$, $\cdots$, $e_k(t-kT)$. As explained in [2] and [7], Corollary 1 implies that the responses corresponding to $e_k(t-kT)$ for $k \in \mathbb{N}$ tend to decay with the decay rate of $\left\| \frac{W_p S_n}{1+\Delta W_u T_n} \right\|_\infty$. Hence, if $\left\| \frac{W_p S_n}{1+\Delta W_u T_n} \right\|_\infty$ is made to be smaller, a better steady-state response can be obtained. However, it is not explained in Corollary 1 that the steady-state tracking error approaches to zero.

4. BOUNDEDNESS OF THE STEADY-STATE TRACKING ERROR

In this subsection, we show that from the viewpoint of the steady-state tracking error, the repetitive control system is more effective than simple feedback control systems. Before advancing a theory, we first show in the following corollary that the repetitive control system satisfying the modified conditions preserves robust stability.

Corollary 2: Consider the repetitive control system in Fig. 2. If the condition (3) and

$$\left\| |W_p^* S_n| + |W_u^* T_n| \right\|_\infty < 1 \tag{18}$$

are satisfied where $W_p^* = W_p/(1-\|1-W_p\|_\infty)$ and $W_u^* = W_u/(1-\|1-W_p\|_\infty)$, the repetitive control system is robustly stable.

Proof: If $W_p = 1$, (18) is the same as condition (2) and the proof is equivalent to the proof of Theorem 1.

Now let us consider the case of $W_p \ne 1$. Condition (3) leads to $\|1-W_p\|_\infty < 1$ and (18) implies that

$$\left\| |W_p S_n| + |W_u T_n| \right\|_\infty < 1 - \|1-W_p\|_\infty ,$$

which also implies that condition (2) is satisfied. Therefore, the considered repetitive control system is robustly stable. □

Using the results in Theorems 1 and 2, we show in the following theorem that the least upper bound of the steady-state tracking error made by the proposed repetitive control system is less than or equal to that generated only by the feedback controller.

Theorem 3: If the conditions (3) and (18) are satisfied, the least upper bound of the steady-state tracking error e_{ss} in the repetitive control system in Fig. 2 is less than or equal to the least upper bound of the steady-state tracking error e_{ss}^0 in the feedback control system in Fig. 1. In other words, if $|e_{ss}| = |\lim_{t\to\infty} e(t)| \le \alpha$ and $|e_{ss}^0| = |\lim_{t\to\infty} e_0(t)| \le \beta$, then $\alpha \le \beta$.

Before proving Theorem 3, we first introduce a useful lemma.

Lemma 2 [15]: For real-valued functions $f(t)$ and $g(t)$, if $f(t) > g(t)$ for all $t \ge 0$ and $\lim_{t\to\infty} f(t)$ and $\lim_{t\to\infty} g(t)$ exist, then $\lim_{t\to\infty} f(t) \ge \lim_{t\to\infty} g(t)$.

Proof of Theorem 3: For the sake of simplicity, let the transfer function $S_n/(1+\Delta W_n T_n)$ be S_e. Then the transfer function $S_n(1-W_p)/(1+\Delta W_u T_n - W_p S_n)$ can be rewritten as $S_e(1-W_p)/(1-W_p S_e)$. The complex Fourier series representation of a periodic reference trajectory $y_r(t)$ with period T is given by $y_r(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$ and $\omega_0 = 2\pi/T$ where c_k is known as the kth Fourier coefficient. From (10) and (14), e_{ss} and e_{ss}^0 are given by

$$e_{ss} = \lim_{t\to\infty} \sum_{k=-\infty}^{\infty} \frac{S_e(jk\omega_0)(1-W_p(jk\omega_0))}{1-W_p(jk\omega_0)S_e(jk\omega_0)} c_k e^{jk\omega_0 t},$$

$$e_{ss}^0 = \lim_{t\to\infty} \sum_{k=-\infty}^{\infty} S_e(jk\omega_0) c_k e^{jk\omega_0 t},$$

