정다각형 배열의 광 마우스를 이용한 이동 로봇의 최소 자승 속도 추정

Least Squares Velocity Estimation of a Mobile Robot Using a Regular Polygonal Array of Optical Mice

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Abstract : This paper presents the velocity estimation of a mobile robot using a regular polygonal array of optical mice that are installed at the bottom of a mobile robot. First, the basic principle of the proposed velocity estimation method is explained. Second, the velocity kinematics from a mobile robot to an array of optical mice is derived as an overdetermined linear system. Third, for a given set of optical mouse readings, the mobile robot velocity is estimated based on the least squares solution to the obtained system. Finally, simulation results are given to demonstrate the validity of the proposed velocity estimation method.

Keywords : mobile robot, velocity estimation, optical mouse, least squares estimation, robustness

I. Introduction

There have been several attempts to employ the PC optical mice for the localization of a mobile robot [1-7]. In fact, the optical mouse is an inexpensive but high performance device equipped with sophisticated image processing engine [8,9]. The velocity estimation using a set of optical mice can be considered as an economic solution, which can overcome the limitations of typical sensors. For instance, encoders are vulnerable to wheel slip, ultrasonic sensors require the line of sight, and cameras usually mandate heavy computation.

For the velocity estimation of a mobile robot traveling on the plane, three variables including two linear components and one angular component need to be determined. Since an individual optical mouse provides two linear movement information, the required number of optical mice should be more than or equal to two. Most of previous research use two optical mice [2-6], while only a single optical mouse is used in[1]. However, few attempt has been made to use more than two optical mice except[7].

In this paper, we present the velocity estimation of a mobile robot using the redundant number of optical mice arranged in a regular polygonal array. This paper is organized as follows. Section II explains the basic principle of the proposed method. Section III derives the velocity kinematics from a mobile robot to an array of optical mice. Given a set of optical mouse readings, Section IV obtains the least squares estimates of the mobile robot velocity. Section V gives simulation results. Finally, the conclusion is made in Section VI.

II. Basic Principle

To explain the basic principle of the proposed velocity estimation method, let us consider a regular triangular array of optical mice attached at the bottom of a mobile robot, as shown in Fig. 1. Generally, the traveling pattern of a mobile robot can be specified in terms of the location of ICR(Instantaneous Center of Rotation) on the plane. For three different traveling patterns of a mobile robot, Fig. 1 shows three linear velocities observed by a regular triangular array of optical mice.

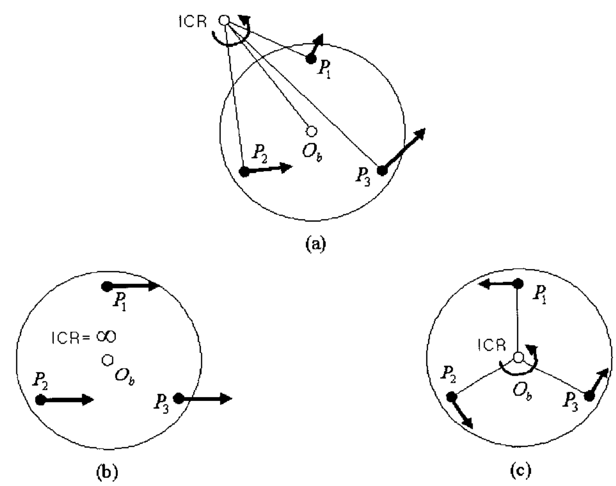

그림 1. 제안된 속도 추정 기법의 기본 원리.

Fig. 1. The basic principle of the proposed velocity estimation method.

When a mobile robot is rotating with ICR apart from the center of a mobile robot as shown in Fig. 1(a), three velocity vectors are different in both direction and magnitude. When a mobile robot is moving straight as shown in Fig. 1(b), corresponding to the case of ICR at infinity, three velocity vectors become the same in both direction and magnitude. When a mobile robot is rotating with ICR coincident with the center of a mobile robot as shown in Fig. 1(c), three velocity vectors become different in direction but the same in magnitude. Theses observations tells that a different traveling pattern of a mobile robot results in a set of different velocity readings of an array of optical mice. Reversely, it is possible to estimate the linear and angular velocities of a traveling mobile robot from the velocity readings of optical mice.

