H_∞ Control for a Class of Singularly Perturbed Nonlinear Systems via Successive Galerkin Approximation

Young-Joong Kim and Myo-Taeg Lim*

Abstract: This paper presents a new algorithm for the closed-loop H_∞ control of a class of singularly perturbed nonlinear systems with an exogenous disturbance, using the successive Galerkin approximation (SGA). The singularly perturbed nonlinear system is decomposed into two subsystems of a slow-time scale and a fast-time scale in the spirit of the general theory of singular perturbation. Two H_∞ control laws are obtained to each subsystem by using the SGA method. The composite control law that consists of two H_∞ control laws of each subsystem is designed. One of the purposes of this paper is to design the closed-loop H_∞ composite control law for the singularly perturbed nonlinear systems via the SGA method. The other is to reduce the computational complexity when the SGA method is applied to the high order systems.

Keywords: Composite control, H_∞ control, nonlinear system, singular perturbation, successive Galerkin approximation.

1. INTRODUCTION

Many real physical systems are described by singularly perturbed nonlinear systems. Singularly perturbed systems include two or multi time scales and have been studied by many researchers [1-3]. In the class of optimal control [4], design of the control law for the singularly perturbed systems has ill-defined numerical problems [2,3]. To avoid these problems, the full order system is decomposed into reduced slow and fast subsystems, and then optimal control laws are designed for each subsystem. Thus, the near-optimal composite control law consists of two optimal sub-control laws.

Recently, robust control is issued and developed by many researchers for linear systems [5-7]. But in the class of nonlinear systems, because conditions for the solvability of the robust H_∞ control design problem are hard, still there are a lot of problems to be developed [8,9]. For nonlinear systems, the H_∞ optimal control problem is reduced to the solution of the Hamilton-Jacobi-Isaac (HJI) equation, which is a nonlinear partial differential equation (PDE) [10]. These equations may be solved analytically for some, but not for all cases of interest. On the other hand, the solution of a nonlinear PDE is extremely difficult to solve and so some researchers have searched for methods of obtaining its approximate solution. In this paper, the approximated solutions are obtained via the SGA method developed in [11,12].

However, the SGA method has the difficulty that the complexity of computations increases according to the system order. Therefore, the full order system is decomposed into the reduced order subsystems via singular perturbation theory and then two robust H_∞ sub-control laws are designed for the corresponding slow and fast nonlinear systems using the SGA method, respectively. Then, the obtained closed-loop H_∞ composite control law is represented by a linear combination of the slow and fast variables. The purpose of this paper is to design the closed-loop H_∞ composite control laws for singularly perturbed nonlinear systems using the SGA method. In order to obtain the closed-loop H_∞ control law for n-th order systems using the SGA method, one must compute n-tuple integrals, and the number of computations increases according to n. Singularly perturbed systems can be decomposed into two subsystems, and we can obtain two sub-control laws for each subsystem through the SGA method. Therefore, n_1 and n_2-tuple integrals are computed and the number of computations are decreased, where $n = n_1 + n_2$. Thus, the near-optimal H_∞ composite control law consists of two optimal H_∞ sub-control laws.

The contents of this paper are as follows. In Section 2, singularly perturbed nonlinear systems with respect

Manuscript received September 6, 2006; revised May 3, 2007; accepted June 5, 2007. Recommended by Editorial Board member Hyungbo Shim under the direction of Editor Jae Weon Choi. This work was supported by a Korea University Grant.

Young-Joong Kim and Myo-Taeg Lim are with the School of Electrical Engineering, Korea University, 1, 5-ka, Anam-dong, Sungbuk-gu, Seoul 136-701, Korea (e-mails: {kyjoong, mlim}@korea.ac.kr)
* Corresponding author.

to performance criteria are studied. We define the Generalized-Hamilton-Jacobi-Isaac (GHJI) equations for each subsystem. The solutions of GHJI equations are obtained using the SGA method and the composite H_∞ control law is designed. We present the new algorithm for H_∞ composite control of singularly perturbed nonlinear systems using the SGA method. Section 3 gives our conclusion.

2. MAIN RESULTS

In this section, the H_∞ sub-control laws are designed for each subsystem using the SGA method, and the closed-loop H_∞ composite control law consists of two optimal control laws for each subsystem.

