Fully Adaptive Feedforward Feedback Synchronized Tracking Control for Stewart Platform Systems

Dongya Zhao, Shaoyuan Li*, and Feng Gao

Abstract: In this paper, a fully adaptive feedforward feedback synchronized tracking control approach is developed for precision tracking control of 6 degree of freedom (6DOF) Stewart Platform. The proposed controller is designed in decentralized form for implementation simplicity. Interconnections among different subsystems and gravity effect are eliminated by the feedforward control action. Feedback control action guarantees the stability of the system. The gains of the proposed controller can be updated on line without requiring any prior knowledge of Stewart Platform manipulator. Thus the control approach is claimed to be fully adaptive. By employing cross-coupling error technology, the proposed approach can guarantee both of position error and synchronization error converge to zero asymptotically. Because the actuators work in synchronous manner, the tracking performances are improved. The corresponding stability analysis is also presented in this paper. Finally, simulation is demonstrated to verify the effectiveness of the proposed approach.

Keywords: Cross-coupling error, decentralized control, fully adaptive control, Stewart platform, synchronized tracking control.

1. INTRODUCTION

As a typical 6DOF parallel manipulator, Stewart Platform has been extensively studied [1-4]. This parallel manipulator can provide better accuracy, rigidity, load to weight ratio and load distribution than serial manipulator. It has been used in the area of low speed and large payload conditions, such as motion base of flight simulator and motion bed of a machine tool [1,5]. How to develop an effective control approach for precise tracking control of Stewart Platform attracts much attention from academe and industry. The tracking control of parallel manipulator can be classified as two kinds of approaches [6]. The first one is independent joint control scheme (nonmodel-based), such as proportional-integral-derivative (PID) control [7,8], fuzzy control [9] and neural network control [10]. Another one is model-based approach, such as robust control [4,6], impedance control [11] and adaptive control [12,13].

The most of existed approach for tracking control of parallel manipulator did not consider the synchronization among control loops. The lack of synchronization will lead to large coupling errors which degrade the performance of overall mechanical system or even damage the machine tool [14-17]. By employing cross-coupling error technology [18-21], some synchronized tracking control approaches were developed for parallel manipulators. Nonmodel-based approach includes integrated saturated PI synchronous control plus PD feedback control approach [22] and feedforward compensation plus cross-coupling error feedback control approach [23]. Model-based approach includes adaptive synchronized control [24] and convex synchronized control [25]. Though mode-based synchronized tracking control approach can improve the performance, its controller structure is complex due to the complicated dynamic model of parallel robot. The PID synchronized tracking control structure is simple without requiring any prior knowledge of plant. However the determination of the PID controller's parameters is difficult in practice.

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Though the approach proposed by literature [23] is an excellent method for parallel robot synchronized tracking control without requiring the explicit use of the system dynamic model, some prior knowledge is required in controller designing. It is difficult to get the prior knowledge of parallel manipulator in practice. These synchronized tracking control approaches mentioned above are all developed for a 3 degree of freedom planar parallel robot, the gravity effect need not to be considered in controller designing.

Stewart Platform is a general configuration of parallel manipulator. With considering the gravity effect, the control approach developed for it can be extended to other type parallel manipulators. Due to the complexity of dynamic model of Stewart Platform, a nonmodel-based synchronized tracking control approach is developed in this paper. Based on the achievements of literature [16-17,23,26], a fully adaptive feedforward feedback synchronized tracking control approach is developed for Stewart Platform. The proposed controller is designed in decentralized form for implementation simplicity. Interconnections among subsystems and gravity effect are eliminated by the feedforward control action. Feedback control action guarantees the stability of the system. The gains of the proposed controller can be updated on line without requiring any prior knowledge of Stewart Platform manipulator. Fully adaptive control means that the desired control objective can be achieved with adaptive control without any prior knowledge of plants [26]. By employing cross-coupling error technology, the proposed approach can guarantee both of position error and synchronization error converge to zero asymptotically. Because the actuators work in synchronous manner, the tracking performances are improved.

The rest of this paper is organized as follows: dynamic model of the Stewart Platform and some of its properties are depicted in Section 2, the definitions of synchronization error and cross-coupling error are also presented in this section. A fully adaptive feedforward feedback synchronized tracking control approach and the corresponding stability analysis are presented in Section 3. In Section 4, a simulation for 6DOF Stewart Platform synchronized tracking control is demonstrated in support of the proposed approach. Finally, concluding remarks are given in Section 5.

2. PROBLEM FORMULATION

As shown in Fig. 1, the Stewart Platform is a 6DOF mechanism with two bodies connected by the six extensible legs. Usually, the inertial frame *O-XYZ* is fixed at the base platform with its origin at the geometry center of the base platform, the body-fixed frame (moving frame) *P-xyz* is attached to the mass

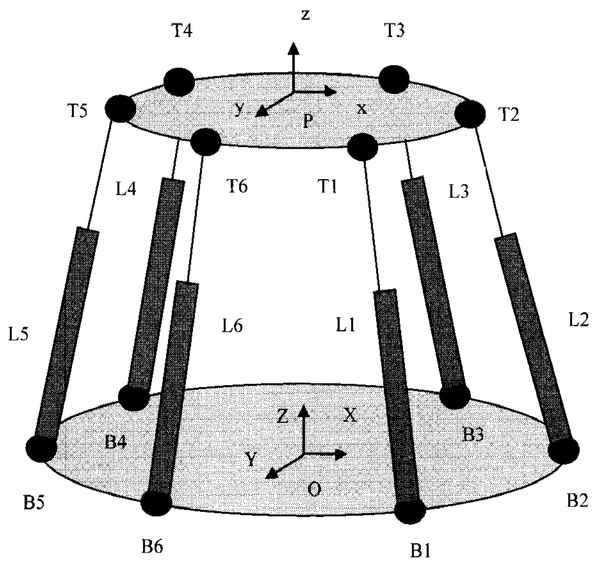

Fig. 1. Coordinates of 6 DOF Stewart Platform.

origin of the moving platform. The six degree of freedom of Stewart Platform are translations along the *X*, *Y*, *Z* axes and the rotations about the axis *O-X*, *O-Y*, *O-Z*.

The work space coordinates of the mass center of moving platform can be written as

$$P=\begin{bmatrix} X & Y & Z & \alpha & \beta & \gamma \end{bmatrix}^T, \tag{1}$$

where *X*, *Y*, *Z* represent the translations and α, β, γ represent the rotations.

The $q=[q_1,\cdots q_n]^T$ is length of legs, which represents generalized coordinates. The relationship between P and q is

$$\dot{P}(t)=(J(t))^{-1}\dot{q}(t), \tag{2}$$

where $J(t)$ is Jacobian matrix.

By using the natural orthogonal complement method, the dynamic model of Stewart Platform can be derived in joint space [1,5,27]:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=\tau, \tag{3}$$

where $q\in\Re^{n\times 1}$, is the generalized coordinate, $M(q)=diag\{M_i(q)\}\in\Re^{n\times n}$ is the inertia matrix, $C(q,\dot{q})=diag\{C_i(q,\dot{q})\}\in\Re^{n\times n}$ is the Coriolis and centrifugal force matrix, $G(q)=[G_1(q),\cdots,G_n(q)]$ $\in\Re^{n\times 1}$ is the gravity force vector, $\tau=[\tau_1,\cdots,\tau_n]^T$ $\in\Re^{n\times 1}$ is the actuating force vector.

Remark 1: Comparing with dynamics of planar parallel robot presented by literature [22-25], the dynamics of Stewart Platform has a gravity force vector, which must be compensated in controller designing. The compensator for gravity effect will be detailed later.

The dynamic model (3) has the following properties that will be used in controller designing [22,23,26,27].

Property 1: The matrix $M(q)$ is a symmetric and positive-definite matrix, which satisfies $\|M(q)\| \le a$ and $\lambda_{\max}(M(q)) \le b$ for some constants $a,b > 0$, where $\lambda_{\max}(\cdot)$ / $\lambda_{\min}(\cdot)$ represents maximum /minimum eigenvalue of the matrix.

