Coprime Factor Reduction of Parameter Varying Controller

Roberd Saragih and Widowati

Abstract: This paper presents an approach to order reduction of linear parameter varying controller for polytopic model. Feasible solutions which satisfy relevant linear matrix inequalities for constructing full-order parameter varying controller evaluated at each polytopic vertices are first found. Next, sufficient conditions are derived for the existence of a right coprime factorization of parameter varying controller. Furthermore, a singular perturbation approximation for time invariant systems is generalized to reduce full-order parameter varying controller via parameter varying right coprime factorization. This generalization is based on solutions of the parameter varying Lyapunov inequalities. The closed loop performance caused by using the reduced order controller is developed. To examine the performance of the reduced-order parameter varying controller, the proposed method is applied to reduce vibration of flexible structures having the transverse-torsional coupled vibration modes.

Keywords: Parameter varying controller, polytopic systems, reduced-order controller, right coprime factorization, singular perturbation method.

1. INTRODUCTION

Design techniques for constructing parameter varying controller with guaranteed H_∞ performance lead to controllers with order equal to the sum of order plant and weighting functions. Controllers with lower order which also stabilize closed-loop systems and provide the same level of closed-loop performance will be investigated. The lower-order controller can be found by controller reduction technique. Coprime factor controller reduction for linear time invariant (LTI) and time varying systems has been published by several authors [8,12,13]. Recently, some authors [7,11] have generalized balanced truncation (BT) corresponding results of LTI systems to reduce the order of unbounded rate linear parameter varying (LPV) model and controller. Balanced singular perturbation approximation (BSPA) to reduce the controller of LTI systems have been published by several authors [6,9]. Further, Widowati, et.al [10] generalized the BSPA method to reduce the model order of unstable LPV systems.

In this paper we propose a generalization of the BSPA method to reduce the order of LPV controller via contractive right coprime factorization (CRCF). Feasible solution for constructing quadratically stable CRCF controller of linear matrix inequality is evaluated at each of polytope vertices. Furthermore, by using a state transformation matrix CRCF parameter varying controller is balanced and then generalized singular perturbation method is applied to obtain reduced-order polytopic LPV controllers.

The paper is outlined as follows. In Section 2 we summarize polytopic model and necessary condition for quadratic stability of parameter varying systems. Design technique for constructing full-order parameter varying controller is presented in Section 3. Section 4 proposes main results regarding the generalization of singular perturbation method for LTI systems to reduce the order of parameter varying controller via right coprime factorization. In Section 5 the validity of the proposed controller reduction method is examined for reducing vibration of flexible structures. Finally, conclusions are given in Section 6.

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2. BRIEF REVIEW OF POLYTOPIC PARAMETER VARYING SYSTEMS

This section presents a brief review of polytopic model and quadratic stability of the LPV systems. Readers are referred to references [2-5,11] for further details.

For a compact set $\mathrm{P} \subset \mathbb{R}^s$, the parameter variation set $\mathbb{F}_\rho$ denotes the set of all piecewise continuous mapping $\mathbb{R}$ (time) into P with a finite number of discontinuity in any interval. $\mathbb{F}_\rho := \{\rho(t) : \mathbb{R} \to \mathrm{P},$

$\rho_{h_{\min}} \le \rho_h \le \rho_{h_{\max}}, h=1,2,\dots,s\}$. A compact set $\mathrm{P} \subset \mathbb{R}^s$, along with continuous functions $A:\mathbb{R}^s \to \mathbb{R}^{n\times n}$, $B:\mathbb{R}^s \to \mathbb{R}^{n\times n_u}$, $C:\mathbb{R}^s \to \mathbb{R}^{n_y\times n}$, $D:\mathbb{R}^s \to \mathbb{R}^{n_y\times n_u}$ represent nth-order LPV systems, $P(\rho)$, whose dynamics evolve as

$$\dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t), \tag{1}$$

$$y(t) = C(\rho(t))x(t) + D(\rho(t))u(t),\ \rho \in \mathbb{F}_\rho. \tag{2}$$

Further, matrix polytopes [3] are defined as the convex hull of a finite number of matrices N_i with the same dimensions, that is,

$$C_o\{N_i : i=1,2,\cdots,l\} := \left\{ \sum_{i=1}^{l} \alpha_i(t) N_i : \alpha_i(t) \ge 0, \sum_{i=1}^{l} \alpha_i(t) = 1 \right\}.$$

If the parameter $\rho(t)$ takes values in a box of $\mathbb{R}^s$ with corners $\{\rho_i\}_{i=1}^{l} (l = 2^s)$, or in other words, $\rho(t)$ varies in a polytope Θ with vertices ρ_1, $\cdots, \rho_l$ then $\rho(t)$ can be written as $\rho(t) \in \Theta := C_o\{\rho_1,\dots,\rho_l\}$, $\forall t \ge 0$. LPV systems are called polytopic when it can be represented by state space matrices $A(\rho(t))$, $B(\rho(t))$, $C(\rho(t))$, $D(\rho(t))$, where the parameter vector $\rho(t)$ ranges over a fixed polytope Θ, and the dependence of $A(.)$, $B(.)$, $C(.)$, $D(.)$ on $\rho(t)$ is affine. Although not fully general, this description encompasses many practical situations [2,3]. From the above characterization, the state space matrices $A(\rho(t))$, $B(\rho(t))$, $C(\rho(t))$, and $D(\rho(t))$ range in a polytope of matrices whose vertices are the images of the vertices $\rho_1, \cdots, \rho_l$, that is,

$$\begin{aligned} \begin{bmatrix} A(\rho(t)) & B(\rho(t)) \\ C(\rho(t)) & D(\rho(t)) \end{bmatrix} &\in C_o\left\{ \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} \right\}_{i=1}^{l} \\ &:= \left\{ \sum_{i=1}^{l} \alpha_i(t) \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} : \alpha_i(t) \ge 0, \sum_{i=1}^{l} \alpha_i(t) = 1 \right\}. \end{aligned} \tag{3}$$