2.1. H_∞ composite control for singularly perturbed nonlinear systems

The infinite-time H_∞ control problem considers a class of singularly perturbed nonlinear systems described by the following differential equations:

$$\dot{\alpha} = f_1(\alpha) + F_1(\alpha)\beta + g_1(\alpha)u + h_1(\alpha)\omega, \tag{1}$$

$$\varepsilon\dot{\beta} = f_2(\alpha) + F_2(\alpha)\beta + g_2(\alpha)u + h_2(\beta)\omega, \tag{2}$$

$$z = \begin{bmatrix} l(\alpha) \\ L(\alpha)\beta \\ Du \end{bmatrix}, \tag{3}$$

$$\alpha(t_0) = \alpha^0, \ \beta(t_0) = \beta^0$$

with respect to the performance criterion:

$$J = \int_0^\infty \left(z^T z - \gamma^2 \omega^T \omega\right) dt, \tag{4}$$

where $\alpha \in \Re^{n_1}$ and $\beta \in \Re^{n_2}$ are state variables, $u \in \Re^m$ is a control input, $\omega \in \Re^p$ is an exogenous disturbance, ε is a small positive parameter, and γ is a positive design parameter. We assume that $f_1 \in \Re^{n_1}$, $f_2 \in \Re^{n_2}$, $F_1 \in \Re^{n_1 \times n_2}$, $F_2 \in \Re^{n_2 \times n_2}$, $g_1 \in \Re^{n_1 \times m}$, $g_2 \in \Re^{n_2 \times m}$, $h_1 \in \Re^{n_1 \times p}$, and $h_2 \in \Re^{n_2 \times p}$ are Lipschitz continuous on a compact set $\Omega \supset B(0)$, and B is a ball around the states $\begin{bmatrix}\alpha^T & \beta^T\end{bmatrix}^T$. We also assume that $f_1(t_0) = 0$ and $f_2(t_0) = 0$. In addition, for simplification of development we assume that $h_2(\beta) = 0$.

The performance criterion (4) can be written in the equivalent form:

$$J = \int_0^\infty \left(l^T l + \beta^T L^T L\beta + u^T D^T Du - \gamma^2 \omega^T \omega\right) dt, \tag{5}$$

where γ is a positive design parameter. In the following, we solve slow and fast robust optimal control problems and combine their solutions to form a composite control:

$$u_c = u_s^* + u_f^*, \tag{6}$$

where u_s^* and u_f^* are the optimal control for slow and fast time scale problems, respectively. A subscript s denotes slow time scale and f denotes fast time scale. The near-optimality of the composite control law (6) is stated in the following theorem.

Theorem 1:

$$u^*(t) = u_c(t) + O(\varepsilon), \quad t \ge t_0, \tag{7}$$

$$\alpha(t) = \alpha_s(t) + O(\varepsilon), \quad t \ge t_0, \tag{8}$$

$$\beta(t) = \beta_s(t) + \beta_f(t) + O(\varepsilon), \quad t \ge t_0. \tag{9}$$

Proof: The proof of this theorem can be drawn from [3]. □

Let us assume that the open-loop system (1)-(2) is a standard singularly perturbed system for every $u \in B(u) \subset R^m$, that is, the equation

$$\beta_s = -F_2^{-1}(\alpha_s)\{f_2(\alpha_s) + g_2(\alpha_s)u_s\} \tag{10}$$

has a unique solution.

The slow time scale problem of order n_1 is defined by eliminating β_f and u_f from (1)-(3) and (5) using (6)-(10). Then the resulting slow time scale problem becomes optimal control of the slow subsystem

$$\dot{\alpha}_s = f_0(\alpha_s) + g_s(\alpha_s)u_s, \quad \alpha_2(t_0) = \alpha^0 \tag{11}$$

with respect to the performance criterion:

$$J_s = \int_0^\infty \left\{l_0(\alpha_s) + 2L_s(\alpha_s)u_s + u_s^T D_s(\alpha_s)u_s\right\} dt, \tag{12}$$

where

$$f_0 = f_1 - F_1F_2^{-1}f_2,$$
$$g_s = g_1 - F_1F_2^{-1}g_2,$$
$$l_0 = l^T l + f_2^T F_2^{-T} L^T L F_2^{-1} f_2,$$
$$L_s = f_2^T F_2^{-T} L^T L F_2^{-1} g_2,$$
$$D_s = D^T D + g_2^T F_2^{-T} L^T L F_2^{-1} g_2.$$

From robust H_∞ control theory [5,6], it is well known that if $J_s^*(\alpha_s)$ is a unique positive-definite solution of the HJI equation:

$$l_s + \frac{\partial J_s^*}{\partial \alpha_s}^T f_s - \frac{1}{4}\frac{\partial J_s^*}{\partial \alpha_s}^T (g_s D_s^{-1} g_s^T - \gamma^{-2} h_1 h_1^T)\frac{\partial J_s^*}{\partial \alpha_s} = 0 \tag{13}$$

with the boundary condition:

$$J_s^*(0) = 0. \tag{14}$$

Then the H_∞ control of the slow time scale problem is given by

$$u_s^* = -D_s^{-1}\left(L_s^T + \frac{1}{2}g_s^T \frac{\partial J_s^*}{\partial \alpha_s}\right) \tag{15}$$

and the exogenous disturbance of the worst case is given by

$$\omega_s^* = \frac{\gamma^{-2}}{2} h_1^T \frac{\partial J_s^*}{\partial \alpha_s}, \tag{16}$$

where

$$f_s = f_0 - g_s D_s^{-1} L_s^T,$$
$$l_s = l_0 - L_s D_s^{-1} L_s^T.$$

The fast time scale problem of order n_2 is defined by freezing the slow variable α_s and shifting the equilibrium of the fast subsystem to the origin.

