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A Fixed Point for Pair of Maps in Intuitionistic Fuzzy Mtric Space

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Abstract

Park, Park and Kwun[6] is defined the intuitionistic fuzzy metric space in which it is a little revised from Park[5]. According to this paper, Park, Kwun and Park[11] Park and Kwun[10], Park, Park and Kwun[7] are established some fixed point theorems in the intuitionistic fuzzy metric space. Furthermore, Park, Park and Kwun[6] obtained common fixed point theorem in the intuitionistic fuzzy metric space, and also, Park, Park and Kwun[8] proved common fixed points of maps on intuitionistic fuzzy metric spaces. We prove a fixed point for pair of maps with another method from Park, Park and Kwun[7] in intuitionistic fuzzy metric space defined by Park, Park and Kwun[6]. Our research are an extension of Vijayaraju and Marudai's result[14] and generalization of Park, Park and Kwun[7], Park and Kwun[10].

Key words : t-norm, t-conorm, Intuitionistic Fuzzy Metric Space, Fixed Point.

1. Introduction

Grabiec [1], Park and Kim[9] are studied a fixed point theorem in a fuzzy metric space. Also, Mishra, Shrama and Singh[4], Subrehmanyam[13] are proved a common fixed point theorem in fuzzy metric spaces. Vijayaraju and Marudai[14] obtained fixed point for pair of maps in fuzzy metric spaces.

Recently, Park[5] is defined the intuitionistic fuzzy metric space, and Park, Park and Kwun[6] is defined the intuitionistic fuzzy metric space in which it is a little revised from Park[5]. According to this paper, Park, Kwun and Park[11] Park and Kwun[10], Park, Park and Kwun[7] are established some fixed point theorems in the intuitionistic fuzzy metric space. Furthermore, Park, Park and Kwun[6] obtained common fixed point theorem in the intuitionistic fuzzy metric space, and also, Park, Park and Kwun[8] proved common fixed points of maps on intuitionistic fuzzy metric spaces.

In this paper, we prove a fixed point for pair of maps in intuitionistic fuzzy metric spaces. Our research are an extension of Vijayaraju and Marudai's result[14] and generalization of Park, Park and Kwun[7], Park and Kwun[10].

2. Preliminaries

We will give some definitions, properties and notation of the intuitionistic fuzzy metric space following by Schweizer and Sklar[12], Grabiec[1] and Park, Park and Kwun[6].

Definition 2.1. ([12]) A operation $* : [0,1]\times[0,1] \to [0,1]$ is continuous $t-$norm if $*$ is satisfying the following conditions:

(a) $*$ is commutative and associative,

(b) $*$ is continuous,

(c) $a * 1 = a$ for all $a \in [0,1]$,

(d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$.

Definition 2.2. ([12]) A operation $\diamond : [0,1]\times[0,1] \to [0,1]$ is continuous $t-$conorm if $\diamond$ is satisfying the following conditions:

(a) $\diamond$ is commutative and associative,

(b) $\diamond$ is continuous,

(c) $a \diamond 1 = a$ for all $a \in [0,1]$,

(d) $a \diamond b \geq c \diamond d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$.

Remark 2.3. ([5]) The following conditions are satisfied :

(a) For any $r_1, r_2 \in (0,1)$ with $r_1 > r_2$, there exist $r_3, r_4 \in (0,1)$ such that $r_1 * r_3 \geq r_2$ and $r_4 \diamond r_2 \leq r_1$.

(b) For any $r_5 \in (0,1)$, there exist $r_6, r_7 \in (0,1)$ such that $r_6 * r_6 \geq r_5$ and $r_7 \diamond r_7 \leq r_5$.

Definition 2.4. ([6]) The $5-$tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous $t-$norm, $\diamond$ is a continuous $t-$conorm and M, N are fuzzy sets on $X^2 \times (0,\infty)$ satisfying the following conditions; for all $x, y, z \in X$, such that

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(a) $M(x, y, t) > 0$,
(b) $M(x, y, t) = 1 \iff x = y$,
(c) $M(x, y, t) = M(y, x, t)$,
(d) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$,
(e) $M(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous,
(f) $N(x, y, t) > 0$,
(g) $N(x, y, t) = 0 \iff x = y$,
(h) $N(x, y, t) = N(y, x, t)$,
(i) $N(x, y, t) \diamond N(y, z, s) \ge N(x, z, t + s)$,
(j) $N(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous.

