Adaptive Observer using Auto-generating B-splines

Dane Baang, Julian Stoev, and Jin Young Choi*

Abstract: This paper presents a new adaptive observer design method for a class of uncertain nonlinear systems by using spline approximation. This scheme leads to a simplified observer structure which requires only fixed number of integrations, regardless of the number of parameters to be estimated. This benefit can reduce the number of integrations of the observer filter dramatically. Moreover, the proposed adaptive observer automatically generates the required spline elements according to the varying output value and, as a result, does not requires the pre-knowledge of upper and lower bounds of the output. This is another benefit of our approach since the requirement for known output bounds have been one of the main drawbacks of practical universal approximation problems. Both of the benefits stem from the local support property, which is specific to splines.

Keywords: Adaptive observers, B-splines, nonlinear systems, splines, uncertain systems.

1. INTRODUCTION

The state observation problem for linear and nonlinear systems arises when the number of measurable plant outputs is limited. In such cases, restoring the state information from the limited number of outputs is required. Linear state observation theory is now mature and well understood.

Often, however, when the plant information is not precise, adaptive state/parameter observation is required. On the other hand, the problem of plant parameter estimation is the topic of system identification theory, which has been explored in great detail for linear systems [1]. The linear adaptive observer problem was explored in [2,3] for applications in the linear adaptive control [4-8].

The problem was later extended to several classes of nonlinear systems. Adaptive observers for large classes of nonlinear systems are discussed in [9-11], often using special forms based on the results from linear systems [12]. The focus was on linear-like systems with nonlinear output injection term and unknown parameters entering linearly [13,14]. The advances in geometric nonlinear control theory [15] make the transformation into such form possible.

It is well known that universal approximators can be used for adaptive control and estimation. A lot of research has been done in this direction, which involves mostly radial basis function networks, fuzzy logic and their combinations. The model structure in traditional nonlinear adaptive control and state estimation may include functions with unknown parameters, but the form of the function is assumed to be known. Situations where the form of the unknown function is also unknown have motivated approaches involving on-line function approximation much of which is classified as neuro control.

A large amount of research has also been done in the area of neuro-control using dynamic and recurrent neural networks [16,17], sigmoidal neural networks [18] and radial basic function networks [19]. Neural network techniques have been found to be particularly useful for controlling highly uncertain, nonlinear and complex systems [20]. The neural network approach was first investigated in off-line environments [21,22]. Initially most studies were based strictly on optimization techniques to derive parameter adaptive laws. Such schemes perform well in many cases, but general difficulties arise in developing analytical results regarding stability, robustness and performance properties of the overall system. Theoretical frameworks for on-line adaptive control using neural networks have been developed for a class of nonlinear systems [18,19,23,24] and they use Lyapunov design methods to guarantee stability.

It is also well known that splines are universal approximators with well established properties. The theory and applications of B-spline have been well developed in various areas including applied numerical analysis, interpolation theory, Neural

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networks, fuzzy systems, signal processing, etc. Splines are used in [25] to approximate probability density function of a continuous random variable, with the measured entropy as its cost function. The potential of the B-spline neural network (BSNN) for the modeling of nonlinear processes is demonstrated in [26] by comparing BSNN to MLP model. An efficient generalized B-spline filtering technique for the signal processing is shown in [27] with a representation of signals in terms of continuous generalized B-spline basis functions. B-splines are adopted in [28] for the design of fuzzy controller and applied to an optimal partition algorithm and linguistic modifications to minimize the size of fuzzy rules. Some recent developments are extended in [29] to image processing applications based on the representation of a signal in terms of continuous generalized B-Spline basis functions. A generalization of B-splines into fuzzy B-splines for modeling uncertain and sparse data can be found in [30]. Smoothing splines [31,32] are used in [33] to develop a method for planning trajectories while minimizing a quadratic cost functional over the control input to a linear system. A connection between B-spline theory and control can be found in [34].

The first motivation for this paper is the fact that as the contemporary control problems deal with increasingly nonlinear and poorly understood plants, the importance of the approximation properties is increasing. The well-established approximation methods, for examples, neural networks, fuzzy logic, have many applications and advantages. But their computational complexity grows very fast as the plant nonlinearity increases. This motivates us to explore splines as alternative approximation technique with less computational complexity. The motivation for this paper is also the relative lack of works related specifically to splines, and of splines used in nonlinear system identification, especially taking into account the large number of such results using other kinds of universal approximators. As a result, there are relatively few works exploring the special properties of splines, which make them more (or less) attractive from the point of view of control engineering.

Another motivation is that, in reality, the information of measurement signal bounds is required in most on-line function approximation problems. This problem often requires pre-operation of the plant or unreliable heuristics to predict measurement bounds. This also motivates us to develop an approximation scheme, which generates basis functions automatically in function approximation for adaptive observer design, as the measurement value changes.

The significance of this paper is the use of splines as universal approximators and the exploration of their advantages when applied to adaptive observer design. In this paper, we consider the problem of adaptive state observation of a large class of nonlinear uncertain systems and show that splines have some special properties, which can lead to simplified observer structure. In particular, the observer filter requires fixed number of integrators, independent of the number of parameters to be estimated. This appears to bring a significant benefit for the observer design, especially compared to other kinds of universal approximators.

We also show that the knowledge of measurement bound is not required in the proposed design since, in our approach, the spline element in approximation is simultaneously generated according to the measurement value of the plant output. This also seems a significant benefit since it removes the necessity of heuristics pre-operations of the plant.

These benefits of the proposed observer design are due to the local support property, which is specific to spline approximation.

In Section 2, we pose an adaptive observer design problem by using universal approximators. In Section 3, we give some basic properties of splines, which are necessary for the understanding of the proposed design. In Section 4, we show a particular form of plant parameterization using splines. Using this result, the observer design is performed in a straightforward way. We design our adaptive observer and analyze it in Sections 5 and 6. In Sections 7 and 8, we finish with a numerical simulation demonstrating the benefits of the proposed design.

2. ADAPTIVE OBSERVER PROBLEM FORMULATION

A large class of SISO systems can be represented in the output feedback form, where the dynamics is linear and the nonlinear terms on the right hand side of the differential equations depend on measurable signals.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + F(y(t), u(t)), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state, $y(t), u(t) \in R$ are the measurable system output and input, respectively. The functions $F(\cdot) \in [f_1(\cdot), \ldots, f_n(\cdot)]^T$ are often partially known. The availability of only partial information about $F(\cdot)$ is the reason why the adaptive observer is appealing. The information about $F(\cdot)$ may be from the physical knowledge of the system or some identification procedure.

- There may be known terms in $F(\cdot)$. No adaptation is necessary for them.
- There may be partially known terms in $F(t)$. They are known up to some unknown constants, which

are linearly combined with some known functions. Adaptation with respect to these unknown constants should be performed.