$$\varepsilon\dot{\beta}_f = F_2(\alpha_s)\beta_f + g_2(\alpha_s)u_f,$$
$$\beta_f(t_0) = \beta^0 + F_2^{-1}(\alpha^0)\left\{f_2(\alpha^0) + g_2(\alpha^0)u_s(t_0)\right\}, \tag{17}$$

where $\beta_f = \beta - \beta_s$. The performance criterion of the fast time scale problem is given by

$$J_f = \int_0^\infty \left\{\beta_f^T L^T(\alpha_s)L(\alpha_s)\beta_f + u_f^T D^T D u_f\right\} dt, \tag{18}$$

where $\alpha_s \in B$ is fixed parameter.

If $J_f^*(\beta_f)$ is a unique positive-definite solution of the HJI equation:

$$\beta_f^T L^T L \beta_f + \frac{\partial J_f^*}{\partial \beta_f}^T F_2 \beta_f - \frac{1}{4}\frac{\partial J_f^*}{\partial \beta_f}^T g_2 (D^T D)^{-1} g_2^T \frac{\partial J_f^*}{\partial \beta_f} = 0 \tag{19}$$

with the boundary condition:

$$J_f^*(0) = 0. \tag{20}$$

Then the H_∞ control of the fast time scale problem is given by

$$u_f^* = -\frac{1}{2}(D^T D)^{-1} g_2^T \frac{\partial J_f^*}{\partial \beta_f}. \tag{21}$$

It is appropriate to consider the following decomposition of feedback controls where $u_s^* = G_0(\alpha_s)$ and $u_f^* = G_2(\beta_f)$ are separately designed for the slow system (11) and fast system (17). A composite control (6) for the full system (1)-(3) might then plausibly be taking as

$$u_c = G_0(\alpha_s) + G_2(\beta_f). \tag{22}$$

However, a realizable composite control requires that the system states α_s and β_f be expressed in terms of the actual system states α and β. This can be achieved by replacing α_s by α and β_f by β so that

$$u_c = G_1(\alpha) + G_2(\beta), \tag{23}$$

where

$$G_1 = G_0 + G_2 F_2^{-1} g_2 G_0 + G_2 F_2^{-1} f_2.$$

2.2. Generalized-Hamilton-Jacobi-Isaac equation

In order to obtain the H_∞ composite control law u_c, we need to find the solutions, $\partial J_s^* / \partial \alpha_s$ and $\partial J_f^* / \partial \beta_f$, using the SGA method.

Assumption 1: Ω is a compact set of $\Re^n$, and all states are bounded on $\left[t_0, t_f\right] \times \Omega$. □

Under Assumption 1, we can define the GHJI equation for singular perturbed nonlinear systems which is defined in the following.

Definition 1: If initial control laws $u_s^{(0)} : \Re^m \times \Omega_s \to \Re^m$ and $u_f^{(0)} : \Re^m \times \Omega_f \to \Re^m$ are admissible, and functions $J_s^{(i)} : \Re^{n_1} \times \Omega_s \to \Re^{n_1}$ and $J_f^{(i)} : \Re^{n_2} \times \Omega_f \to \Re^{n_2}$ satisfy the following GHJI equations, written by $GHJI(J_s^{(i)}, u_s^{(i)}) = 0$, namely

$$\frac{\partial J_s^{(i)}}{\partial \alpha_s}^T f_s + \frac{1}{4}\frac{\partial J_s^{(i-1)}}{\partial \alpha_s}^T \left(g_s D_s^{-1} g_s^T - \gamma^{-2} h_1 h_1^T\right)\frac{\partial J_s^{(i-1)}}{\partial \alpha_s} - \frac{1}{2}\frac{\partial J_s^{(i)}}{\partial \alpha_s}^T \left(g_s D_s^{-1} g_s^T - \gamma^{-2} h_1 h_1^T\right)\frac{\partial J_s^{(i-1)}}{\partial \alpha_s} + l_s = 0 \tag{24}$$

with boundary condition:

$$J_s^{(i)}(0) = 0, \tag{25}$$

then i-th slow control law is

$$u_s^{(i)} = -D_s^{-1}\left(L_s^T + \frac{1}{2}g_s^T \frac{\partial J_s^{(i-1)}}{\partial \alpha_s}\right), \tag{26}$$