respectively. Hence, the following inequalities are obtained:

$$
\begin{aligned}
|e_{ss}| &= \left|\lim_{t\to\infty} e(t)\right| \\
&\le \lim_{t\to\infty}\sum_{k=-\infty}^{\infty}\left|\frac{S_e(jk\omega_0)(1-W_p(jk\omega_0))}{1-W_p(jk\omega_0)S_e(jk\omega_0)}\right|\left|c_k e^{jk\omega_0 t}\right| \\
&\le \alpha, \\
|e_{ss}^0| &= \left|\lim_{t\to\infty} e_0(t)\right| \\
&\le \lim_{t\to\infty}\sum_{k=-\infty}^{\infty}\left|S_e(jk\omega_0)\right|\left|c_k e^{jk\omega_0 t}\right| \le \beta,
\end{aligned}
$$

where α and β are the least upper bounds of e_{ss} and e_{ss}^0, respectively. By showing that $\left|S_e(jk\omega_0)(1-W_p(jk\omega_0))/(1-W_p(jk\omega_0)S_e(jk\omega_0))\right|$ is less than $|S_e(jk\omega_0)|$, we prove that α is less than or equal to β.

Condition (18) implies that

$$|1-W_p(j\omega)| < 1-|W_p(j\omega)S_e(j\omega)|,\ \forall\omega. \quad (19)$$

We have

$$
\begin{aligned}
1 &= |1+W_p(j\omega)S_e(j\omega)-W_p(j\omega)S_e(j\omega)| \\
&\le |W_p(j\omega)S_e(j\omega)|+|1-W_p(j\omega)S_e(j\omega)|,\ \forall\omega
\end{aligned}
$$

and therefore

$$1-|W_p(j\omega)S_e(j\omega)| \le |1-W_p(j\omega)S_e(j\omega)|,\ \forall\omega. \quad (20)$$

Finally, from (19) and (20), the following inequality

$$|1-W_p(j\omega)| < |1-W_p(j\omega)S_e(j\omega)|,\ \forall\omega$$

is obtained, which implies that

$$\left|\frac{S_e(j\omega)(1-W_p(j\omega))}{1-W_p(j\omega)S_e(j\omega)}\right| < |S_e(j\omega)|,\ \forall\omega.$$

The above inequality leads to

$$\left|\frac{S_e(jk\omega_0)(1-W_p(jk\omega_0))}{1-W_p(jk\omega_0)S_e(jk\omega_0)}\right| < |S_e(jk\omega_0)|,\ \forall k. \quad (21)$$

By (21) and Lemma 2, it is clear that $\alpha \le \beta$. □

Theorem 2 shows that the steady-state tracking error will be zero if we design the feedback controller $C(s)$ satisfying the robust performance condition (6) under $W_p(s)=1$. In the meantime, Theorem 3 explains the case that the controller design is not achievable with $W_p(s)=1$. If we select $W_p(s)\neq 1$ satisfying both (2) and (3) and solve the modified robust performance condition (18), the repetitive control system composed of $W_p(s)$ and the designed controller $C(s)$ can ensure that the steady-state tracking error is less than or equal to that only by the feedback controller.

5. ILLUSTRATIVE EXAMPLES

To show the feasibility of the proposed method, we present two illustrative examples.

Example 1 (Case I: $W_p(s)=1$ **):** Consider the following plant, performance weighting transfer function, and uncertainty weighting function:

$$
\begin{aligned}
G_n(s) &= \frac{0.9s^2+24s+50}{s^2+25s+400}, \\
W_p(s) &= 1,\ W_u(s)=\frac{0.75s+10}{s+100}.
\end{aligned}
$$

