# Trust-Tech based Parameter Estimation and its Application to Power System Load Modeling

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**Abstract** – Accurate load modeling is essential for power system static and dynamic analysis. By the nature of the problem of parameter estimation for power system load modeling using actual measurements, multiple local optimal solutions may exist and local methods can be trapped in a local optimal solution giving possibly poor performance. In this paper, Trust-Tech, a novel methodology for global optimization, is applied to tackle the multiple local optimal solutions issue in measurement-based power system load modeling. Multiple sets of parameter values of a composite load model are obtained using Trust-Tech in a deterministic manner. Numerical studies indicate that Trust-Tech along with conventional local methods can be successfully applied to power system load model parameter estimation in measurement-based approaches.

**Keywords:** Global optimization, Measurement-based approach, Multiple local minima, Parameter estimation, Power system load modeling, Trust-Tech

#### 1. Introduction

Given a load model structure, the measurement-based approach is currently one of the well-recognized methods for deriving model parameter values [2]. The measurement-based approach derives parameter values of given load model structures by using field measurement data taken at load buses for which load models are to be developed. This approach has the advantage of directly measuring actual load behaviors during system disturbances so that accurate parameter values can be obtained in the form needed for existing power system analysis and control programs [2], [3].

The task of load model parameter estimation using actual measurements is usually formulated as a (constrained) nonlinear least squares problem, where the mismatch between the simulated load model output and the actual measurement is minimized [4-10]. However, it is known that the objective function may have multiple local optimal solutions by nature of the problem [5], [11, 12]. Hence, local methods such as gradient-based optimization techniques can potentially lead to a local optimal solution. The obtained local optimal solution may not satisfy model validation criteria. Even if the local optimal solution satisfies the validation criteria, it still demonstrates inferior modeling performance to a global optimal solution.

The multiple local optimal solutions issue in measurement-based power system load modeling has not been discussed in detail in the literature. Several papers briefly address the multiple local optimal solution issue; the authors in [11] mention the issue of multiple sets of parameters for different initial values in parameter estimation of an exponential type static load model (Type 2B in [11]). The authors in [12] suggest global optimization techniques such as the adaptive simulated annealing (ASA) to solve the issue and the authors in [15] apply genetic algorithms (GA) for finding optimal solutions for parameter values of a composite dynamicstatic model (CDSM). However, most of the approaches reported in the literature to the load model parameter estimation are stochastic approaches, the results of which may not be reproduced with the same initial condition.

In this paper, Trust-Tech [17], a novel methodology for a global optimization, is introduced and applied to a parameter estimation task for two composite load models; ZIP-induction motor (ZIP-IM) model where a static ZIP model is connected to an induction motor in parallel. The parameter estimation task is formulated as a special constrained nonlinear optimization problem where inequality conditions for certain parameters should be satisfied during the complete parameter estimation process. The Trust-Tech method searches feasible parameter space for local optimal solutions in a tier-by-tier manner with the aid of a local method. The numerical results show that Trust-Tech can be successfully applied to load model parameter estimation problems for the purpose of

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obtaining superior optimal solutions by guiding local methods.

# 2. Problem Preliminaries

The key objective of load model parameter identification using measurement data is to identify accurate parameter values whereby the model output is as close to the measured output as possible. An overall procedure for identifying a load model is described in the following:

- Step 1. Obtain a set of input-output data derived from a set of measurements.
- Step 2. Select a load model structure.
- Step 3. Estimate its parameters using a suitable method and estimation criterion.
- Step 4. Validate the derived model with the parameters obtained in Step 3.
- Step 5. If the validation criterion is not met, take remedial actions; for example, try another estimation method, or try another model structure and return to Step 3.