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III. Velocity Kinematics

Assume that N optical mice are installed at the vertices, $P_i, i=1,\cdots,N$, of a regular polygon centered at the center, O_b, of a mobile robot traveling on the xy plane. Fig. 2. shows an example of a regular triangular array of optical mice with $N=3$. Let $\mathbf{u}_x=[1\ 0]'$ and $\mathbf{u}_y=[1\ 0]'$ be the unit vectors along the x axis and the y axis, respectively. The position vector, $\mathbf{p}_i=[p_{ix}\ p_{iy}]'$, $i=1,\cdots,N$, from O_b, to P_i, can be expressed as

$$\mathbf{p}_i=\begin{bmatrix} p_{ix} \\ p_{iy} \end{bmatrix}=\begin{bmatrix} r\cos\{\theta+(i-1)\times\frac{2\pi}{N} \\ r\sin\{\theta+(i-1)\times\frac{2\pi}{N} \end{bmatrix} \quad (1)$$

where θ represents the heading angle of a mobile robot with the forwarding direction of $\mathbf{p}_1$, and r represents the distal distance to each optical mouse. Due to the regular polygonal arrangement of optical mice, it holds that

$$\sum_{i=1}^{N} p_{ix}=\sum_{i=1}^{N} p_{iy}=0 \quad (2)$$

regardless of the heading angle θ. For notational convenience, let $\mathbf{q}_i$, $i=1,2,3$, be the vector obtained by rotating $\mathbf{p}_i$ by 90° counterclockwise.

Let $\mathbf{v}_b=[\upsilon_{bx}\ \upsilon_{by}]'$ and ω_b be the linear velocity and the angular velocity at the center O_b of a mobile robot, respectively. And, let $\mathbf{v}_i=[\upsilon_{ix}\ \upsilon_{iy}]'$, $i=1,\cdots,N$, be the linear velocity at the vertex P_i, which corresponds to a pair of velocity readings of the i^{th} optical mouse. Then, we have

$$\mathbf{v}_b+\omega_b\mathbf{q}_i=\mathbf{v}_i \quad (3)$$

Premultiplied by $\mathbf{u}_x'$ and $\mathbf{u}_y'$, (3) gives

$$\mathbf{u}_x'\mathbf{v}_b+\omega_b\mathbf{u}_x'\mathbf{q}_i=\mathbf{u}_x'\mathbf{v}_i \quad (4)$$

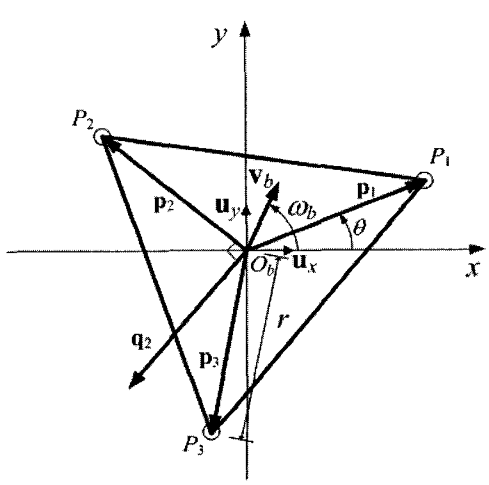

그림 2. 정삼각형으로 배열된 광 마우스의 예(N=3).

Fig. 2. An example of a regular triangular array of optical mice with N=3.

$$\mathbf{u}_y'\mathbf{v}_b+\omega_b\mathbf{u}_y'\mathbf{q}_i=\mathbf{u}_y'\mathbf{v}_i \quad (5)$$

respectively. Referring to Fig. 2, (4) and (5) can be rewritten as

$$\upsilon_{bx}-\omega_b\times p_{iy}=\upsilon_{ix} \quad (6)$$

$$\upsilon_{by}-\omega_b\times p_{ix}=\upsilon_{iy} \quad (7)$$

From (6) and (7), the velocity kinematics from a mobile robot to an array of optical mice can be represented as

$$\mathbf{A}\dot{\mathbf{x}}=\dot{\Theta} \quad (8)$$

where $\dot{\mathbf{x}}=[\mathbf{v}_b\omega_b]'\in\mathbf{R}^{3\times1}$ is the velocity vector of a mobile robot, $\dot{\Theta}=[\upsilon_{1x}\ \upsilon_{1y}\ \upsilon_{2x}\ \upsilon_{2y}\cdots\upsilon_{Nx}\ \upsilon_{Ny}]'\in\mathbf{R}^{2N\times1}$ is the velocity vector consisting of the movement information from n optical mice, and $\mathbf{A}$ is the Jacobian matrix mapping $\dot{\mathbf{x}}$ to $\dot{\Theta}$, given by

$$\mathbf{A}=\begin{bmatrix} 1 & 0 & -p_{1y} \\ 0 & 1 & p_{1y} \\ 1 & 0 & -p_{2y} \\ 0 & 1 & p_{2x} \\ \vdots & \vdots & \vdots \\ 1 & 0 & -p_{Ny} \\ 0 & 1 & p_{Nx} \end{bmatrix}\in\mathbf{R}^{2N\times3} \quad (9)$$