Using the μ-Analysis and Synthesis Toolbox of Matlab [16], the feedback controller

$$C(s)=\frac{1.6889(s+100)(s^2+28.54s+237)}{(s+86.6)(s+23.11)(s+2.763)}$$

can be obtained, which leads to

$$\left\| |W_pS_n|+|W_uT_n| \right\|_\infty = 0.846$$

as shown in Fig. 4. Therefore, not only the robust performance of the feedback control system but also the robust stability of the proposed repetitive control system are guaranteed. In simulations, let the reference trajectory be a periodic trapezoidal signal in Fig. 5. The repetitive controller is turned on at 2 sec. Fig. 5 shows the reference trajectory (dash-dot) and the tracking output (solid), and Fig. 6 shows the tracking error. After turning on the repetitive controller, the tracking error is abruptly reduced and

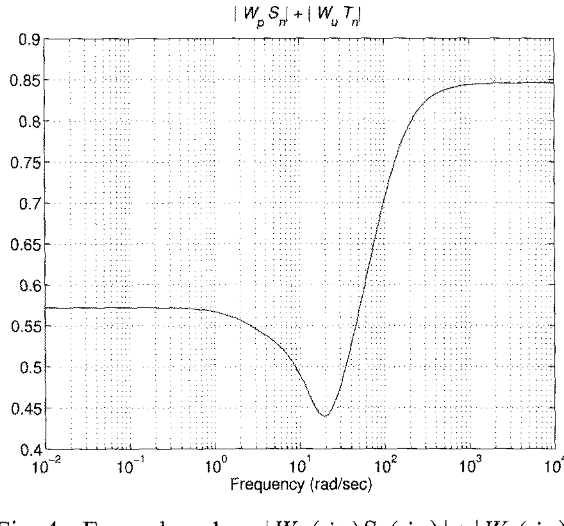

Fig. 4. Example 1: $|W_p(j\omega)S_n(j\omega)|+|W_u(j\omega)\cdot T_n(j\omega)|$ versus frequency.

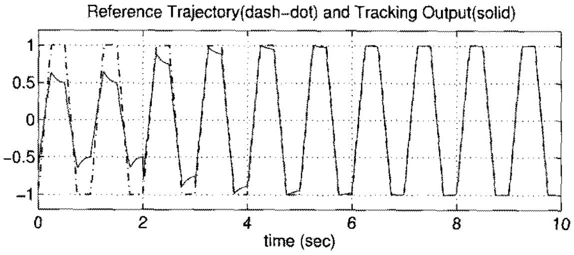

Fig. 5. Example 1: Reference trajectory (dash-dot) and tracking output (solid).

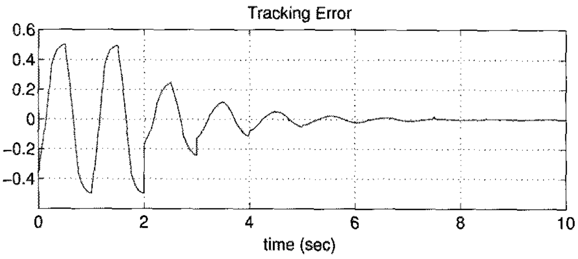

Fig. 6. Example 1: Tracking error.

approaches to zero since $W_p(s)=1$.

Example 2 (Case II: $W_p(s)\neq 1$**):** Consider the following plant, performance weighting transfer function, and uncertainty weighting function:

$$G_n(s)=\frac{24s+50}{s^2+25s+400},$$

$$W_p(s)=\frac{50}{s+50},\ W_u(s)=\frac{0.75s+10}{s+100}.$$

Using the μ -Analysis and Synthesis Toolbox of Matlab [16], the feedback controller

$$C(s)=\frac{4357(s+100)(s+26.81)(s+9.939)}{(s+4782)(s+62.8)(s+50)(s+1.468)}$$

can be obtained, which leads to

$$\left\| |W_pS_n|+|W_uT_n| \right\|_\infty = 0.686$$

as shown in Fig. 7. Therefore, both the robust performance of the feedback control system and the robust stability of the repetitive control system are guaranteed. As Example 1, the reference trajectory is the trapezoidal signal and the repetitive controller is turned on at 2 sec. Fig. 8 shows the reference trajectory (dash-dot) and the tracking output (solid), and Fig. 9 shows the tracking error. After the repetitive controller is turned on, the tracking error is apparently reduced. However, unlike Example 1, the tracking error cannot approach to zero since $W_p(s)\neq 1$. It is verified in Fig. 9 that the tracking error by simulation approaches to the steady-state tracking error estimated by (10) as $t\to\infty$.