In Step 2, physically based load model structures are usually adopted since good insights into model structures can be often obtained from domain knowledge. A physically based load model can be generally represented as follows:

$$\dot{z}(t,p) = f(z(t,p), u(t), p), \quad z_0 = z(t_0) = \Gamma(p)$$
 (1)

$$y(t, p) = g(z(t, p), u(t), p)$$
 (2)

where z is a vector of state variables; u is a model input vector; y is model output; p is an  $n_p$  -dimensional parameter vector; and  $f, g, \Gamma$  are vectors of differentiable nonlinear (or linear) functions of p and t. In case of a static load model, the dynamic-algebraic equations (DAEs) are reduced to only algebraic equation (2) without dynamic states.

In Step 3, the parameter estimation task is often formulated as a constrained nonlinear least squares problem using an output error function that can be computed from measurement output and model output. The (output) error function is minimized as follows:

$$\min_{p \in \mathbb{Z}} \varepsilon(p) = \min_{p \in \mathbb{Z}} \frac{1}{2} \sum_{k=1}^{N} \left( y_m(k) - y(k, p) \right)^2$$
s.t. 
$$\begin{cases} h_i(p) = 0 & \text{for } i \in E = \{1, \dots, n_e\} \\ h_j(p) \le 0 & \text{for } j \in I = \{1, \dots, n_i\} \end{cases}$$

where  $p \in \Re^{n_p}$  denotes a parameter vector to be estimated; N and Z are the total number of samples used for estimation and the feasible parameter space respectively;  $y_m(k)$  and y(k) are the measured value and the model output at the  $k^{th}$  sample, respectively; and  $h_i(p)$  and  $h_j(p)$  represent equality and inequality constraints on p.

It is clear that the objective error function  $\varepsilon(p)$  in (3) is nonlinear and can be non-convex [20]. Hence, multiple local optimal solutions may exist, which implies that local methods such as the gradient-based local optimization techniques can be trapped in a local minimum.

The inequality constraints  $h_j(p)$  in (3) can be transformed into equality constraints by introducing slack variables  $s = \{s_i\}$  with  $j = 1, \dots, n_i$ :

$$h_i(p) + s_i^2 = 0$$
 for  $j \in I = \{1, \dots, n_i\}$  (4)

By combining the slack variables with parameters, the original problem can be represented as follows:

$$\min_{x \in \Omega} \varepsilon(x) = \min_{x \in \Omega} \frac{1}{2} \sum_{k=1}^{N} \left( y_m(k) - y(k, x) \right)^2$$
s.t.  $H_i(x) = 0$  for  $i \in E = \{1, \dots, n_e + n_i\}$ 

where 
$$x \in \mathbb{R}^n = [p, s]^T = [p_1, \dots, p_{n_p}, s_1, \dots, s_{n_i}]^T$$
 and  $n = n_p + n_i$ . Let us define  $H(x) = [H_1(x), \dots, H_{n_i + n_i}(x)]^T$ .

Evaluation of the objective function is the most basic but important task in solving the parameter estimation problem. For static models, it is straightforward to obtain model output with given parameters and model inputs. It is, however, necessary to solve a set of differential equations for dynamic or composite load models to evaluate objective function value. The initial state  $z_0$  can be obtained from steady state condition, which is represented by a set of functions of parameters. Constraints on certain parameters are necessary to enforce for the existence of initial state  $z_0$ . Note that the constraints for the existence of initial state  $z_0$  should always be satisfied during the entire parameter estimation process; otherwise evaluation of objective function value will fail.

#### 3. Trust-Tech Methodology

The Trust-Tech methodology provides a framework and computational schemes for locating multiple local

optimal solutions in a systematic manner. With a suitable mathematical transformation, the Trust-Tech methodology achieves this capability with the aid of knowledge of stability boundaries and completely stable dynamical systems [17 - 19]. Compared to other stochastic type algorithms developed for identifying multiple solutions, Trust-Tech based algorithms have an appealing feature of deterministically locating multiple local optimal solutions in a tier-by-tier manner.