Property 2: The matrix $C(q,\dot{q})$ satisfies $\|C(q,\dot{q})\| \le c$ for some constant $c > 0$.

Property 3: The vector $G(q)$ satisfies $\|G(q)\| \le d$ for some constant $d > 0$.

Property 4: The matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric matrix.

To formulate the decentralized control system structure, the *i*th subsystem of dynamic equation (3) can be written as the following decentralized form

$$M_i(q)\ddot{q}_i + C_i(q,\dot{q})\dot{q}_i + G_i(q) = \tau_i \quad i = 1,\cdots,n, \quad (4)$$

where τ_i represents *i*th control input for leg *i*.

For parallel manipulator, the synchronization error can provide each actuated joint with motion information both from itself and other actuated joints. As a result, this error represents the degree of coordination among the actuated joints [16,17]. By employing cross-coupling error technology, the synchronized control approach can improve tracking performance of Stewart Platform.

Define the position error

$$\Delta q_i = q_i^d - q_i, \quad (5)$$

where Δq_i is position error, q_i^d is the desired coordinate in joint space, $i = 1,\cdots,n$.

The coordinated actuators of Stewart Platform are subjected to the following synchronization function [16,23].

$$f(q_1,\cdots q_n): c_1 q_1 = c_2 q_2 = \cdots = c_n q_n, \quad (6)$$

where c_i is the coupling coefficient of *i*th actuator and is assumed to be nonzero. This synchronization function represents a new task requirement in kinematics. The function (6) is valid for all desired coordinates $q_i^d(t)$, namely

$$f(q_1^d,\cdots q_n^d): c_1 q_1^d = c_2 q_2^d = \cdots = c_n q_n^d, \quad (7)$$

From (6) and (7), the following synchronization goal is defined as [16,23]

$$c_1\Delta q_1 = c_2\Delta q_2 = \cdots = c_n\Delta q_n. \quad (8)$$

The synchronization goal (8) can be divided into n sub-goals such as $c_i\Delta q_i = c_{i+1}\Delta q_{i+1}$, when $i = n$, $i+1 = 1$. The position synchronization errors of Stewart Platform can be defined as follows [16,23]:

$$\begin{cases} \varepsilon_1 = c_1\Delta q_1 - c_2\Delta q_2 \\ \varepsilon_2 = c_2\Delta q_2 - c_3\Delta q_3 \\ \quad\vdots \\ \varepsilon_n = c_n\Delta q_n - c_1\Delta q_1, \end{cases} \quad (9)$$

where ε_i represents the synchronization error of *i*th joint. When $\varepsilon_i = 0$ for all $i = 1,\cdots,n$, the actuated joints will work in a synchronous manner.

For developing a synchronized tracking controller, the cross-coupling error is defined as Sun *et al.* [16,17, 23]:

$$e_i = c_i\Delta q_i + \mu\int_0^t(\varepsilon_i(\omega) - \varepsilon_{i-1}(\omega))d\omega, \quad (10)$$

where μ is a positive coupling parameter, ω is a variable from time zero to t, when $i = 1$, $i-1 = n$.

Remark 2: The position error and synchronization error are included in cross-coupling error expression. By designing an appropriate controller, both of them can converge to zero asymptotically.

Remark 3: The coupling parameter μ plays an important role in the synchronized tracking control. As μ increases, the synchronization error decreases.

Differentiating e_i with respect to time yields

$$\dot{e}_i(t) = \dot{c}_i\Delta q_i + c_i\Delta\dot{q}_i + \mu(\varepsilon_i - \varepsilon_{i-1}). \quad (11)$$

The command vector u_i [23]

$$u_i = c_i\dot{q}_i^d + \dot{c}_i\Delta q_i + \mu(\varepsilon_i - \varepsilon_{i-1}) + \Lambda e_i. \quad (12)$$

The generalized error r_i [23]

$$r_i = u_i - c_i\dot{q}_i = \dot{e}_i + \Lambda e_i, \quad (13)$$

where $\Lambda \in \Re^{n\times n}$ is a feedback gain matrix.

Substituting (13) into (4), the dynamic equation can be rewritten as

$$M_i(q)c_i^{-1}\dot{r}_i + C_i(q,\dot{q})c_i^{-1}r_i = -\tau_i + \xi_i, \quad (14)$$

$$\xi_i = M_i(q)_i c_i^{-1}(\dot{u}_i - \dot{c}_i\dot{q}_i) + C_i(q,\dot{q})_i c_i^{-1}u_i + G_i(q)_i, \quad (15)$$

where $i=1,\cdots,n$ for all of the expressions above.

Remark 4: ξ_i represents the interconnections among different subsystems. The synchronized tracking control is then to find a class of fully adaptive feedforward law to compensate the effect of ξ_i and fully adaptive feedback law to converge $r_i \to 0$ and $\varepsilon_i \to 0$ as $t \to \infty$.

Remark 5: The control law proposed by literature [23] is

$$\begin{aligned}\tilde{\tau}_i &= K_i^M c_i^{-1}\left(\dot{u}_i - \dot{c}_i \dot{q}_i\right) \\ &+ K_i^C c_i^{-1} u + sign\left(c_i^{-1} r_i\right) K_i^N{}_i \\ &+ K_{ri} c_i^{-1} r_i + c_i^T K_{\varepsilon i}\left(\varepsilon_i - \varepsilon_{i-1}\right),\end{aligned} \tag{16}$$

$$K_i^N = D_i^M \left\|c_i^{-1}\left(\dot{u}_i - \dot{c}_i \dot{q}_i\right)\right\| + D_i^C \left\|c_i^{-1} u_i\right\|, \tag{17}$$

where $K_i^M, K_i^C > 0$ are positive feedforward control gains, $K_{ri}, K_{\varepsilon i} > 0$ are positive feedback gains, $D_i^M, D_i^C > 0$ are scalars. These gains are required to satisfy the following conditions:

(1) $\lambda_{\min}\left(K_{ri}\right) \ge \lambda_{\max}\left(M_i(q)\dot{c}_i^{-1} c_i\right)$

(2) $D_i^M \ge \left\|K_i^M - M_i(q)\right\|$, $D_i^C \ge \left\|K_i^C - C_i(q,\dot{q})\right\|$.

It is obvious that the control law proposed by [23] does not consider gravity effect. Some prior knowledge is required by this control law.

The control objective is to design a fully adaptive feedforward feedback synchronized tracking control input τ_i to converge $r_i \to 0$ and $\varepsilon_i \to 0$ as $t \to \infty$. From expressions (9)~(13), one can see $\Delta q_i \to 0$ as $r_i \to 0$ and $\varepsilon_i \to 0$. Unlike the conventional non-synchronized tracking control approach which considers the position error only, the synchronized tracking control can guarantee both position error and synchronization error converge to zero asymptotically.

3. CONTROLLER DESIGN

In this section, a fully adaptive feedforward feedback synchronized tracking controller is developed for 6DOF Stewart Platform.

Without loss of generality, several technical assumptions are made to pose the problem in a tractable manner.

Assumption 1: The feedback gain Λ is constant, diagonal and positive-definite.

Assumption 2: Desired joint position trajectory $q_i^d(t)$ is a real number and nonzero. $q_i^d(t)$ and the time derivatives $\dot{q}_i^d(t)$, $\ddot{q}_i^d(t)$ are bounded.

Based on the achievements of literature [23,26], the fully adaptive feedforward feedback synchronized tracking controller is designed as

$$\begin{aligned}\tau_i &= K_i^M c_i^{-1}\left(\dot{u}_i - \dot{c}_i \dot{q}_i\right) + K_i^C c_i^{-1} u_i \\ &+ K_i^G + sign\left(c_i^{-1} r_i\right)\hat{K}_i^N(t) \\ &+ \hat{K}_{ri}(t) c_i^{-1} r_i + c_i^T K_{\varepsilon i}\left(\varepsilon_i - \varepsilon_{i-1}\right),\end{aligned} \tag{18}$$

for $i=1,\cdots,n$, where the $K_i^M, K_i^C > 0$ are the feedforward gains to compensate the effects of $M(q)_i c_i^{-1}\left(\dot{u}_i - \dot{c}_i \dot{q}_i\right)$ and $C(q,\dot{q})_i s_i^{-1} u_i$, respectively. $K_i^G > 0$ is the gravity compensator. $\hat{K}_{ri}(t) > 0$ and $K_{\varepsilon i} > 0$ are positive feedback control gains.