and $GHJI(J_f^{(i)}, u_f^{(i)}) = 0$, namely

$$\frac{\partial J_f^{(i)}}{\partial \beta_f}^T F_2\beta_f + \frac{1}{4}\frac{\partial J_f^{(i-1)}}{\partial \beta_f}^T g_2(D^TD)^{-1}g_2^T \frac{\partial J_f^{(i-1)}}{\partial \beta_f}$$
$$-\frac{1}{2}\frac{\partial J_f^{(i)}}{\partial \beta_f}^T g_2(D^TD)^{-1}g_2^T \frac{\partial J_f^{(i-1)}}{\partial \beta_f} + \beta_f^T L^T L\beta_f = 0 \tag{27}$$

with boundary condition:

$$J_f^{(i)}(0) = 0, \tag{28}$$

then i-th fast control law is

$$u_f^{(i)} = -\frac{1}{2}(D^TD)^{-1}g_2^T \frac{\partial J_f^{(i-1)}}{\partial \beta_f}, \tag{29}$$

where i is an iteration number. □

2.3. Galerkin projections of the Generalized-Hamilton-Jacobi-Isaac equations

Given an initial control $u_s^{(0)}$, we compute an approximation to its cost $J_{sN}^{(0)} = \mathbf{c}_{sN}^{(0)T}\Phi_{sN}$, where $\mathbf{c}_{sN}^{(0)}$ is the solution of Galerkin approximation of GHJB equation (24), i.e.,

$$A_s^{(0)}\mathbf{c}_{sN}^{(0)} + b_s^{(0)} = 0, \tag{30}$$

where

$$A_s^{(0)} = \langle \nabla\Phi_{sN} f_s, \Phi_{sN}\rangle_{\Omega_s} + \left\langle \nabla\Phi_{sN}(g_s u_s^{(0)} + h_1\omega^{(0)}, \Phi_{sN}\right\rangle_{\Omega_s},$$
$$b_s^{(0)} = \langle l_s, \Phi_{sN}\rangle_{\Omega_s} + \left\langle u_s^{(0)T} D_s u_s^{(0)} - \gamma^2\omega^{(0)T}\omega^{(0)}, \Phi_{sN}\right\rangle_{\Omega_s}.$$

We can compute the updated control law and the exogenous disturbance which are based on the approximated solution, $J_{sN}^{(i-1)}$.

$$\tilde{u}_s^{(i)} = -\frac{1}{2}D_s^{-1}g_s^T\frac{\partial J_s^{(i-1)}}{\partial \alpha_s} = -\frac{1}{2}D_s^{-1}g_s^T\nabla\Phi_{sN}^T\mathbf{c}_{sN}^{(i-1)},$$
$$\omega^{(i)} = \frac{\gamma^{-2}}{2}h_1^T\frac{\partial J_s^{(i-1)}}{\partial \alpha_s} = \frac{\gamma^{-2}}{2}h_1^T\nabla\Phi_{sN}^T\mathbf{c}_{sN}^{(i-1)}.$$

Then we can obtain the approximation

$$J_{sN}^{(i)} = \mathbf{c}_{sN}^{(i)T}\Phi_{sN}, \tag{31}$$

where $\mathbf{c}_{sN}^{(i)}$ is the solution of

$$A_s^{(i)}\mathbf{c}_{sN}^{(i)} + b_s^{(i)} = 0, \tag{32}$$

where

$$A_s^{(i)} = \langle \nabla\Phi_{sN} f_s, \Phi_{sN}\rangle_{\Omega_s} - \frac{1}{2}\left\langle \nabla\Phi_{sN}(g_s D_s^{-1} g_s^T - \gamma^{-2}h_1h_1^T)\nabla\Phi_{sN}^T\mathbf{c}_{sN}^{(i-1)}, \Phi_{sN}\right\rangle_{\Omega_s},$$
$$b_s^{(i)} = \langle l_s, \Phi_{sN}\rangle_{\Omega_s} + \frac{1}{4}\left\langle \mathbf{c}_{sN}^{(i-1)T}\nabla\Phi_{sN}(g_s D_s^{-1} g_s^T - \gamma^{-2}h_1h_1^T)\nabla\Phi_{sN}^T\mathbf{c}_{sN}^{(i-1)}, \Phi_{sN}\right\rangle_{\Omega_s},$$

and i is an iteration number.

Similarly, given an initial control $u_f^{(0)}$, we can compute an approximation to its cost $J_{fN}^{(0)} = \mathbf{c}_{fN}^{(0)T}\Phi_{fN}$, where $\mathbf{c}_{fN}^{(0)}$ is the solution of Galerkin approximation of GHJB equation for the fast-time case. A detailed description of Galerkin approximation can be founded in [12-15].

2.4. A new SGA method for H_∞ composite control

The following algorithm shows that the H_∞ composite control can be designed by two closed-loop control laws of fast-and slow-subsystems using the SGA method for singularly perturbed nonlinear systems.