The above equation indicates that $\begin{bmatrix} A(\rho(t)) & B(\rho(t)) \\ C(\rho(t)) & D(\rho(t)) \end{bmatrix}$ is a convex combination from systems matrices $\begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}_{i=1}^{l}$. The following proposition provides a necessary condition for quadratic stability and L_2-induced norm bound γ of LPV systems.

Proposition 1 [5,11]: The LPV systems $P(\rho)$ with state space matrices in (1-2) is quadratically stable and satisfies

$$\|P(\rho)\|_{i,2} := \sup_{\rho(t)\in\mathbb{F}_\rho} \sup_{u\neq 0, u\in L_2} \frac{\|y\|_2}{\|u\|_2} \le \gamma, \tag{4}$$

if there exists a constant positive-definite matrix X such that

$$\begin{aligned} &XA(\rho(t)) + A^T(\rho(t))X + C^T(\rho(t))C(\rho(t)) + \\ &\left(XB(\rho(t)) + C^T(\rho(t))D(\rho(t))\right)\times \\ &\left(\gamma^2 I - D^T(\rho(t))D(\rho(t))\right)^{-1}\times \\ &\left(XB(\rho(t)) + C^T(\rho(t))D(\rho(t))\right)^T < 0, \forall \rho(t) \in \mathbb{F}_\rho. \end{aligned} \tag{5}$$

3. FULL-ORDER PARAMETER VARYING CONTROLLER

In this section, we discuss design technique for constructing full-order parameter varying controller which developed by Apkarian [1]. The technique is formulated as Linear Matrix Inequality (LMI) problems, that is, we find symmetric positive definite matrices which solve LMI expressions. Suppose the parameter varying generalized plant $G(\rho)$ is described as follows

$$\begin{aligned} \dot{x}(t) &= A(\rho(t))x(t) + B_1(\rho(t))w(t) \\ &\quad + B_2(\rho(t))u(t), \end{aligned} \tag{6}$$

$$\begin{aligned} z(t) &= C_1(\rho(t))x(t) + D_{11}(\rho(t))w(t) \\ &\quad + D_{12}(\rho(t))u(t), \end{aligned} \tag{7}$$

$$\begin{aligned} y(t) &= C_2(\rho(t))x(t) + D_{21}(\rho(t))w(t) \\ &\quad + D_{22}(\rho(t))u(t), \end{aligned} \tag{8}$$

where w is the exogenous inputs, u is the control inputs, y is the measured outputs, and z is the controlled outputs.

The generalized plant can be written in polytopic form

$$\begin{bmatrix} A(\rho(t)) & B_1(\rho(t)) & B_2(\rho(t)) \\ C_1(\rho(t)) & D_{11}(\rho(t)) & D_{12}(\rho(t)) \\ C_2(\rho(t)) & D_{21}(\rho(t)) & D_{22}(\rho(t)) \end{bmatrix} \in$$

$$C_o\left\{\begin{bmatrix} A_i & B_{1i} & B_{2i} \\ C_{1i} & D_{11i} & D_{12i} \\ C_{2i} & D_{21i} & D_{22i} \end{bmatrix}\right\}_{i=1}^{l}. \quad (9)$$

Furthermore, we will find a full-order parameter varying controller, $K(\rho)$, with state space realizations is

$$\dot{x}_k(t) = A_k(\rho(t))x_k(t) + B_k(\rho(t))y(t), \quad (10)$$

$$u(t) = C_k(\rho(t))x_k(t) + D_k(\rho(t))y(t), \quad (11)$$

which satisfies H_∞ performance criterion [1], i.e., the parameter varying closed-loop system (6-11) is quadratically stable over Θ and the L_2 gain of the parameter varying closed-loop system is bounded by γ, $\gamma > 0$ for all possible parameter trajectories. The basic characterization of the full-order parameter varying controller is given in the next theorem.

Theorem 1 [1]: Consider the generalized LPV plant with polytopic form (9). There exists a polytopic LPV controller enforcing quadratic stability and a bound γ, (γ > 0), on the L_2 gain of the closed-loop system, whenever there exist symmetric positive definite matrices Y and Z and quadruples $(\bar{A}_{ki}, \bar{B}_{ki}, \bar{C}_{ki}, \bar{D}_{ki})$ such that the following LMI problems is feasible

$$\begin{bmatrix} YA_i + \bar{B}_{ki}C_{2i} + (*) & * & * & * \\ \bar{A}_{ki}^T & A_iZ + B_{2i}\bar{C}_{ki} + (*) & * & * \\ (YB_{1i} + \bar{B}_{ki}D_{21i})^T & (B_{1i} + B_{2i} + \bar{D}_{ki}D_{21i})^T & -\gamma I & * \\ C_{1i} + D_{12i}\bar{D}_{ki}C_{2i} & C_{1iZ+D_{21i}}\bar{C}_{ki} & D_{11i} + D_{12i}\bar{D}_{ki}D_{21i} & -\gamma I \end{bmatrix} < 0, \quad (12)$$

$$\begin{bmatrix} Y & I \\ I & Z \end{bmatrix} > 0, \quad (13)$$

where $i = 1, 2, \ldots, l$, terms denoted $*$ will be induced by symmetry.