To compensate for the effect due to the errors between the feedforward control gains and the modeling parameters, a saturated control utilizing the sign function $sign\left(c_i^{-1} r_i\right)\hat{K}_i^N(t)$ is employed, $\hat{K}_i^N(t)$ can be written as

$$\begin{aligned}\hat{K}_i^N(t) &= \hat{D}_i^M(t)\left\|c_i^{-1}\left(\dot{u}_i - \dot{c}_i \dot{q}_i\right)\right\| \\ &+ \hat{D}_i^C(t)\left\|c_i^{-1} u_i\right\| + \hat{D}_i^G(t),\end{aligned} \tag{19}$$

for $i=1,\cdots,n$ where $\hat{D}_i^M(t), \hat{D}_i^C(t), \hat{D}_i^G(t) \ge 0$.

Assume that $K_{ri}^*, D_i^{M*}, D_i^{C*}, D_i^{G*} \ge 0$ are the desired value of $\hat{K}_{ri}(t), \hat{D}_i^M(t), \hat{D}_i^C(t), \hat{D}_i^G(t)$, which satisfy the following expression.

$$\begin{cases}\lambda_{\min}\left(K_{ri}^*\right) \ge \lambda_{\max}\left(M_i(q)\dot{c}_i^{-1} c_i\right) \\ D_i^{M*} \ge \left\|K_i^M - M_i(q)\right\| \\ D_i^{C*} \ge \left\|K_i^C - C_i(q,\dot{q})\right\| \\ D_i^{G*} \ge \left\|K_i^G - G_i(q)\right\|.\end{cases} \tag{20}$$

Define the estimation errors of $\hat{K}_{ri}(t)$, $\hat{D}_i^M(t)$, $\hat{D}_i^C(t)$, $\hat{D}_i^G(t)$ as

$$\begin{cases}\tilde{K}_{ri}(t) = \hat{K}_{ri}(t) - K_{ri}^* \\ \tilde{D}_i^M(t) = \hat{D}_i^M(t) - D_i^{M*} \\ \tilde{D}_i^C(t) = \hat{D}_i^C(t) - D_i^{C*} \\ \tilde{D}_i^G(t) = \hat{D}_i^G(t) - D_i^{G*}.\end{cases} \tag{21}$$

To cope with uncertainty of dynamics of Stewart Platform, the adaptive laws are designed as follows:

$$\begin{cases} \dot{\tilde{K}}_{ri}(t) = \dot{\hat{K}}_{ri}(t) = k_{ri} \left\| c_i^{-1} r_i \right\|^2 \\ \dot{\tilde{D}}_i^M(t) = \dot{\hat{D}}_i^M(t) = d_i^M \left\| c_i^{-1} r_i \right\| \left\| c_i^{-1}(\dot{u}_i - \dot{c}_i \dot{q}_i) \right\| \\ \dot{\tilde{D}}_i^C(t) = \dot{\hat{D}}_i^C(t) = d_i^C \left\| c_i^{-1} r_i \right\| \left\| c_i^{-1} u_i \right\| \\ \dot{\tilde{D}}_i^G(t) = \dot{\hat{D}}_i^G(t) = d_i^G \left\| c_i^{-1} r_i \right\|, \end{cases} \tag{22}$$

for $i = 1, \cdots, n$, where $\hat{K}_{ri}(0)$, $\hat{D}_i^M(0)$, $\hat{D}_i^C(0)$, $\hat{D}_i^G(0) > 0$ are the initial value and the $k_{ri}, d_i^M, d_i^C, d_i^G > 0$ are adaptive gains.

Remark 6: From comparing the control law (18)~(22) with (16)~(17), one can find that the gravity compensator K_i^G and $\hat{D}_i^G(t)$ are employed by the proposed approach of this paper to eliminate the effect of gravity.

Remark 7: Control gains $\hat{K}_{ri}(t)$, $\hat{D}_i^M(t)$, $\hat{D}_i^C(t)$, $\hat{D}_i^G(t)$ of the proposed approach of this paper can be updated online without requiring any prior knowledge of the Stewart Platform, while the corresponding gains of the control law proposed by literature [23] are required to satisfy some conditions which are required to be determined a prior.

Theorem 1: Under Assumptions 1 and 2, consider the error dynamics of Stewart Platform (14)~(15) subject to the proposed control law (18)~(22). If the gains $K_i^M, K_i^C, K_i^G, K_{\varepsilon i} > 0$, the system is asymptotic stability, i.e., $\Delta q_i(t) \to 0$ and $\varepsilon_i(t) \to 0$ as time $t \to \infty$.

Proof: Define a Lyapunov function candidate as

$$\begin{aligned} V = &\sum_{i=1}^{n} \left[\frac{1}{2} r_i^T c_i^{-T} M_i(q_i) c_i^{-1} r_i + \frac{1}{2} \varepsilon_i^T K_{\varepsilon i} \varepsilon_i \right] \\ &+ \frac{1}{2} \left(\int_0^t \sum_{i=1}^{n} (\varepsilon_i(w) - \varepsilon_{i-1}(w)) dw \right)^T \\ &\times \Lambda \mu K_{\varepsilon i} \left(\int_0^t \sum_{i=1}^{n} (\varepsilon_i(w) - \varepsilon_{i-1}(w)) dw \right) \\ &+ \frac{1}{2} \sum_{i=1}^{n} \left[k_{ri}^{-1} \tilde{K}_{ri}^2 + d_i^{M-} \left(\tilde{D}_i^M\right)^2 \right. \\ &\left. + d_i^{C-} \left(\tilde{D}_i^C\right)^2 + d_i^{G-} \left(\tilde{D}_i^G\right)^2 \right]. \end{aligned} \tag{23}$$

Differentiating V with respect to time along the trajectories of the system defined by (14) and (18)

$$\begin{aligned} \dot{V} = &\sum_{i=1}^{n} \left[r_i^T c_i^{-T} M_i(q_i) c_i^{-1} \dot{r}_i \right. \\ &+ r_i^T c_i^{-T} M_i(q_i) \dot{c}_i^{-1} r_i \\ &\left. + \frac{1}{2} r_i^T c_i^{-T} \dot{M}_i(q_i) c_i^{-1} r_i + \varepsilon_i^T K_{\varepsilon i} \dot{\varepsilon}_i \right] \\ &+ \sum_{i=1}^{n} (\varepsilon_i - \varepsilon_{i-1})^T \Lambda \mu K_{\varepsilon i} \\ &\times \left(\int_0^t \sum_{i=1}^{n} (\varepsilon_i(w) - \varepsilon_{i-1}(w)) dw \right) \\ &+ \sum_{i=1}^{n} \left[k_{ri}^{-1} \tilde{K}_{ri} \dot{\tilde{K}}_{ri} + d_i^{M-} \tilde{D}_i^M \dot{\tilde{D}}_i^M \right. \\ &\left. + d_i^{C-} \tilde{D}_i^C \dot{\tilde{D}}_i^C + d_i^{G-} \tilde{D}_i^G \dot{\tilde{D}}_i^G \right]. \end{aligned} \tag{24}$$

