

## Reliable $H_\infty$ Controller Design for a Class of Uncertain Linear Systems with Actuator Failures

Shi-Lu Dai and Jun Zhao\*

**Abstract:** This paper is concerned with the reliable  $H_\infty$  controller design problem for uncertain linear systems against actuator failures. In the design, the  $H_\infty$  performance of the closed-loop system is optimized during normal operation (without failures) while the system satisfies a prescribed  $H_\infty$  performance level in the case of actuator failures. Single and parameter-dependent Lyapunov function approaches are applied in designing suboptimal reliable  $H_\infty$  controllers. Simulation studies are presented to demonstrate the effectiveness of the proposed design procedures.

**Keywords:** Actuator failures,  $H_\infty$  control, parameter-dependent Lyapunov function, reliable control.

### 1. INTRODUCTION

Reliable control system design problems have attracted considerable attention [1-4], since failures of control components (e.g., sensor failures or actuator failures) often occur in applications. A control system designed to tolerate failures of control components, while maintaining an acceptable level of the closed-loop systems stability/performance, is called a reliable control system [1], in which a methodology was introduced for the design of reliable linear control systems such that the resulting control systems are stability and satisfy an acceptable  $H_\infty$  disturbance attenuation level not only when all actuators or sensors are operational but also in the case of some admissible actuator or sensor outages. A robust and reliable  $H_\infty$  control scheme was developed in [3] via state-feedback for uncertain linear systems with actuator failures. In [4], a more practical model of sensor and actuator failures than outages was considered to develop the reliable  $H_\infty$  controller design methods. Nevertheless, in these reliable design methodologies, the optimization of normal perfor-

mance is not explicitly addressed when the normal performance is sacrificed for the reliability (stability) goal. The reliable design should also seek to optimize the normal performance as systems are operating under normal conditions in most of the time [5]. Recently, reliable control design methodologies were developed for linear systems [5-7], which optimized the normal performances of the closed-loop systems while maintaining acceptable low-level performances of the systems in the event of actuator failures.

In this paper, we explore the problem of the reliable  $H_\infty$  control for a class of uncertain linear systems with external disturbances. Differently from the aforementioned results of reliable design, actuator failures are described as polytopic uncertainty, which covers the cases of normal operation, partial degradations, and outages. During most reliable design processes, the normal performances of the closed-loop systems are usually sacrificed for the reliability goal. It is, however, undesirable to sacrifice significantly the normal performances for the occasional failure cases in practice. Motivated by this fact, we design a stabilizing state-feedback controller such that the normal  $H_\infty$  performance is optimized while satisfying a prescribed  $H_\infty$  performance level in the case of actuator failures. Two efficient procedures for designing a suboptimal reliable  $H_\infty$  controller are presented in terms of linear matrix inequalities (LMIs). In the first design procedure, a single Lyapunov function is used for the actuator failure cases. However, the procedure may be conservative by the use of the single Lyapunov function. To reduce its conservatism, the second design procedure employs a parameter-dependent Lyapunov function to develop a new design technique for the actuator failure cases.

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**2. PROBLEM STATEMENT**

Consider the uncertain linear system described by

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) + G(t)\omega(t), \\ z(t) &= C(t)x(t) + D(t)u(t) + D_\omega(t)\omega(t), \end{aligned} \tag{1}$$

where  $x(t) \in \mathbf{R}^n$  is the state vector,  $u(t) \in \mathbf{R}^m$  is the control input,  $\omega(t) \in L_2[0, \infty)$  is the external disturbance signal, and  $z(t) \in \mathbf{R}^p$  is the controlled output. Suppose that system matrices have the following form

$$\begin{aligned} \begin{bmatrix} A(t) & B(t) & G(t) \\ C(t) & D(t) & D_\omega(t) \end{bmatrix} &= \begin{bmatrix} A & B & G \\ C & D & D_\omega \end{bmatrix} \\ &+ \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} F(t) [E_1 \ E_2 \ E_3], \end{aligned} \tag{2}$$

where  $A, B, G, C, D, D_\omega, H_i$  ( $i=1, 2$ ), and  $E_i$  ( $i=1, 2, 3$ ) are constant matrices of appropriate dimensions, and  $F(t)$  is an unknown matrix function satisfying  $F^T(t)F(t) \leq I$ , with the elements being Lebesgue-measurable.