Example:

$$\begin{bmatrix} M + N + (*) & * \\ Q & P \end{bmatrix} := \begin{bmatrix} M + M^T + N + N^T & Q^T \\ Q & P \end{bmatrix}.$$

When the above LMI problem holds a polytopic LPV controller can be obtained by using the following procedure

1. Obtain N and M which satisfy

$$I - YZ = NM^T.$$

2. Construct A_{ki}, B_{ki}, C_{ki}, D_{ki} with

$$A_{ki} = N^{-1}\left(\bar{A}_{ki} - (\tilde{A}_{ki})^T - Y(\tilde{B}_{ki})Z - \tilde{C}_{ki}\right)M^{-T},$$

where $\tilde{A}_{ki} = A_i + B_{2i}\bar{D}_{ki}C_{2i}$, $\tilde{B}_{ki} = A_i - B_{2i}\bar{D}_{ki}C_{2i}$,

$$\tilde{C}_{ki} = \bar{B}_{ki}C_{2i}Z + YB_{2i}\bar{C}_{ki}.$$

$$B_{ki} = N^{-1}(\bar{B}_{ki} - YB_{2i}\bar{D}_{ki}),$$

$$D_{ki} = \bar{D}_{ki}, i = 1, 2, \ldots l.$$

State-space realizations of the polytopic parameter varying controller are obtained as follows

$$\begin{bmatrix} A_k(\rho(t)) & B_k(\rho(t)) \\ C_k(\rho(t)) & D_k(\rho(t)) \end{bmatrix} := \sum_{i=1}^{l} \alpha_i(t) \begin{bmatrix} A_{ki} & B_{ki} \\ C_{ki} & D_{ki} \end{bmatrix}, \quad \alpha_i(t) \geq 0, \sum_{i=1}^{l} \alpha_i(t) = 1, \quad (14)$$

which ensures quadratic stability and bound γ, $(\gamma > 0)$ on the L_2 gain of the closed-loop systems over the entire parameter polytope Θ.

4. CRCF CONTROLLER REDUCTION USING GENERALIZED BSPA

Our aim in this section is to propose results regarding the generalization of balanced singular perturbation approximation (SPA) for time invariant systems to reduce the order of parameter varying controller via contractive right coprime factorizations (CRCF). For the definition of CRCF refers to Wood, et. al [11]. The following lemma is required to derive CRCF of an LPV controller $K(\rho)$ where the dependence of parameter vectors on t has been omitted for notation simplicity.

Lemma 1 [11]: Let $K(\rho)$ have a continuous, quadratically stabilizable, and quadratically detectable state space realization. Let $F_k(\rho)$ and $L_k(\rho)$ such that $A_k(\rho) + B_k(\rho)F_k(\rho)$ and $A_k(\rho) + L_k(\rho)C_k(\rho)$ are quadratically stable for all $\rho \in \Theta$. Define

$$\begin{bmatrix} U(\rho) & \tilde{Y}(\rho) \\ V(\rho) & \tilde{X}(\rho) \end{bmatrix} := \left[\begin{array}{c|cc} \hat{A}_k(\rho) & B_k(\rho) & -L_k(\rho) \\ \hline \hat{C}_k(\rho) & D_k(\rho) & I \\ F_k(\rho) & I & 0 \end{array}\right],$$

where

$$\hat{A}_k(\rho) = A_k(\rho) + B_k(\rho)F_k(\rho),$$

$$\hat{C}_k(\rho) = C_k(\rho) + D_k(\rho)F_k(\rho).$$

$$\begin{bmatrix} X(\rho) & Y(\rho) \\ \tilde{V}(\rho) & -\tilde{U}(\rho) \end{bmatrix} := \left[\begin{array}{c|cc} \breve{A}_k(\rho) & L_k(\rho) & -\breve{B}_k(\rho) \\ \hline F_k(\rho) & 0 & I \\ C_k(\rho) & I & -D_k(\rho) \end{array}\right],$$

where

$$\breve{A}_k(\rho) = A_k(\rho) + L_k(\rho)C_k(\rho),$$
$$\breve{B}_k(\rho) = B_k(\rho) + L_k(\rho)D_k(\rho),$$

then

$$\begin{bmatrix} X(\rho) & Y(\rho) \\ \tilde{V}(\rho) & -\tilde{U}(\rho) \end{bmatrix}\begin{bmatrix} U(\rho) & \tilde{Y}(\rho) \\ V(\rho) & \tilde{X}(\rho) \end{bmatrix} = I.$$

Definition 1 [11]: The ordered pair $\begin{bmatrix} U(\rho) & V(\rho) \end{bmatrix}$ represents a CRCF of $K(\rho)$ if

1. $K(\rho) = U_\rho V_\rho^{-1}$,
2. There exist $X(\rho)$, $Y(\rho)$ such that $X(\rho)U(\rho) + Y(\rho)V(\rho) = I$,
3. $\begin{bmatrix} U^T(\rho) & V^T(\rho) \end{bmatrix}$ is a contraction in the following sense

$$\sup_{\rho(t)\in\mathbb{F}_\rho} \sup_{u\in L_2:\|u\|_2\le 1} \left\| \begin{bmatrix} U(\rho) \\ V(\rho) \end{bmatrix} u \right\| \le 1. \quad (15)$$

Consider the CRCF existence of the m-order parameter varying controller $K(\rho)$, in the next theorem.