- There may be the terms in $F(t)$, about which we do not know any functional relation, or the functional relation available is nonlinear in the unknown constant parameters. Universal approximators can be used to model these terms. Several classes of universal approximators, which are linear in parameters can be used. These include radial basis functions, fuzzy logic, and among others, spline based representations.

This paper considers the third case, the most general one. This problem has been addressed by neural network and fuzzy communities using the theoretical background from nonlinear control. Since the fuzzy systems and radial basis functions are universal approximators, it is well known that the minimum approximation error of this term can be made arbitrary small by using more rules and RBF neurons [35]. With the complexity of the universal approximator increasing, the number of parameters to be updated is also increasing very quickly and this makes the application of such methods problematic in the real world. Furthermore, approximation procedure often requires the knowledge of the bound of measurement in advance, to determine the area where neurons are to be built.

In this paper, we show that the use of spline approximation can overcome these two difficulties, when applied to adaptive observers for a class of nonlinear systems.

3. SPLINE-BASED FUNCTION APPROXIMATION

The book [36] has the most important developments in the area of spline approximators. We give some of the B-spline definitions and properties. All functions here are chosen to be right-continuous. Other choices are also possible.

Definition 1: For a nondecreasing knot sequence $\Upsilon := \{y_i, i=1,\dots,m, y_i \le y_{i+1}\}$, the B-spline of order 1 over the domain $y_i \le y < y_{i+1}$ is defined as

$$B_{i,1}(y) \equiv X_i(y) := \begin{cases} 1, & \text{if } y_i \le y < y_{i+1} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Higher-order B-splines are defined recursively as follows and an example is given in Fig. 1.

$$\begin{aligned} B_{i,k}(y) &:= \omega_{i,k}(y) B_{i,k-1}(y) + (1-\omega_{i+1,k}(y)) B_{i+1,k-1}(y), \\ \omega_{i,k}(y) &:= \begin{cases} \dfrac{y-y_i}{y_{i+k-1}-y_i}, & \text{if } y_i \ne y_{i+k-1} \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \tag{3}$$

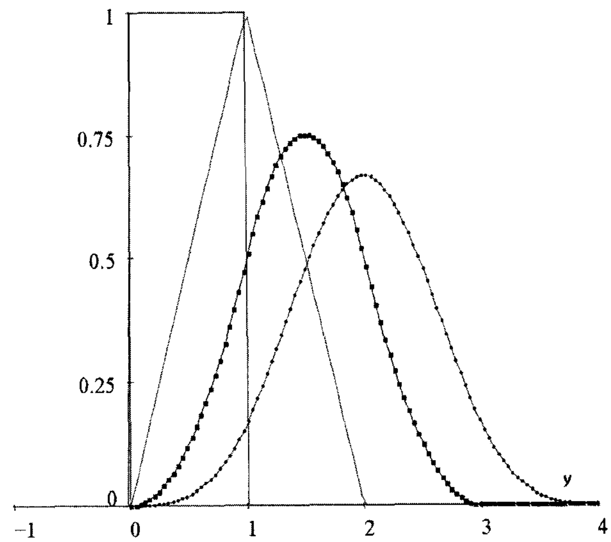

Fig. 1. Example of B-splines $B_{1,1}, B_{1,2}, B_{1,3}, B_{1,4}$.

Remark 1: From the above definitions, it is clear that $B_{i,k}(y)$ is defined only on $k+1$ knots and it has non-zero value only on $y_i \le y < y_{i+k}$.

Corollary 1: $B_{i,k}(y)$ can be represented as a polynomial in y of degree $k-1$ with switching parameters. For some particular y, (3) is equivalent to

$$\begin{aligned} B_{i,k}(y) &= [\beta_{i,j,0} \cdots \beta_{i,j,k-1}] \phi_k(y), \\ \phi_k(y) &= [y^0 \; y^1 \; \cdots \; y^{k-1}]^T, \end{aligned} \tag{4}$$

where $\beta_{i,j,k,}$ $j=1,\dots,k,$ are the polynomial coefficients for the piecewise segments of $B_{i,k}(y)$, which can be calculated recursively using (2) and (3).

A spline of order k with a knot sequence Υ is a linear combination of B-splines $B_{i,k}$ associated with the knot sequence Υ. In this paper, B-splne is used as a spline.

We next provide an important theorem which shows that B-splines can approximate any continuous functions with arbitrary precision on a compact set. Because B-splines can be alternatively represented as piecewise polynomial functions (as shown in [36]), this property is related to the classical Weierstrass approximation theorem, which assures us that polynomial approximation can get arbitrarily close to any continuous function as the polynomial order is increased. However, we prefer to use a more general version of this theorem.

Theorem 1 (Spline Universal Approximation): Suppose that the input universe of discourse U is a compact set in R^n. Then, for any given real continuous function $g(x)$ on U and arbitrary small positive $\varepsilon > 0$, there exist a spline $f(x)$ such that

$$\sup_{x \in U} | f(x) - g(x) | < \varepsilon.$$

That is, the spline functions are universal approximators.

Proof [of Theorem 1, outline]: Let Y be the set of all spline functions. It is shown in [36], that they can be alternatively represented as a piecewise polynomial functions on the same knot sequence. Using this piecewise polynomial property, it can be easily shown that three conditions required in the Stone-Weierstrass Theorem are satisfied. It follows from the Stone-Weierstrass Theorem [37] that Splines are universal approximators. □

4. SPLINE-BASED PLANT PARAMETERIZATION

Consider the sub-class of the system (1) with the structure

$$\begin{aligned} \dot{x}_i &= x_{i+1} + f_i(y), 1 \le i \le n-m-1, 0 \le m, m < n, \\ \dot{x}_i &= x_{i+1} + f_i(y) + g_i(y)u, n-m \le i \le n-1, \\ \dot{x}_n &= f_n(y) + g_n(y)u, \\ y &= x_1. \end{aligned} \tag{5}$$

Assumption 1: The $f_i(y), 1 \le i \le n$ are partially known smooth functions. In such case, $f_i(y)$ can be decomposed into $f_i(y) = v_i(y) + \overline{f}_i(y)$, where $v_i(y)$ contains the known terms in $f_i(y)$ and $\overline{f}_i(y)$ are uncertain - perhaps with unknown functional structure or nonlinear dependence on physical parameters. This design formulation permits to use the available prior physical or expert information about $f_i(y)$, and $\overline{f}_i(y)$ can be approximated by neural networks, or more generally, by any kind of universal approximator. Since the functions $\overline{f}_i(y)$, $1 \le i \le n$ use the same input variable y, a single network with multiple outputs may be used. In this paper, we use B-splines. Define

$$\overline{F}(y) = [\overline{f}_1(y), \ldots, \overline{f}_n(y)]^T. \tag{6}$$

Assumption 2: All unknown functions in the vector (6) will be approximated using B-splines defined on the same knot sequence Υ.