Multiplying both sides of (14) by $\left(c_i^{-1} r_i\right)^T$ and substituting the result into (24), one can get

$$\begin{aligned} \dot{V} = &-\sum_{i=1}^{n} \left[\left(c_i^{-1} r_i\right)^T \left(\hat{K}_{ri} - M_i(q_i) \dot{c}_i^{-1} c_i \right) c_i^{-1} r_i \right] \\ &- \sum_{i=1}^{n} \left[\left(c_i^{-1} r_i\right)^T N_i + \left\| c_i^{-1} r_i \right\| \hat{K}_i^N \right] \\ &- \sum_{i=1}^{n} \left[r_i^T K_{\varepsilon i} (\varepsilon_i - \varepsilon_{i-1}) \right] + \sum_{i=1}^{n} \varepsilon_i^T K_{\varepsilon i} \dot{\varepsilon}_i \\ &+ \sum_{i=1}^{n} (\varepsilon_i - \varepsilon_{i-1})^T \Lambda \mu K_{\varepsilon i} \\ &\times \left(\int_0^t \sum_{i=1}^{n} (\varepsilon_i(w) - \varepsilon_{i-1}(w)) dw \right) \\ &+ \sum_{i=1}^{n} \left[k_{ri}^{-1} \tilde{K}_{ri} \dot{\tilde{K}}_{ri} + d_i^{M-} \tilde{D}_i^M \dot{\tilde{D}}_i^M \right. \\ &\left. + d_i^{C-} \tilde{D}_i^C \dot{\tilde{D}}_i^C + d_i^{G-} \tilde{D}_i^G \dot{\tilde{D}}_i^G \right]. \end{aligned} \tag{25}$$

Substitute the adaptive law (22) into (25), one can get

$$\begin{aligned} \dot{V} \le &-\sum_{i=1}^{n} \left[\left(c_i^{-1} r_i\right)^T \left(K_{ri}^* - M_i(q_i) \dot{c}_i^{-1} c_i \right) c_i^{-1} r_i \right] \\ &- \sum_{i=1}^{n} \left\| c_i^{-1} r_i \right\| \left[\left(D_i^{M*} - \left\| K_i^M - M_i(q) \right\| \right) \right. \\ &\times \left\| c_i^{-1} (\dot{u}_i - \dot{c}_i \dot{q}_i) \right\| \\ &+ \left(D_i^{C*} - \left\| K_i^{C*} - C_i(q_i, \dot{q}_i) \right\| \right) \left\| c_i^{-1} u_i \right\| \\ &\left. + \left(D_i^{G*} - \left\| K_i^{G*} - G_i(q) \right\| \right) \right] \end{aligned}$$

$$-\sum_{i=1}^{n}\left[r_i^T K_{\varepsilon i}\left(\varepsilon_i-\varepsilon_{i-1}\right)\right]+\sum_{i=1}^{n}\varepsilon_i^T K_{\varepsilon i}\dot{\varepsilon}_i +\sum_{i=1}^{n}\left(\varepsilon_i-\varepsilon_{i-1}\right)^T \Lambda\mu K_{\varepsilon i} \times\left(\int_0^t\sum_{i=1}^{n}\left(\varepsilon_i(w)-\varepsilon_{i-1}(w)\right)dw\right). \quad (26)$$

According to (20), (26) can be written as

$$\dot{V}\leq-\sum_{i=1}^{n}\left[r_i^T K_{\varepsilon i}\left(\varepsilon_i-\varepsilon_{i-1}\right)\right]+\sum_{i=1}^{n}\varepsilon_i^T K_{\varepsilon i}\dot{\varepsilon}_i +\sum_{i=1}^{n}\left(\varepsilon_i-\varepsilon_{i-1}\right)^T \Lambda\mu K_{\varepsilon} \times\left(\int_0^t\sum_{i=1}^{n}\left(\varepsilon_i(w)-\varepsilon_{i-1}(w)\right)dw\right). \quad (27)$$

From the expression (9)~(13), one can get [23]

$$\begin{aligned}&\sum_{i=1}^{n} r_i^T K_{\varepsilon i}\left(\varepsilon_i-\varepsilon_{i-1}\right)\\ &=\sum_{i=1}^{n}\dot{\varepsilon}_i^T K_{\varepsilon i}\varepsilon_i+\sum_{i=1}^{n}\varepsilon_i^T\Lambda\mu K_{\varepsilon i}\varepsilon_i\\ &+\sum_{i=1}^{n}\left(\varepsilon_i-\varepsilon_{i+1}\right)^T\mu K_{\varepsilon i}\\ &\times\sum_{i=1}^{n}\int_0^t\sum_{i=1}^{n}\left(\varepsilon_i(w)-\varepsilon_{i+1}(w)\right)dw\\ &+\sum_{i=1}^{n}\left(\varepsilon_i-\varepsilon_{i+1}\right)^T\mu K_{\varepsilon i}\sum_{i=1}^{n}\left(\varepsilon_i-\varepsilon_{i+1}\right).\end{aligned} \quad (28)$$

Substituting (28) into (27) yields

$$\dot{V}\leq-\sum_{i=1}^{n}\varepsilon_i^T\Lambda\mu K_{\varepsilon i}\varepsilon_i -\sum_{i=1}^{n}\left(\varepsilon_i-\varepsilon_{i+1}\right)^T\mu K_{\varepsilon i}\sum_{i=1}^{n}\left(\varepsilon_i-\varepsilon_{i+1}\right). \quad (29)$$

It is obvious that $\dot{V}\leq 0$ from expression (29). From the Barbălat's Lemma [28], $r_i\to 0$ and $\varepsilon_i\to 0$ as time $t\to\infty$. From (13), one further has $e_i\to 0$ as $t\to\infty$. The result means that the synchronization goal is achieved.

From (9) and (10), one can get $\Delta q_i=0$ as $\varepsilon_i=0$ and $e_i=0$. Therefore, the closed loop system is asymptotically stable. This is the end of proof. □

Remark 8: From Property 1~Property 3 and Assumption 2, one can see that $\lambda_{\max}\left(M_i(q)\dot{c}_i^{-1}c_i\right)$, $\left\|K_i^M-M_i(q)\right\|$, $\left\|K_i^C-C_i(q,\dot{q})\right\|$, $\left\|K_i^G-G_i(q)\right\|$ are independent of K_{ri}^*, D_i^{M*}, D_i^{C*} and D_i^{G*}, respectively. Thus, inequality (27) can be derived from (26) when $K_{ri}^*, D_i^{M*}, D_i^{C*}$ and D_i^{G*} are large enough.

Remark 9: The approach in Theorem 1 can be implemented without any prior knowledge of Stewart Platform manipulator, such as initial conditions and parameters in expression (3).

Remark 10: When all knowledge of Stewart Platform manipulator and desired trajectory are provided a prior, the bounds of $\lambda_{\max}\left(M_i(q)\dot{c}_i^{-1}c_i\right)$, $\left\|K_i^M-M_i(q)\right\|$, $\left\|K_i^C-C_i(q,\dot{q})\right\|$, $\left\|K_i^G-G_i(q)\right\|$ can be estimated in advance, and then the desired K_{ri}^*, D_i^{M*}, D_i^{C*} and D_i^{G*} can be determined by making inequality (20) satisfied. Substituting these determined desirable values into (18) and (19) to replace online updated gains, the approach in Theorem 1 becomes the one proposed by literature [23] plus gravity compensation, which can be guaranteed by the similar procedure in poof of Theorem 1.

4. SIMULATION RESULTS AND DISCUSSION

Simulations were performed in Matlab SimMechnics toolbox. The parameters of 6DOF Stewart Platform were given as: the mass and mass moment of inertia values of upper platform are m = 1216.9kg, $I_X, I_Y\left(I_Z\right)=304.48(608.46)\text{kg}\cdot\text{m}^2$, the mass of upper/lower part of ith leg are $\left(m_u\right)_i/\left(m_d\right)_i$ $=51.81/92.11\text{kg}$, the mass moment of inertia values of upper and lower part of ith leg are, $I_{uX}, I_{uY}\left(I_{uZ}\right)=24.17(0.023)\text{kg}\cdot\text{m}^2$ and I_{dX}, I_{dY} $\left(I_{dZ}\right)=43.02(0.156)\text{kg}\cdot\text{m}^2$.