Seen from (9), the expression of the Jacobian matrix $\mathbf{A}$ is quite simple as a function of the position vector $\mathbf{p}_i=[p_{ix}\ p_{iy}]'$, $i=1,\cdots,N$. It should be mentioned that such simplicity of $\mathbf{A}$ is effective not only for a regular polygonal array of optical mice, but also for a general polygonal array.

IV. Velocity Estimation

In the case of $N=1$, (8) represents two equations of three unknowns, including two linear velocity components, υ_{bx} and υ_{by}, and one angular velocity component, ω_b. Thus, (8) becomes an underdetermined system, for which the mobile robot velocity cannot be uniquely determined from the optical mouse readings. However, for $N\geq2$, (8) becomes a overdetermined system consisting of $2N$ equations, for which the least squares solution is obtained by

$$\dot{\mathbf{x}}=\mathbf{B}\dot{\Theta} \quad (10)$$

where

$$\mathbf{B}=(\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}'\in\mathbf{R}^{3\times2N} \quad (11)$$

which is the inverse Jacobian matrix.

For a general polygonal array of optical mice, from (9), it can be shown that

$$\mathbf{A}'\mathbf{A}=\begin{bmatrix} N & 0 & -\sum_{i=1}^{N}p_{iy} \\ 0 & N & \sum_{i=1}^{N}p_{ix} \\ -\sum_{i=1}^{N}p_{iy} & \sum_{i=1}^{N}p_{ix} & \sum_{i=1}^{N}\|\mathbf{p}_i\|^2 \end{bmatrix} \quad (12)$$

It should be noted that the inverse of $\mathbf{A}'\mathbf{A}$ always exists independent of the heading angle θ of a mobile robot, which guarantees the observability of the velocity kinematic model, given by (8). Especially for a regular general polygonal array of optical mice, using (1), (2), (9), and (12), the inverse Jacobian matrix $\mathbf{B}$ can be obtained as

$$\mathbf{B}=\begin{bmatrix} \frac{1}{N} & 0 & \frac{1}{N} & 0 & \cdots & \frac{1}{N} & 0 \\ 0 & \frac{1}{N} & 0 & \frac{1}{N} & \cdots & 0 & \frac{1}{N} \\ -\frac{\sin\theta}{Nr} & \frac{\cos\theta}{Nr} & -\frac{\sin(\theta+\frac{2\pi}{N})}{Nr} & \frac{\cos(\theta+\frac{2\pi}{N})}{Nr} & \cdots & -\frac{\sin\{\theta+\frac{(N-1)2\pi}{N}\}}{Nr} & \frac{\cos\{\theta+\frac{(N-1)2\pi}{N}\}}{Nr} \end{bmatrix}. \quad (13)$$

Finally, for a given set of the velocity readings from a regular polygonal array of N optical mice, $\dot{\Theta}=[\upsilon_{1x}\ \upsilon_{1y}\ \upsilon_{2x}\ \upsilon_{2y}\cdots\upsilon_{Nx}\ \upsilon_{Ny}]'$, the linear and angular velocities of a mobile robot, $\dot{\mathbf{x}}=[\upsilon_{bx}\ \upsilon_{by}\ \omega_b]'$, can be determined, from (10) and (13), as follows:

$$\upsilon_{bx}=\frac{1}{N}\sum_{i=1}^{N}\upsilon_{ix}$$
$$\upsilon_{by}=\frac{1}{N}\sum_{i=1}^{N}\upsilon_{iy} \quad (14)$$
$$\upsilon_{b}=\frac{1}{N}\sum_{i=1}^{N}\omega_{i}$$

where

$$\frac{1}{r}\left[-\sin\left\{\{\theta+\frac{(i-1)2\pi}{N}\}\right\}\times\upsilon_{ix}+\cos\left\{\{\theta+\frac{(i-1)2\pi}{N}\}\times\upsilon_{iy}\right\}\right] \quad (15)$$

which represents the angular velocity experienced by the i^{th} optical mouse.

Seen from (14), each of the least squares velocity estimates of a mobile robot is determined as the simple average of the corresponding velocity components read from N optical mice. It should be mentioned that such computational simplicity is attributed to the arrangement of N optical mice in a regular polygonal array centered at the center of a mobile robot.