Algorithm 1:

Initial Step

Compute

$$A_s^{(0)} = \langle \nabla\Phi_{sN} f_s, \Phi_{sN}\rangle_{\Omega_s} + \left\langle \nabla\Phi_{sN}(g_s u_s^{(0)} + h\omega^{(0)}), \Phi_{sN}\right\rangle_{\Omega_s},$$
$$b_s^{(0)} = \langle l_s, \Phi_{sN}\rangle_{\Omega_s} + \left\langle u_s^{(0)T} D_s u_s^{(0)} - \gamma^2\omega^{(0)T}\omega^{(0)}, \Phi_{sN}\right\rangle_{\Omega_s},$$

and

$$A_f^{(0)} = \langle \nabla\Phi_{fN} F_2\beta_f, \Phi_{fN}\rangle_{\Omega f} + \left\langle \nabla\Phi_{fN} g_2 u_f^{(0)}, \Phi_{fN}\right\rangle_{\Omega f},$$
$$b_s^{(0)} = \left\langle \beta_f^T L^T L\beta_f, \Phi_{fN}\right\rangle_{\Omega f} + \left\langle u_f^{(0)T} D^T D u_f^{(0)}, \Phi_{fN}\right\rangle_{\Omega f}.$$

Find $\mathbf{c}_{sN}^{(0)}$ and $\mathbf{c}_{fN}^{(0)}$ satisfying the following linear equations:

$$A_s^{(0)}\mathbf{c}_{sN}^{(0)} + b_s^{(0)} = 0,$$
$$A_f^{(0)}\mathbf{c}_{fN}^{(0)} + b_f^{(0)} = 0.$$

Set $i=1$.
Iterative Step
Improved controllers are given by

$$\tilde{u}_{sN}^{(i)} = -\frac{1}{2}D_s^{-1}g_s^T\nabla\Phi_{sN}^T\mathbf{c}_{sN}^{(i-1)},$$
$$u_{fN}^{(i)} = -\frac{1}{2}(D^TD)^{-1}g_2^T\nabla\Phi_{fN}^T\mathbf{c}_{fN}^{(i-1)}.$$

Compute

$$A_s^{(i)} = \langle\nabla\Phi_{sN}f_s,\Phi_{sN}\rangle_{\Omega_s} - \frac{1}{2}\Big\langle\nabla\Phi_{sN}(g_sD_s^{-1}g_s^T - \gamma^{-2}h_1h_1^T)\nabla\Phi_{sN}^T\mathbf{c}_{sN}^{(i-1)},\Phi_{sN}\Big\rangle_{\Omega_s},$$
$$b_s^{(i)} = \langle l_s,\Phi_{sN}\rangle_{\Omega_s} + \frac{1}{4}\Big\langle\mathbf{c}_{sN}^{(i-1)T}\nabla\Phi_{sN}(g_sD_s^{-1}g_s^T - \gamma^{-2}h_1h_1^T)\nabla\Phi_{sN}^T\mathbf{c}_{sN}^{(i-1)},\Phi_{sN}\Big\rangle_{\Omega_s},$$

and

$$A_f^{(i)} = \langle\nabla\Phi_{fN}F_2\beta_f,\Phi_{fN}\rangle_{\Omega_f} - \frac{1}{2}\Big\langle\nabla\Phi_{fN}g_2(D^TD)^{-1}g_2^T\nabla\Phi_{fN}^T\mathbf{c}_{fN}^{(i-1)},\Phi_{fN}\Big\rangle_{\Omega_f},$$
$$b_f^{(i)} = \langle\beta_f^TL^TL\beta_f,\Phi_{fN}\rangle_{\Omega_f} + \frac{1}{4}\Big\langle\mathbf{c}_{fN}^{(i-1)T}\nabla\Phi_{fN}g_2(D^TD)^{-1}g_2^T\nabla\Phi_{fN}^T\mathbf{c}_{fN}^{(i-1)},\Phi_{fN}\Big\rangle_{\Omega_f},$$

Find $\mathbf{c}_{sN}^{(i)}$ and $\mathbf{c}_{fN}^{(i)}$ satisfying the following linear equations:

$$A_s^{(i)}\mathbf{c}_{sN}^{(i)} + b_s^{(i)} = 0,$$
$$A_f^{(i)}\mathbf{c}_{fN}^{(i)} + b_f^{(i)} = 0.$$

Set $i=i+1$.
Final Step
The realizable H_∞ composite control law is

$$u_c = -D_s^{-1}\left(L_s^T + \frac{1}{2}g_s^TG_s\alpha\right) - \frac{1}{2}(D^TD)^{-1}g_2^TG_f \times\left[\beta + F_2^{-1}f_2 - F_2^{-1}g_2D_s^{-1}\left(L_s^T + \frac{1}{2}g_s^TG_s\alpha\right)\right],$$

where $\nabla\Phi_{sN}^T\mathbf{c}_{sN} = G_s\alpha_s$ and $\Phi_{fN}^T\mathbf{c}_{fN} = G_f\beta_f$. □

The following theorem demonstrates that the approximate H_∞ composite control law, u_{cN}, designed by the proposed algorithm, converges to the H_∞ optimal control law, u^*.