As a result of Assumption 2, the same B-spline basis can be used to represent each uncertain function in $\overline{F}(y)$ as a linear combination of B-splines. Let $a \in R^{qn}$ be a vector containing the unknown spline weights. Then

$$\begin{aligned} \overline{F}(y) &\approx \Psi_a(y)a + \tilde{F}(y), \\ \Psi_a(y) &= [B_{1,k}(y)I \ \ldots \ B_{q,k}(y)I], \end{aligned} \tag{7}$$

where I is $n \times n$ identity matrix and $\tilde{F}(y)$ is the inherent error existing in any approximation. As mentioned in Remark 1, for some particular value of y, where $y_i \le y < y_{i+1}$, only k (spline order) number of $B_{i,k}(y)$, $1 \le i \le q$ are active (non-zero).

Assumption 3: $g_{k+n-m}(y), 0 \le k \le m$ are partially known functions. It is assumed, that $g_{k+n-m}(y)$ can be represented as $g_{k+n-m}(y) = b_{m-k}\sigma(y)$, $0 \le k \le m$ with b_{m-k}, unknown constants, and $\sigma(y): R \mapsto R$ a known function.

Let $b = [b_m, \ldots, b_0]^T \in R^{m+1}$ be vectors of unknown constant optimal parameters related to $g_i(y)$. The general class of systems (5) can be represented in the form (1).

$$\begin{aligned} \dot{x}(t) &= Ax(t) + v(y) + \tilde{f}(y,u) + \Psi_a(y)a + \begin{bmatrix} 0 \\ b \end{bmatrix} \sigma(y)u, \\ y &= Cx, \\ A &= \begin{bmatrix} 0 & 1 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 1 \\ 0 & 0 & \ldots & 0 \end{bmatrix}, \\ v(y) &= [v_1(y) \ldots v_n(y)]^T, \tilde{f}(y,u) = [\tilde{f}_1(y,u) \ldots \tilde{f}_n(y,u)]^T, \\ C &= [1 \ 0 \ \ldots \ 0], \end{aligned} \tag{8}$$

where $\tilde{f}_i(y,u)$ are uncertain terms containing the approximating and modelling errors. The functions $\tilde{f}(y,u)$ in (8) is defined in Definition 3 and can be interpreted as the minimum approximation error, which represents the minimum possible deviation between the uncertain functions, $f_i(y)$ and $g_{k+n-m}(y)$, and their on-line approximations. Let $\theta := \begin{bmatrix} b \\ a \end{bmatrix} \in R^{qn+m+1}$ and construct the matrix

$$\Psi(y,u) = \left[\begin{bmatrix} 0_{(\rho-1)\times(m+1)} \\ I_{m+1} \end{bmatrix} \sigma(y)u \quad \Psi_a(y) \right], \tag{9}$$

where ρ is the relative degree of the system (5).

Assumption 4: For $\forall y \in R$ and $\forall u \in R$, when using the optimal weights θ, the minimum approximation error is bounded.

Definition 2: Define the approximation error $\tilde{f}(y,u,\hat{\theta})$ for some estimated values $\hat{\theta}$ as

$$\tilde{f}(y,u,\hat{\theta}) := \begin{bmatrix} f_1(y) \\ . \\ . \\ . \\ f_n(y) \end{bmatrix} + \begin{bmatrix} 0 \\ . \\ . \\ . \\ 0 \\ g_{n-m}(y)u \\ . \\ . \\ . \\ g_n(y)u \end{bmatrix} - v(y) - \Psi(y,u)\hat{\theta}. \quad (10)$$

Remark 2: The optimal weights θ are an artificial constant quantities introduced for analytical purposes. They minimize the approximation error

$$\theta := \arg_{\hat{\theta}\in R^{qn+m+1}} \min\left\{\sup_{y\in R, u\in R} \| \tilde{f}(y,u,\hat{\theta}) \|\right\}.$$

The values of θ are not needed for the implementation.

Definition 3: Define the minimum approximation error $\tilde{f}(y,u) = \tilde{f}(y,u,\theta)$ as the approximation error (10) obtained when the optimal weights θ are used.

5. SPLINE-BASED SYSTEM APPROXIMATION

The core property of the spline-based approximator is that the active polynomial parameters are changing abruptly when the measured plant output is moving across different areas. This occurs, for example, when y goes from $y_i \le y < y_{i+1}$ to $y_{i+1} \le y < y_{i+2}$, or to $y_{i-1} \le y < y_i$. These switches occur at some time moments t_j and we assume that it is possible to detect the occurrence of such events and the time at which they occur. These times are strictly increasing sequence, $\{t_j\}$, $\lim_{j\to\infty} t_j = \infty$.

Remark 3: Because of the spline structure of $\Psi_a(y)$ in (7) and Remark 1, it is possible to factor $\Psi_a(y)$ as

$$\Psi_a(y) = \left[0_{n\times n(i-k)} \;\; M_{i\mapsto t_j} \mathrm{P}(y) \;\; 0_{n\times n(q-i)}\right], \quad (11)$$

where

$$\mathrm{P}(y) = [y^0 I \; y^1 I \; \dots \; y^{k-1} I],$$

and the matrix $M_{i\mapsto t_j}$ depends on the piecewise index i, such that $y_i \le y < y_{i+1}$. This matrix is performing the necessary switching and is constant for $t_j \le t < t_{j+1}$ (when the index i is not changing). The values in this matrix $M_{i\mapsto t_j}$ are easy to be determined using the definition of B-splines and (4).

This computation can be performed only once off-line because $M_{i\mapsto t_j}$ depends only on the index i and can be calculated using i and the spline node sequence Υ. We can now formulate the following Lemma, which assumes zero approximation errors for simplicity. These errors, however, will be included later in the adaptive observer stability analysis.

Lemma 1: Assuming that there are no approximation errors, if the vectors a and b are known, the plant (8) can be represented as

$$x(t) = x_v(t) + x_b(t) + x_a(t), \quad (12)$$
$$\dot{x}_v(t) = (A-KC)x_v(t) + v(y) + Ky(t), \quad (13)$$
$$x_b(t) = \Omega_b(t)b, \quad (14)$$
$$\dot{\Omega}_b(t) = (A-KC)\Omega_b(t) + \begin{bmatrix} 0_{(\rho-1)\times(m+1)} \\ I_{m+1} \end{bmatrix} \sigma(y)u, \quad (15)$$
$$x_a(t) = \Omega_a(t)a, \quad (16)$$
$$\Omega_a(t) = \Omega_{a,0}(t) + \Omega_{a,t_j}(t), \quad (17)$$
$$\Omega_{a,0}(t) = \Phi(t)\Omega_{a,0}(t_j), \quad (18)$$
$$\dot{\Phi}(t) = (A-KC)\Phi(t), \quad (19)$$
$$\Phi(t_j) = I_{n\times n}, \quad (20)$$
$$\Omega_{a,0}(t_j) = \Omega_a(t_j^-), \quad (21)$$
$$\Omega_{a,t_j}(t) = \begin{bmatrix} 0^T_{n\times n(i-k)} \\ (M_{i\mapsto t_j}\Xi(t))^T \\ 0^T_{n\times n(q-i)} \end{bmatrix}^T, \quad (22)$$
$$\dot{\Xi}(t) = (A-KC)\Xi(t) + \mathrm{P}(y), \quad (23)$$
$$\Xi(t_j) = 0_{n\times nk}, \quad (24)$$

where K is a feedback gain matrix such that $(A-KC)$ is stable, and t^- denotes the limit $t^- = \lim_{\varepsilon\to 0}(t-\varepsilon)$.