V. Simulation

To evaluate the performance of the proposed velocity estimation method, three different regular polygonal arrays of optical mice are compared, including $N=2,3,4,$ as shown in Fig. 3. For all cases, the distal distance of each optical mouse is set to be the same as r = 0.3m. To simulate the measurement noise, a certain level of random noise is added independently to each of three optical mouse readings.

Assume that a mobile robot stands still without moving, $\upsilon_{bx}=\upsilon_{by}=0.0$[m/sec] and $\omega_b=0.0$[rad/sec]. Fig. 4 shows the plots of the velocity readings from the i^{th} optical mouse which are corrupted by random noise corresponding to ±2.5% the nominal velocity. Using $2N$ optical mouse readings, three velocity components of a mobile robot velocities, denoted by $\hat{\upsilon}_{bx}$, $\hat{\upsilon}_{by}$, and $\hat{\omega}_b$, are estimated, based on the least squares estimation, given by (14).

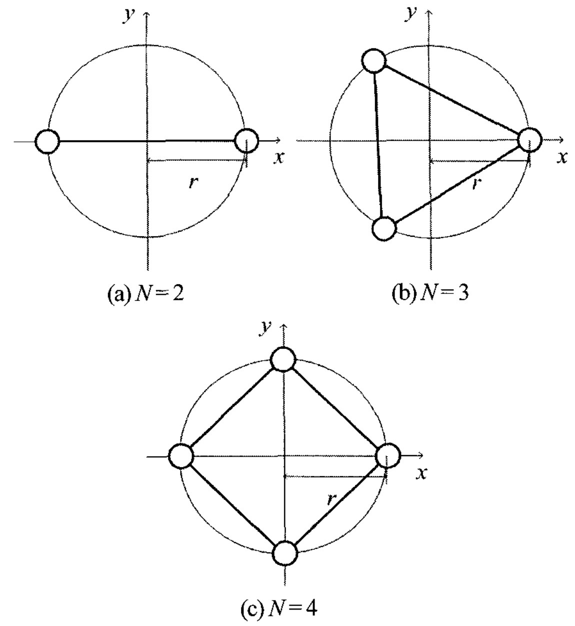

그림 3. 정다각형 배열의 광 마우스.

Fig. 3. Three regular polygonal array of optical mice.

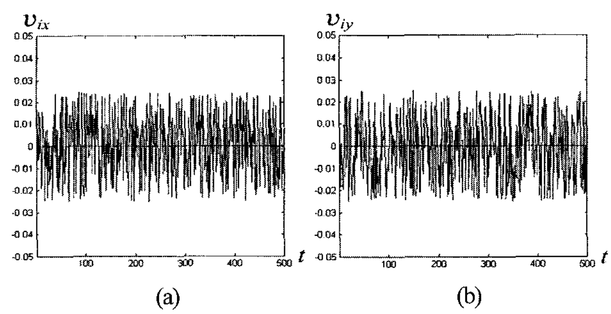

그림 4. 랜덤 노이즈가 섞인 i번째 광 마우스의 속도 출력.

Fig. 4. The velocity readings of the i^{th} optical mouse corrupted by random noise.

표 1. 추정 속도의 표준 편차.

Table 1. The standard deviation of the velocity estimates.

N	$\hat{\upsilon}_{bx}$	$\hat{\upsilon}_{by}$	$r\hat{\omega}_b$
2	0.0100	0.0101	0.0106
3	0.0082	0.0087	0.0082
4	0.0071	0.0074	0.0070