The Trust-Tech constrained methodology considers the constrained nonlinear programming problem described in (3) or (5). The difficulties of solving problem (5) are well recognized. First, the feasible region may be composed of several disconnected feasible components in the entire search space. Second, there may be multiple local optimal solutions inside each feasible component.

Instead of directly solving the constrained optimization problem (5), Trust-Tech solves this problem by defining two respective dynamical systems: "Quotient Gradient System" (QGS) and "Projected Gradient System" (PGS) [17, 18]. The QGS is designed to locate multiple feasible components. The PGS is designed to locate multiple local optimal solutions lying within each feasible component. Through some trajectories of these two dynamical systems, the Trust-Tech method can locate multiple local optimal solutions in each disconnected feasible component.

#### 3.1 Phase I (QGS Phase)

The quotient gradient system (QGS) is defined as follows:

$$\dot{x}(t) = -J_{H}(x)^{T} \cdot H(x) \tag{6}$$

where  $J_H(x)$  is the Jacobian matrix of the constant vector function H(x). It is proved that a stable equilibrium manifold of (6) corresponds to a feasible component of (5). And if  $\Sigma$  is a stable equilibrium manifold of (6), then it is a local optimal solution of the following optimization problem:

$$\min E(x) = \min \frac{1}{2} ||H(x)||^2$$
 (7)

where  $x \in \mathbb{R}^n$ . E(x) is called the energy function of QGS.

Therefore, all or multiple disconnected feasible components of problem (5) can be identified through locating all or multiple stable equilibrium manifolds of QGS. The conceptual QGS algorithm can be briefly described as follows:

**Step 1**. Create a path leading away from the initial stable equilibrium manifold to reach the stability boundary.

- **Step 2**. Follow the stability boundary to identify the unstable equilibrium manifold separating the initial and target stable equilibrium manifolds.
- **Step 3**. Go outwards to cross the stability boundary from the identified unstable equilibrium manifold and follow the trajectory to locate the adjacent stable equilibrium manifold.

# 3.2 Phase II (PGS Phase)

To overcome the difficulty of searching multiple local optimal solutions within each identified feasible component by Phase I, we explore some trajectories of the following projected gradient system (PGS):

$$\dot{x}(t) = -P_H(x(t)) \cdot \nabla f(x(t)) \tag{8}$$
where  $P_H(x) = (I - J_H(x))^T \cdot [J_H(x)J_H(x)]^{-1} \cdot J_H(x)$  with identity matrix  $I \in \Re^{n \times n}$  and 
$$\nabla f(x(t)) = [\partial f / \partial x_1, \dots, \partial f / \partial x_n]^T.$$

The basic idea of the projected gradient system (8) is to apply the projection matrix to restrict the trajectory to follow the direction of the minus gradient vector projected onto the tangent space of the stable equilibrium manifold. With the projection matrix, the trajectory will stay inside the same stable equilibrium manifold. The minus gradient vector and the projection matrix ensure that the trajectory converges to a stable equilibrium point (SEP) inside the stable equilibrium manifold. This SEP will be a local optimal solution of (5). Hence, it is sufficient to consider the optimal solutions of (5) through the SEPs of the PGS.

To identify the target SEP from an initial SEP, it suffices to develop a mechanism to penetrate the stability boundary from the initial stability region to reach the target stability region with the aid of the so-called 'decomposition point' on the boundary [17]. This leads to the following conceptual algorithm:

- **Step 1**. Create a path moving away from the initial SEP to reach the stability boundary.
- **Step 2.** Follow the stability boundary to identify the decomposition point separating the initial and the corresponding SEPs.
- **Step 3.** Go outwards to cross the stability boundary from the identified decomposition point and follow the trajectory to locate the adjacent SEP.

It is noted that Step 2 is computationally complex and can be very time consuming. In this study, a simpler but efficient algorithm called "exit point method" is used. In the exit point method, once the boundary is detected at Step 1, the search procedure can directly move across the

boundary and enter the stability region of the corresponding SEP. Afterwards, the procedure follows the trajectory to search for the corresponding SEP.