The failure model is described by

$$u^F(t) = \alpha u(t), \quad \alpha = \text{diag}\{\alpha_1, \dots, \alpha_m\}, \tag{3}$$

$$0 \leq \underline{\alpha}_j \leq \alpha_j \leq \bar{\alpha}_j \leq 1, \quad j = 1, 2, \dots, m \tag{4}$$

where  $\underline{\alpha}_j$  and  $\bar{\alpha}_j$  represent the lower and upper bounds, respectively. Denote the matrix set

$$\begin{aligned} \Omega_\beta &= \{\beta_L \mid \beta_L = \text{diag}\{\beta_{L1}, \beta_{L2}, \dots, \beta_{Lm}\}, \\ &\beta_{Lj} = \underline{\alpha}_j \text{ or } \bar{\alpha}_j, j = 1, 2, \dots, m\}, \end{aligned} \tag{5}$$

where  $L = 1, 2, \dots, n_\beta$ ,  $n_\beta = 2^l$ , and  $l \leq m$  is the number of actuator failures. It is clear that the matrix  $\alpha$  resides within the matrix polytope with the vertices  $\Omega_\beta$ , i.e.,  $\alpha \in \Omega_\alpha$  with

$$\Omega_\alpha = \{\alpha \mid \alpha = \sum_{L=1}^{n_\beta} \theta_L \beta_L, \theta_L \geq 0, \sum_{L=1}^{n_\beta} \theta_L = 1\}. \tag{6}$$

Consider system (1) with the following state-feedback controller

$$u(t) = Kx(t), \tag{7}$$

where  $K \in \mathbf{R}^{m \times n}$  is a constant matrix gain to be designed. Then, the closed-loop system with actuator failures (3) is given by

$$\begin{aligned} \dot{x}(t) &= A_c(t, \alpha)x(t) + G(t)\omega(t), \\ z(t) &= C_c(t, \alpha)x(t) + D_\omega(t)\omega(t), \end{aligned} \tag{8}$$

where  $A_c(t, \alpha) = A(t) + B(t)\alpha K$  and  $C_c(t, \alpha) = C(t)$

+  $D(t)\alpha K$ . Obviously, when  $\alpha = I$ , system (8) becomes the following system

$$\begin{aligned} \dot{x}(t) &= A_c(t)x(t) + G(t)\omega(t), \\ z(t) &= C_c(t)x(t) + D_\omega(t)\omega(t), \end{aligned} \tag{9}$$

with  $A_c(t) = A(t) + B(t)K$  and  $C_c(t) = C(t) + D(t)K$ . For convenience, system (8) and system (9) are said to be the reliable and standard closed-loop system, respectively.

**The reliable  $H_\infty$  control problem:** Consider system (1). Given constant scalars  $\gamma_f \geq \gamma_0 > 0$ , design a controller (7) such that:

- i) the standard closed-loop system (9) is asymptotically stable and  $\|z(t)\|_2 < \gamma_0 \|\omega(t)\|_2$  holds for all  $z(t), \omega(t)$  satisfying (9), where  $\|\cdot\|_2$  denotes the usual  $L_2(0, \infty)$ -norm;
- ii) the reliable closed-loop system (8) is asymptotically stable and under the zero initial condition,  $\|z(t)\|_2 < \gamma_f \|\omega(t)\|_2$  holds for all  $z(t), \omega(t)$  satisfying (8) and all probable  $\alpha \in \Omega_\alpha$ .

It is worth pointing out there might exist a set of controllers such that above reliable control problem is solvable. Here, the objective of this paper is to design a controller (7) such that:

- i) the above reliable control problem is solvable;
- ii) for a given constant scalar  $\gamma_f > 0$  in the actuator failure cases, the level  $\gamma_0 > 0$  for the closed-loop system in the normal case is minimized.