For comparison purpose, three control algorithms including the proposed approach, adaptive synchronized control (A-S control) [24] and PID control were used to control the system respectively. The A-S control [24] is expressed as follows.

$$\tau_a=Y(q,\dot{q},\ddot{q})\hat{\theta}(t)-K_r r(t)-K_e e^*(t)$$

The adaptation law is

$$\dot{\hat{\theta}}(t)=-P(t)Y(q,\dot{q},\ddot{q})^T\left[r(t)+Y(q,\dot{q},\ddot{q})\tilde{\theta}\right],$$
$$d\left(P^{-1}\right)/dt=-\lambda P^{-1}(t)+Y(q,\dot{q},\ddot{q})^T Y(q,\dot{q},\ddot{q}),$$
$$\lambda(t)=\lambda_0\left(1-\|P\|/k_0\right),$$

Table 1. Control gains of the three controllers.

Controllers	Gains
The proposed controller	$\mu = 10$, $\Lambda = diag\{10\}$, $K^M = diag\{1\}$, $K^C = diag\{1\}$, $K^G = diag\{300\}$, $d_i^M = 1$, $d_i^C = 1$, $d_i^G = 1$, $\hat{D}_i^M(0) = 0.8$, $\hat{D}_i^C(0) = 0.8$, $\hat{D}_i^G(0) = 200$, $k_{ri} = 200000$, $\hat{K}_i(0) = 50000$, $K_{\varepsilon i} = diag\{50000\}$
A-S control	$K_r = diag\{200000\}$, $K_e = diag\{50000\}$
PID	$K_p = diag\{2000000\}$, $K_i = diag\{10000\}$, $K_d = 4500$

where $K_r \in R^{6\times6}$, $K_e \in R^{6\times6}$ are positive diagonal gain matrices, $P(t) \in R^{6\times6}$ is the positive definite diagonal gain matrix of the estimator, λ_0 and k_0 are positive constants.

Table 1 lists the selected control gains of the three control algorithms used in the simulation, where the control gains of PID control were chosen the values used in Matlab Demo directly.

The desired trajectories in work space are

$$X(t) = \begin{cases} 0.1 & 0 \le t \le 1 \\ 0.2t - 0.1 & 1 \le t \le 2 \\ 0.3 & 2 \le t \le 3, \\ -0.2t + 0.9 & 3 \le t \le 4 \\ 0.1 & 4 \le t \end{cases} \quad (30)$$

$$Y(t) = \begin{cases} 0.1 & 0 \le t \le 2 \\ 0.2t - 0.3 & 2 \le t \le 3 \\ 0.3 & 3 \le t \le 4, \\ -0.2t + 1.1 & 4 \le t \le 5 \\ 0.1 & 5 \le t \end{cases} \quad (31)$$

$$Z(t) = \begin{cases} 0.5t + 2.5 & 0 \le t \le 1 \\ 3 & 1 \le t \end{cases}, \quad (32)$$

$$\alpha(t) = \begin{cases} 0 & 0 \le t \le 1 \\ 0.025t - 0.025 & 1 \le t \le 5, \\ 0.1 & 5 \le t \end{cases} \quad (33)$$

$$\beta(t) = 0,\ \gamma(t) = 0, \quad (34)$$

Figs. 2 and 3 illustrate the tracking performance in work space and joint space, respectively. The dotted lines are the desired trajectory, the solid lines are the performance with the proposed control, the dashed lines are the performance with A-S control, the dashdotted lines are the performance with PID control. From Fig. 2, one can see that all of the three control-

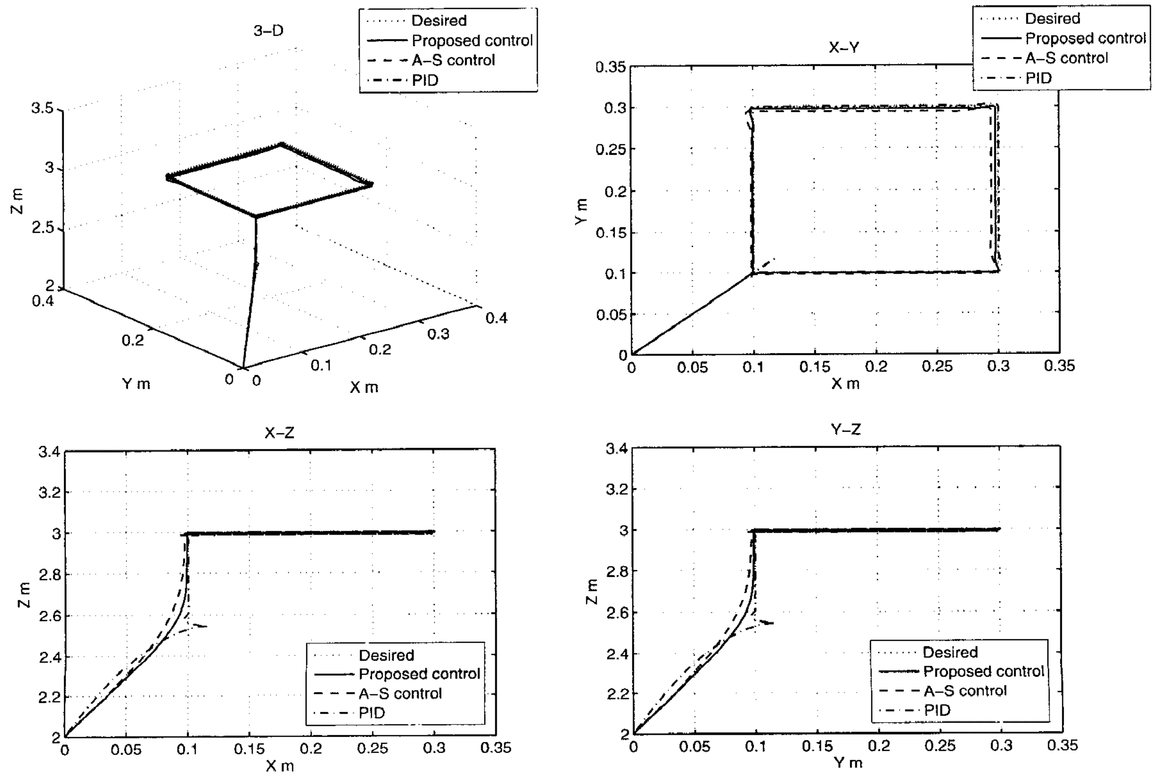

Fig. 2. Tracking performance in workspace.