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2.5. ([11]) In an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, $M(x,\ y,\ \cdot)$ is nondecreasing and $N(x,\ y,\ \cdot)$ is nonincreasing for all $x,\ y \in X$.

Throughout the paper, we shall use $\mathbf{N}$ to denote the set of natural numbers and X to denote an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ with the following properties:

$$\lim_{t \to \infty} M(x, y, t) = 1, \quad \lim_{t \to \infty} N(x, y, t) = 0 \ \text{ for all } x, y \in X.$$

Definition 2.6. ([10]) Let X be an intuitionistic fuzzy metric space.

(a) A sequence $\{x_n\}$ in a X is called Cauchy sequence iff $\lim_{n \to \infty} M(x_{n+p}, x_n, t) = 1$, $\lim_{n \to \infty} N(x_{n+p}, x_n, t) = 0$ for each $p \in \mathbf{N},\ t > 0$.

(b) A sequence $\{x_n\}$ in a X is convergent to x in X iff $\lim_{n \to \infty} M(x_n, x, t) = 1$, $\lim_{n \to \infty} N(x_n, x, t) = 0$ for each $t > 0$.

(c) X is said to be complete if every Cauchy sequence in X is convergent in X.

Lemma 2.7. ([10]) Let $\{x_n\}$ be a sequence in an intuitionistic fuzzy metric space X. If there exists a positive number k, $0 < k < 1$ such that

$$\begin{aligned} M(x_{n+2}, x_{n+1}, kt) &\ge M(x_{n+1}, x_n, t), \\ N(x_{n+2}, x_{n+1}, kt) &\le N(x_{n+1}, x_n, t), \quad t > 0,\ n \in \mathbf{N}. \end{aligned}$$

Then $\{x_n\}$ is a Cauchy sequence.

Lemma 2.8. ([10]) If x, y are any two points in an intuitionistic fuzzy metric space X and k is a positive number with $k < 1$, and

$$M(x, y, kt) \ge M(x, y, t), \quad N(x, y, kt) \le N(x, y, t),$$

then $x = y$.

Lemma 2.9. ([10]) Let X be a complete intuitionistic fuzzy metric space and T be a self map of X satisfying

$$M(Tx, Ty, kt) \ge M(x, y, t),\ N(Tx, Ty, kt) \le N(x, y, t)$$

for all $x, y \in X$ and $0 < k < 1$. Then T has a unique fixed point in X.

3. Main Results

In this section, we prove a fixed point for pair of maps with another method from Park, Park, Kwun[7] in intuitionistic fuzzy metric space defined by Park, Park, Kwun[6]. Our research are an extension of Vijayaraju, Marudai's result[14] and generalization of Park, Park, Kwun[7], Park, Kwun[10].

Lemma 3.1. ([10]) Let $\{x_n\}$ is a sequence in an intuitionistic fuzzy metric space X. If

$$\begin{aligned} M(x_n, x_{n+1}, t) &\ge M(x_0, x_1, \frac{t}{\alpha^n}), \\ N(x_n, x_{n+1}, t) &\le N(x_0, x_1, \frac{t}{\alpha^n}), \end{aligned}$$

where α is a positive number with $0 < \alpha < 1$ and $n \in \mathbf{N}$, then $\{x_n\}$ is a Cauchy sequence.

Lemma 3.2. ([10]) If X is an intuitionistic fuzzy metric space and $\{x_n\}$ is a sequence in X such that

$$\begin{aligned} M(x_{i+1}, x_{i+2}, kt) &\ge M(x_i, x_{i+1}, t) * M(x_{i+1}, x_{i+2}, t) \\ N(x_{i+1}, x_{i+2}, kt) &\le N(x_i, x_{i+1}, t) \diamond N(x_{i+1}, x_{i+2}, t), \end{aligned}$$

where $0 < k < 1,\ i = 0, 1, 2, \cdots$ and $t > 0$, then

$$\begin{aligned} M(x_{i+1}, x_{i+2}, kt) &\ge M(x_i, x_{i+1}, t) \\ N(x_{i+1}, x_{i+2}, kt) &\le N(x_i, x_{i+1}, t). \end{aligned}$$