Theorem 2: Let $K(\rho)$ has a continuous, quadratically stabilizable, quadratically detectable realizations, then CRCF of $K(\rho)$ is given by a right coprime factorization (RCF) of the

$$K(\rho) = U(\rho)V^{-1}(\rho),$$

$$\begin{bmatrix} U(\rho) \\ V(\rho) \end{bmatrix} = \left[\begin{array}{c|c} \hat{A}_k(\rho) & B_k(\rho) \\ \hline \hat{C}_k(\rho) & D_k(\rho) \\ F_k(\rho) & I \end{array}\right], \quad (16)$$

where is $F_k(\rho) = -B_k^T(\rho)X$, $X = X^T > 0$ a feasible solution of the following inequality

$$XA_k(\rho) + A_k^T(\rho)X - XB_k(\rho)B_k^T(\rho)X < 0,$$
$$\forall \rho \in \Theta.$$

Proof: We will use Proposition 1 to show that the RCF parameter varying controller (12) is quadratically stable and contractive. We need to show that there exists a matrix $X = X^T > 0$ such that

$$\begin{aligned} &X\hat{A}_k(\rho) + \hat{A}_k^T(\rho)X + \hat{C}_k^T(\rho)\hat{C}_k(\rho) + \\ &F_k^T(\rho)F_k(\rho) + \left\{XB_k(\rho) + \hat{C}_k^T(\rho)D_k(\rho) + F_k^T(\rho)\right\} \\ &\times\left(\gamma^2 I - D_k^T(\rho)D_k(\rho)\right)^{-1} \times \\ &\left\{XB_k(\rho) + \hat{C}_k^T(\rho)D_k(\rho) + F_k^T(\rho)\right\}^T < 0, \quad (17) \\ &\forall \rho \in \Theta. \end{aligned}$$

Firstly, observe that by using $F_k(\rho) = -B_k^T(\rho)X$, we have

$$\begin{aligned} &X\hat{A}_k(\rho) = XA_k(\rho) - XB_k(\rho)B_k^T(\rho), \\ &\left\{XB_k(\rho) + \hat{C}_k^T(\rho)D_k(\rho) + F_k^T(\rho)\right\} \\ &\quad = \left\{XB_k(\rho) + \hat{C}_k^T(\rho)D_k(\rho) - XB_k(\rho)\right\} \quad (18) \\ &\quad = \left(C_k(\rho) + D_k(\rho)F_k(\rho)\right)^T D_k(\rho) \\ &\quad = C_k^T(\rho)D_k(\rho) - XB_k(\rho)D_k^T(\rho)D_k(\rho). \end{aligned}$$

Hence

$$\begin{aligned} &\left\{XB_k(\rho) + \hat{C}_k^T(\rho)D_k(\rho) + F_k^T(\rho)\right\} \times \\ &\left(-D_k^T(\rho)D_k(\rho)\right)^{-1} \times \\ &\left\{XB_k(\rho) + \hat{C}_k^T(\rho)D_k(\rho) + F_k^T(\rho)\right\}^T \\ &= \left\{C_k^T(\rho)D_k(\rho) - XB_k(\rho)D_k^T(\rho)D_k(\rho)\right\} \times \\ &\quad \left(-D_k^T(\rho)D_k(\rho)\right)^{-1} \times \quad (19) \\ &\quad \left\{C_k^T(\rho)D_k(\rho) - XB_k(\rho)D_k^T(\rho)D_k(\rho)\right\}^T \\ &= -\left\{C_k^T(\rho) - XB_k(\rho)D_k^T(\rho)\right\}D_k(\rho)D_k^{-1}(\rho) \times \\ &\quad D_k^{-T}(\rho)D_k^T(\rho)\left\{C_k(\rho) - D_k(\rho)B_k^T(\rho)X\right\} \\ &= -\left(C_k(\rho) + D_k(\rho)F_k(\rho)\right)^T \times \\ &\quad \left(C_k(\rho) + D_k(\rho)F_k(\rho)\right). \end{aligned}$$

Now, by using (18), (19), and $F_k(\rho) = -B_k^T(\rho)X$, (17) is equivalent to the following expression

$$XA_k(\rho) + A_k^T(\rho)X - XB_k(\rho)B_k^T(\rho)X < 0, \quad (20)$$
$$\forall \rho \in \Theta.$$

The above expression indicates that there exists $X = X^T > 0$ such that (17) is negative definite. Based on Proposition 1 and Definition 1, the system (16) is quadratically stable and contractive. These imply that $U(\rho)$ and $V(\rho)$ are both quadratically stable with $V(\rho)$ invertible. We can construct $K(\rho)=U(\rho)V^{-1}(\rho)$. Moreover, using state-space realization in Lemma 1 and the detectability of $K(\rho)$ we can find a left inverse for $K(\rho)$, i.e., there exists $X(\rho)$ and $Y(\rho)$ such that $X(\rho)U(\rho)+Y(\rho)V(\rho)=I$. From these results, it can be seen that the ordered pair $\begin{bmatrix}U(\rho) & V(\rho)\end{bmatrix}$ represents the CRCF of $K(\rho)$ and the proof is completed.

Now let symmetric positive definite matrices P and Q be the controllability and observability Gramians of the $\begin{bmatrix}U(\rho)\\ V(\rho)\end{bmatrix}$.