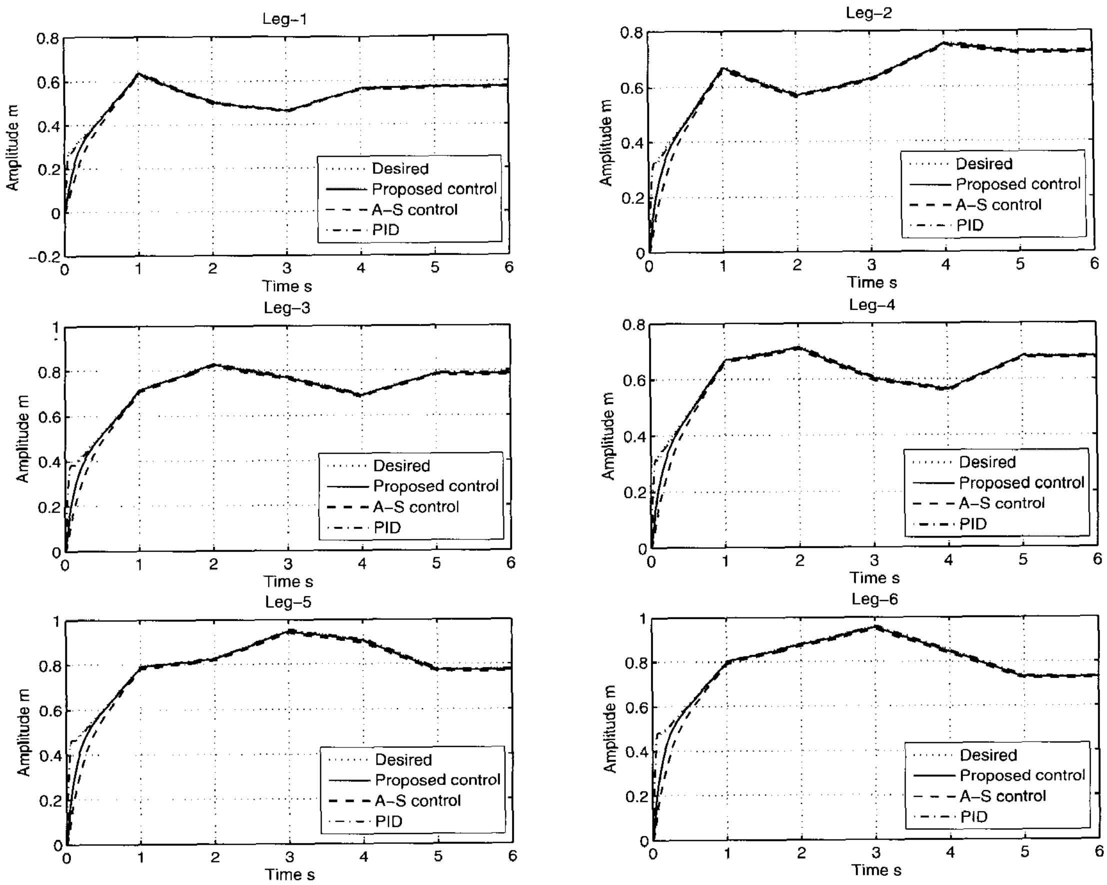

Fig. 3. Tracking performance in joint space.

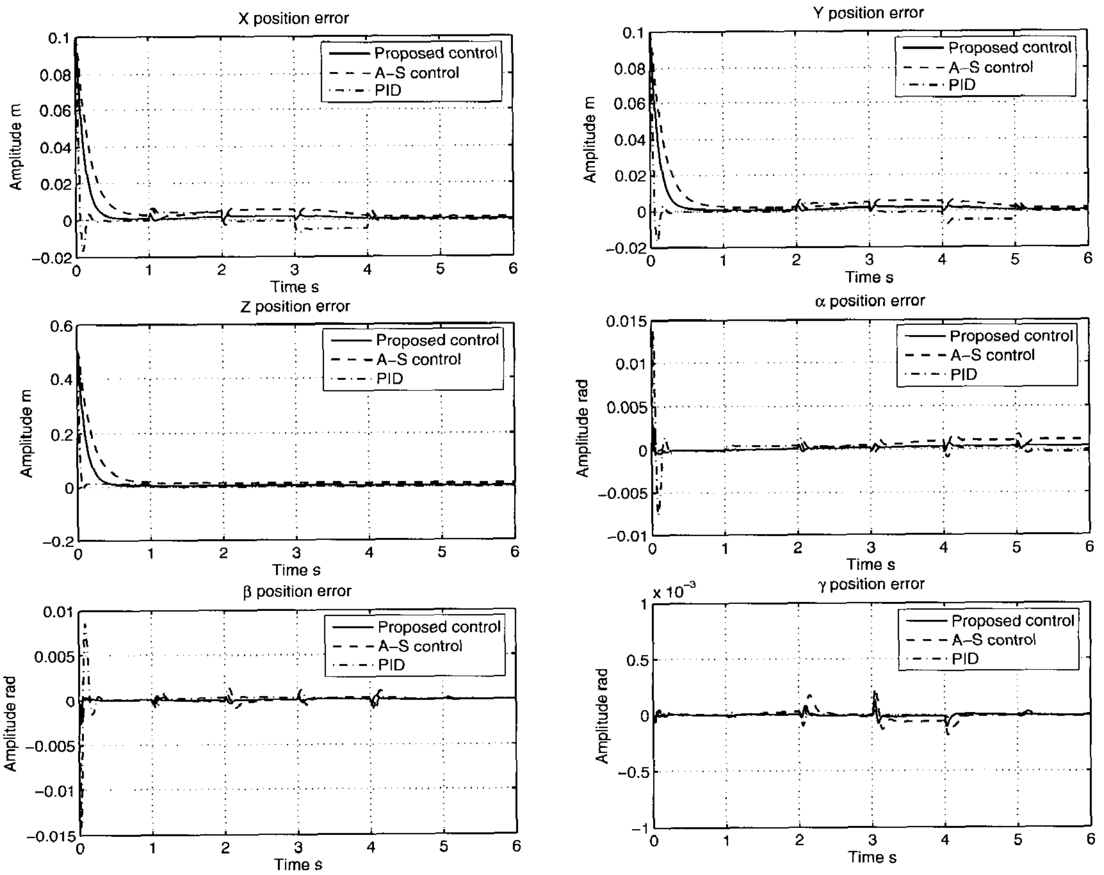

Fig. 4. Attitude errors of the moving platform.

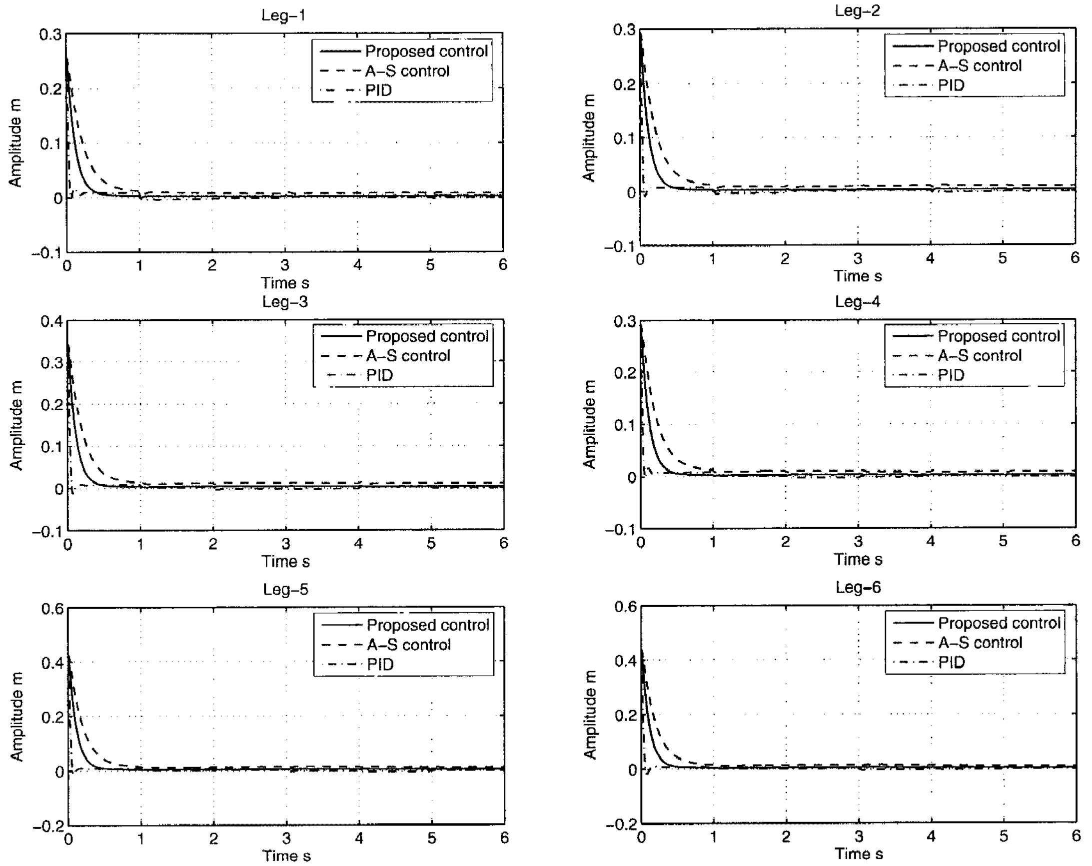

Fig. 5. Joint position errors.