If the above objective is achieved, controller (7) is said to be a suboptimal reliable  $H_\infty$  controller.

The following lemma will be used in the development of the main results.

**Lemma 1** [8]: For given matrices  $Y, M$  and  $N$  of appropriate dimensions and a scalar  $\eta > 0$ ,

$$Y + M \Xi N + N^T \Xi^T M^T < 0$$

holds for all  $\Xi$  satisfying  $\Xi^T \Xi \leq \eta I$  if and only if there exists a constant  $\varepsilon > 0$  such that

$$Y + \varepsilon M M^T + \frac{\eta}{\varepsilon} N^T N < 0.$$

**3. RELIABLE  $H_\infty$  CONTROLLER DESIGN**

This section presents two efficient procedures for designing suboptimal reliable  $H_\infty$  controller for system (1). In the design procedures, single Lyapunov function and parameter-dependent Lyapunov function are employed, respectively, to develop the procedures

for the actuator failure cases. The suboptimal reliable controllers can be obtained through solving convex optimization problems with LMI constraints.

3.1. Single Lyapunov function approach

Following bounded real lemma (BRL) [9], it is clear that for system (9) with a scalar  $\gamma_0 > 0$ , if there exists a matrix  $X = X^T > 0$  such that

$$\begin{bmatrix} A_c(t)X + XA_c^T(t) & G(t) & XC_c^T(t) \\ * & -\gamma_0 I & D_\omega^T(t) \\ * & * & -\gamma_0 I \end{bmatrix} < 0, \quad (10)$$

where the symbol \* represents the transposed elements in the symmetric positions, then system (9) is asymptotically stable and satisfies  $\|z(t)\|_2 < \gamma_0 \|\omega(t)\|_2$ . For a given scalar  $\gamma_f > 0$ , if there exists a matrix  $X_f = X_f^T > 0$  such that

$$\begin{bmatrix} \Phi(t, \alpha) & G(t) & X_f C_c^T(t, \alpha) \\ * & -\gamma_f I & D_\omega^T(t) \\ * & * & -\gamma_f I \end{bmatrix} < 0, \quad (11)$$

where  $\Phi(t, \alpha) = A_c(t, \alpha)X_f + X_f A_c^T(t, \alpha)$ , then system (8) is asymptotically stable and satisfies  $\|z(t)\|_2 < \gamma_f \|\omega(t)\|_2$ .

In order to obtain reliable  $H_\infty$  controllers, we need to solve inequalities (10) and (11). However, inequalities (10) and (11) are not convex. To solve this non-convex problem, let us set

$$X = \lambda X_f, \quad (12)$$

where  $\lambda > 0$  is a scalar variable. Since inequality (11) is infinite-dimensional, in general, the problem of direct testing its feasibility is not tractable. Using the characteristic of the polytopic uncertainty, this problem can be transformed into the test only at the vertices  $\Omega_\beta$ . The following result gives a solution to the reliable  $H_\infty$  control problem.

**Theorem 1:** Consider the closed-loop system (8). For given scalars  $\gamma_f \geq \gamma_0 > 0$ , the controller (7) solves the reliable  $H_\infty$  control problem if there exists a solution  $(\varepsilon_1, \varepsilon_2, \lambda_1, X, Y)$  to the following:

$$\begin{bmatrix} M_{11} & G & M_{13} & XE_1^T + Y^T E_2^T \\ * & -\gamma_0 I & D_\omega^T & E_3^T \\ * & * & M_{33} & 0 \\ * & * & * & -\varepsilon_1 I \end{bmatrix} < 0, \quad (13)$$