Theorem 3.3. Let X be a complete intuitionistic fuzzy metric space. If T, S are self maps on X such that

$$\begin{aligned} M(Tx, Sy, \beta t) &\ge M(x, Tx, t) * M(y, Sy, t) \\ N(Tx, Sy, \beta t) &\le N(x, Tx, t) \diamond N(y, Sy, t) \end{aligned}$$

for all $x, y \in X$ and $0 < \beta < \frac{1}{2}$, then T and S have a unique common fixed point in X.

Proof. Let $x_0 \in X$ be fixed . We define a sequence $\{x_n\} \subset X$ by

$$x_{n+1} = \begin{cases} Tx_n & \text{if } n \text{ is even} \\ Sx_n & \text{if } n \text{ is odd.} \end{cases}$$

Now, we will prove that $M(x_n, x_{n+1}, (\frac{\beta}{1-\beta})^n t) \ge M(x_0, x_1, t)$ and $N(x_n, x_{n+1}, (\frac{\beta}{1-\beta})^n t) \le N(x_0, x_1, t)$.

$$\begin{aligned}
& M(x_1, x_2, (\frac{\beta}{1-\beta})t) \\
= \ & M(Tx_0, Sx_1, \beta\frac{t}{1-\beta}) \\
\geq \ & M(x_0, x_1, \frac{t}{1-\beta}) * M(x_1, x_2, \frac{t}{1-\beta}) \\
\geq \ & M(x_0, x_1, \frac{t}{1-\beta}), \ \text{(by Lemma3.2)} \\
\geq \ & M(x_0, x_1, t), \ \text{(because of } \frac{t}{1-\beta} > t), \\
& N(x_1, x_2, (\frac{\beta}{1-\beta})t) \\
= \ & N(Tx_0, Sx_1, \beta\frac{t}{1-\beta}) \\
\leq \ & N(x_0, x_1, \frac{t}{1-\beta}) \diamond N(x_1, x_2, \frac{t}{1-\beta}) \\
\leq \ & N(x_0, x_1, \frac{t}{1-\beta}) \\
\leq \ & N(x_0, x_1, t), \ \text{(because of } \frac{t}{1-\beta} > t).
\end{aligned}$$

Thus the result is true for $n = 1$.

Suppose that the result is true for $n = k$, that is,

$$M(x_k, x_{k+1}, (\frac{\beta}{1-\beta})^k t) \geq M(x_0, x_1, t)$$
$$N(x_k, x_{k+1}, (\frac{\beta}{1-\beta})^k t) \leq N(x_0, x_1, t).$$

Without loss of generality, let us assume that k is even,

$$\begin{aligned}
& M(x_{k+1}, x_{k+2}, (\frac{\beta}{1-\beta})^{k+1} t) \\
= \ & M(Tx_k, Sx_{k+1}, \beta(\frac{\beta}{1-\beta})^k \cdot \frac{t}{1-\beta}) \\
\geq \ & M(x_k, x_{k+1}, (\frac{\beta}{1-\beta})^k \cdot \frac{t}{1-\beta}) \\
& \quad * M(x_{k+1}, x_{k+2}, (\frac{\beta}{1-\beta})^k \cdot \frac{t}{1-\beta}), \\
& N(x_{k+1}, x_{k+2}, (\frac{\beta}{1-\beta})^{k+1} t) \\
= \ & N(Tx_k, Sx_{k+1}, \beta(\frac{\beta}{1-\beta})^k \cdot \frac{t}{1-\beta}) \\
\leq \ & N(x_k, x_{k+1}, (\frac{\beta}{1-\beta})^k \cdot \frac{t}{1-\beta}) \\
& \quad \diamond N(x_{k+1}, x_{k+2}, (\frac{\beta}{1-\beta})^k \cdot \frac{t}{1-\beta}).
\end{aligned}$$