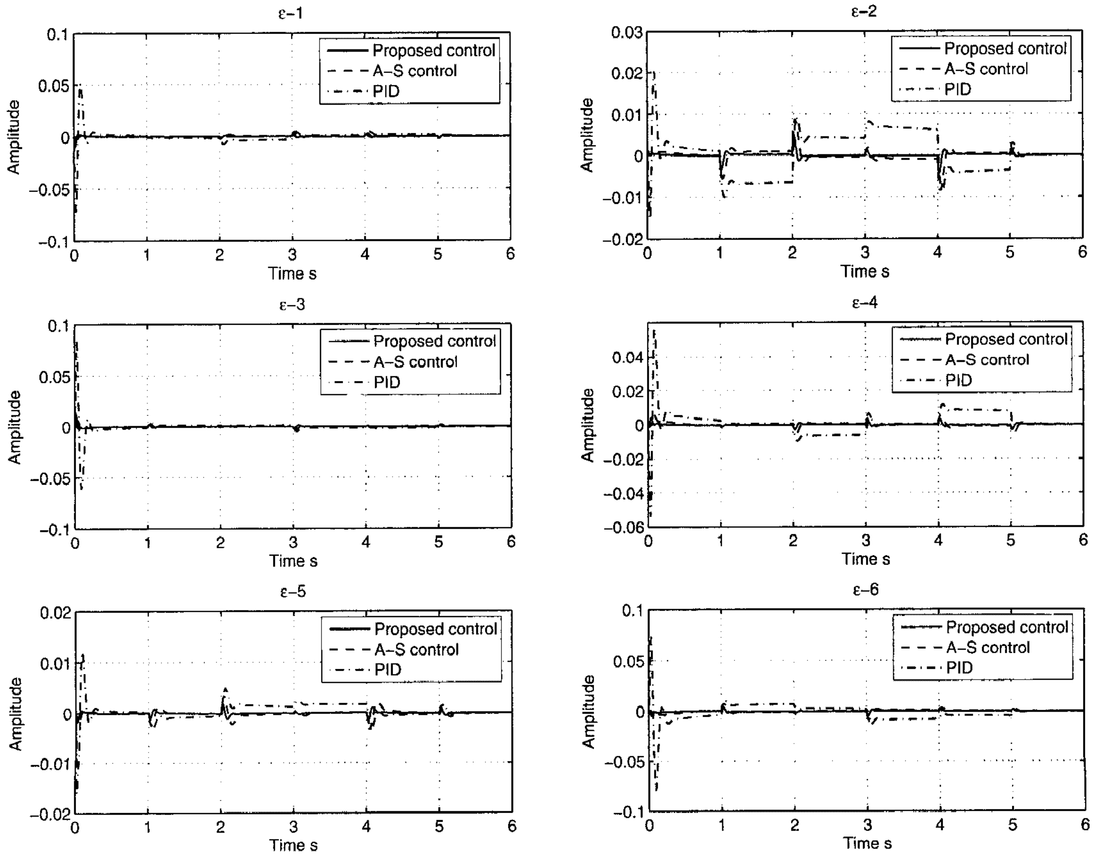

Fig. 6. Synchronization errors.

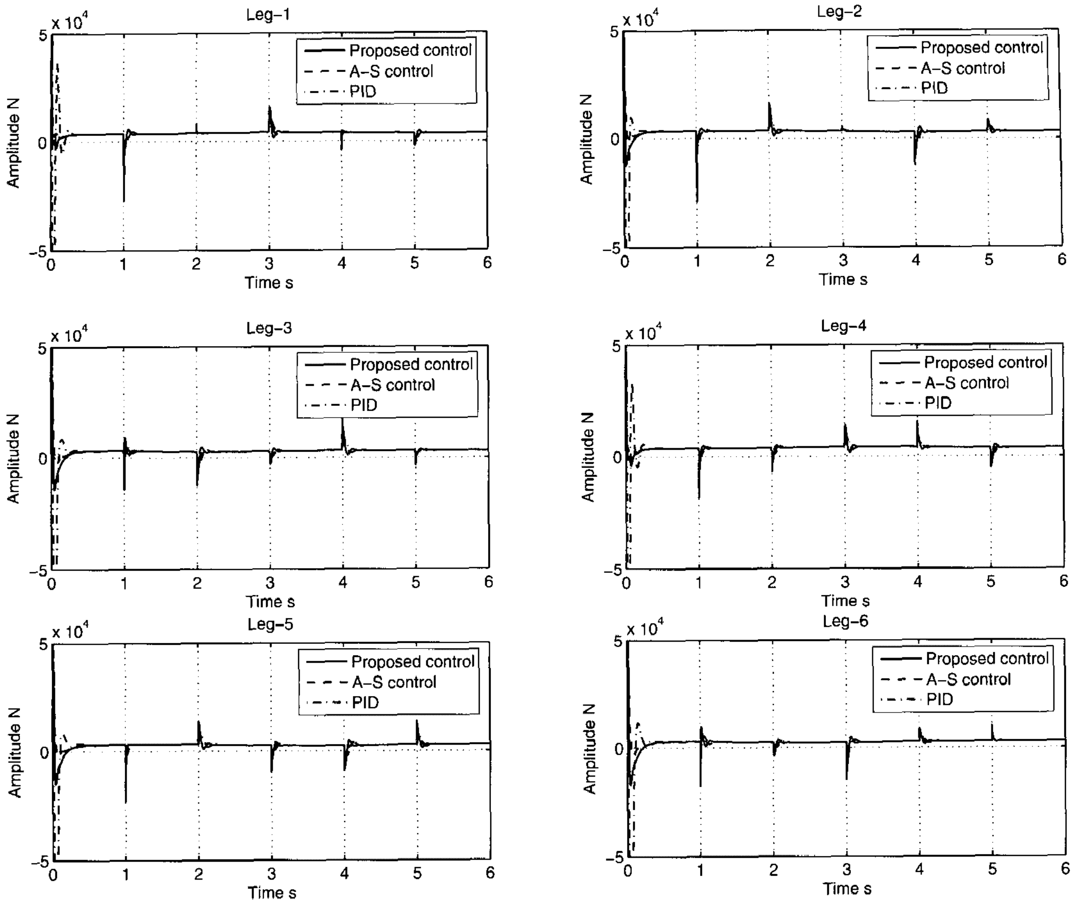

Fig. 7. Torque of each leg.