Equation (13) represents the part of the system, which is independent of uncertainties. Equations (23) and (19) represent the part of the system, which is approximated by B-splines, because (17) holds only when splines are used. The specific form of (22) is due to the B-spline approximation. The large zero-filled blocks are due to the local support property of the B-splines, as noted in Remark 1. Equation (15) represents the part of the system, which exactly depends on some linear parameters without switching.

Remark 4: The number of required integrators in of the representation (12)-(24) is fixed. Only (13), (15), (19), and (23) require integrators for implementation. In the proposed approach, the constant number of integrators, regardless of approximation accuracy, is obtained because of the

- switching performed in (22) and (21),
- resets of the initial conditions of (19) and (23), performed in (20) and (24), respectively.

This property is specific to B-spline approximation because of its local support property. In (11), there are large zero blocks. The size of the nonzero block $M_{i\mapsto t_j}\mathrm{P}(y)$ is constant and does not depend on the number of B-splines and parameters to be estimated. This is not the case when using other universal approximators without local support property. When universal approximators without local support property are used, the large zero blocks in (11) cannot be guaranteed to exist and, as a result, the number of integrators increases with the number of basis functions. This reduced computational complexity of the adaptive observer for system (8), when using B-spline approximation, appears to be one of the main contributions of this paper.

Remark 5: Equation (19) represents a calculation of a transition matrix. It is possible to further reduce the number of required integrators of the representation (12)-(24) by analytical calculation of the symbolic expression for $\Phi(t)$, given some initial conditions. This symbolic calculation is always possible if the eigenvalues of $(A-KC)$ in (14) are known. This is very often the case when the matrix K is calculated using pole-placement design.

Proof [of Lemma1]**:** The proof will be given in the form of constructive derivation of (12)-(24). The original system (8) can be rewritten as

$$\dot{x}(t) = (A-KC)x(t) + v(t) + Ky(t) + \Psi(y,u)\theta, \quad (25)$$

where K is some feedback matrix such that $(A-KC)$ is stable. There are two types of "external" signals in (25):

Parameter independent signal: $v(y) + Ky(t)$, which corresponds to

$$\dot{x}_v(t) = (A-KC)x_v(t) + v(y) + Ky(t).$$

This reflects directly (13).

Parameter dependent signal: $\Psi(y,u)\theta$ which corresponds to

$$\dot{x}_\theta(t) = (A-KC)x_\theta(t) + \Psi(y,u)\theta. \quad (26)$$

Assume that there exists a matrix $\Omega(t)$, such that

$$x_\theta(t) = \Omega(t)\theta. \quad (27)$$

If $\Omega(t)$ is generated using

$$\dot{\Omega}(t) = (A-KC)\Omega(t) + \Psi(y,u), \quad (28)$$

then we have the equivalence of (26) and (27). In this way, $x_\theta(t)$ can be represented as a linear combination of the signals in the filter (28). This approach is known as K-filter representation [40]. The next step is to use the special properties of $\Psi(y,u)$, which are present in the case of spline-based approximation

$$\Omega(t) = [\Omega_b(t) \mid \Omega_a(t)].$$

Observe that $\Omega(t)$ is equivalently generated by the filter (28), where the input of this filter is given by

$$\Psi(y,u) = [\Psi_b(y,u) \;\; \Psi_a(y)],$$
$$\Psi_b(y,u) = \begin{bmatrix} 0_{(\rho-1)\times(m+1)} \\ I_{m+1} \end{bmatrix} \sigma(y)u,$$

where $\Psi_a(y)$ is defined in (7).

The first part, $\Omega_b(t)$, is represented in (15).

The second part, $\Omega_a(t)$, can be expressed as

$$\dot{\Omega}_a(t) = (A-KC)\Omega_a(t) + \Psi_a(y). \quad (29)$$

This is a valid traditional approach, which was proved very effective when the plant is modelled based on the physical knowledge with a relatively small number of linear parameters. However, this approach has several important drawbacks with universal approximators. A very important property of the universal approximators is that the minimum approximation error can be made arbitrary small by increasing the complexity of the approximator. With the complexity of the universal approximator increasing, the number of elements in the corresponding parameter vector and the related columns in $\Omega_a(t)$ also incearses very quickly, which increases the dynamic order of the filters and makes the application of such methods problematic in the real world. We can see that $\Psi_a(y)$, which is part of $\Psi(y,u)$ in (9), has a special spline structure (7). As mentioned before, a large part of $\Psi_a(y)$ is guaranteed to contain zero elements. An equivalent expression for $\tilde{F}(y)$ containing only the non-zero part $\Psi_a(y)$ can be used as (11). It is natural to use this large sparsity in $\Psi_a(y)$. When applied to (29), this can be rewritten as

$$\begin{aligned}
&\Omega_a(t) = \Omega_{a,0}(t) + \Omega_{a,t_j}(t), \\
&\Omega_{a,0}(t) = \Phi_{A-KC}(t)\Omega_a(t_j), \\
&\Omega_{a,t_j}(t) = M_{i\mapsto t_j}\int_{t_j}^{t} \Phi(-\tau)P(y)d\tau, \\
&\dot{\Phi}_{(A-KC)}(t) = (A-KC)\Phi_{(A-KC)}(t),
\end{aligned} \quad (30)$$

$$\Phi_{A-KC}(t_j) = I_{n \times n},$$

where $\Phi_{(A-KC)}(t)$ is the transition matrix of $(A-KC)$, which is reset at t_j and $\Omega_a(t_j)$ is the state of the filter at time t_j. These equations correspond to (17)-(21). It is possible to implement $\Omega_{a,t_j}(t)$ as

$$\Omega_{a,t_j}(t) = [0_{n \times n(i-k)} \;\; M_{i \mapsto t_j} \Xi(t) \;\; 0_{n \times n(q-i)}],$$
$$\dot{\Xi}(t) = (A-KC)\Xi(t) + \mathrm{P}(y),$$
$$\Xi(t_j) = 0_{n \times nk},$$

which corresponds to (22)-(24). □

6. ADAPTIVE OBSERVER DESIGN AND STABILITY PROPERTIES

First, we will analyze the stability properties with the assumption that the plant is persistently excited. Then we will relax this assumption and will modify the adaptation to accommodate the possible lack of persistent excitation.