A disadvantage of this algorithm is that it is possible to locate the same SEP along different search directions. However, compared to the computational complexity of Step 2, the exit point method can be more efficient than the original algorithm.

Step 3 involves integration of PGS with an initial point obtained from Step 2. Instead of integrating PGS until a SEP is reached, an efficient local optimization solver can be employed to solve the original nonlinear programming problem (5).

To appreciate a feature of constrained Trust-Tech, the details of the Step 1 procedure are presented in the following. Given an initial SEP  $x_s^1$  and initial search direction  $\vec{d}$ , the goal of this procedure is to generate a sequence of points to approach the corresponding stability boundary and to identify the corresponding "exit point". It is numerically implemented as follows: Initialization. Set  $x_0 = x_s^1$  and i = 0.

Step 1. 
$$y_{i+1} = x_i + \tau \cdot \vec{d}$$
.  
Step 2.  $x_{i+1} = x_i + P_H(x_i) \cdot (y_{i+1} - x_i)$ .  
Step 3. If  $f(x_{i+1}) \ge f(x_i)$ , then set  $\vec{d} = (x_{i+1} - x_s^1) / ||x_{i+1} - x_s^1||$ ,  $i = i+1$  and go to Step 1; otherwise,  $x_{exi} = x_{i+1}$ .

In each iteration, one point  $y_i$  is generated from a feasible point  $x_i$  with a small distance  $\tau$  along the search direction  $\vec{d}$  in Step 1. The generated point will lie outside the stable equilibrium manifold (or equivalently, the constraint surface or feasible region of problem (5)). In Step 2, a new feasible point  $x_{i+1}$  on the stable equilibrium manifold of QGS is created from  $y_i$  by applying projection matrix  $P_{H}(x_{i})$ . Thereafter,  $\vec{d}$  is updated as a unit vector with the direction pointing towards  $x_{i+1}$  with initial point  $x_s^1$  in Step 3. Such update ensures that  $x_{i+1}$  moves away from  $x_s^1$  and the generated sequence  $\{x_i\}$  gradually approaches the stability boundary. This procedure terminates once function f(x) starts to decrease. The point at which f(x) decreases is the exit point. The term "exit point" is used to indicate that this point is a good approximation to the point where the procedure exits the stability boundary.

# 4. A Composite Load Model Structure

In this section, the ZIP-IM composite load model is described before numerical studies for parameter estimation using actual measurements.

## 4.1 ZIP-IM Model

Aggregate power system loads are often represented by a composite load model to capture the static and dynamic behaviors of various load components. The ZIP-IM model consists of a static ZIP model and third-order induction motor model [13] as follows:

$$\begin{cases} T_0' \frac{dE'}{dt} = -\frac{X}{X'} E' + \frac{X - X'}{X'} \cdot V \cdot \cos \delta \\ \frac{d\delta}{dt} = \omega - \omega_s - \frac{X - X'}{X'} \cdot \frac{V \cdot \sin \delta}{T_0' \cdot E'} \\ M \frac{d\omega}{dt} = -\frac{V \cdot E' \cdot \sin \delta}{X'} - T_m \end{cases}$$
(9)

$$\begin{cases}
P = P_{ZIPo} \left( P_Z \left( \overline{V} \right)^2 + P_I \left( \overline{V} \right) + P_P \right) - VE' \sin \delta / X' \\
Q = Q_{ZIPo} \left( Q_Z \left( \overline{V} \right)^2 + Q_I \left( \overline{V} \right) + Q_Q \right) + V(V - E' \cdot \cos \delta) / X'
\end{cases} \tag{10}$$