According to the $\begin{bmatrix}U(\rho)\\ V(\rho)\end{bmatrix}$ as in (16), P and Q satisfy the following Lyapunov inequalities

$$\hat{A}_k(\rho)P+P\hat{A}_k^T(\rho)+B_k(\rho)B_k^T(\rho)<0,\ \forall\rho\in\Theta, \tag{21}$$

$$\begin{aligned}&\hat{A}_k^T(\rho)Q+Q\hat{A}_k(\rho)+\hat{C}_k^T(\rho)\hat{C}_k(\rho)\\ &\quad+F_k^T(\rho)F_k(\rho)<0,\ \forall\rho\in\Theta.\end{aligned} \tag{22}$$

By Schur complement and changing variable $\tilde{P}=P^{-1}$ and $\tilde{Q}=Q^{-1}$, the above inequality is equivalent to the following LMIs

$$\begin{bmatrix}\tilde{P}\hat{A}_k(\rho)+\hat{A}_k^T(\rho)\tilde{P} & \tilde{P}B_k(\rho)\\ B_k^T(\rho)\tilde{P} & -I\end{bmatrix}<0, \tag{23}$$

$$\begin{bmatrix}\tilde{Q}\hat{A}_k^T(\rho)+\hat{A}_k(\rho)\tilde{Q} & \tilde{Q}\tilde{C}_k^T(\rho)\\ \tilde{C}_k(\rho)\tilde{Q} & -I\end{bmatrix}<0, \tag{24}$$

$$\begin{bmatrix}\tilde{P} & I\\ I & \tilde{Q}\end{bmatrix}>0, \tag{25}$$

where $\tilde{C}_k(\rho)=\begin{bmatrix}C_k(\rho)+D_k(\rho)F_k(\rho)\\ F_k(\rho)\end{bmatrix}$. The solutions of (21) and (22) can be found by taking the inversion of the feasible solutions of the above LMIs (23-25).

By using a balancing state transformation matrix we obtain the transformed controllability and observability Gramians

$\tilde{P}=\tilde{Q}=\Sigma=diag(\Sigma_1,\Sigma_2)$, with $\tilde{P}=T^{-1}PT^{-T}$, $\tilde{Q}=T^TQT$, $\Sigma_1=diag(\sigma_1,\cdots,\sigma_r)$, $\Sigma_2=diag(\sigma_{r+1},\cdots,\sigma_m)$, $\sigma_r>\sigma_{r+1}$, and

$$\sigma_j=\sqrt{\lambda_j(PQ)},\ \sigma_j>\sigma_{j+1},\quad j=1,2,\cdots,r,r+1,\cdots,m.$$

Further, we call σ_i as ϑ-singular values.

A balanced parameter varying CRCF of $K(\rho)$ can be expressed by

$$\begin{bmatrix}U(\rho)\\ V(\rho)\end{bmatrix}=\left[\begin{array}{c|c}\bar{A}_k(\rho)+\bar{B}_k(\rho)\bar{F}_k(\rho) & \bar{B}_k(\rho)\\ \hline \bar{C}_k(\rho)+D_k(\rho)\bar{F}_k(\rho) & D_k(\rho)\\ \bar{F}_k(\rho) & I\end{array}\right], \tag{26}$$

where

$$\bar{A}_k(\rho)=T^{-1}A_k(\rho)T,\ \bar{B}_k(\rho)=T^{-1}B_k(\rho),$$
$$\bar{C}_k(\rho)=C_k(\rho)T,\ \bar{F}_k(\rho)=-\bar{B}_k^T(\rho)X.$$

Partition the balanced parameter varying CRCF conformably with $\Sigma=diag(\Sigma_1,\Sigma_2)$ as follows

$$\begin{bmatrix}U(\rho)\\ V(\rho)\end{bmatrix}=\left[\begin{array}{cc|c}\bar{A}_{k1}(\rho) & \bar{A}_{k2}(\rho) & \bar{B}_{k1}(\rho)\\ \bar{A}_{k3}(\rho) & \bar{A}_{k4}(\rho) & \bar{B}_{k2}(\rho)\\ \hline \bar{C}_{k11}(\rho) & \bar{C}_{k12}(\rho) & D_k(\rho)\\ \bar{F}_{k1}(\rho) & \bar{F}_{k2}(\rho) & I\end{array}\right], \tag{27}$$

where

$$\bar{A}_{k1}(\rho)=\bar{A}_{k11}(\rho)+\bar{B}_{k1}(\rho)\bar{F}_{k1}(\rho),$$
$$\bar{A}_{k2}(\rho)=\bar{A}_{k12}(\rho)+\bar{B}_{k1}(\rho)\bar{F}_{k2}(\rho),$$
$$\bar{A}_{k3}(\rho)=\bar{A}_{k21}(\rho)+\bar{B}_{k2}(\rho)\bar{F}_{k1}(\rho),$$
$$\bar{A}_{k4}(\rho)=\bar{A}_{k22}(\rho)+\bar{B}_{k2}(\rho)\bar{F}_{k2}(\rho),$$
$$\bar{C}_{k11}(\rho)=\bar{C}_{k1}(\rho)+D_k(\rho)\bar{F}_{k1}(\rho),$$
$$\bar{C}_{k12}(\rho)=\bar{C}_{k2}(\rho)+D_k(\rho)\bar{F}_{k2}(\rho),$$

with

$\bar{A}_{k11}\in\mathbb{R}^{r\times r}$, $\bar{A}_{k12}\in\mathbb{R}^{r\times(m-r)}$, $\bar{A}_{k21}\in\mathbb{R}^{(m-r)\times r}$, $\bar{A}_{k22}\in\mathbb{R}^{(m-r)\times(m-r)}$, $\bar{B}_{k1}\in\mathbb{R}^{r\times m_y}$, $\bar{B}_{k2}\in\mathbb{R}^{(m-r)\times m_y}$, $\bar{C}_{k1}\in\mathbb{R}^{m_u\times r}$, $\bar{C}_{k2}\in\mathbb{R}^{m_u\times(m-r)}$, $\bar{F}_{k1}\in\mathbb{R}^{m_y\times r}$, $\bar{F}_{k2}\in\mathbb{R}^{m_y\times(m-r)}$.