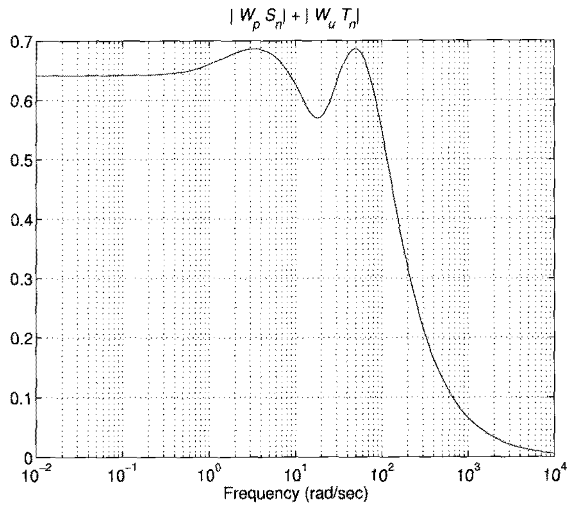

Fig. 7. Example 2: $|W_p(j\omega)S_n(j\omega)|+|W_u(j\omega)\cdot T_n(j\omega)|$ versus frequency.

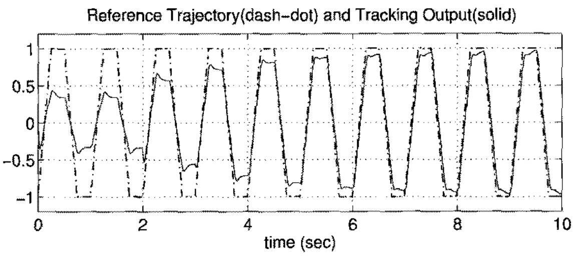

Fig. 8. Example 2: Reference trajectory (dash-dot) and tracking output (solid).

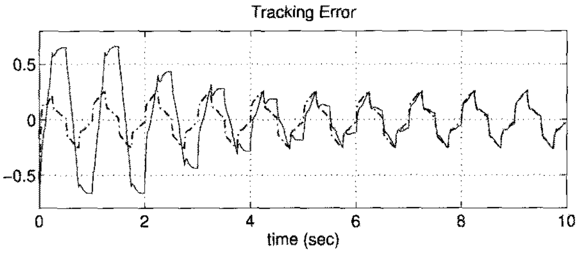

Fig. 9. Example 2: The tracking error by simulation (solid) and the steady-state tracking error

6. CONCLUDING REMARKS

In this paper, it was shown that the robust stability of the repetitive control system is equivalent to the robust performance of the feedback control system. If the performance weighting function $W_p(s)$ is used as a filter in the repetitive controller and the feedback controller $C(s)$ satisfying the robust performance condition is designed using several tools from robust control theory, the robust stability of the repetitive control system is immediately guaranteed. Moreover, properties of the steady-state tracking error of the repetitive control system were investigated. The steady-state tracking error was described in a simple form without time-delay element. Then we obtained a sufficient condition ensuring that the least upper bound of the steady-state tracking error in the repetitive control system is less than or equal to the least upper bound of the steady-state tracking error in the feedback control system. Finally, simulations were performed and the results were presented to validate the effectiveness of the proposed method.

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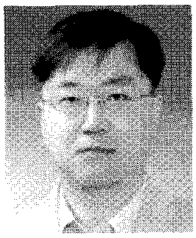

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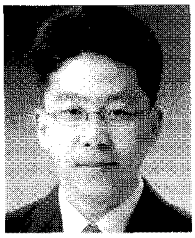

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