where  $\overline{V} = V/V_0$ . E',  $\delta$ : voltage magnitude, angle behind transient reactance.  $\omega_s$ ,  $\omega$ : angular velocity of stator and rotor [rad/s].  $X_m$ ,  $X_s$ ,  $X_r$ : magnetizing, stator and rotor reactances.  $X' = X_s + X_m X_r / (X_m + X_r)$ : transient reactance.  $X = X_s + X_m$ .  $T_0' = (X_r + X_m)/\omega_s R_r$ : transient open-circuit time constant.  $R_r$ : rotor resistances. M: motor inertia.  $T_m$ : load torque constant.  $V \angle \theta$ : terminal voltage of ZIP-IM model.  $P_{ZIP0}$ ,  $Q_{ZIP0}$ : initial real and reactive powers consumed by ZIP model. Equations (9) and (10) represent the state equations and the model output of the ZIP-IM model, respectively. It is noted that  $P_z + P_I + P_P = 1$  and  $Q_z + Q_I + Q_Q = 1$  at  $V = V_0$ .

In developing a composite load model including induction motor load, we usually introduce a parameter  $K_{pm}$  which denotes the percentage of induction motor loads at a steady state. Once  $K_{pm}$  is set, then the induction motor can be initialized and  $Q_{IM0}$ , the initial reactive power consumed by the induction motor, can be computed. Hence, the initial power consumed by ZIP model,  $P_{ZIP0}$ ,  $Q_{ZIP0}$  is directly computed. Also  $P_I$  and  $Q_I$  are eliminated using two equality constraints at the steady state. Hence, the minimal parameter vector to be estimated is then reduced to nine and listed as follows:

$$p = [M, T_0, X, X, X, K_{pm}, P_Z, P_P, Q_Z, Q_O]^T$$
(11)

Fig. 1 shows an equivalent circuit of the ZIP-IM load model.

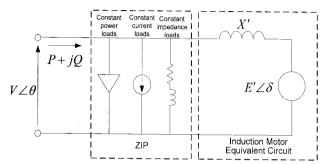


Fig. 1. An equivalent circuit of ZIP-IM model

#### 4.2 Constraints

In order to simulate the output of the ZIP-IM model with a given set of parameters and measurements, initial states of the induction motor,  $z_0 = [E_0', \delta_0, \omega_0]^T$  should be feasible. The initial states are functions of model parameters. For instance, initial angle of induction motor  $\delta_0$  is computed using the following equation:

$$\delta_0 = -\frac{1}{2} \sin^{-1} \left( \frac{2XX'K_{pm}P_0}{(X - X')V_0^2} \right)$$
 (12)

As shown in (12), the initial rotor angle  $\delta_0$  can be defined only when the following condition is satisfied:

$$-1 \le \frac{2XX'K_{pm}P_0}{(X - X')V_0^2} \le 1 \tag{13}$$

It is noted that this nonlinear inequality constraint should be satisfied during the whole optimization process since without the constraints (13) satisfied, model output cannot be simulated and the optimization process cannot proceed because of numerical ill-conditioning. This strong requirement of parameter feasibility is a special feature of the parameter estimation of composite load models associated with induction motors. In this regard, Trust-Tech can be effectively applied to this problem since the PGS phase guarantees the parameter feasibility.

From the physical arguments, the following linear constraint and box constraints are also applied.

$$\begin{cases} X' - X < 0.0 \\ 0.001 \le M \le 10.0 \\ 0.001 \le T'_0 \le 4.0 \\ 0.001 \le X \le 4.0 \\ 0.001 \le X' \le 4.0 \end{cases} \text{ and } \begin{cases} 0.1 \le K_{pm} \le 0.95 \\ 0.1 \le P_z \le 0.9 \\ 0.1 \le P_p \le 0.9 \\ 0.1 \le Q_z \le 0.9 \\ 0.1 \le Q_Q \le 4.0 \end{cases}$$
 (14)

The inequality constraints in  $(13) \sim (14)$  are transformed into appropriate equality forms by introducing slack variables as in the problem definition in (5).