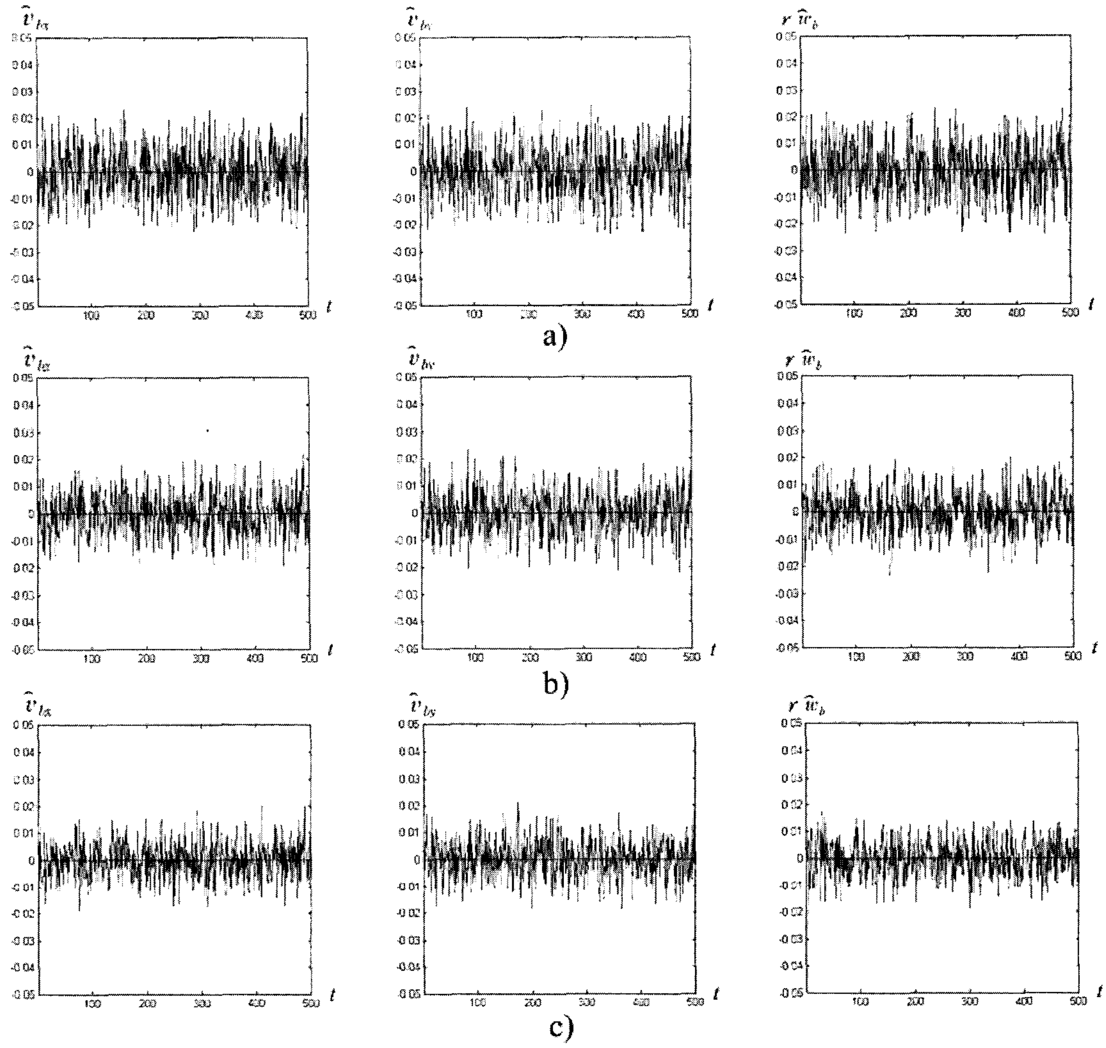

그림 5. 이동 로봇의 추정 속도; a) $N=2$, b) $N=3$, 그리고 c) $N=4$.
Fig. 5. The velocity estimates of a mobile robot; a) $N=2$, b) $N=3$, and c) $N=4$.

For $N=2,3,4$, Fig. 5 shows the plots of three velocity estimates of a standing mobile robot. Note that $r\hat{\omega}_b$, instead of $\hat{\omega}_b$, is plotted for the comparison with $\hat{\upsilon}_{bx}$ and $\hat{\upsilon}_{by}$. From Fig. 5, it can be observed that the velocity estimation error of a mobile robot decreases as the number of optical mice increases. Table 1 lists the standard deviation of three velocity estimates for the cases of $N=2,3,4$. From Table 1, it should be noticed that the increased number of optical mice strictly reduces the standard deviation of three velocity estimates, and the percent improvement is more significant for the smaller number of optical mice.

VI. Conclusion

In this paper, we proposed to estimate the velocity of a traveling mobile robot using a regular polygonal array of optical mice that are installed at the bottom of a mobile robot. First, the basic principle of the proposed velocity estimation method was explained. Second, the velocity kinematics from a mobile robot to an array of optical mice was derived as an overdetermined system. Third, for a given set of optical mouse readings, the least squares velocity estimates of a mobile robot was obtained as the simple average of the corresponding velocity components. Finally, simulation results were given to demonstrate the validity of the proposed method.

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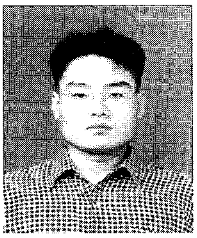

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