When the system is balanced, states corresponding to ϑ-smaller singular values $(\Sigma_2(\rho))$ represent the fast dynamics of the systems (i.e. its states have very fast transient dynamics and decay rapidly to certain steady value). Based on the concept of the singular perturbation method [6,9], we set the derivative of all states corresponding Σ_2 equal to zero. Moreover, the generalized singular perturbation method can be applied to approximate balanced CRCF (27) where state space matrices are evaluated at each of the polytope vertices as follows.

$$
\begin{aligned}
A_{k_r i} &= \bar{A}_{k11i} + \bar{B}_{k1i}\bar{F}_{k1i} - \left(\bar{A}_{k12i} + \bar{B}_{k1i}\bar{F}_{k2i}\right)\times \\
&\quad \left(\bar{A}_{k22i} + \bar{B}_{k2i}\bar{F}_{k2i}\right)^{-1}\left(\bar{A}_{k21i} + \bar{B}_{k2i}\bar{F}_{k1i}\right), \\
B_{k_r i} &= \bar{B}_{k1i} - \left(\bar{A}_{k12i} + \bar{B}_{k1i}\bar{F}_{k2i}\right)\times \\
&\quad \left(\bar{A}_{k22i} + \bar{B}_{k2i}\bar{F}_{k2i}\right)^{-1}\bar{B}_{k2i}, \\
C_{ku_r i} &= \bar{C}_{k1i} + D_k\bar{F}_{k1i} - \left(\bar{C}_{k2i} + D_k\bar{F}_{k2i}\right)\times \\
&\quad \left(\bar{A}_{k22i} + \bar{B}_{k2i}\bar{F}_{k2i}\right)^{-1}\left(\bar{A}_{k22i} + \bar{B}_{k2i}\bar{F}_{k2i}\right)^{-1}\times \\
&\quad \left(\bar{A}_{k21i} + \bar{B}_{k2i}\bar{F}_{k1i}\right), \\
C_{kv_r i} &= \bar{F}_{k1i} - \bar{F}_{k2i}\left(\bar{A}_{k22i} + \bar{B}_{k2i}\bar{F}_{k2i}\right)^{-1}\times \\
&\quad \left(\bar{A}_{k21i} + \bar{B}_{k2i}\bar{F}_{k1i}\right), \\
D_{ku_r i} &= D_{ki} - \left(\bar{C}_{k2i} + D_{ki}\bar{F}_{k2i}\right)\times \\
&\quad \left(\bar{A}_{k22i} + \bar{B}_{k2i}\bar{F}_{k2i}\right)^{-1}\bar{B}_{k2i}, \\
D_{kv_r i} &= I - \bar{F}_{k2i}\left(\bar{A}_{k22i} + \bar{B}_{k2i}\bar{F}_{k2i}\right)^{-1}\bar{B}_{k2i},
\end{aligned}
$$

by assuming $\left(\bar{A}_{k22i} + \bar{B}_{k2i}\bar{F}_{k2i}\right)$ invertible for all $i = 1, 2, \ldots, l$. State space realizations of $U(\rho)$ and $V(\rho)$ are expressed as follows

$$
\begin{aligned}
U_r(\rho) &= \left[\begin{array}{cc} A_{k_r}(\rho) & B_{k_r}(\rho) \\ C_{ku_r}(\rho) & D_{ku_r}(\rho) \end{array}\right] \\
&:= \sum_{i=1}^{l}\alpha_i(t)\left[\begin{array}{cc} A_{k_r i} & B_{k_r i} \\ C_{ku_r i} & D_{ku_r i} \end{array}\right], \qquad (28) \\
&\alpha_i(t) \ge 0,\ \sum_{i=1}^{l}\alpha_i(t) = 1, \\
V_r(\rho) &= \left[\begin{array}{cc} A_{k_r}(\rho) & B_{k_r}(\rho) \\ C_{kv_r}(\rho) & D_{kv_r}(\rho) \end{array}\right] \\
&:= \sum_{i=1}^{l}\alpha_i(t)\left[\begin{array}{cc} A_{k_r i} & B_{k_r i} \\ C_{kv_r i} & D_{kv_r i} \end{array}\right], \qquad (29) \\
&\alpha_i(t) \ge 0,\ \sum_{i=1}^{l}\alpha_i(t) = 1
\end{aligned}
$$

and we find reduced-order polytopic parameter varying controller , r-order, in the form

$$
\begin{aligned}
K_r(\rho) &= U_r(\rho)V_r^{-1}(\rho) \\
&= \left[\begin{array}{cc} A_{k_r}(\rho) - B_{k_r}(\rho)D_{kv_r}^{-1}(\rho)C_{kv_r}(\rho) & B_{k_r}(\rho)D_{kv_r}^{-1}(\rho) \\ C_{ku_r}(\rho) - D_{ku_r}(\rho)D_{kv_r}^{-1}(\rho)C_{kv_r}(\rho) & D_{ku_r}(\rho)D_{kv_r}^{-1}(\rho) \end{array}\right] \qquad (30) \\
&:= \sum_{i=1}^{l}\alpha_i(t)\left[\begin{array}{cc} A_{k_r i} - B_{k_r i}D_{kv_r i}^{-1}C_{kv_r i}(\rho) & B_{k_r i}D_{kv_r i}^{-1} \\ C_{ku_r i} - D_{ku_r i}D_{kv_r i}^{-1}C_{kv_r i} & D_{ku_r i}D_{kv_r i}^{-1} \end{array}\right], \\
&\alpha_i(t) \ge 0,\ \sum_{i=1}^{l}\alpha_i(t) = 1.
\end{aligned}
$$

The closed loop performance caused by using the reduced order controller is given in the following theorem.