Then by Lemma 3.2, we have

$$\begin{aligned}
& M(x_{k+1}, x_{k+2}, (\frac{\beta}{1-\beta})^{k+1} t) \\
\geq \ & M(x_k, x_{k+1}, (\frac{\beta}{1-\beta})^k \cdot \frac{t}{1-\beta}) \\
\geq \ & M(x_k, x_{k+1}, (\frac{\beta}{1-\beta})^k t) \\
\geq \ & M(x_0, x_1, t), \\
& N(x_{k+1}, x_{k+2}, (\frac{\beta}{1-\beta})^{k+1} t) \\
\leq \ & N(x_k, x_{k+1}, (\frac{\beta}{1-\beta})^k \cdot \frac{t}{1-\beta}) \\
\leq \ & N(x_k, x_{k+1}, (\frac{\beta}{1-\beta})^k t) \\
\leq \ & N(x_0, x_1, t).
\end{aligned}$$

Hence the result is true for all n. Therefore

$$M(x_n, x_{n+1}, (\frac{\beta}{1-\beta})^n t) \geq M(x_0, x_1, t),$$
$$N(x_n, x_{n+1}, (\frac{\beta}{1-\beta})^n t) \leq N(x_0, x_1, t),$$

which can be written as

$$M(x_n, x_{n+1}, t) \geq M(x_0, x_1, (\frac{1-\beta}{\beta})^n t),$$
$$N(x_n, x_{n+1}, t) \leq N(x_0, x_1, (\frac{1-\beta}{\beta})^n t).$$

By Lemma 3.1, $\{x_n\}$ is a Cauchy sequence in X. Since X is complete, $\{x_n\}$ converges to a point x in X. That is,

$$\lim_{n\to\infty} M(x_n, x, t) = 1, \quad \lim_{n\to\infty} N(x_n, x, t) = 0.$$

Now, by Definition 2.3 and assumption of this theorem

$$\begin{aligned}
& M(x, Tx, t) \\
\geq \ & M(x, x_n, \frac{t}{2}) * M(x_n, Tx, \frac{t}{2}) \\
= \ & M(x, x_n, \frac{t}{2}) * M(Sx_{n-1}, Tx, \frac{t}{2}) \\
\geq \ & M(x, x_n, \frac{t}{2}) * M(x, Tx, \frac{t}{2\beta}) * M(x_{n-1}, x_n, \frac{t}{2\beta}), \\
& N(x, Tx, t) \\
\leq \ & N(x, x_n, \frac{t}{2}) \diamond N(x_n, Tx, \frac{t}{2}) \\
= \ & N(x, x_n, \frac{t}{2}) \diamond N(Sx_{n-1}, Tx, \frac{t}{2}) \\
\leq \ & N(x, x_n, \frac{t}{2}) \diamond N(x, Tx, \frac{t}{2\beta}) \diamond N(x_{n-1}, x_n, \frac{t}{2\beta}).
\end{aligned}$$

Taking limit as $n \to \infty$, we get

$$\begin{aligned} M(x,Tx,t) &\geq 1 * M(x,Tx,\frac{t}{2\beta}) * 1 \\ &= M(x,Tx,\frac{t}{2\beta}), \\ N(x,Tx,t) &\leq 0 \diamond N(x,Tx,\frac{t}{2\beta}) \diamond 0 \\ &= N(x,Tx,\frac{t}{2\beta}). \end{aligned}$$

By lemma 2.9, $Tx = x$.

Similarly, $Sx = x$.

Now, we will show that x is a unique common fixed point of T and S in X.

Assume that there exist another fixed point y in $X(Ty = Sy = y)$. Then

$$\begin{aligned} M(x,y,t) &= M(Tx,Sy,t) \\ &\geq M(x,Tx,\frac{t}{\beta}) * M(y,Sy,\frac{t}{\beta}) = 1, \\ N(x,y,t) &= N(Tx,Sy,t) \\ &\leq N(x,Tx,\frac{t}{\beta}) \diamond N(y,Sy,\frac{t}{\beta}) = 0. \end{aligned}$$

Therefore $M(x,y,t) = 1$ and $N(x,y,t) = 0$. Hence $x = y$. Thus x is a unique common fixed point of T and S in X. □

Corollary 3.4. ([10]) If T is a self map on a complete intuitionistic fuzzy metric space X and if there exists a positive number β with $0 < \beta < \frac{1}{2}$ such that

$$\begin{aligned} M(Tx,Ty,\beta t) &\geq M(x,Tx,t) * M(y,Ty,t), \\ N(Tx,Ty,\beta t) &\leq N(x,Tx,t) \diamond N(y,Ty,t) \end{aligned}$$

for all $x, y \in X$ and $t \geq 0$, then T has a unique fixed point in X.