Using Lemma 1 and assuming that the values of the parameters a and b are known, it is possible to construct a non-adaptive observer for the system (8) in the form

$$\hat{x}(t) = \hat{x}_v(t) + \hat{x}_b(t) + \hat{x}_a(t),$$
$$\dot{\hat{x}}_v(t) = (A-KC)\hat{x}_v(t) + v(t) + Ky(t),$$
$$\hat{x}_b(t) = \Omega_b(t)b,$$
$$\hat{x}_a(t) = \Omega_a(t)a.$$

If a and b are not known, it is often necessary to develop an adaptive observer to restore the information of plant states.

The idea of B-spline auto generation is introduced. Consider the following picture.

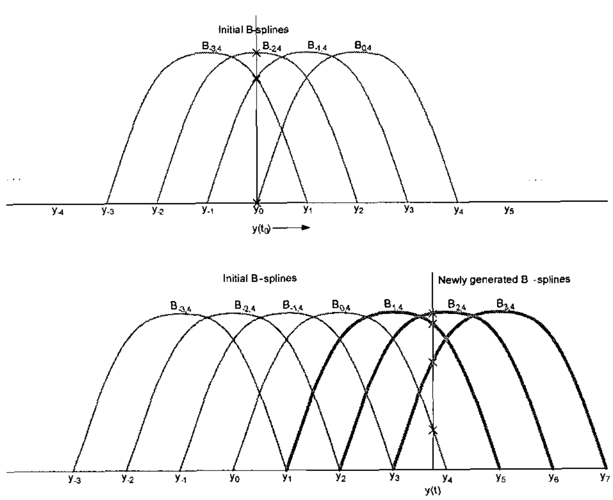

Fig. 2. B-spline auto generation with $y(t)$.

Assume that B-spline order is determined to be B-spline order k. (In Fig. 2, k is assumed to be 4.) At t_0, only k number of B-splines around $y(t_0)$ are generated because they have nonzero corresponding weights (see the upper part of Fig. 2). The adaptation for only these k number of B-splines is necessary. The other B-splines with zero corresponding weights need not to be built at this stage. As the output $y(t)$ grows up (see the lower part of Fig. 2), the additional B-splines whose weights become nonzero are generated. When a new spline is generated, the dimensions of parameters, observer filter, and some other related variables also increase. This auto generation of B-splines according to $y(t)$ is guaranteed to be stopped in a finite time due to the compactness assumption in universal approximation theorem. It is natural to adopt this idea to reduce calculation effort and, furthermore, to avoid the requirement for the knowledge of measurement bounds.

Assumption 5: Assume that $\Psi(y,u)$ is persistently exciting so that

$$\int_t^{t+T} \Omega^T(\tau) C^T C \Omega(\tau) d\tau \geq \delta I, \tag{31}$$

where δ and T are some positive constants.

In the following, we define some sets to decide when to generate a new B-spline, and propose an adaptive observer for the system (8) with B-spline auto-generation property.

Knot sequence:

Let the knot sequence be given as $\Upsilon' := \{y_i, \vartheta \leq i \leq \upsilon,\ y_i \leq y_{i+1},\ y_\vartheta \leq y(t) < y_\upsilon, t \geq t_0\}$, where $\{\vartheta, \upsilon\}$ are unknown (possibly negative) integers with the meaning of lower/upper bounds of the knot index i.

Update algorithm for $N(t)$:

If $i_c \notin M(t_{i_c})$, then $\begin{cases} M(t^+_{i_c}) = M(t_{i_c}) \cup \{i_c\}, \\ N(t^+_{i_c}) = N(t_{i_c}) + 1, \end{cases}$

where $t_{i_c} \geq t_0$ is every time instance when $y(t_{i_c}) = y_{i_c}$ (i.e., the time when $y(t)$ meets any knot), $\vartheta \leq i_c \leq \upsilon$ is the index from the definition of Υ' when $y(t_{i_c}) = y_{i_c}$, $\mathrm{M}(t)$ is the set containing the knot indices of Υ' that $y(t)$ has occupied up to time t with the initial $\mathrm{M}(t_0) = \{0\}$, $N(t)$ is the number of B-splines built up to time t with the initial $N(t_0) = k$.

Remark 6: The update algorithm for $N(t)$ determines when to generate a new B-spline. When $y(t)$ meets any knot $(y(t_{i_c}) = y_{i_c})$, it is checked if the knot has been occupied by $y(t)$ in the past, or not. If it is a new knot, then the algorithm saves the current knot index i_c and generates one new B-spline. In result, related dimensions of the adaptive observer (32) is also increased.

Adaptive observer:

$$\begin{aligned}
&\hat{x}(t) = \hat{x}_v(t) + \Omega(t)\hat{\theta}(t),\\
&\dot{\hat{x}}_v(t) = (A-KC)\hat{x}_v(t) + v(y) + Ky(t),\\
&\dot{\Omega}(t) = (A-KC)\Omega(t) + \Psi(y,u),\\
&\dot{\hat{\theta}}(t) = P(t)\varepsilon(t)\Omega^T(t)C^T, \qquad (32)\\
&\dot{P}(t) = \begin{cases} \bar{P}(t), & if\ \|P(t)\| \le R_0 \\ 0, & otherwise, \end{cases}\\
&\dot{\bar{P}}(t) = \beta P(t) - \frac{P(t)\Omega^T(t)C^TC\Omega(t)P(t)}{1+\alpha C\Omega(t)\Omega^T(t)C^T},
\end{aligned}$$

where α, β are some positive scalar values, $P(0) = P_0$, $\|P(t)\| \le R_0$ for some symmetric positive definite matrix P_0 and some positive scalar R_0, and the time-varying dimensions of $\hat{\theta}(t)$, $\Omega(t)$, $\Psi(y,u)$, and $P(t)$ are defined as follows depending on the scalar variable $N(t)$.

$$\begin{aligned}
&\hat{\theta}(t) \in R^{N(t)n+m+1},\\
&\Omega(t) \in R^{n\times(N(t)n+m+1)},\\
&\Psi_a(y,u) \in R^{n\times(N(t)n)},\\
&\Psi(y,u) = [\Psi_b(y,u)\ \Psi_a(y)] \in R^{n\times(N(t)n+m+1)},\\
&P(t) \in R^{(N(t)n+m+1)\times(N(t)n+m+1)}.
\end{aligned}$$

Theorem 2: For the knot sequence Υ', under the Assumption 5 and the update algorithm for $N(t)$, the adaptive observer (32) for the plant (5) represented in the form (8), has arbitrary small parameter and state estimation errors.