Theorem 3: Let $K(\rho)$ be the full-order controller and $K_r(\rho)$ is the reduced-order LPV controller using BSPA. If $\tilde{T}_{zwq}(\rho)$ is a transfer function of closed loop system with reduced-order controller $K_r(\rho)$, $\tilde{T}_{zw}(\rho)$ is balanced reduced transfer fuction of closed loop system using singular perturbation approach, and $T_{zw}(\rho)$ is a transfer function of closed loop system with the full order controller $K(\rho)$ then $\tilde{T}_{zwq}(\rho) = \tilde{T}_{zw}(\rho)$, $\tilde{T}_{zwq}(\rho)$ is quadratically stable, and $\left\|T_{zw}(\rho) - \tilde{T}_{zwq}(\rho)\right\|_{i,2} \le 2\sum_{j=r+1}^{m}\delta_j$, where δ_j is the singular values of the balanced transfer function of closed-loop system.

Proof: Proof of the theorem is not so difficult, but since too long, we give the outline i.e., the transfer function of the closed-loop system with the full-order controller $T_{zw}(\rho)$ is balanced, and then it is truncated by using the singular perturbation, we obtain $\tilde{T}_{zw}(\rho)$. By algebra manipulation, we can show that $\tilde{T}_{zwq}(\rho) = \tilde{T}_{zw}(\rho)$. Based on the balanced truncation property [13], the stability of $\tilde{T}_{zwq}(\rho)$ and $\left\|T_{zw}(\rho) - \tilde{T}_{zwq}(\rho)\right\|_{i,2} \le 2\sum_{j=r+1}^{m}\delta_j$, can be obtained.

5. SIMULATION RESULTS

To examine the capability of the proposed method, a simulation is carried out by applying it to reduce the vibration of flexible structures. The structure has four stories and is tower-like in shape. Each story is modeled such that it has a single-degree-of-freedom in the transverse direction (the same direction as the excitation) and one more degree-of-freedom in the angle of torsion around the centroid of the story. Thus, the whole structure has 8 degrees-of-freedom. Due to an auxiliary mass on the right side at the third layer, the structure has the long sides and the short sides symmetric with respect to the central axis, which thereby creates a coupling between the transverse and torsional vibration. At the base, the structure is connected to a shaker where the entire structure is shaken in the direction of its short sides.

The matrix equation of motion of the structure can be expressed as [9]:

$$\begin{aligned} M_p(\rho)\ddot{x}_0(t) + C_p(\rho)\dot{x}_0(t) \\ + K_p(\rho)x_0(t) + d_p\ddot{z}(t) + b_p f(t) = 0, \end{aligned} \tag{31}$$

where M_p, C_p, and K_p are inertia, damping and stiffness matrices of the structure, respectively. d_p is disturbance matrix for the excitation acceleration $\ddot{z}$ and b_p is input matrix for the control force f. In this case, the variation of parameters lies on the M_p. Except for the 3rd story, the mass distribution of each story is homogeneous and the stiffnesses of 4 columns are supposed to be the same in the direction of the excitation at all stories. Therefore distance from centroid to the spring of right and left sides of 3rd story become unequal, the cross terms have certain value, and the structure possesses transverse torsional coupled vibration modes. Two active dynamic absorbers are installed on the left and the right sides of the top layer symmetrically with respect to the long side, which enable the application of control force along the direction of the short side. Each dynamic vibration absorber consists of a moving mass, a supporting roller bearing, and a coil.

For control analysis and design purposes, the model of structure, absorbers, and strokes are transformed into the state space form as is given in (6-8). The model dynamic of structure can be affected by operating conditions such that the linear parameter varying framework can be used to represent structural dynamic. The framework represents the dynamic with dependency on operating parameter as a set of state space matrices that are affine functions of those parameters, as given in (3).

5.1. Full-order controller design

The purpose of the controller design is to flatten the peaks of the open-loop transfer function from the excitation acceleration to the right acceleration output on the first and second modes. For this purpose we utilize the 4th-order frequency weighting high-pass filter. The state equation of the frequency weighting filter is

$$W_H = Lev \times \frac{s^4 + 4\xi_l\omega_l s^3 + 2\left(2\xi_l^2 + 1\right)\omega_l^2 s^2 + 4\xi_l\omega_l^3 s + \omega_l^4}{s^4 + 4\xi_h\omega_h s^3 + 2\left(2\xi_h^2 + 1\right)\omega_h^2 s^2 + 4\xi_h\omega_h^3 s + \omega_h^4}, \tag{32}$$

where $Lev = 400$, $\omega_l = 6\pi$, $\xi_l = 0.6$, $\omega_h = 36\pi$, $\xi_h = 0.6$.