Theorem 2: For any small positive constant σ, we can choose N for a sufficiently large i to satisfy that:

$$\left\|u^* - u_{cN}^{(i)}\right\| < \sigma. \tag{33}$$

Proof: It was proved that u^* converges to u_N pointwise on Ω for finite N in [11], where u_N is a control law designed using the SGA. It implies that for a sufficiently large i, we can choose N satisfying $\left\|u_c - u_{cN}^{(i)}\right\| < \tilde{\sigma}$, where u_c is the composite control law obtained by the reduced order scheme for singularly perturbed nonlinear systems and $\tilde{\sigma}$ is a small positive constant. By the help of singular perturbation theory, $u_c = u^* + O(\varepsilon)$. This implies that for any small positive constant σ, we can choose N for a sufficiently large i satisfying (33). □

3. A NUMERICAL EXAMPLE

Now, we apply the proposed algorithm to a numerical example. Consider the fifth-order numerical example which is the standard singularly perturbed nonlinear system (1)-(3). The states variables are $\alpha = [x_1 \quad x_2 \quad x_3]^T$ and $\beta = [x_4 \quad x_5]^T$, and the control variable is $u = [u_1 \quad u_2]^T$. The problem matrices have the following values:

$$f_1(\alpha) = \begin{bmatrix} -0.04611x_1 \\ -2.149x_1 - x_1x_3 \\ x_1x_2 - 2.146x_3 \end{bmatrix},$$
$$f_2(\alpha) = \begin{bmatrix} 0.146x_2 + 0.068x_1x_3 \\ -0.068x_1x_2 + 0.146x_3 \end{bmatrix},$$
$$F_1(\alpha) = \begin{bmatrix} -16.6x_3 & 16.6x_2 \\ 0.146 & 0 \\ 0 & 0.146 \end{bmatrix},$$
$$F_2(\alpha) = \begin{bmatrix} -0.00225 & 0 \\ 0 & -0.00225 \end{bmatrix},$$
$$g_1(\alpha) = 0,\quad g_2(\alpha) = 0.0399I_2,$$
$$h_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T,\quad \varepsilon = 0.00262.$$

In this example, we assume that the exogenous disturbance $\omega = 130\sin(148\pi t)$. The simulation results are presented in Figs. 1-6 with initial states $x_0 = [10 \quad -0.07 \quad 0.04 \quad 15 \quad 47]^T$. The dashed lines

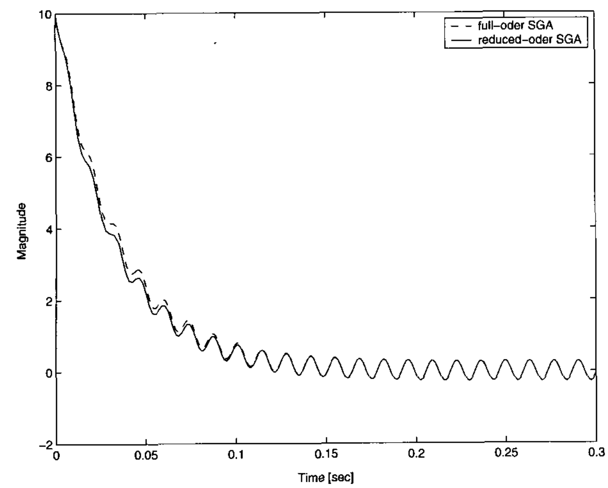


Fig. 1. Trajectories of x_1.

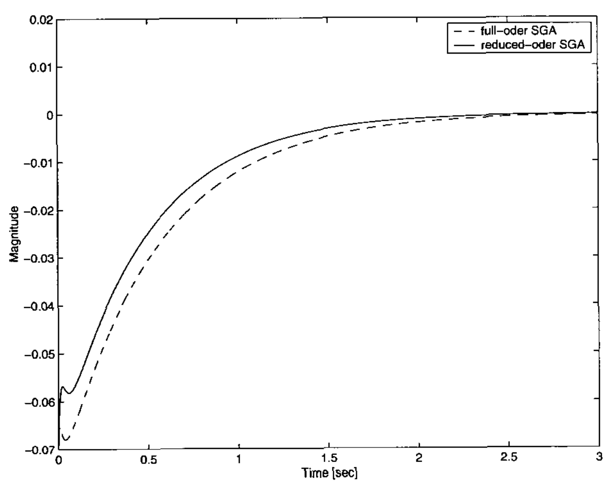


Fig. 1. Trajectories of x_2.