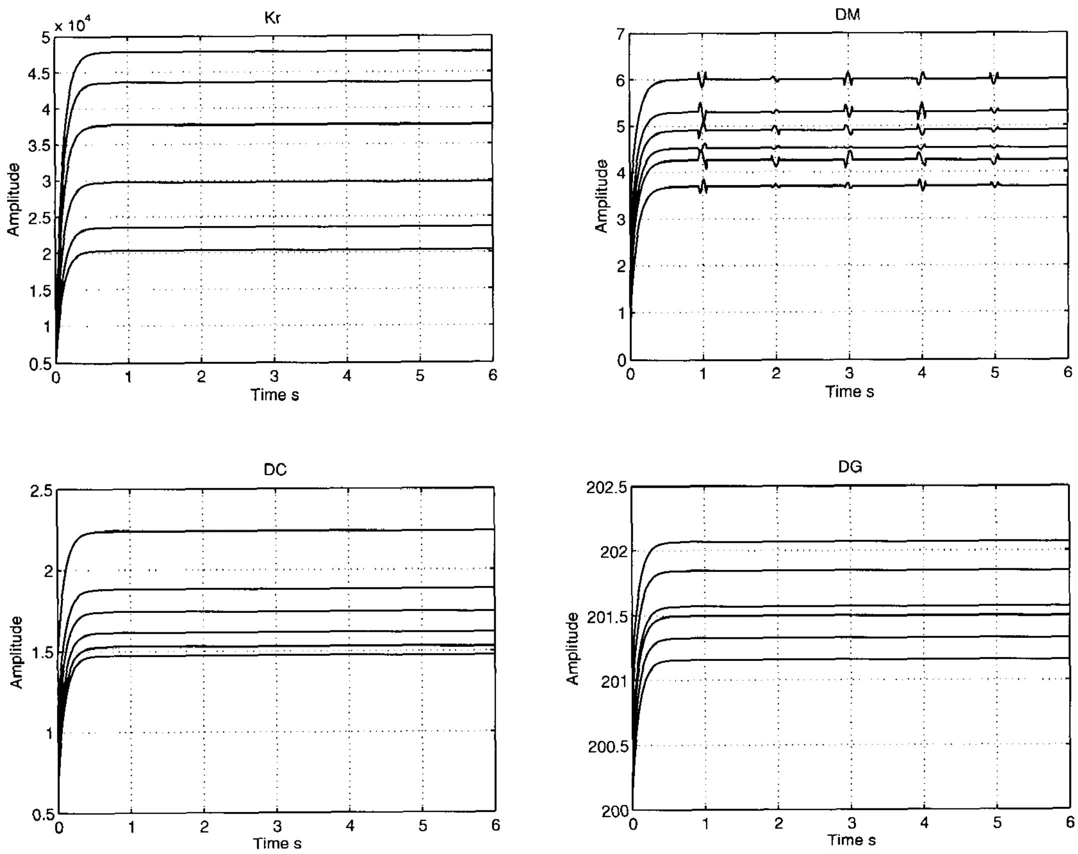

Fig. 8. Adaptive laws of the proposed control.

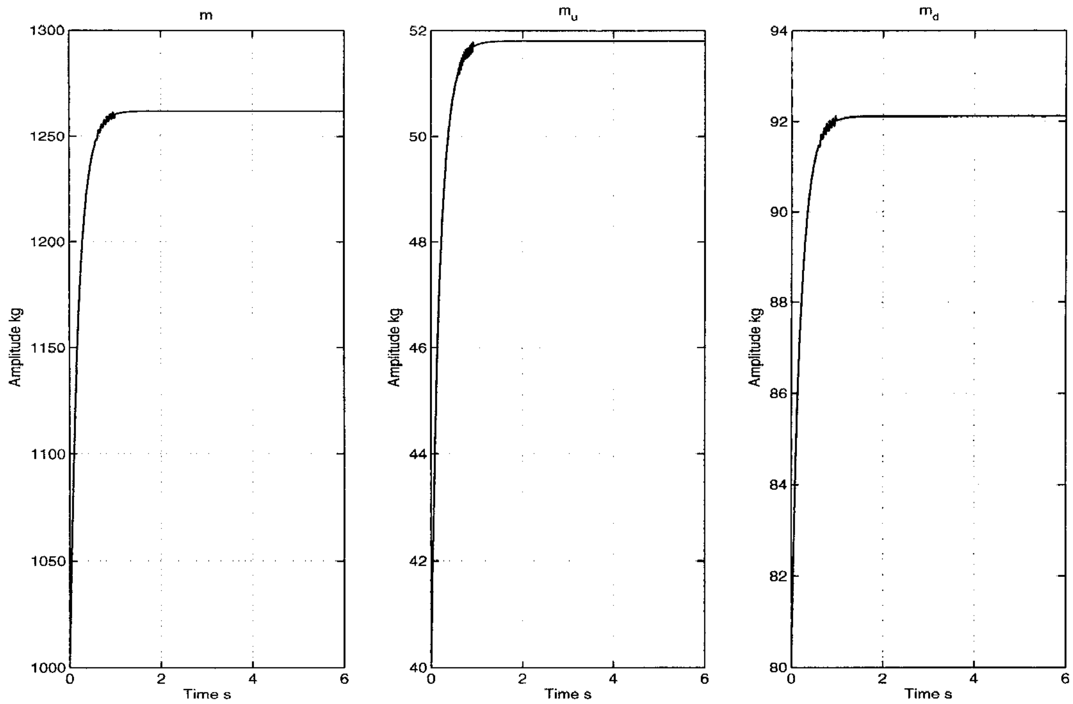

Fig. 9. Adaptive laws of A-S control.

lers can track the desired trajectory well. The PID control has overshoot during the response process, this phenomena is not expected in industry application. The corresponding joint tracking performances are illustrated in Fig. 3, which show the desired joint trajectory $q_i^d \neq 0$.

Figs. 4, 5 and 6 illustrate the attitude errors of upper platform, position errors of legs and synchronization errors, respectively. The solid lines are the errors with the proposed control, the dashed lines are the errors with A-S control, the dashdotted lines are the errors with PID control. From these figures, one can see that the tracking performances are improved by using the proposed control approach. It is important that the synchronization errors with the proposed approach and A-S control are much smaller than the ones with PID control. Large synchronization errors may damage the manipulator in practice. These three figures indicate that the system enters steady state after $t \geq 1s$. Table 2 lists the maximum absolute values of upper platform attitude errors, leg position errors and synchronization errors after $t \geq 1s$. It can be seen from these results that the proposed approach and A-S control has better performance than PID.

The torque output by each leg is shown in Fig. 7. From this figure one can see that the control input of three control algorithms are alike and bounded. Adaptive law of the proposed control is shown in Fig. 8. The mass of moving platform, the mass of upper part of leg, the mass of lower part of leg are treated as estimated parameters with initial values $\theta = [1000, 40, 80]^T$ in A-S control. The estimated parameters converge to their true values $\theta = [1261.9, 51.81, 92.11]^T$ in Fig. 9. One can see that the adaptive laws are also bounded from Figs. 8 and 9.

Table 2. Maximum absolute errors.

Maximum absolute errors	Proposed control	A-S	PID
X (m)	0.0030	0.0064	0.0074
Y (m)	0.0028	0.0062	0.0076
Z (m)	0.0056	0.0170	0.0095
α (rad)	0.0009	0.0019	0.0014
β (rad)	0.0005	0.0010	0.0014
γ (rad)	0.0001	0.0002	0.0001
Δq_1 (m)	0.0043	0.0115	0.0073
Δq_2 (m)	0.0048	0.0121	0.0072
Δq_3 (m)	0.0053	0.0133	0.0075
Δq_4 (m)	0.0049	0.0124	0.0077
Δq_5 (m)	0.0059	0.0152	0.0072
Δq_6 (m)	0.0061	0.0156	0.0072
ε_1	0.0028	0.0047	0.0069
ε_2	0.0049	0.0087	0.0099
ε_3	0.0028	0.0048	0.0054
ε_4	0.0034	0.0064	0.0118
ε_5	0.0019	0.0034	0.0049
ε_6	0.0049	0.0085	0.0139

From these comparisons, one can see that, the performance of the proposed control and A-S control are better than PID control. Especially, the synchronization errors of the proposed control and A-S control are much smaller than PID control. The large synchronization errors are undesired in practice. The proposed control is developed without any prior knowledge of plants but the A-S control is model based. Due to the complexity of dynamics of Stewart Platform system, the structure of A-S control is more complex than the proposed control. Though the performance of A-S control is almost as good as the one of the proposed control, the proposed control is more applicable than A-S control in practice.

5. CONCLUSIONS

A fully adaptive feedforward feedback synchronized tracking control approach is developed for precision control of 6DOF Stewart Platform. By incorporating cross-coupling error technology, the proposed approach can guarantee both of position error and synchronization error converge to zero asymptotically. The tracking performances are improved for the actuators working in coordinating manner. The gains of the proposed controller can be updated on line without requiring any prior knowledge of Stewart Platform manipulator. The corresponding stability analysis is also presented in this paper. The proposed controller is designed in decentralized form for implementation simplicity. It can be implemented in practice easily. Finally, performances of the proposed approach are compared with performances of A-S control and PID control through simulation. The results verify the effectiveness of the proposed approach.

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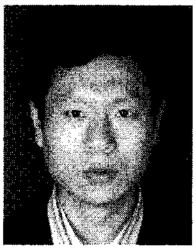

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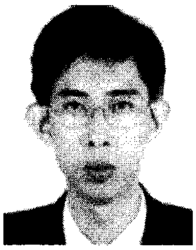

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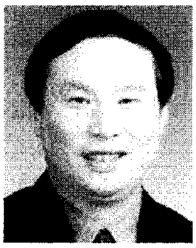

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