$$N_L := \begin{bmatrix} N_{11L} & \lambda_1 G & N_{13L} & XE_1^T + Y^T E_2^T \\ * & -\lambda_1 \gamma_f I & \lambda_1 D_\omega^T & \lambda_1 E_3^T \\ * & * & N_{33L} & 0 \\ * & * & * & -\lambda_1 \varepsilon_2 I \end{bmatrix} < 0, \quad L = 1, 2, \dots, n_\beta, \quad (14)$$

where  $M_{11} = AX + XA^T + BY + Y^T B^T + \varepsilon_1 H_1 H_1^T$ ,

$$M_{13} = XC^T + Y^T D^T + \varepsilon_1 H_1 H_2^T,$$

$$M_{33} = -\gamma_0 I + \varepsilon_1 H_2 H_2^T,$$

$$N_{11L} = AX + XA^T + B\beta_L Y + Y^T \beta_L B^T + \varepsilon_2 H_1 H_1^T,$$

$$N_{13L} = XC^T + Y^T \beta_L D^T + \varepsilon_2 H_1 H_2^T,$$

$$N_{33L} = -\lambda_1 \gamma_f I + \varepsilon_2 H_2 H_2^T.$$

In this case, the state-feedback gain is given by

$$K = YX^{-1}. \quad (15)$$

**Proof:** By applying Lemma 1 and Schur complement formula and introducing the new variable  $Y = KX$ , it is easy to verify that inequality (10) holds if inequality (13) holds.

By performing congruence transformation to (11) by  $\text{diag}\{\lambda_1^{1/2} I, \lambda_1^{1/2} I, \lambda_1^{1/2} I\}$ , considering (12) and (15), and using Lemma 1, we can conclude that a sufficient condition for (11) is

$$N_c(\alpha) := \begin{bmatrix} N_{c11}(\alpha) & \lambda_1 G & N_{c13}(\alpha) & N_{c14}(\alpha) \\ * & -\lambda_1 \gamma_f I & \lambda_1 D_\omega^T & \lambda_1 E_3^T \\ * & * & N_{c33} & 0 \\ * & * & * & -\lambda_1 \varepsilon_2 I \end{bmatrix} < 0, \quad (16)$$

with

$$N_{c11}(\alpha) = AX + XA^T + B\alpha Y + Y^T \alpha B^T + \varepsilon_2 H_1 H_1^T,$$

$$N_{c13}(\alpha) = XC^T + Y^T \alpha D^T + \varepsilon_2 H_1 H_2^T,$$

$$N_{c14}(\alpha) = XE_1^T + Y^T \alpha E_2^T,$$

$$N_{c33} = -\lambda_1 \gamma_f I + \varepsilon_2 H_2 H_2^T.$$

Since  $\alpha \in \Omega_\alpha$ , we have

$$N_c(\alpha) = \sum_{L=1}^{n_\beta} \theta_L N_L. \quad (17)$$

Then, it is clear that inequality (16) holds if inequality (14) holds. This completes the proof.

**Remark 1:** Consider system (8). If we choose a proper value of  $\gamma_f$ , then suboptimal reliable  $H_\infty$  controller of the form (7) can be determined by solving the following convex optimization problem:

$$\begin{aligned} & \min_{\varepsilon_1, \varepsilon_2, \gamma_f, X, Y, \lambda_1} \gamma_0 \\ & \text{subject to (13), (14), } X > 0. \end{aligned}$$

The reliable controller gain is given by (15).

**Remark 2:** Remark 1 contains the tuning scalar parameter  $\lambda_1$ . The optimum value can be ascertained by the approach in [10]. Applying a numerical optimization algorithm, such as the program *fminsearch* in the optimization toolbox of Matlab, a numerical solution to this problem is obtained.

Since the single Lyapunov matrix  $X_f = \lambda_1^{-1}X$  solves inequality (14) for all values of  $\alpha$ , the single Lyapunov function  $V(x) = x^T(t)X_f^{-1}x(t)$  works for all probable  $\alpha \in \Omega_\alpha$ . The design procedure above is simple but might be conservative due to employing the single Lyapunov function  $V(x) = x^T(t)X_f^{-1}x(t)$ . To reduce this conservatism, we will resort to a parameter-dependent Lyapunov function for the actuator failure cases in the next subsection. It should be mentioned that parameter-dependent Lyapunov functions have been extensively used to solve the  $H_\infty$  control problem for continuous-time polytopic systems in the literature, see, e.g., [10, 11] and references therein, however, these studies neither consider the uncertainty form (2), nor tackle the reliable control issue.