Proof. The proof follows immediately from Theorem 3.3 by putting $T = S$. □

Theorem 3.5. Let X be a complete intuitionistic fuzzy metric space. Also, let T and S be two self maps on X such that

(a)$M(Tx,Sy,\alpha t) \geq M(x,y,t)$, $N(Tx,Sy,\alpha t) \leq N(x,y,t)$, where $0 < \alpha < 1$, $x, y \in X$, $x \neq y$,

(b)S is a contraction on X. That is, there exists β with $0 < \beta < 1$ such that $M(Sx,Sy,\beta t) \geq M(x,y,t)$, $N(Sx,Sy,\beta t) \leq N(x,y,t)$ for all $x, y \in X$, and

(c)there exists $x_0 \in X$ such that

$$x_{n+1} = \begin{cases} Tx_n & \text{if } n \text{ is even} \\ Sx_n & \text{if } n \text{ is odd} \end{cases}$$

with $x_m \neq x_l$ if $m \neq l$.

Then T and S have a unique common fixed point in X.

Proof. If x_1, x_2 are two distinct points in X, then it is impossible that $Tx_1 = x_1$ and $Sx_2 = x_2$. For if $Tx_1 = x_1$ and $Sx_2 = x_2$, then by (a),

$$\begin{aligned} M(x_1,x_2,\alpha t) &= M(Tx_1,Sx_2,\alpha t) \geq M(x_1,x_2,t), \\ N(x_1,x_2,\alpha t) &= N(Tx_1,Sx_2,\alpha t) \leq N(x_1,x_2,t). \end{aligned}$$

This is a contradiction from Remark 2.5. Since S is contraction, S has a unique fixed point say x in X from Lemma 2.9. Therefore if T has a fixed point, it is unique and must coincide with x. If $x_0 = x_1$, since $x_1 = Tx_0 = x_0 = Sx_0$, assume that $x_0 \neq x_1$. Let x_1, x_2 be any two members of $\{x_n\}$ defined by (c). Then from (a),

$$\begin{aligned} M(x_1,x_2,t) &\geq M(x_0,x_1,\frac{t}{\alpha}), \\ N(x_1,x_2,t) &\leq N(x_0,x_1,\frac{t}{\alpha}). \end{aligned}$$

Similarly, from

$$\begin{aligned} M(x_2,x_3,\alpha t) = M(Sx_1,Tx_2,\alpha t) \geq M(x_1,x_2,t), \\ N(x_2,x_3,\alpha t) = N(Sx_1,Tx_2,\alpha t) \leq N(x_1,x_2,t), \end{aligned}$$

we have

$$\begin{aligned} M(x_2,x_3,t) &= M(Sx_1,Tx_2,\alpha t) \geq M(x_0,x_1,\frac{t}{\alpha^2}), \\ N(x_2,x_3,t) &= N(Sx_1,Tx_2,\alpha t) \leq N(x_0,x_1,\frac{t}{\alpha^2}), \\ \cdots\cdots & \;\cdots\; \cdots\cdots \\ M(x_n,x_{n+1},t) &\geq M(x_0,x_1,\frac{t}{\alpha^n}), \\ N(x_n,x_{n+1},t) &\leq N(x_0,x_1,\frac{t}{\alpha^n}). \end{aligned}$$

Hence by Lemma 3.1 and Lemma 2.7, $\{x_n\}$ is a Cauchy sequence. Since X is complete, it converges to y_0 in X. Therefore it satisfied the Definition 2.6(b).