## 5. Parameter Estimation using Trust-Tech

In this section, a Trust-Tech based parameter estimation algorithm is presented for the parameter estimation of the ZIP-IM composite load model. The key features of this specific parameter estimation problem involving the ZIP-IM model under study are summarized as follows:

- A set of differential equations need to be solved to obtain model output.
- Nonlinear constraints for feasible initial dynamic states should always be satisfied during the complete parameter estimation process.
- Despite the feasible parameter set, model outputs can go to infinity (i.e. system goes unstable), which means that it is difficult to obtain gradient information.

In case that model output goes to infinity along a search direction, it is assumed that an exit point doesn't exit along the search direction and the algorithm switches to another search direction. In the integration of the PGS, it is possible that a predicted point  $x_{k+1}$  from the current point  $x_k$  may violate the nonlinear inequality constraints (13). In this case, integration step size is reduced by half. If several attempts at step size shrinking fails, correction of infeasible point  $x_{k+1}$  is performed, i.e., the QGS is integrated with  $x_{k+1}$  as an initial point until a feasible point is found. Then integration of the PGS is continued. This special feature indicates that parameter estimation problem of composite load models is different from the pure static nonlinear programming problem.

In this study, the Trust-Tech methodology with the 'exit point method' is adopted for computational efficiency. Since the integration of PGS from an exit point to a local optimal solution can be time consuming, conventional local methods such as trust-region method and sequential quadratic programming (SQP) method are employed near a local optimal solution.

The overall procedure of finding a global optimal solution for composite load model parameter estimation using Trust-Tech is summarized as follows:

- **Step 1**. Find a feasible component by performing QGS phase.
- **Step 2**. From a feasible initial point, locate a local optimal solution by integrating PGS and using a local solver.
- **Step 3**. Generate search directions from the obtained local solution using a proper method.

Step 4. Along a search direction, detect an exit point by integrating PGS. If the detection of an exit point fails, start Step 4 over using another search direction. If all search directions have been explored, terminate the algorithm.

Step 5. Compute

$$q_0 = x_s + \overrightarrow{x_s x_{exit}} + 0.1 \times P_H(x_{exit}) \cdot \overrightarrow{x_s x_{exit}}$$
 and sufficiently integrate PGS using  $q_0$  as an initial point.

- **Step 6**. Call a local solver to find a local optimal solution using the end point of the PGS integration as an initial point.
- **Step 7**. Check if the obtained local optimal solution is a new one. If it is, add it to the local optimal solution set. If all search directions are explored, then terminate the algorithm.

In Step 3, search directions can be generated using Hessian matrix at a local optimal solution. In this study, eigenvectors of the Hessian including several user defined search directions are used as search directions. In Step 4, an exit point may not be found along a certain direction when system states go unstable which makes model output and objective function value go infinite. In this case, the algorithm terminates searching for an exit point and tries another search direction.

In Step 6, a local solver can be directly called with  $q_0$  without integrating PGS, but infeasible point can be encountered during the optimization process when the employed local solver does not guarantee the feasibility during the intermediate iterations. Two approaches can be used in order to solve this numerical problem; one is that a feasible direction method is employed to ensure the parameter feasibility during the whole optimization process. However, feasible direction methods can be numerically expensive. The other approach is to integrate the PGS until sufficiently approaching a local optimal solution. Near a local optimal solution, a local solver has less possibility of suffering from parameter infeasibility during the optimization process.

Note that the employed local solvers do not guarantee parameter feasibility during the optimization process; hence the algorithm may fail to find a local optimal solution since the algorithm is terminated when an infeasible point is encountered during the iterations. This case is actually observed in the numerical studies.

#### 6. Numerical Studies

Multiple sets of parameter values of the ZIP-IM composite load model are found using the Trust-Tech based algorithm. Actual measurements taken from a power system are used in this study. Two sets of

measurement data (SP1 and SL2 in [5]) are used in deriving and validating parameter values, respectively.