Remark 7: From the main properties of modified least-squares update law with forgetting factor (32), $P(t)$ in (32) is guaranteed to be bounded and positive definite for $\forall t \ge 0$, as shown in [6, p. 199].

The proof of Theorem 2 requires the following two Lemmas, which can easily be obtained from the work [41] with slight modification.

Lemma 2: Let $\chi(t) := C\Omega(t)$ and $P(t)$ be from (32). If there exist positive constants T, c_1, c_2 such that $\forall t$

$$c_1 I \le \int_t^{t+T} \chi(\tau)^T \chi(\tau) d\tau \le c_2 I, \qquad (33)$$

then the system

$$\dot{z}(t) = -P(t)\chi^T(t)\chi(t)z(t) \qquad (34)$$

is globally exponentially stable.

Lemma 3: If the autonomous linear time varying system

$$\zeta(t) = F(t)\zeta(t) \qquad (35)$$

is globally exponentially stable and $u(t)$ is bounded by some $\Pi > 0$, then $z(t)$, driven by $u(t)$ of the following system

$$\dot{z}(t) = F(t)z(t) + u(t) \qquad (36)$$

is also bounded. Moreover, if Π can be designed arbitrary small, then $z(t)$ can be driven to arbitrary small region around the origin.

By using Lemmas 2 and 3, Theorem 2 is proved as follows.

Proof [of Theorem 2]: For stability analysis, the dimensions of the matrices $\hat{\theta}(t)$, $\Omega(t)$, $\Psi(y,u)$, and $P(t)$ are increased up to their maximum values. This augmentation does not affect their convergence properties since it uses only additional zeros. Let

$$\bar{\hat{\theta}}(t) := \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ \hat{\theta}(t) \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \in R^{(\upsilon-\vartheta+1)n+m+1},$$

$$\begin{aligned}
&\bar{\Omega}(t) := [0 \ldots 0\ \Omega(t) 0 \ldots 0] \in R^{n\times((\upsilon-\vartheta+1)n+m+1)},\\
&\bar{\Psi}(y,u) := [\Psi_b(y,u)\ 0 \ldots 0\ \Psi_a(y) 0 \ldots 0] \in R^{n\times((\upsilon-\vartheta+1)n+m+1)},\\
&\bar{P}(t) \in R^{((\upsilon-\vartheta+1)n+m+1)\times((\upsilon-\vartheta+1)n+m+1)},\\
&\tilde{\theta}(t) := \bar{\hat{\theta}}(t) - \theta,\\
&\tilde{x}(t) := \hat{x}(t) - x,\\
&\tilde{x}_v(t) := \hat{x}_v(t) - x_v,
\end{aligned}$$

and note that $\dot{\theta} = 0$. Then using the matrix form (8), we obtain

$$\begin{aligned}
\dot{\tilde{x}}(t) = &(A-KC)\tilde{x}(t) + \bar{\Psi}(y,u)\tilde{\theta}(t)\\
&+ \bar{\Omega}(t)\dot{\tilde{\theta}}(t) + \tilde{f}(y,u,\theta).
\end{aligned}$$

Consider the following linear combination.

$$\tilde{x}_v(t) = \tilde{x}(t) - \bar{\Omega}(t)\tilde{\theta}(t).$$

Differenciating this combination leads to

$$\dot{\tilde{x}}_v(t) = (A-KC)(\tilde{x}_v(t)+\bar{\Omega}(t)\tilde{\theta}(t)) + \bar{\Psi}(y,u)\tilde{\theta}(t) - \dot{\bar{\Omega}}(t)\tilde{\theta}(t) + \tilde{f}(y,u,\theta)$$
$$= (A-KC)\tilde{x}_v(t) + \tilde{f}(y,u,\theta) + [(A-KC)\bar{\Omega}(t) + \bar{\Psi}(y,u) - \dot{\bar{\Omega}}(t)]\tilde{\theta}(t).$$

Because $\dot{\bar{\Omega}}(t) = (A-KC)\bar{\Omega}(t) + \bar{\Psi}(y,u)$ holds, we simply have

$$\dot{\tilde{x}}_v(t) = (A-KC)\tilde{x}_v(t) + \tilde{f}(y,u,\theta), \quad (37)$$

where $\tilde{f}(y,u,\theta)$ is bounded and can be designed arbitrary small according to properties of Universal Approximation Theorem 1. Therefore, by using Lemma 3, $\tilde{x}_v(t)$ is guaranteed to be bounded and arbitrary small. Now we study the behavior of $\tilde{\theta}(t)$. As $\dot{\theta} = 0$, we have

$$\dot{\tilde{\theta}}(t) = \bar{P}(t)\bar{\Omega}^T(t)C^T(y - C\hat{x}(t))$$
$$= -\bar{P}(t)\bar{\Omega}^T(t)C^T C(\tilde{x}_v(t) + \bar{\Omega}(t)\tilde{\theta}(t))$$
$$= -\bar{P}(t)\bar{\Omega}^T(t)C^T C\bar{\Omega}(t)\tilde{\theta}(t) - \bar{P}(t)\bar{\Omega}^T(t)C^T C\tilde{x}_v(t).$$

According to Lemma 2 and Assumption 5, setting $\chi(t) = C\bar{\Omega}(t)$ makes the homogeneous part of this system globally exponentially stable. Now from Lemma 3 and the fact that $\bar{\Omega}(t), C, \bar{P}(t), \tilde{x}_v(t)$ are bounded and especially that $\tilde{x}_v(t)$ can be designed to be arbitrary small, we conclude that $\tilde{\theta}(t)$ is bounded and can be designed arbitrary small. As a result, $\tilde{x}(t) = \tilde{x}_v(t) + \bar{\Omega}(t)\tilde{\theta}(t)$ is also bounded and can be designed arbitrary small. □

Remark 8: The initial dimensions of $\hat{\theta}(t)$, $\Omega(t)$, $\Psi(y,u)$, and $P(t)$ are $kn+m+1, n\times(kn+m+1)$, $n\times(kn+m+1)$, and $(kn+m+1)\times(kn+m+1)$, respectively, since only k number of B-splines are built at $t=t_0$. Their dimensions increase as $y(t)$ changes or, more precisely, the knot sequence index i changes. Note that their initial values depend on only n,m, and pre-defined B-spline order k. This increment is guaranteed to be stopped in a finite time because of the compactness assumptions in the universal approximation theorem.

The B-spline based approximation proposed in this paper has three important advantages compared to other universal approximators without local support properties. These advantages are described in Remarks 9, 10, and 11.