3.2. Parameter-dependent Lyapunov function approach

Let us give an equivalent representation of inequality (10).

**Lemma 2:** Given a constant  $\gamma_0 > 0$ , the following statements are equivalent.

- i) There exists a matrix  $X = X^T > 0$  satisfying inequality (10).
- ii) There exists a scalar  $\delta > 0$ , matrices  $Q$  and  $X = X^T > 0$  such that

$$\begin{bmatrix} \Psi(t) & X - Q^T + \delta A_c(t)Q & G(t) & Q^T C_c^T(t) \\ * & -\delta(Q + Q^T) & 0 & \delta Q^T C_c^T(t) \\ * & * & -\gamma_0 I & D_\omega^T(t) \\ * & * & * & -\gamma_0 I \end{bmatrix} < 0, \tag{18}$$

where  $\Psi(t) = A_c(t)Q + Q^T A_c^T(t)$ .

**Proof:** ii)  $\Rightarrow$  i): It is clear that multiplying (18) by  $J_1$  on the left and by  $J_1^T$  on the right, respectively, yields (10), where

$$J_1 = \begin{bmatrix} I & A_c(t) & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & C_c(t) & 0 & I \end{bmatrix}$$

i)  $\Rightarrow$  ii): If there exists a matrix  $X > 0$  such that (10) holds, then there exists a sufficiently small scalar

$\delta > 0$  such that

$$\begin{bmatrix} A_c(t)X + XA_c^T(t) & G(t) & XC_c^T(t) \\ * & -\gamma_0 I & D_\omega^T(t) \\ * & * & -\gamma_0 I \end{bmatrix} + \frac{\delta}{2} \begin{bmatrix} A_c(t)X \\ 0 \\ C_c(t)X \end{bmatrix} X^{-1} \begin{bmatrix} A_c(t)X \\ 0 \\ C_c(t)X \end{bmatrix}^T < 0,$$

which is equivalent to

$$\begin{bmatrix} \Phi(t) & X - X^T + \delta A_c(t)X & G(t) & XC_c^T(t) \\ * & -\delta(X + X) & 0 & \delta XC_c^T(t) \\ * & * & -\gamma_0 I & D_\omega^T(t) \\ * & * & * & -\gamma_0 I \end{bmatrix} < 0,$$

where  $\Phi(t) = A_c(t)X + XA_c^T(t)$ . Hence, there exist a scalar  $\delta > 0$  and matrix  $Q = X$  such that (18) holds. This completes the proof.

Unlike in (10), with the introduction of the extra variables  $Q$  and  $\delta$ , the matrix  $X$  is separated from the matrices  $A_c(t)$  and  $C_c(t)$  in (18). This property is useful and critical, which can be used to design controller for the actuator failure cases by employing a parameter-dependent Lyapunov function. This is shown in the following theorem.

**Theorem 2:** For some given scalars  $\gamma_f \geq \gamma_0 > 0$ , the controller (7) solves the reliable  $H_\infty$  control problem if there exists a solution  $(\varepsilon_3, \varepsilon_4, \lambda_2, \delta_0, \delta_f, X, Q, Y, X_L, L = 1, 2, \dots, n_\beta)$  to the following:

$$\begin{bmatrix} \bar{M}_{11} & \bar{M}_{12} & G & \bar{M}_{14} & \bar{M}_{15} \\ * & -\delta_0(Q + Q^T) & 0 & \bar{M}_{24} & \delta_0 \bar{M}_{15} \\ * & * & -\gamma_0 I & D_\omega^T & 0 \\ * & * & * & \bar{M}_{44} & 0 \\ * & * & * & * & -\varepsilon_3 I \end{bmatrix} < 0, \tag{19}$$