A feasible point can be obtained by performing Phase I (QGS Phase) or can be determined based on domain knowledge. Solution No. 1 in Table 1 is obtained by integrating the PGS from a feasible point and by a local solver. Then, nine search directions are generated using a Hessian matrix at solution No. 1. In addition, three userdefined search directions are also added to the set of search directions. A total of nine local optimal solutions are obtained using the Trust-Tech based approach and are listed in Table 1. Table 1 indicates that solutions No. 1, 4-7, and 9 show similar modeling performances ( $\varepsilon_p$  and  $\varepsilon_o$ , the relative mismatch error (%) in real and reactive loads) while solutions No. 2, 3, and 8 show different (worse) modeling performance from the others. Among the solutions showing similar modeling performance, several parameter values such as  $Q_{\scriptscriptstyle \mathcal{Q}}$  and  $Q_{\scriptscriptstyle \mathcal{Z}}$  are quite different from one another.

Figure 2 shows the variation of objective function values along search directions 2 and 10 in Table 2. Fig. 2 demonstrates the procedure of a local optimal solution along the search directions; first an exit point is found by detecting the first local maximum of objective function value. Second, from the exit point, the PGS is integrated for a while to approach a local optimal solution associated with the exit point. Finally a local solver is called to obtain the exact local optimal solutions.

Table 2 presents the relationship between search directions and solutions listed in Table 1. In Table 2, '\*' in search directions 2 and 4 indicates that a local solver encounters infeasible points during the optimization process. In this case, a very large objective function value is intentionally set in this study.

It is not always possible to find a local optimal solution along each search direction. As noted in the previous section, two numerical problems can occur during the PGS phase; first, objective function value can go to infinity resulting in the failure of the exit point finding. Figure 4 shows the case where objective function value goes to infinity along search direction 5. Second, conventional gradient-based local solvers fail to converge when an infeasible point for the constraints (11) is encountered since initial states cannot be defined, thereby objective function value cannot be evaluated. In this numerical study, both of the numerical problems are observed.

Derived parameter values should be validated using unseen measurement data. The nine sets of parameter values in Table 1 are validated using a different measurement data SL2 in [5], which is of summer light loading condition. Table 3 shows validation errors of the nine sets of parameter values. Solutions No. 1 and No. 4 through No. 9 show similar validation errors in real load modeling while solutions No. 2 and No. 3 give slightly

more validation errors than the others. In reactive load modeling, solutions No. 9, No. 7, and No. 5 give better validation errors. From Tables 1 and 3, solution No. 9 appears to be a good choice for a representative set of parameter values for the ZIP-IM model. Figure 3 shows the simulated real and reactive loads by the ZIP-IM model

with the parameter values of solution No. 9 in case of SL2 measurement data. Figure 3 indicates that the derived ZIP-IM model can quite accurately model the dynamic behaviors of real and reactive loads during disturbances.

Table 1. Multiple sets of parameter values for ZIP-IM model obtained by the Trust-Tech based approach (SP1 measurement data in [5])

Sol.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9
M	0.0015	0.0024	4.7646	0.0016	0.0015	0.0010	0.0015	0.0015	0.0018
$T_{o}$	0.1963	1.0852	1.2844	0.0402	0.2524	0.1294	0.2702	0.2331	0.4246
X	1.2901	1.9606	3.9603	0.1434	1.6960	1.3634	1.8553	2.9249	2.5739
X'	0.1228	0.3837	0.1022	0.0700	0.1256	0.1243	0.1261	0.0990	0.1261
$K_{pm}$	0.5024	0.2102	0.1000	0.4998	0.5024	0.5449	0.5049	0.6469	0.4907
$P_z$	0.5365	0.6153	0.1000	0.5312	0.5352	0.7485	0.6471	0.6161	0.5273
$P_{_{P}}$	0.1008	0.6177	0.3840	0.1000	0.1000	0.2297	0.1915	0.1040	0.1273
$\overline{Q_z}$	0.1000	0.1000	0.9000	0.5913	0.1000	0.1204	0.1000	0.1000	0.1000
$\overline{\mathcal{Q}_{\scriptscriptstyle \mathcal{Q}}}$	2.3497	1.2660	1.2975	0.1000	3.0032	2.9980	3.2525	3.3122	3.8614
E <sub>p</sub> (%)	1.2421	1.7943	2.2862	1.2457	1.2421	1.2002	1.2402	1.4190	1.2596
$\mathcal{E}_{\mathcal{Q}}(\%)$	6.7509	22.7817	5.0289	6.6160	6.7580	7.6647	6.8145	11.4788	6.5054