Remark 9: For any universal approximators, the error can be made arbitrary small by increasing the number of basis functions. However, the increased number of basis functions results in high-dimension of observer filters which requires more integrators for implementation. This increasing complexity is avoided in the proposed design. As mentioned in Remark 4, the number of integrators in the filters (12)-(24) is constant and does not depend on the number of B-splines used for approximation. As a result of this important property, increasing the number of B-splines can be used to decrease the approximation error without increasing the computational complexity related to the observer filters. The parameterization proposed in this paper breaks the link between the number of integrators in the filter and the number of basis functions, and much better approximation can be obtained with near-constant complexity related to the integrator numbers in the filters (12)-(24).

Remark 10: The second advantage is that the observer (32) reduces calculation effort by avoiding unnecessary adaptations for B-splines with zero weights. This is possible since it starts with only k number of B-splines around the initial output $y(t_0)$, and then increase the dimension of itself as $y(t)$ varies. In this auto-generations, only the minimal number of B-splines with nonzero weights are generated and the calculation for the B-splines with zero weight is not performed.

Remark 11: In many function approximation problems in adaptive observer design, it is commonly assumed that the bounds of input/output measurement is known, since the basis functions of universal approximators are usually to be built between the upper and lower bounds of measurement values. Therefore, often, at least one process before control and/or identification, is required to obtain upper and lower bounds of $y(t)$. Practically this process causes additional consumption of resources depending on the system size or complexity. When this pre-process takes too much resource, heuristics to predict the output bounds are necessary. This problem can be avoided by using the proposed design. The observer (32) increases its dimension as the value of $y(t)$ changes and this increment is to be stopped when $y(t)$ finally reaches its unknown bounds. In other words, it automatically detects the lower and upper bounds of $y(t)$ by on-line generation of additional B-splines. In result, the proposed observer (32) does not require the pre-knowledge of upper and lower bounds of $y(t)$.

Remark 12: From the proof of Theorem 2, it is clear to improve the performance of the observer, taking into account the approximation errors, which are inherently present in the model (8). According to the Universal Approximation Theorem 1, the approximation error can be made arbitrary small by using finite number of B-splines, so it is possible to reduce the approximation error by increasing the number of B-splines used.

- Another way to decrease the approximation error is to increase the order of B-splines, which makes the order of the local polynomial approximation higher, thus increasing the approximation precision.
- The B-spline approximation error can be reduced by a proper selection of the knot sequence Υ' [36].
- Looking at (36), one can see that the use of a strong feedback gain matrix also decreases the influence of the approximation errors on $\tilde{x}_v(t)$, which improves the convergence of the observer states.

Remark 13: In the case of other universal approximators without local support property, like radial-basis function, the properties in Remarks 8-10 cannot be obtained since any of the weights cannot be guaranteed to be zero and B-spline auto-generation scheme can not be applied.

Remark 14: Assumption 5 is restrictive, but necessary to obtain bounded parameter estimates in the presence of approximation errors. Several approaches are known to modify the observer and obtain bounded parameter estimates even without persistently exciting plant input/output. These methods are discussed in depth in [5,6]. The ε-modification can be used, but other choices are also possible.

Remark 15: The methods to improve the observer convergence properties mentioned in Remark 12 can be applied in this case too.

7. SIMULATION

A numerical simulation was performed to verify the proposed design. The system we consider is a single-link robot arm coupled to a DC motor with a flexible joint.

$$\begin{aligned}
\frac{d\phi_1(t)}{dt} &= \omega_1(t),\\
\frac{d\omega_1(t)}{dt} &= \frac{-mgd}{J_1}\sin(\phi_1(t)) - \frac{F_1}{J}\omega_1(t) - \frac{K}{J_1}(\phi_1(t) - \frac{\phi_2(t)}{N}),\\
\frac{d\phi_2(t)}{dt} &= \omega_2(t),\\
\frac{d\omega_2(t)}{dt} &= \frac{-F_2}{J_2}\omega_2(t) - \frac{K_t}{J_2}i(t) - \frac{K}{J_2 N}(\phi_1(t) - \frac{\phi_2(t)}{N}),\\
\frac{di(t)}{dt} &= -\frac{R}{L}i(t) - \frac{K_b}{L}\omega_2(t) + \frac{1}{L}u(t),
\end{aligned} \tag{38}$$

where ϕ_1, ω_1, and ϕ_2, ω_2 are the angular positions and velocities of the arm and the motor shaft, i and u are the motor armature current and voltage, J_1, J_2, F_1, F_2 are the inertia and viscous friction coefficients, K is a spring constant, K_t and K_b are the torque and e.m.f. constants related with the DC motor, R is the armature resistance and L is the armature inductance, m is the arm mass, d is the position of the arm's center of gravity, N is the gear ratio and g is acceleration of gravity. System output is defined as $y(t) = \phi_1(t)$ and the reference signal is random. The system physical parameters for the simulation are selected as

$$\begin{aligned}
&F_1 = 0.5kg/s,\ J_1 = 3kg.m^2,\ F_2 = 0.5kg/s,\\
&J_2 = 1kg.m^2,\ m = 2kg,\ g = 9.8m/s^2,\\
&d = 0.5m,\ K = 100,\ K_t = 10,\ R = 1\Omega,\\
&L = 0.1H,\ N = 1.
\end{aligned}$$

The considered system is relaxed at $t_0 = 0$. This system is feedback linearizable and transformable into output feedback adaptive form (5). The interested readers are referred to [38, p. 315] for details. We assume only that it is possible to transform the system into form (5), which was shown to be possible. The objective of this simulation is the design of B-spline based adaptive observer that tracks system states by using only the information of input/output signal, where $\overline{F}(y)$ (sinusoidal function of y) in (6), and $b_0(=1/L)$ in Assumption 3 are assumed to be unknown.

We then proceed to apply the B-spline design as presented. The step of the knot sequence for the observer is selected as $\Upsilon = [k\frac{\pi}{4}]$, where k are integers starting from 0. On this knot sequence, B-splines order was set to be 5. Note that the knowledge of the bounds of $y(t)$ is not necessary in this simulation. Uniformly random signal is used as reference and the output feedback gains for the K-filter both for the physical observer and for the B-spline observer are selected so that the poles are at [-5,-5,-5,-5,-5]. The simulation was performed for 200 seconds for the plant (38) and uniformly random signal with maximum value of 2π was applied as the reference. The input-output data shown in Fig. 3 was selected to be persistently exciting, so (32) are used in this simulation. Similar input/output shapes repeated after 30 seconds.

Actual and observed states produced by the adaptive B-spline based observer are shown on Fig. 4. At about 90 seconds, the estimated states are starting to converge close to the real ones quite fast. Especially the output of the plant, which is the first state, converges almost immediately. After 100 seconds, all the states from the adaptive observer are very close to the real states of the plant in output feedback form and its convergence improves very little.