$$\begin{bmatrix} \bar{N}_{11L} & \bar{N}_{12L} & \lambda_2 G & \bar{N}_{14L} & \bar{N}_{15L} \\ * & \bar{N}_{22L} & 0 & \bar{N}_{24L} & \delta_f \bar{N}_{15L} \\ * & * & -\lambda_2 \gamma_f I & \lambda_2 D_\omega^T & 0 \\ * & * & * & \bar{N}_{44L} & 0 \\ * & * & * & * & -\lambda_2 \varepsilon_4 I \end{bmatrix} < 0,$$

where  $\bar{M}_{11} = AQ + Q^T A^T + BY + Y^T B^T + \varepsilon_3 H_1 H_1^T$ ,  $\bar{M}_{12} = X - Q^T + \delta_0(AQ + BY)$ ,

$$\begin{aligned} \bar{M}_{14} &= Q^T C^T + Y^T D^T + \varepsilon_3 H_1 H_2^T, \\ \bar{M}_{15} &= Q^T E_1^T + Y^T E_2^T, \quad \bar{M}_{24} = \delta_0(Q^T C^T + Y^T D^T), \\ \bar{M}_{44} &= -\gamma_0 I + \varepsilon_3 H_2 H_2^T, \\ \bar{N}_{11L} &= A Q + Q^T A^T + B \beta_L Y + Y^T \beta_L B^T + \varepsilon_4 H_1 H_1^T, \\ \bar{N}_{12L} &= X_L - Q^T + \delta_f(A Q + B \beta_L Y), \\ \bar{N}_{14L} &= Q^T C^T + Y^T \beta_L D^T + \varepsilon_4 H_1 H_2^T, \\ \bar{N}_{15L} &= Q^T E_1^T + Y^T \beta_L E_2^T, \quad \bar{N}_{22L} = -\delta_f(Q + Q^T), \\ \bar{N}_{24L} &= \delta_f(Q^T C^T + Y^T \beta_L D^T), \\ \bar{N}_{44L} &= -\lambda_2 \gamma_f I + \varepsilon_4 H_2 H_2^T. \end{aligned}$$

In this case, the state-feedback gain is given by

$$K = YQ^{-1}. \tag{21}$$

**Proof:** The proof is similar to that of Theorem 1 by Lemma 1, and is, thus, omitted for conciseness.

**Remark 3:** Consider system (8). Choosing a proper value of  $\gamma_f$ , the suboptimal reliable  $H_\infty$  controller of the form (7) can be determined by solving the following convex optimization problem:

$$\begin{aligned} &\min_{\varepsilon_3, \varepsilon_4, \gamma_f, X, X_L, Q, Y, \lambda_2, \delta_0, \delta_f} \gamma_0 \\ &\text{subject to (19), (20), } X > 0, X_L > 0, L = 1, 2, \dots, n_\beta. \end{aligned}$$

The reliable controller gain is given by (21).

#### 4. SIMULATION STUDIES

To illustrate the effectiveness of the proposed design procedures, reliable controllers are applied to the lateral control of an AV-8A Harrier VTOL aircraft in a hover model. The linearized unstable lateral dynamic model with parameters for this aircraft is taken from [12]. The state-space model for this aircraft is given by

$$\dot{x}(t) = (A + \Delta A(t))x(t) + Bu(t) + G\omega(t),$$

where  $\Delta A(t) = 0.02 \sin(t)A$ .

Define the controlled output:

$$z(t) = Cx(t) + Du(t),$$

where

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

For comparison purpose, we design three different controllers: the optimal standard  $H_\infty$  controller; and the suboptimal reliable  $H_\infty$  controllers by the first and second design procedures given in Section 3, respectively. By the approach proposed in [8], the obtained minimum disturbance attenuation level is  $\gamma_s^* = 0.2255$  for standard controller. Next, reliable

controllers are designed to cater both the normal and the actuator failure cases. Here, our main purpose is to design suboptimal reliable  $H_\infty$  controllers such that the lateral stick perturbation is tolerable to failure and its control effectiveness can vary from the normal case (100% effectiveness) to the complete outage case (0% effectiveness).