Table 2. Search directions and corresponding solution numbers

	Search Directions											
	1	2	3	4	5	6	7	8	9	10	11	12
Sol. No.	2	3*	inf**	4*	inf	5	6	inf	inf	7	8	9

<sup>\*</sup> A local solver encounters infeasible points during the optimization process. In this case, a very large objective function value is intentionally set.

<sup>\*\*</sup> Note that the 'inf' denotes that objective function value goes to infinity along the search direction resulting in a termination of the algorithm.

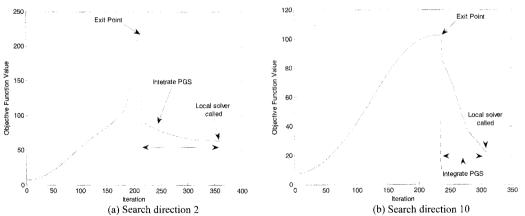
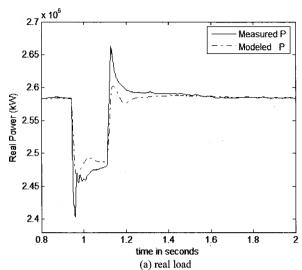
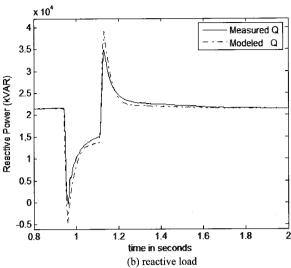


Fig. 2. Variation of objective function values along the search directions 2 and 10 in Table 2

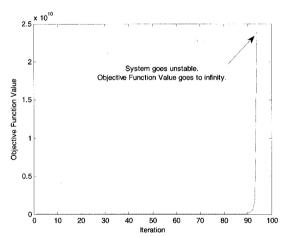
Table 3. Validation errors of the local minima in Table 1 using a different measurement data (SL2 measurement data in [5])

Sol.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9
$\mathcal{E}_{p}$ (%)	0.5835	0.8171	0.9473	0.5799	0.5838	0.5565	0.5734	0.5451	0.5836
$\mathcal{E}_{_{Q}}(\%)$	7.7342	11.4762	12.2767	15.2905	6.2652	8.8609	5.7851	6.6987	4.8954





**Fig. 3.** Modeled real and reactive loads using solution No. 9 in Table 1 and different measurement data (SL2 data in [5]). The modeling errors in real and reactive loads are 0.5836% and 4.8954%, respectively (see Table 3).



**Fig. 4.** Variation of objective function values along the search direction 5

## 7. Concluding remarks

In this paper, Trust-Tech, a novel methodology for a global optimization is introduced and applied to a parameter estimation task for the ZIP-induction motor (ZIP-IM) composite model. The parameter estimation task is formulated as a nonlinear constrained optimization problem. The main results and key observations of this paper are summarized as follows:

- Trust-Tech is successfully applied to a composite load model parameter estimation.
- Multiple sets of parameter values of the ZIP-IM are obtained in a deterministic manner.
- Due to strong feasibility requirement during the parameter estimation, robust feasible local solvers need to be employed.

- An efficient correction method using the QGS is proposed when an infeasible point is encountered.
- Local solvers can converge to the starting point due to the non-convexity of the objective function.

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