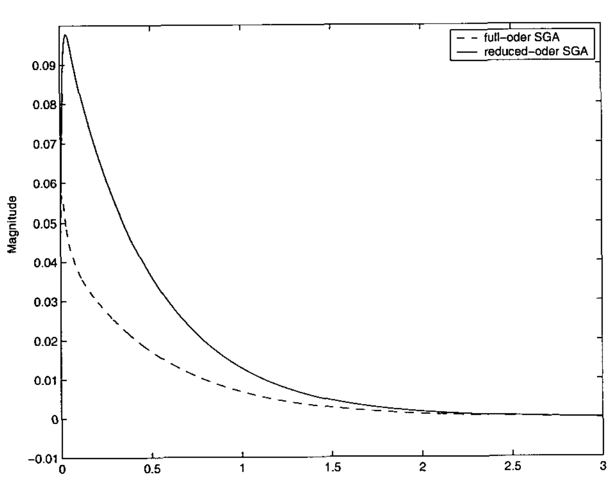


Fig. 3. Trajectories of x_3.

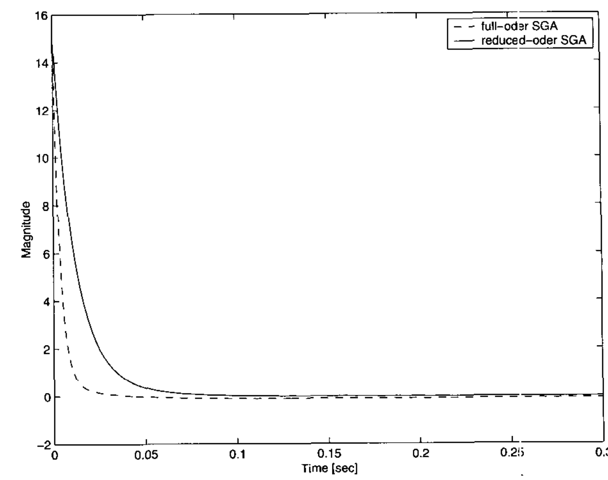


Fig. 4. Trajectories of x_4.

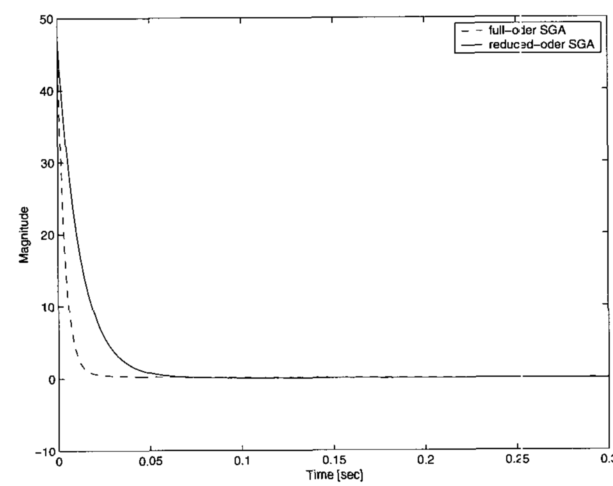


Fig. 5. Trajectories of x_5.

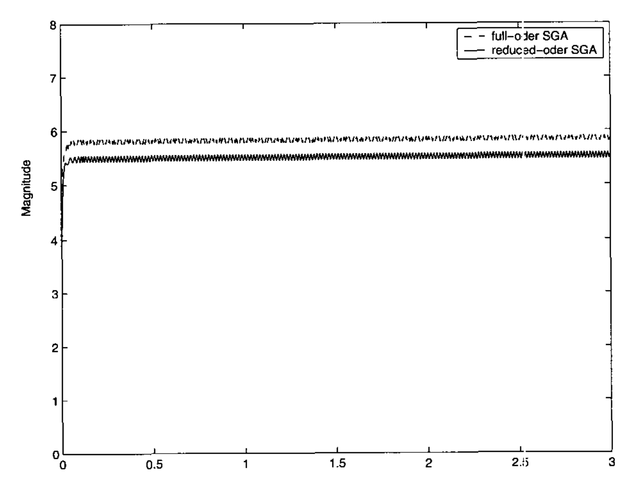


Fig. 6. Trajectories of the performance criterion.

(- -) are the trajectories obtained from the full-order SGA method and the sold lines (—) are the trajectories obtained from the proposed algorithm. Fig. 6 indicates that the performance criterion trajectory of the proposed algorithm is better than that of the full-order SGA method, because errors of the full-order SGA method are greater than those of the proposed algorithm. In the full-order SGA method, ten-dimensional basis $\Phi_{10} = \{x_1^2, x_1x_2, x_2^2, x_2x_3, x_3^2, x_3x_4,$

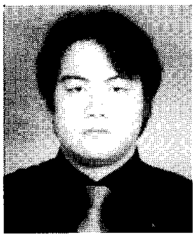
$x_4^2, x_4x_5, x_5^2, x_1x_5\}$ are used and five-dimensional integrals of $10\times(1+10+100)=1110$ times are performed. However, in the proposed algorithm, we can use only six-dimensional basis $\Phi_6=\{x_1^2, x_1x_2, x_2^2, x_2x_3, x_3^2, x_1x_3\}$ computing three-dimensional integrals of $6\times(1+6+36)=248$ times for slow-time scale subsystems, and two-dimensional integrals of $3\times(1+3+9)=39$ times based on three-dimensional basis $\Phi_3=\{x_4^2, x_4x_5, x_5^2\}$ for fast-time scale subsystems in parallel. Therefore, the computational complexity is greatly reduced.