Estimated parameters representing the B-spline

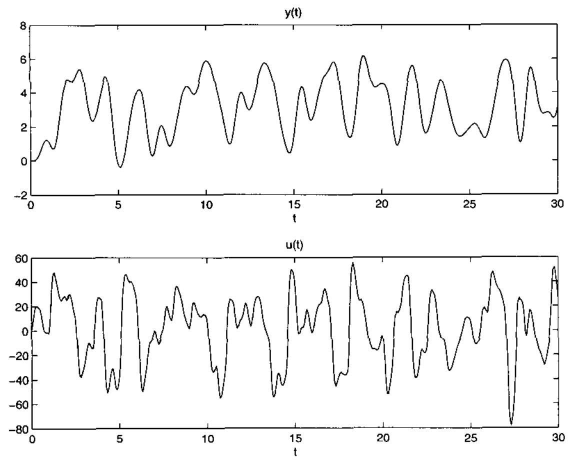

Fig. 3. Plant output $y(t)$ and output $u(t)$ used in the simulation $(0 \le t \le 30)$.

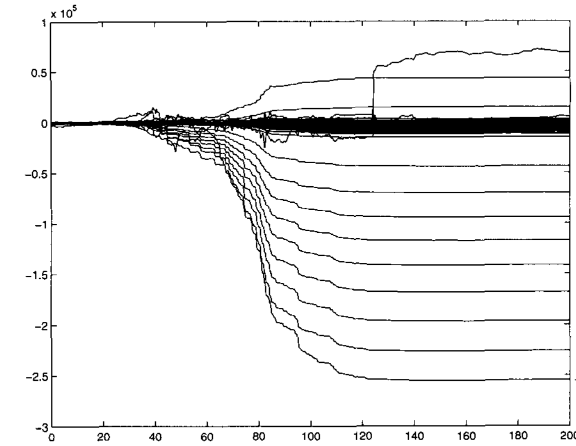

Fig. 5. B-spline weights $(0 \le t \le 200)$.

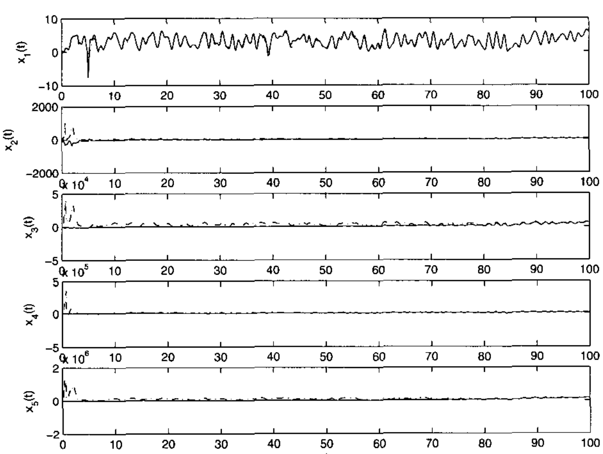

Fig. 4. States $x_1 = y = \phi_1$, $x_2 = \omega_1$, $x_3 = \phi_2$, $x_4 = \omega_2$, $x_5 = i$, from the B-spline observer in solid lines and actual states in dashed/dotted lines $(0 \le t \le 100)$.

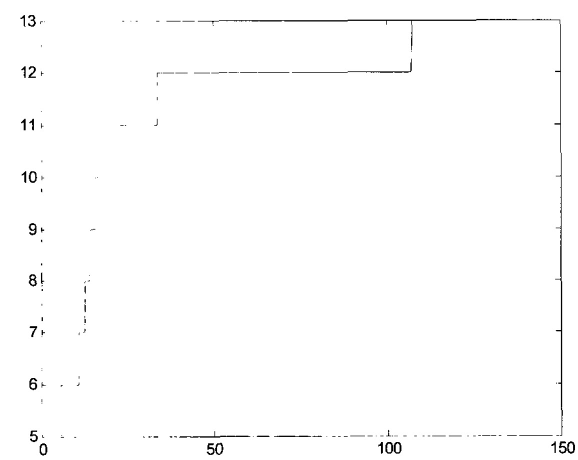

Fig. 6. Number of automatically generated B-splines $(0 \le t \le 150)$.

weights of the adaptive B-spline based observer are shown on Fig. 5. The B-spline weights dramatically change at about 90 seconds in this figure. So it is reasonable that the estimated states in Fig. 4 also converge to real ones quite fast at this time point.

The number of automatically generated B-splines is described in Fig. 6. At $y(t_0 = 0)$, five number of B-splines $[B_{-4,5}(y), B_{-3,5}(y), B_{-2,5}(y), B_{-1,5}(y), B_{0,5}(y)]$ around zero are initially generated since the B-spline degree was defined as 5. As $y(t)$ varies, the number of B-splines with nonzero weights increases up to 13. No more increment occurred after 150 seconds. Therefore, the lastly generated B-spline is $B_{8,5}(y)$ and, in result, the total set of the B-splines generated in the simulation is

$$[B_{-4,5}(y), \ldots, B_{0,5}(y), \ldots, B_{8,5}(y)].$$

From the knot sequence definition $\Upsilon = [k\frac{\pi}{4}]$, one can see that the B-splines $B_{i,5}(y)$, $i \ge 9$ were not generated in this simulation because the maximum values of $y(t)$ is about 6.35, which is less than $\frac{9\pi}{4}$. Therefore, it is verified that the proposed adaptive observer automatically detects the bounds of $y(t)$ and generates minimal number of necessary B-splines on-line. In result, the proposed method does not require the pre-knowledge of the output bounds.

8. CONCLUSION

Using B-splines as universal approximators, we have obtained a plant parameterization, which permits the construction of an adaptive observer. The first particular property of this parameterization is that the number of integrators in the observer filters in this design does not depend on the number of parameters in the plant parameterization. This appears to be a beneficial property especially because the number of such parameters tends to be very high for the approximator based designs. The second particular property is that it automatically generates minimal number of necessary B-splines as the output value varies. By this property, the proposed observer also reduces calculation effort by neglecting unnecessary spline adaptation with zero weights, and, furthermore, does not require the pre-knowledge of output bounds, which often consumes additional efforts in system identification problems. Possible direction for future work includes the exploration of approximation errors influence on the observer and auto-positioning of B-spline centers.

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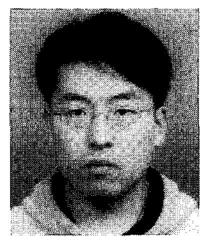

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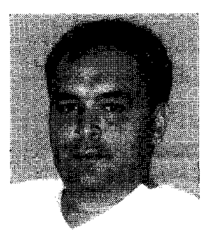

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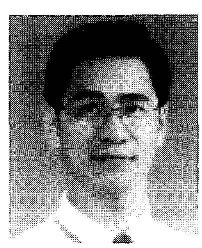

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