The controller can also change with the operating parameters so that the state space matrices of this controller can be written as an affine combination of matrices multiplied by the element of parameters, as given in (14). The advantage of the LPV controller is that the closed-loop system be stabilized for any value of parameter and also for any time-varying trajectory of parameter. In addition, the closed-loop system satisfies an H_∞-norm bound on the worst case gain from disturbances to the errors for any parameter. The full-order controller is designed according to the design procedure developed by Apkarian, et al. [1] and we obtain the 30th-order controller.

5.2. Reduced-order controller

By using (30), the full-order controller is reduced. The frequency response of the open-loop, the closed-loop with the high-order controller, and the closed-loop with the 7th-order controller are given in Fig. 1. From this figure, we see that the first mode's gain the open-loop transfer function from the excitation to the right acceleration output can be reduced by about 15 dB and 8 dB for the second mode by using the full-order controller. This performance can be maintained although the full-order controller is reduced up to 7th-order except for the 8th-order controller which yields unstable in the closed-loop system. The performance of the closed-loop system with the full-order controller and the reduced-order controllers are compared, as shown in Table 1. From Table 1, we can see that the performance of the full-order controller is nearly same as that of the reduced-order controllers.

Table 1. H_∞-norms of closed-loop system with the full-order controller and the reduced-order controller.

r	20	15	12	10	9	7
$\left\|T_{zw}(\rho)\right\|_\infty - \left\|\tilde{T}_{zwr}(\rho)\right\|_\infty$	0.0027	0.0074	0.0067	0.0072	0.0075	0.0054

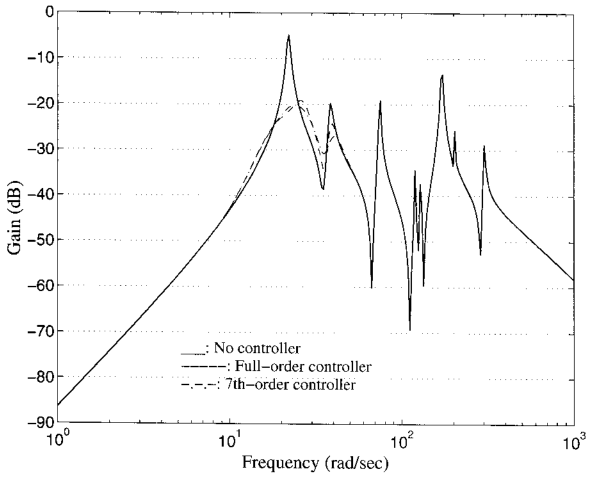

Fig. 1. Frequency response of the full-order and reduced order controller.

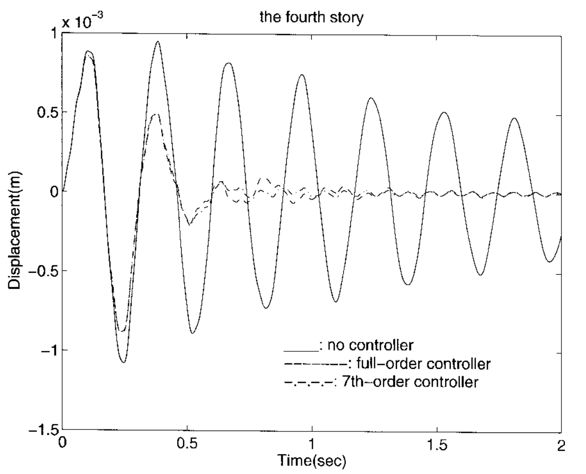

Fig. 2. Transverse response of the full-order and reduced-order controller.

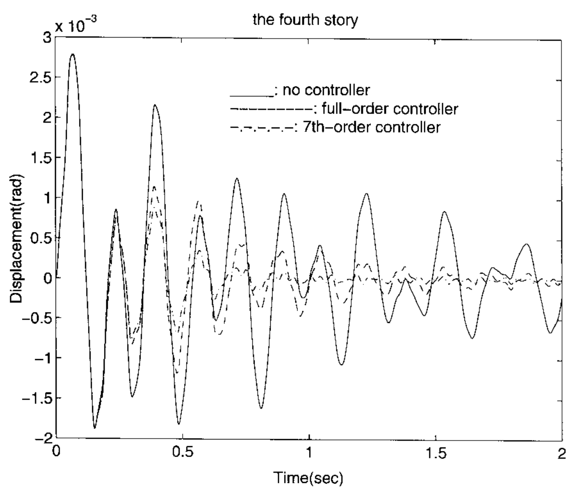

Fig. 3. Torsional response of the full-order and reduced-order controller.

The impulse response of the transverse and torsional displacements for the forth story is given in Figs. 2 and 3, respectively. From those figures, we can conclude that the performance of the reduced-order controller is identical to that of the full-order controller.

6. CONCLUSIONS

We have derived sufficient conditions for the CRCF existence of the parameter varying controller. We also have generalized the singular perturbation approximation to reduce the order of the polytopic parameter varying controller based on contractive right coprime factorizations. The reduced-order controller can be obtained by setting to zero the derivative of all states corresponding to the $\vartheta -$ smaller singular values of the contractive right coprime factorizations and the closed loop performance caused by using the reduced order controller is obtained. The reduced order controller is applied to reduce the vibration of flexible structures. From the simulation results, the proposed method can be reduced the order of controller into the 7th-order.

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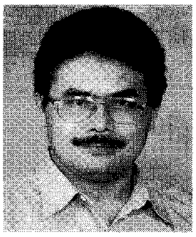

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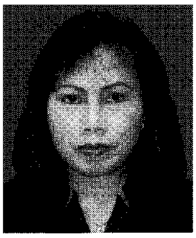

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