4. CONCLUSION

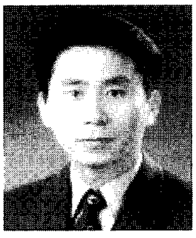
In this paper, we have presented the closed-loop H_∞ composite control scheme for a class of singularly perturbed nonlinear systems using the SGA method. The difficulty of the SGA method is a computational complexity, but in the proposed algorithm, n-tuple integrals are reduced to n_1- and n_2-tuple integrals in parallel. Moreover, the computational complexity according to n state variables is decreased to n_1 and n_2 state variables. The presented simulation results show that the performance of the proposed algorithm is better than those of the full order SGA method.

REFERENCES

[1] J. Chow and P. Kokotovic, "A decomposition of near-optimum regulators for the systems with slow and fast modes," *IEEE Trans. on Automatic Control*, vol. AC-21, pp. 701-706, 1976.
[2] J. Chow and P. Kokotovic, "A two-stage Lyapunov-Bellan feedback design of a class of nonlinear systems," *IEEE Trans. on Automatic Control*, vol. AC-26, no. 3, pp. 656-663, 1981.
[3] P. Kokotovic, H. Khalil, and J. O'Reilly, *Singular Perturbation Methods in Control: Analysis and Design,* Academic Press, Orlando, 1986.
[4] D. Kirk, *Optimal Control Theory*, Prentice-Hall, Englewood Cliffs, 1970.
[5] M. Green and D. Limebeer, *Linear Robust Control*, Prentice-Hall, New Jersey, 1995.
[6] K. Zhou, *Essentials of Robust Control*, Prentice-Hall, New Jersey, 1998.
[7] H. Lee, S. Kau, Y. Liu, C. Fang, J. Chen, M. Tsai, and L. Lee, "An improvement on robust H_∞ control for uncertain continuous-time descriptor systems," *International Journal of Control, Automation, and Systems*, vol. 4, no. 3, pp. 271-280, 2006.
[8] A. Isidori, *Nonlinear Control Systems*, Springer-Verlag, London, 1995.
[9] H. Khalil, *Nonlinear Systems*, Prentice-Hall, New Jersey, 1996.
[10] A. van der Schaft, "L_2-gain analysis of nonlinear systems and nonlinear state-feedback H_∞ control," *IEEE Trans. on Automatic Control,* vol. AC-37, no. 6, pp. 770-784, 1992.
[11] R. Beard, *Improving the Closed-Loop Performance of Nonlinear Systems*, Ph.D. thesis, Rensselaer Polytechnic Institute, Troy, NY, 1995.
[12] R. Beard, G. Saridis, and J. Wen, "Galerkin approximation of the generalized Hamilton-Jacobi-Bellman equation," *Automatica*, vol. 33, no. 12, pp. 2159-2177, 1996.
[13] Y. J. Kim, B. S. Kim, and M. T. Lim, "Composite control for singularly perturbed nonlinear systems via successive Galerkin approximation," *DCDIS, Series B: Applications and Algorithms*, vol. 10, no. 2, pp. 247-258, 2003.
[14] Y. J. Kim, B. S. Kim, and M. T. Lim, "Composite control for singularly perturbed bilinear systems via successive Galerkin approximation," *IEE Proc. - Control Theory and Application*, vol. 150, no. 5, pp. 483-488, 2003.
[15] Y. J. Kim and M. T. Lim, "Parallel robust H_∞ control for weakly coupled bilinear systems with parameter uncertainties using successive Galerkin approximation," *International Journal of Control, Automation, and Systems*, vol. 4, no. 6, pp. 689-696, 2006.



Young-Joong Kim received the B.S., M.S., and Ph.D. degrees in Electrical Engineering from Korea University, Seoul, in 1999, 2001, and 2006, respectively. Since 2006, he has been a Postdoctoral Fellow in the School of Electrical Engineering at Korea University. His research interests include optimal control, robust control, and visual control of autonomous mobile robots. He is a Member of KIEE.



Myo-Taeg Lim received the B.S. and M.S. degrees in Electrical Engineering from Korea University in Seoul, in 1985 and 1987, respectively. He also received the M.S. and Ph.D. degrees in Electrical Engineering from Rutgers University, U.S.A. in 1990 and 1994, respectively. Since 1996, he has been a Professor in the School of Electrical Engineering at Korea University. His research interests include robust control, multivariable system theory, and computer-aided control systems design. He is a Member of KIEE.