Let  $\gamma_f = 10$  and the initial value  $\lambda_{10} = 0.1$ . Applying Remark 1 and the *fminsearch* in Remark 2 gives the minimum disturbance attenuation level  $\gamma_{01}^* = 0.4588$  with  $\lambda_1 = 0.0617$ . Let the initial value  $(\delta_{00}, \lambda_{20}, \delta_{f0}) = (0.3, 0.2, 2.0)$ . Applying Remark 3 yields  $\gamma_{02}^* = 0.4129$ , which is better than that given by the first design procedure.

Assume the external disturbance signal  $\omega(t) = [0 \ 0 \ 0 \ 0 \ \omega_1]^T$ ,  $\omega_1 = 10e^{-0.15t} \sin(0.4\pi t + 0.25\pi)$ . Under the initial state variables  $x_0 = [-2 \ 0.2 \ 2 \ 0.5 \ 0.2]^T$ , Figs. 1 and 2 show the state responses for the corresponding closed-loop systems with standard controller and reliable controller in the case of lateral stick perturbation

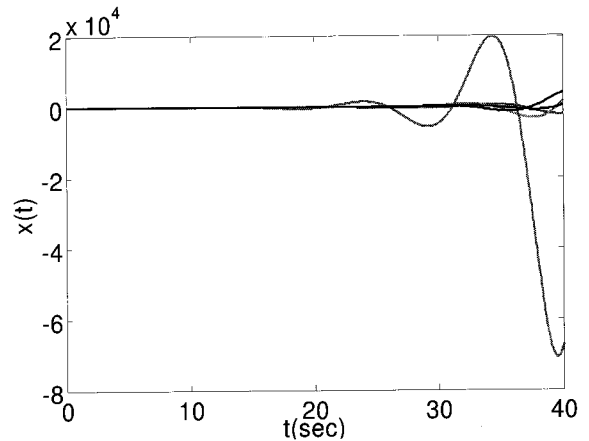


Fig. 1. The state responses of standard controller with failure.

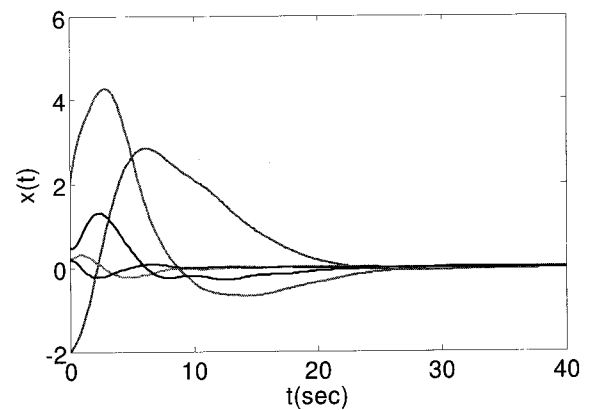


Fig. 2. The state responses of reliable controller with failure.

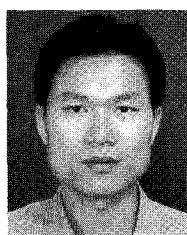
complete outage. From Fig. 1, it can be seen that standard controller can not maintain the closed-loop system stability, while reliable controller maintains the resulting closed-loop system stability (see Fig. 2).

## 5. CONCLUSIONS

In this paper, the suboptimal reliable  $H_\infty$  controller design procedures have been proposed for uncertain linear systems with external disturbances. Two efficient procedures for designing a suboptimal reliable  $H_\infty$  controller have been presented. The suboptimal reliable controller design procedures for the AV-8A Harrier VTOL aircraft with lateral unstable dynamic model parameters have been given to illustrate the applicability of the design procedures proposed. Simulation results further demonstrate the necessity and effectiveness of reliable controllers.

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