Suppose that n is even integer. Then

$$\begin{aligned} M(y_0,Ty_0,t) &\geq M(y_0,x_n,\frac{t}{2}) * M(x_n,Ty_0,\frac{t}{2}) \\ &= M(y_0,x_n,\frac{t}{2}) * M(Sx_{n-1},Ty_0,\frac{t}{2}) \\ &\geq M(y_0,x_n,\frac{t}{2}) * M(x_{n-1},y_0,\frac{t}{2\alpha}), \\ N(y_0,Ty_0,t) &\leq N(y_0,x_n,\frac{t}{2}) \diamond N(x_n,Ty_0,\frac{t}{2}) \\ &= N(y_0,x_n,\frac{t}{2}) \diamond N(Sx_{n-1},Ty_0,\frac{t}{2}) \\ &\leq N(y_0,x_n,\frac{t}{2}) \diamond N(x_{n-1},y_0,\frac{t}{2\alpha}). \end{aligned}$$

Taking limit as $n \to \infty$, we get

$$M(y_0,Ty_0,t) \geq 1 * 1 = 1, \quad N(y_0,Ty_0,t) \leq 0 * 0 = 0.$$

Thus $y_0 = Ty_0$. We know that y_0 is a fixed point of T. Therefore $y_0 = x$. Hence T and S have a unique common fixed point in X. □

Corollary 3.6. ([7]) (Intuitionistic fuzzy Banach contraction theorem) Let X be a complete intuitionistic fuzzy metric space and $T : X \to X$ be a mapping satisfying

$$M(Tx, Ty, \alpha t) \geq M(x, y, t), \quad N(Tx, Ty, \alpha t) \leq N(x, y, t)$$

where $0 < \alpha < 1$, $x, y \in X$ and all $t > 0$. Then T has a unique fixed point in X.

Proof. We proved this corollary from Theorem 3.5 by putting $T = S$. Also, in this proof, we used another method with respect to [7]. □

4. Example

Example 4.1. Let (X, d) be a metric space in $X = [0, 1]$. Denote $x * y = \min\{x, y\}$, $x \diamond y = \max\{x, y\}$ for all $x, y \in X$ and let M_d, N_d be fuzzy sets on $X^2 \times (0, \infty)$ as follows:

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}, \quad N_d(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

where for any $x, y \in X$, $t > 0$, $d(x, y) = |x - y|$.

Define maps $T, S : X \to X$ by $Tx = 1 - x$, $Sx = \frac{3}{4} - \frac{x}{2}$ for all $x \in X$. Then $(X, M_d, N_d, *, \diamond)$ is an intuitionistic fuzzy metric space.

Also,

$$\begin{aligned} M_d(Sx, Sy, t) &= \frac{t}{t + d(Sx, Sy)} = \frac{t}{t + \frac{1}{2}|y - x|} \\ &\geq \frac{t}{t + \frac{2}{3}|y - x|} = M_d(x, y, \frac{t}{\frac{2}{3}}), \\ N_d(Sx, Sy, t) &= \frac{d(Sx, Sy)}{t + d(Sx, Sy)} = \frac{\frac{1}{2}|y - x|}{t + \frac{1}{2}|y - x|} \\ &\leq \frac{\frac{2}{3}|y - x|}{t + \frac{2}{3}|y - x|} = N_d(x, y, \frac{t}{\frac{2}{3}}). \end{aligned}$$

Clearly, $T(\frac{1}{2}) = \frac{1}{2} = S(\frac{1}{2})$ and $\frac{1}{2}$ is the only fixed point of both T and S in X.

But since

$$\begin{aligned} M_d(Tx, Sy, t) &= \frac{t}{t + |\frac{y}{2} - x + \frac{1}{4}|}, \\ N_d(Tx, Sy, t) &= \frac{|\frac{y}{2} - x + \frac{1}{4}|}{t + |\frac{y}{2} - x + \frac{1}{4}|}. \end{aligned}$$

If $x = \frac{3}{4}, y = \frac{1}{2}$, then

$$\begin{aligned} M_d(Tx, Sy, t) &= \frac{t}{t + \frac{1}{2}} = M_d(x, y, t), \\ N_d(Tx, Sy, t) &= \frac{\frac{1}{2}}{t + \frac{1}{2}} = N(x, y, t). \end{aligned}$$

Hence we can know that Theorem 3.5 gives only some sufficient conditions for which T and S have a common unique fixed point in X.

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